IMPERFECT DARK ENERGY OF KINETIC GRAVITY BRAIDING

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THIS TALK IS BASED ON WORK IN PROGRESS AND

- ***Imperfect Dark Energy from Kinetic Gravity Braiding***
  arXiv:1008.0048 [hep-th], JCAP 1010:026, 2010

- ***The Imperfect Fluid behind Kinetic Gravity Braiding***
  arXiv:1103.5360 [hep-th]

IN COLLABORATION WITH

Cédric Deffayet, Oriol Pujolàs
& Ignacy Sawicki

RÉSUMÉ I

- **Scalar field ϕ with a non-canonical *action* can "behave" like imperfect fluid: on general (not exactly isotropic and homogeneous FRW) background:**

$$T_{\mu\nu} \neq \mathcal{E} u_\mu u_\nu - \perp_{\mu\nu} \mathcal{P}$$

- **Fluid elements (particles) are shift-charges: charges with respect to: $\phi \longrightarrow \phi + c$**

- **ϕ kinetically mixes / "braids" with the metric:**

$$(\partial\phi)^2 \, \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right) \text{ c.f. } F^{(1)}_{\mu\nu} \left(A^\alpha \right) F^{(2)\mu\nu} \left(B_\beta \right)$$

RÉSUMÉ II

- **Manifestly stable (*no ghosts* and *no gradient instabilities*) and *large* violation of the Null Energy Condition (NEC) is possible even in theories *minimally coupled to gravity*: *healthy Phantom* with $w < -1$**

- **Vanishing shift-charge, $n = 0$, corresponds to cosmological attractors similar to *Ghost Condensate* / "bad" *k-Inflation*. But here these attractors can be manifestly stable (*no ghosts* and *no gradient instabilities*) and their exact properties depend on external matter. Through Euler relation $\mathcal{E} = mn - P_0$ these attractors generically evolve to de Sitter in late time asymptotic.**

  Interesting for DE!

ISOTROPY AND HOMOGENEITY OF THE FRIEDMANN UNIVERSE

$$T_{\mu\nu} = (\mathcal{E} + P)\, u_\mu u_\nu - P g_{\mu\nu}$$

A perfect fluid and a “simple” single scalar field have this EMT for

“*all*” backgrounds

interesting for DE and Inflation when $P \approx -\mathcal{E}$

CAN ONE GO BEYOND THE PERFECT FLUID, BUT STILL KEEPING ONLY ONE SINGLE DEGREE OF FREEDOM ?

YES WE CAN

We Can Do It!
WAR PRODUCTION CO-ORDINATING COMMITTEE

BRAIDING METRIC WITH A SCALAR FIELD-

Kinetic Gravity Braiding

WHAT IS KINETIC GRAVITY BRAIDING?

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\left[K\left(\phi, X\right) + G\left(\phi, X\right)\Box\phi\right]$$

where $X \equiv \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$

Minimal coupling to gravity $S_{\rm tot} = S_\phi + S_{\rm EH}$

However, derivatives of the metric are coupled to the derivatives of the scalar, provided $G_X \neq 0$

shift-symmetry: $\phi \to \phi + c$

theory is **not** "parity" symmetric: $\phi \nrightarrow -\phi$

ACTION FOR KINETIC GRAVITY BRAIDING IS SIMILAR TO EINSTEIN-HILBERT ACTION

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  NO new degrees of freedom -
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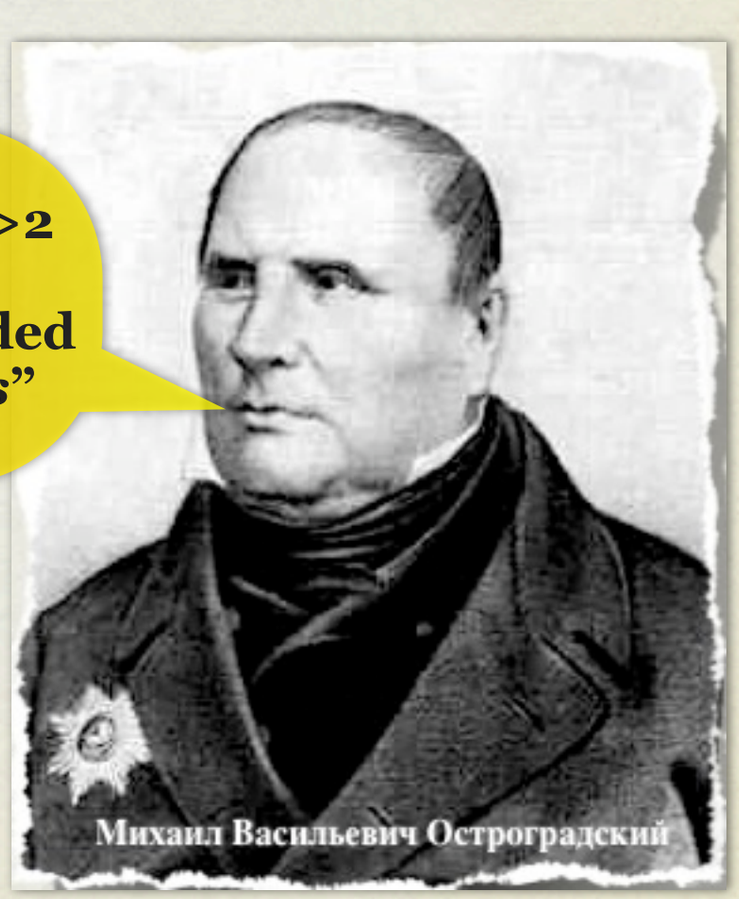
order of equations of motion >2 implies Hamiltonian unbounded from below - "*ghosts*"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KINETIC GRAVITY BRAIDING IS SIMILAR TO GALILEON

(©*Nicolis, Rattazzi, Trincherini 2008*)

BUT

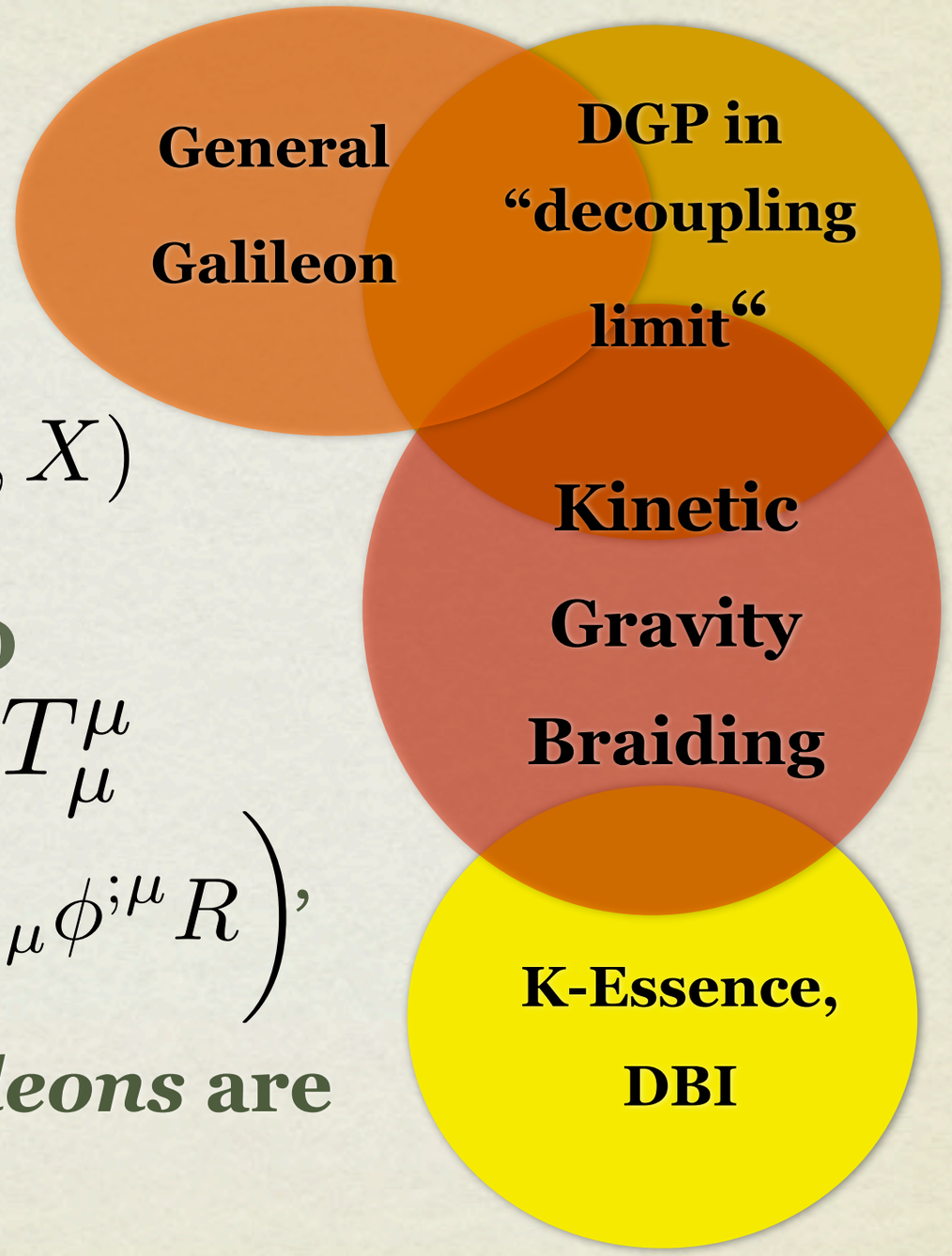
- Does **not require** the Galilean symmetry: $\partial_\mu \phi \to \partial_\mu \phi + c_\mu$ & $\phi \to \phi + c$

- General functions $K(\phi, X)$ & $G(\phi, X)$

- **Minimal coupling to gravity, NO NO higher order therms like** ϕT^μ_μ

$$\phi_{;\lambda}\phi^{;\lambda}\left((\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu} - \frac{1}{4}\phi_{;\mu}\phi^{;\mu}R\right),$$

Indeed, manifestly healthy *Galileons* are NEVER *Galilean* symmetric !!!



EQUATION OF MOTION I

$$L^{\mu\nu}\nabla_\mu\nabla_\nu\phi + (\nabla_\alpha\nabla_\beta\phi)\,Q^{\alpha\beta\mu\nu}\,(\nabla_\mu\nabla_\nu\phi) + \quad \textbf{Braiding}$$
$$+Z - G_X R^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = 0$$

EOM (and Einstein equations) is of the second order: $L_{\mu\nu}\,, Q^{\alpha\beta\mu\nu}\,, Z$ **constructed from field and it's first derivatives.**

$Q^{\alpha\beta\mu\nu}$ **is such that EOM is a 4D Lorentzian generalization of the Monge-Ampère Equation, always *linear* in** $\ddot{\phi}$

EQUATION OF MOTION II

- **Shift-Charge Current:** J_μ

- **New Equivalent Lagrangian:** $\mathcal{P}$

- **Equation of motion is a “conservation law”:**

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$$J_\mu = (\mathcal{L}_X - 2G_\phi)\,\nabla_\mu\phi - G_X\nabla_\mu X$$

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$$\mathcal{P} = K - 2XG_\phi - G_X\nabla^\lambda\phi\nabla_\lambda X$$

- **Equation of motion is a "conservation law":**

$$\boxed{\nabla_\mu J^\mu = \mathcal{P}_\phi}$$

IMPERFECT FLUID FOR TIMELIKE GRADIENTS

- **Four velocity :** $u_\mu \equiv \dfrac{\nabla_\mu \phi}{\sqrt{2X}}$ → ϕ is an ***internal clock***

- **projector:** $\perp_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$

- **Time derivative:** $(\dot{}) \equiv \dfrac{\mathrm{d}}{\mathrm{d}\tau} \equiv u^\lambda \nabla_\lambda$

- **Expansion :** $\theta \equiv \perp^\lambda_\mu \nabla_\lambda u^\mu = \dot{V}/V$

comoving volume

EFFECTIVE MASS & CHEMICAL POTENTIAL

- charge density: $n \equiv J^\mu u_\mu = n_0 + \kappa\theta$

"Braiding"

- energy density: $\mathcal{E} \equiv T_{\mu\nu} u^\mu u^\nu = \mathcal{E}_0 + \theta m \kappa$
- effective mass per shift-charge / chemical potential:

$$m \equiv \left(\frac{\partial \mathcal{E}}{\partial n} \right)_{V,\phi} = \sqrt{2X} = \dot{\phi}$$

SHIFT-CURRENT AND DIFFUSION

$$J_\mu = n u_\mu - \frac{\kappa}{m} \perp^\lambda_\mu \nabla_\lambda m$$

"Diffusion"

§ 59, *L&L, vol. 6*

$$\kappa \equiv 2 X G_X$$

Is a "diffusion"/ transport coefficient

IMPERFECT FLUID ENERGY-MOMENTUM TENSOR

- **Pressure** $\mathcal{P} \equiv -\frac{1}{3} T^{\mu\nu} \perp_{\mu\nu} = P_0 - \kappa \dot{m}$
- **Energy Flow** $q_\mu \equiv \perp_{\mu\lambda} T^\lambda_\nu u^\nu = m \perp_{\mu\nu} J^\nu$

$$q_\mu = -\kappa \perp^\nu_\mu \nabla_\nu m$$

No Heat Flux!

- **Energy Momentum Tensor**

$$T_{\mu\nu} = \mathcal{E} u_\mu u_\nu - \perp_{\mu\nu} \mathcal{P} + 2 u_{(\mu} q_{\nu)}$$

Solving for $\dot{m}$ *for small gradients or small* κ *one obtains "bulk viscosity"*

DIFFUSION OF CHARGE

For incompressible motion $\theta \equiv 0$ and equation of motion is:

$$\dot{n} = -\overline{\boldsymbol{\nabla}}_\mu \left(\mathfrak{D} \overline{\boldsymbol{\nabla}}^\mu n \right) + \mathfrak{D} a^\mu \overline{\boldsymbol{\nabla}}_\mu n$$

where the diffusion constant: $\mathfrak{D} \equiv -\dfrac{\kappa}{n_m m}$

4-acceleration: $a^\mu \equiv \dot{u}^\mu$

spatial gradient: $\overline{\boldsymbol{\nabla}}_\mu \equiv \perp^\nu_\mu \nabla_\nu$

ENERGY CONSERVATION IN COMOVING VOLUME

Energy conservation: $u_\nu \nabla_\mu T^{\mu\nu} = 0$

$$\mathrm{d}E = -\mathcal{P}\mathrm{d}V + m\mathrm{d}\mathcal{N}_{\mathrm{dif}}$$

Euler relation: $\mathcal{E} = mn - P_0$

Momentum conservation:

$$\perp_{\mu\nu} \nabla_\lambda T^{\lambda\nu} = 0$$

VACUUM-ATTRACTORS

Euler relation: $\mathcal{E} = mn - P_0$

$\downarrow$

for no particles: $n_* = 0$

$\downarrow$

$$\mathcal{E}_* = -\mathcal{P}_* - \kappa_* \dot{m}_*$$

almost dS!

COSMOLOGY

$$q_\mu = 0 \quad \textbf{and} \quad \theta = 3H$$

Friedmann Equation:

$$H^2 = \kappa m H + \frac{1}{3}\left(\mathcal{E}_0 + \rho_{\rm ext}\right)$$

$$r_{\rm c}^{-1} = \kappa m$$

"crossover" scale in DGP

CHARGE CONSERVATION

$$\dot{n} + 3Hn = \mathcal{P}_\phi$$

If there is shift-symmetry then

$$\mathcal{P}_\phi = 0$$

$$n \propto a^{-3}$$

INFLATION BRINGS THE SCALAR TO ATTRACTOR

$$n_* = 0$$

EXAMPLE: SIMPLEST IMPERFECT DARK ENERGY

Only one free parameter μ

- Lagrangian

$$\mathcal{L} = X\left(-1 + \mu\Box\phi\right)$$

- shift-charge density

$$n = m\left(3\mu H m - 1\right)$$

NONTRIVIAL ATTRACTOR

No Particles: $n_* = 0$

$$m_* = (3\mu H)^{-1}$$

$$H_*^2 = \frac{1}{6}\rho_{\rm ext}\left(1+\sqrt{1+\frac{2}{3}\left(\mu\rho_{\rm ext}\right)^{-2}}\right)$$

STABLE

$$m_* = 0$$

$$H_*^2 = \frac{1}{3}\rho_{\rm ext}$$

GHOSTY

DARK ENERGY

assume today $\sqrt{\frac{3}{2}}\mu\rho_{\rm ext} \ll 1$ ⇒ $H_*^2 \simeq \frac{1}{6}\sqrt{\frac{2}{3}}\mu^{-1}$

$\Lambda_* \simeq \frac{1}{2}\sqrt{\frac{2}{3}}\mu^{-1} \simeq 3\rho_{\rm CDM}$ ⇒ $\sqrt{\frac{3}{2}}\rho_{\rm CDM}\mu \simeq \frac{1}{6} \ll 1$

$$\text{Mass Scale} \sim \mu^{-1/3} \sim (H_0^2 M_{\rm Pl})^{1/3} \sim 10^{-13}\text{eV}$$

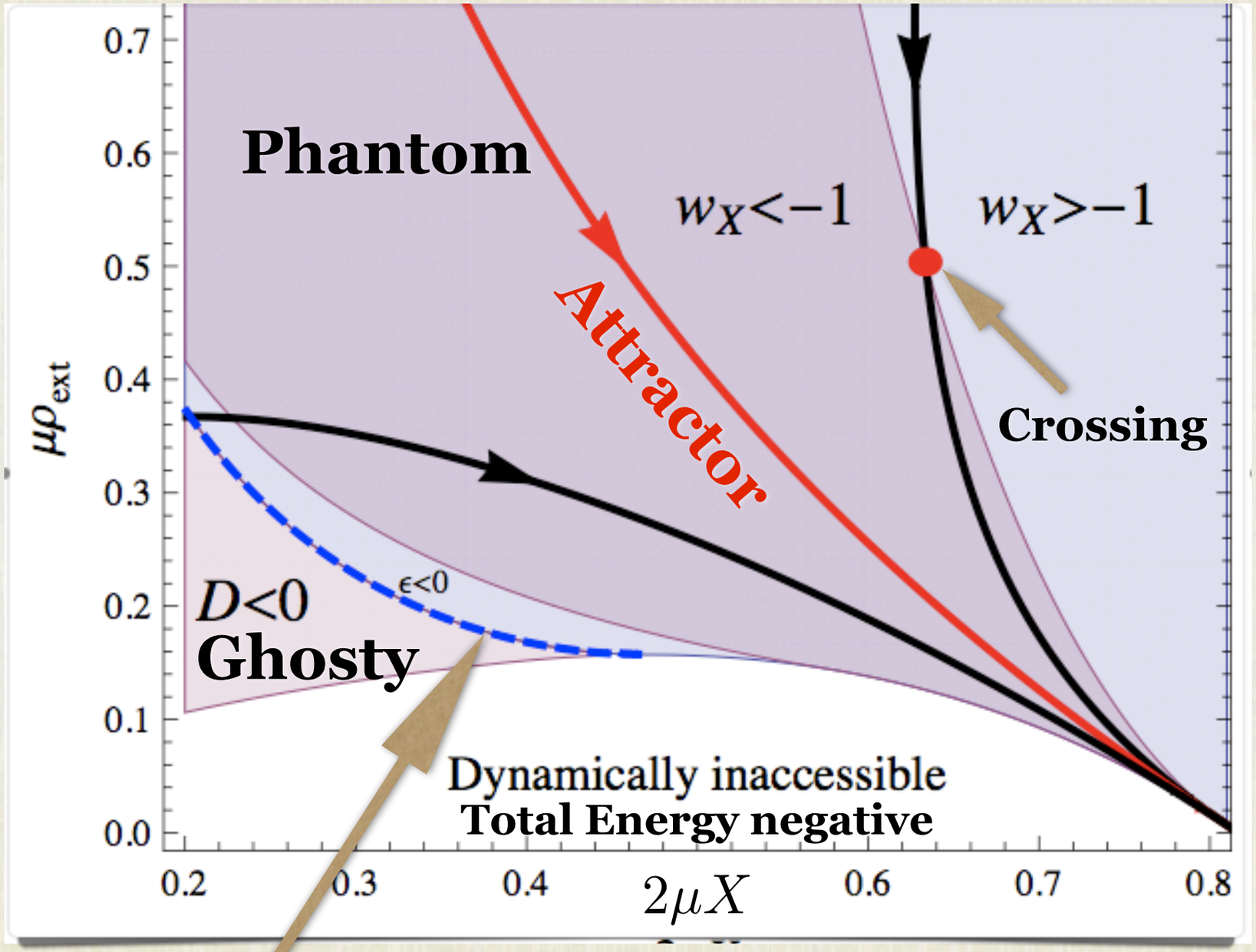
Length Scale: **1000 km**

In Quintessence - the size of the universe

SOUND SPEED & GHOSTS

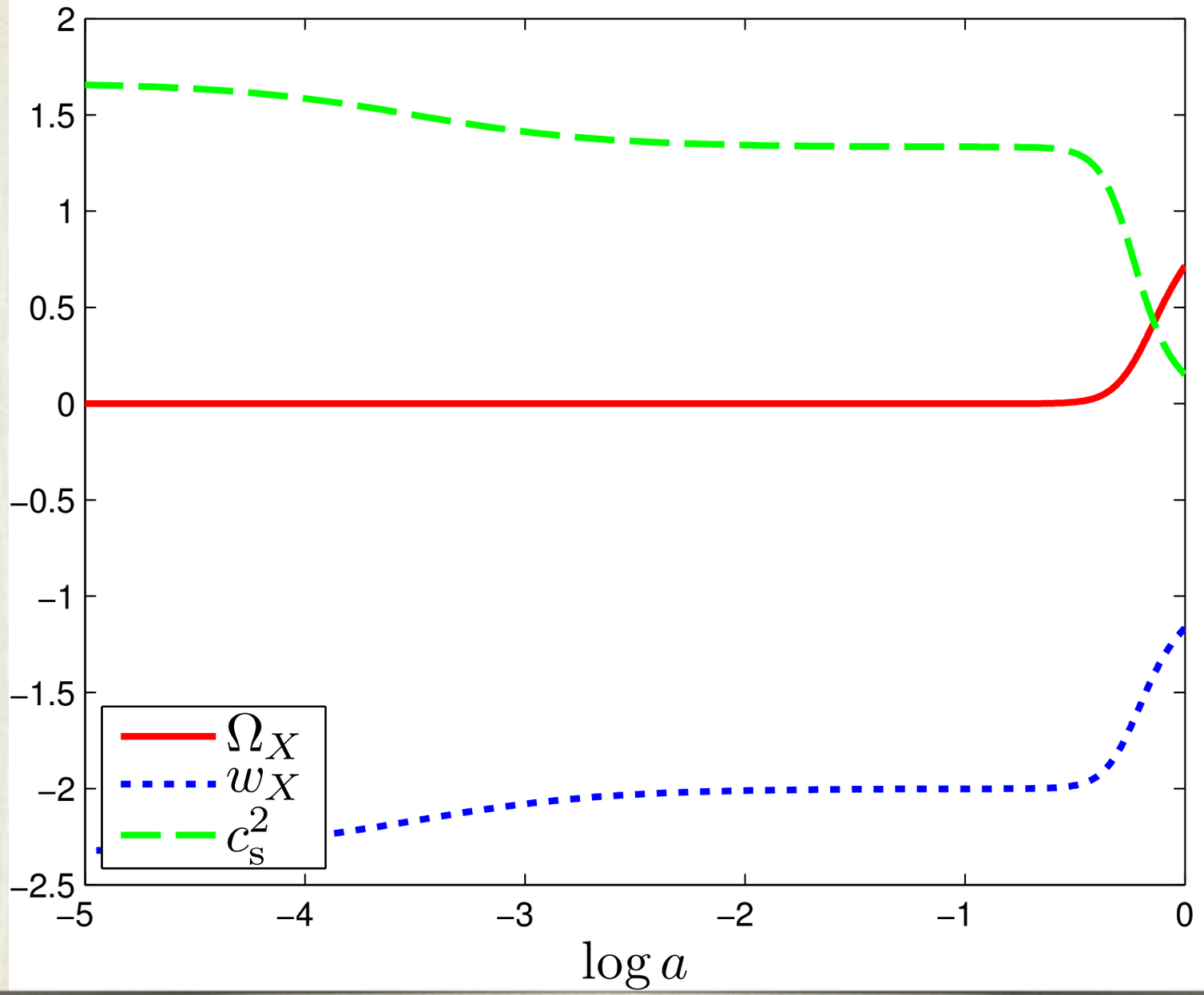
$$c_{\rm s}^2 = \frac{\mathcal{P}_m + 2\dot{\kappa} + \kappa\,(4H - \kappa m/2)}{\mathcal{E}_m - 3\kappa\,(H - \kappa m/2)} \neq \frac{\dot{\mathcal{P}}}{\dot{\mathcal{E}}}$$

$$\Leftrightarrow \quad D = \frac{\mathcal{E}_m - \kappa\theta}{m} + \frac{3}{2}\kappa^2 > 0$$

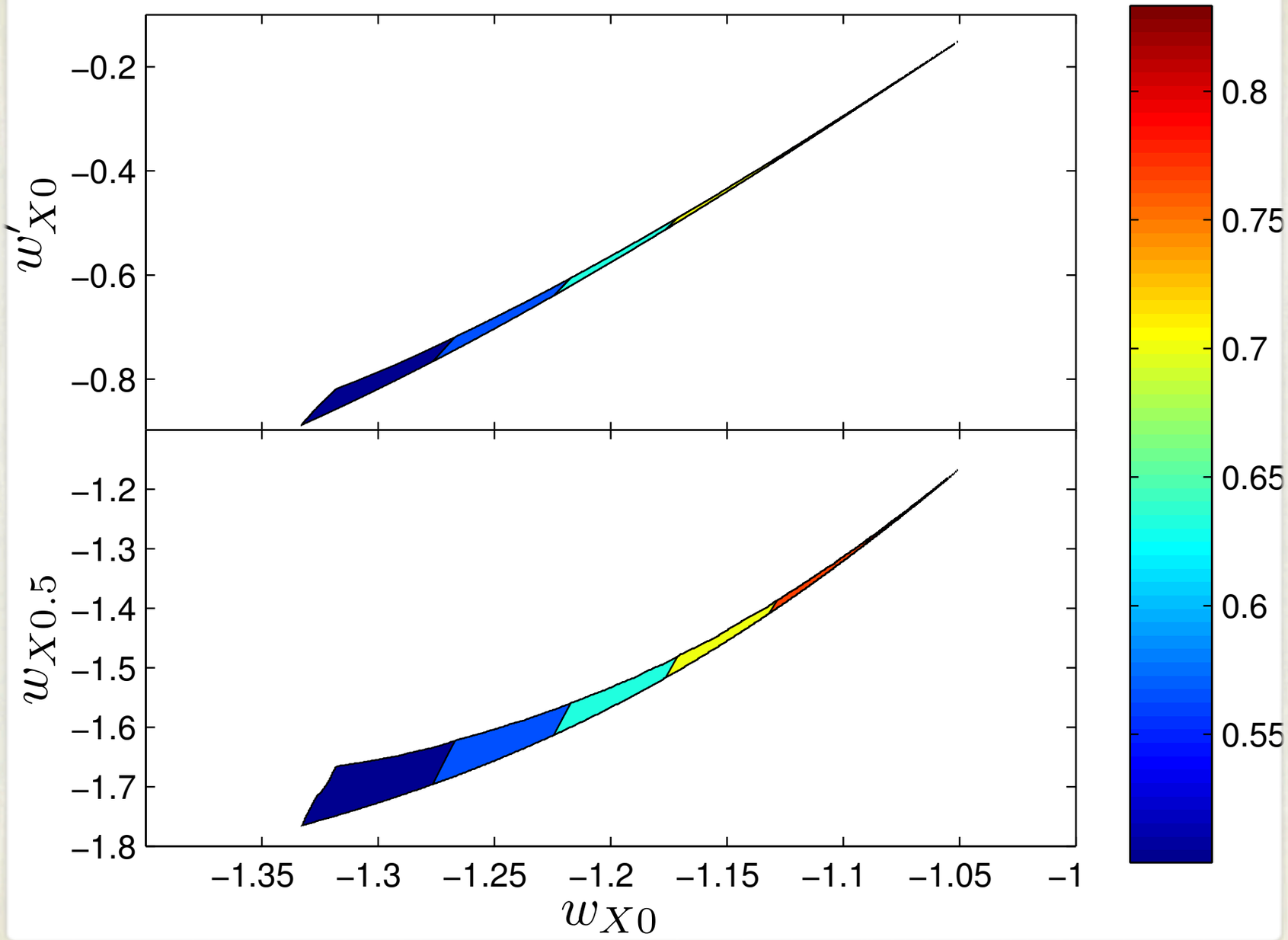


Pressure singularity

Phase portrait for scalar field & dust



Evolution of dark energy properties in the Friedmann universe also containing dust and radiation. The scalar evolves on its attractor throughout the presented period. During matter domination $w_X = -2$, while $w_X = -7/3$ during radiation domination. The sound speed is superluminal when the scalar energy density is subdominant, becoming subluminal when $\Omega_X \approx 0.1$ and $w_X \approx -1.4$

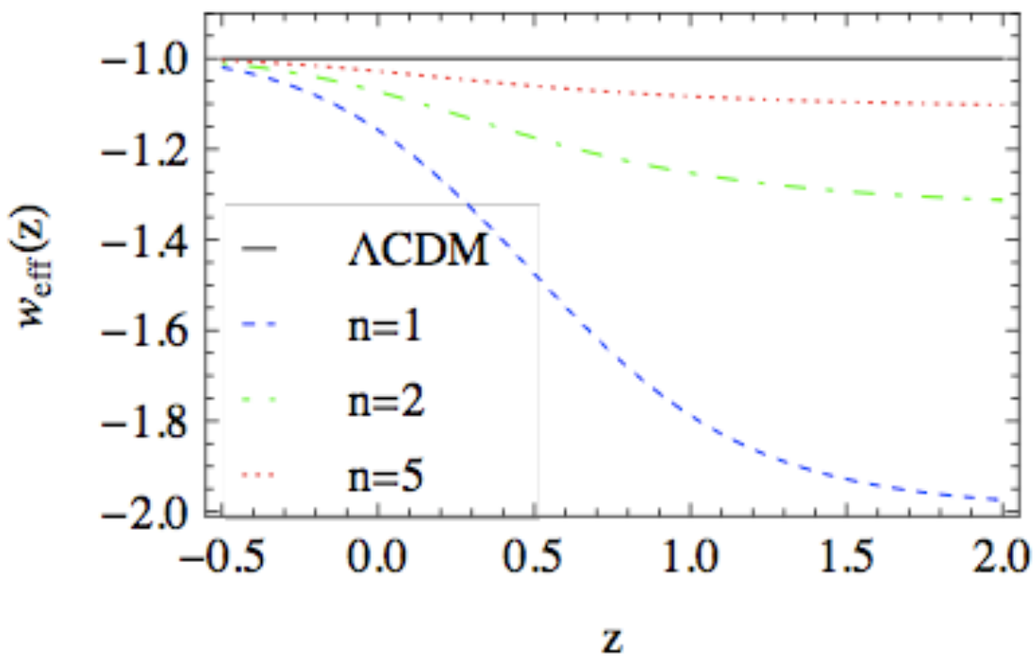


$0.1 < \Omega_{\rm m} < 0.5$ and $\Omega_{X\rm eq} < 0.1$. The shading contours correspond to the energy density of DE today Ω_{X0}. Two parameterisations of DE behaviour are shown: w_X and w'_X evaluated today, and w_X evaluated today and at $z = 1/2$. The requirement that the energy density in DE at matter-radiation equality be small, $\Omega_X^{\rm eq} < 0.1$ forces the value of the shift charge to be small today $Q_0 < 10^{-2}$. This means that in the most recent history, the evolution has effectively been on attractor or very close to it and the permitted value of w_X is very restricted and determined to all intents and purposes by Ω_X^0.

FURTHER DEVELOPMENT

Kinetic Gravity Braiding with $G \propto X^n$

arXiv:1011.2006v2 [astro-ph.CO], *Rampei Kimura, Kazuhiro Yamamoto*



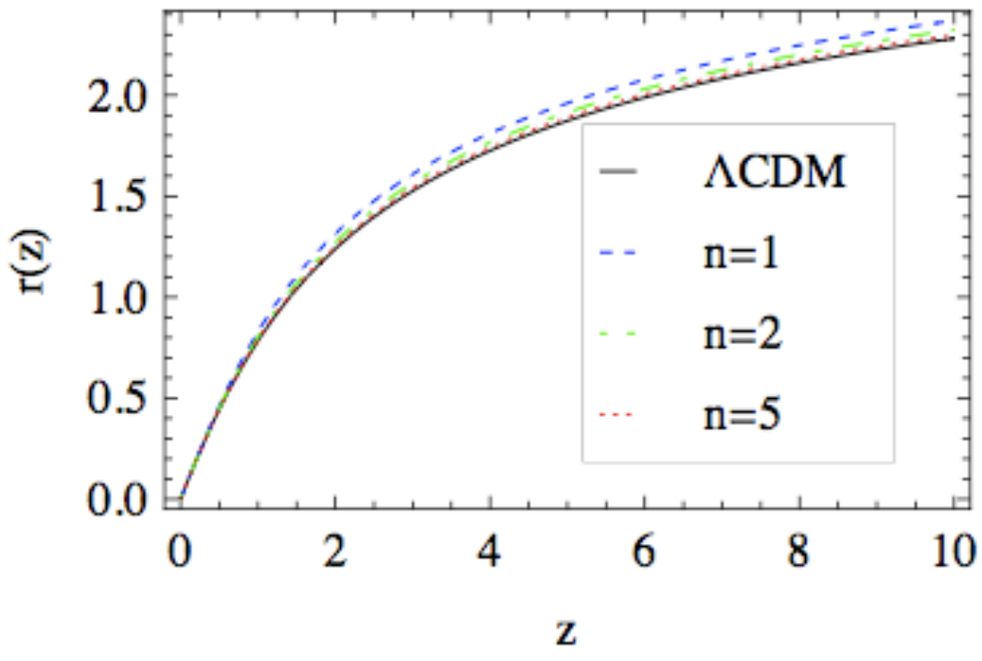


FIG. 1: Left panel: The effective equation of state $w_{\rm eff}$ as a function of redshift for ΛCDM (solid curve) and the kinetic braiding mode with $n = 1$ (dashed curve), $n = 2$ (dash-dotted curve), and $n = 5$ (dotted curve), respectively. Right panel: The comoving distance $r(z)$, normalised by H_0, as a function of redshift for ΛCDM and this model.

CONSTRAINTS FROM CMB AND SN IA

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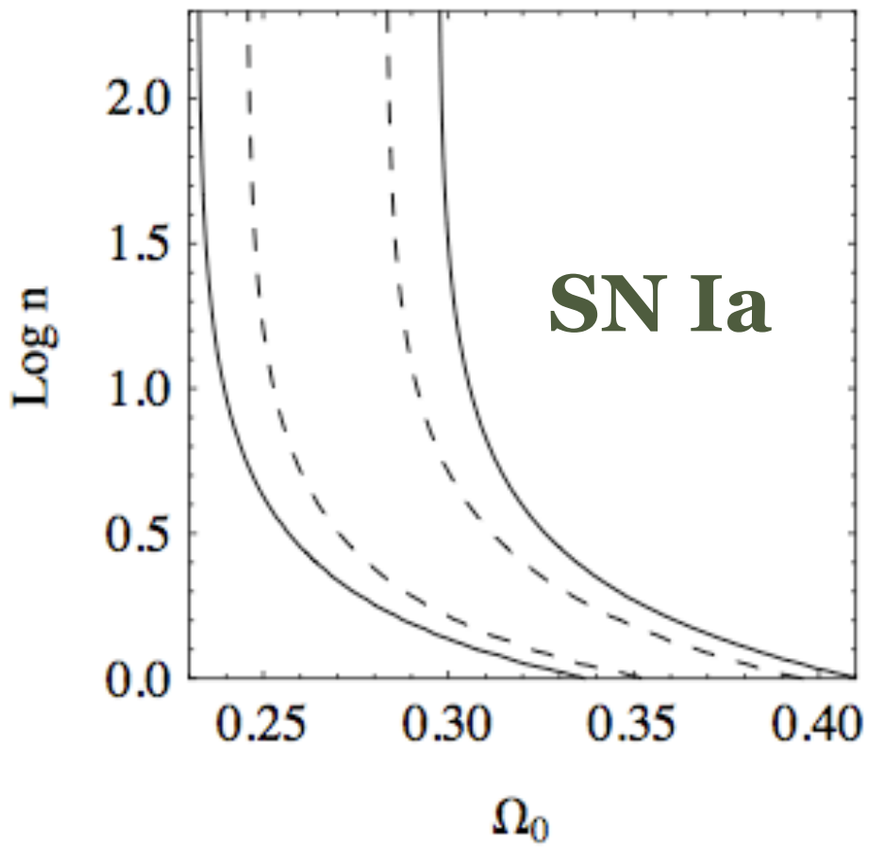

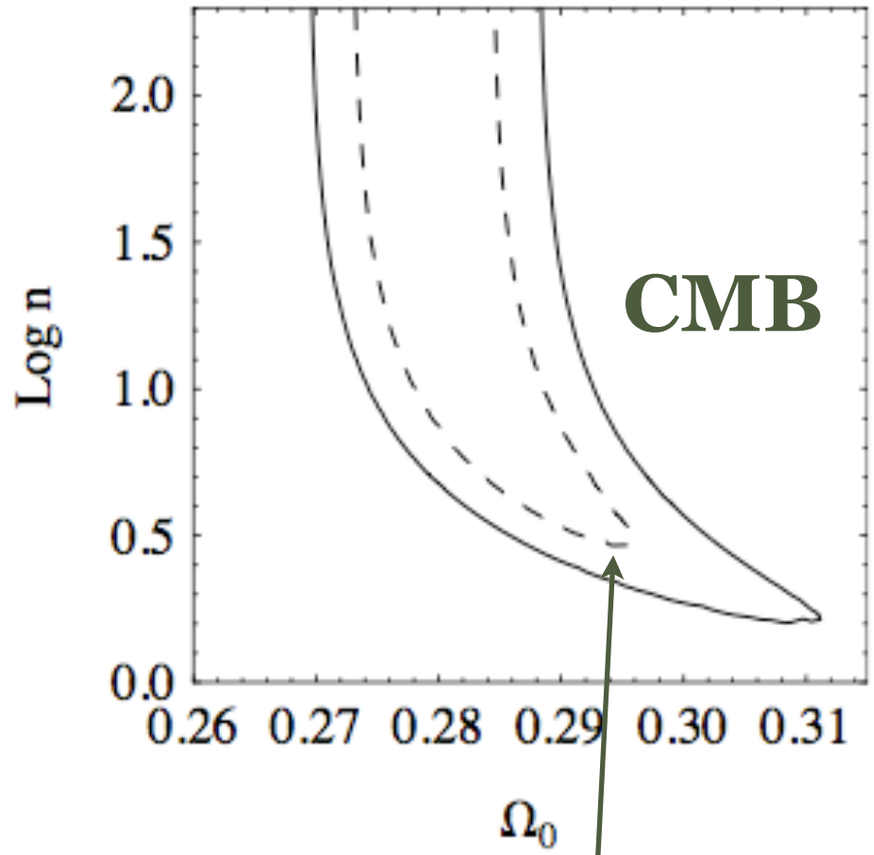

FIG. 3: The left panel is the contour of $\chi^2_{\rm SN}$ on the plane Ω_0 and n for the kinetic braiding model. The dashed curve and the solid curve are the 1 σ and 2 σ contours, respectively. The right panel is the same but of $\chi^2_{\rm CMB}$.

The SCP Union2 Compilation is a collection of 557 type Ia supernovae data whose range of the redshift is 0.015 < z < 1.4

Thus $n \gtrsim 3$ mass scale $\sim 10^{-3}$eV

Length Scale: **1/10 mm**

GROWTH FACTOR

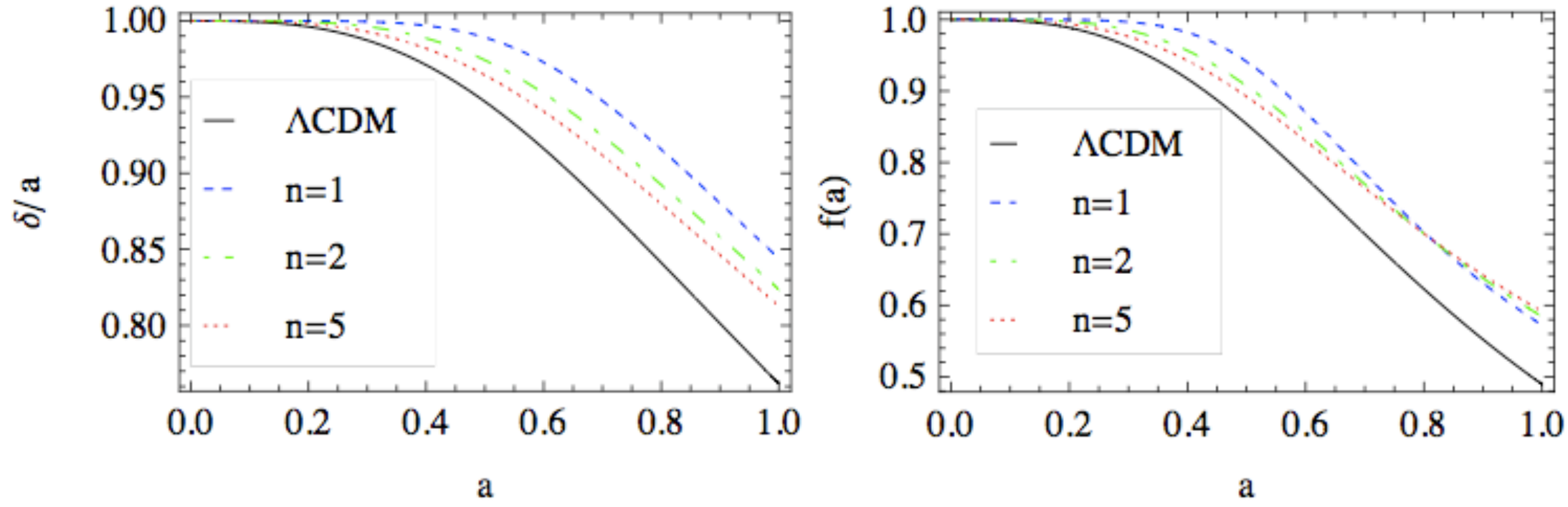

FIG. 4: Left panel: The growth factor divided by scale factor as a function of scale factor for the Λ CDM model (solid curve) and the kinetic braiding model $n = 1$ (dashed curve), $n = 2$ (dash-dotted curve), and $n = 5$ (dotted curve), respectively. Right panel: The linear growth rate as a function of scale factor.

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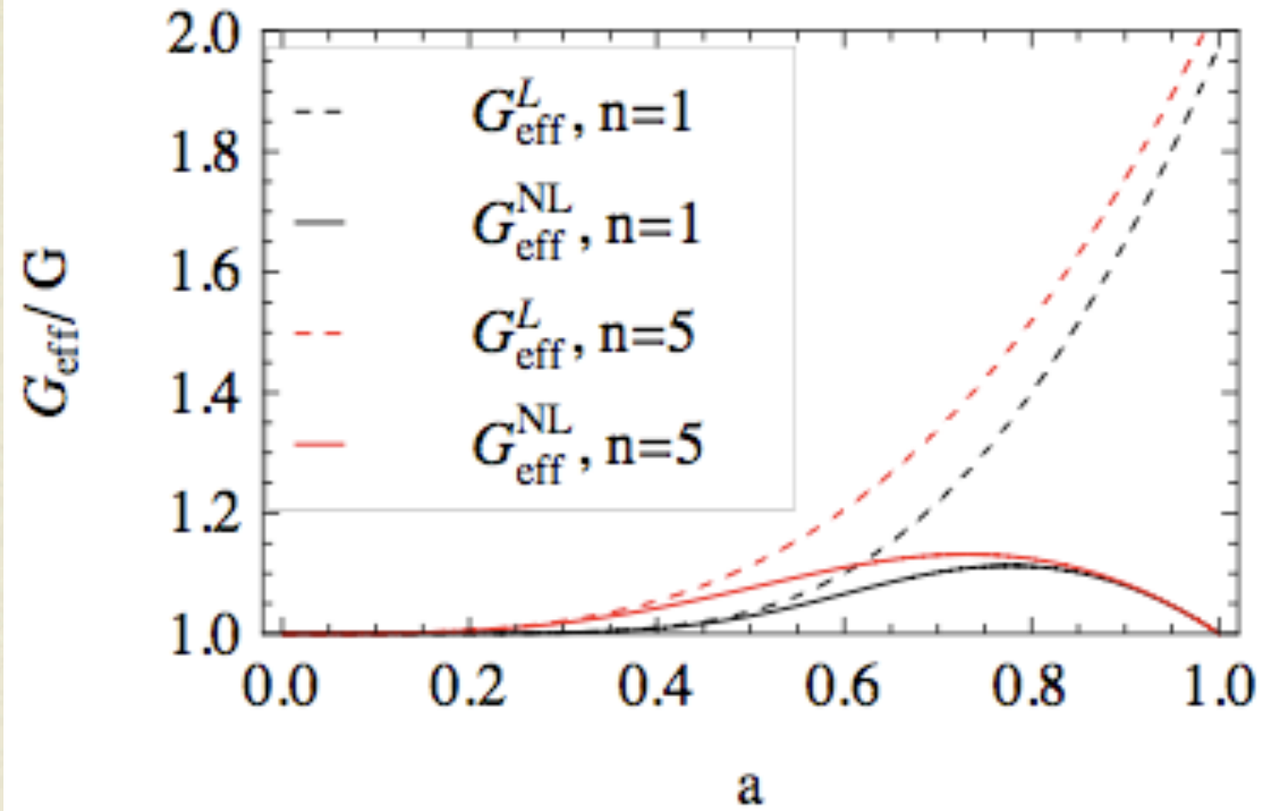
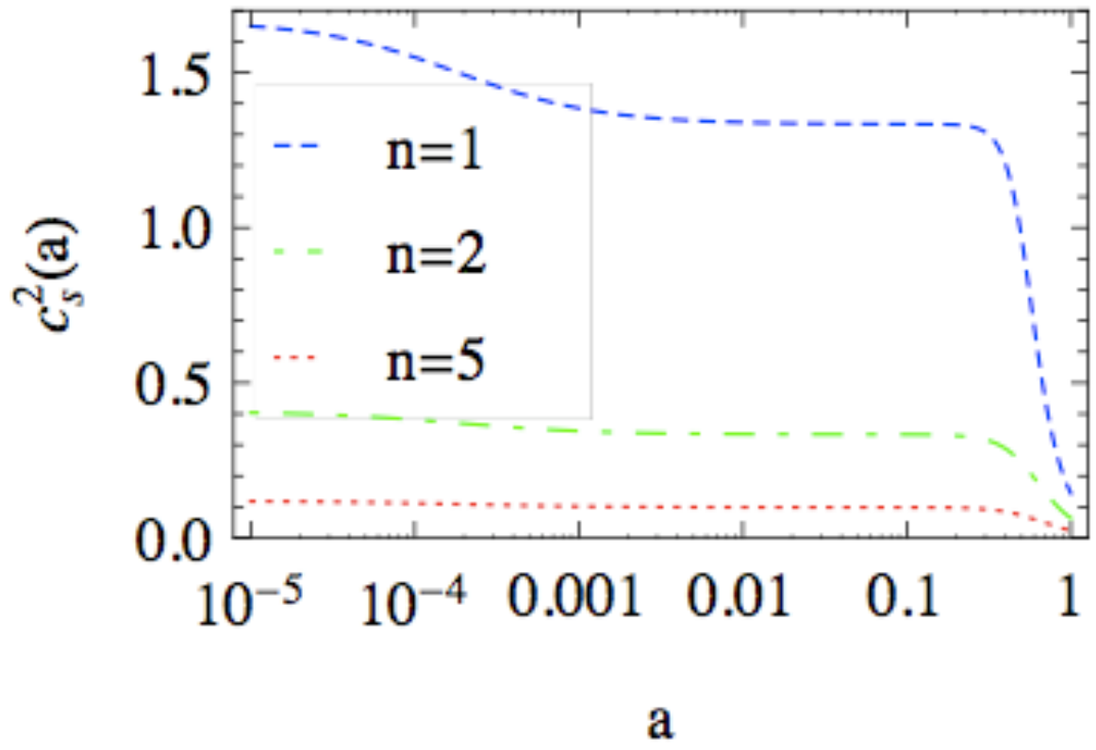
EFFECTIVE NEWTON CONSTANT FOR PERTURBATIONS AND THE SOUND SPEED





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THANKS A LOT FOR YOUR ATTENTION!