Loop Quantum Cosmology with vector fields

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Outline

- Motivation

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- GR with the vector field domination

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- Embedding of vector fields in Loop Quantum Cosmology

The Ashtekar formalism

Let us assume the perfect fluid with the vector field. Then

$$A_\mu = A(t)\delta^z{}_\mu, \qquad ds^2 = dt^2 - a^2(dx^2 + dy^2) - b^2 dz^2. \tag{1}$$

We construct the scalar constraint (Hamiltonian)

$$\mathcal{C} = -\frac{1}{\gamma^2 p_1 \sqrt{p_2}}(c_1^2 p_1^2 + 2c_1 c_2 p_1 p_2) + p_1 \sqrt{p_2}\rho, \tag{2}$$

where Ashtekar variables are defined by

$$p_1 = L_1 L_2 ab, \qquad p_2 = L_1^2 a^2, \qquad c_1 = \gamma L_1 \dot{a}, \qquad c_2 = \gamma L_2 \dot{b}. \tag{3}$$

Physical interpretation

a^2, ab - areas in anisotropic Universe

c_1, c_2 - external curvature along x, y and z axes

Cosmology with vector fields

From Hamilton equations

$$\dot{p}_1 = \frac{\gamma}{2}\{p_1, \mathcal{C}\}, \dot{p}_2 = \gamma\{p_2, \mathcal{C}\}, \qquad \dot{c}_1 = \frac{\gamma}{2}\{c_1, \mathcal{C}\}, \dot{c}_2 = \gamma\{c_2, \mathcal{C}\}, \tag{4}$$

one obtains Einstein's equations

$$H(H + 2\mathcal{H}) = \rho = \rho_f + \rho_A, \tag{5}$$

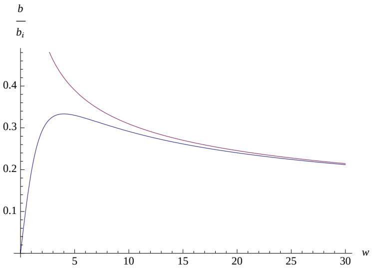
$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + H\mathcal{H} = -p_\perp = -p_f - p_A, \tag{6}$$

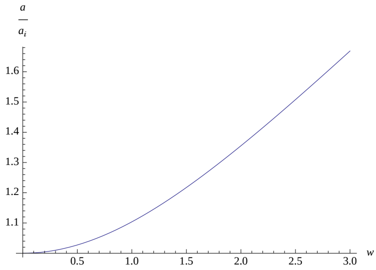
$$2\frac{\ddot{a}}{a} + H^2 = -p_\| = -p_f + p_A, \tag{7}$$

where $H = \dot{a}/a$, $\mathcal{H} = \dot{b}/b$ and
$\rho_A = (\dot{A}^2 + m^2A^2)/b^2,\ \ p_A = (\dot{A}^2 - m^2A^2)/b^2$

Massless vector field domination

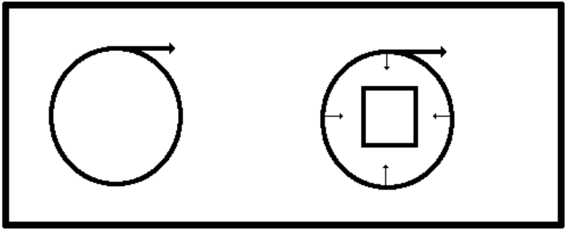
For $m = 0$ one obtains $b = E\dot{a}$, where $E = const > 0$. The Big Bang has finite energy density ρ_I





Loop Quantum Cosmology in FRW

In isotropic case $a = b$, so $p_1 = p_2$ and $c_1 = c_2$



The loop correction appears due to Δ - the minimal value of the area operator. This changes c to

$$c \to \sin\left(\bar{\mu} c\right), \quad (8)$$

where $\bar{\mu} = \sqrt{\Delta / p}$
Hamiltonian of matter fields and p remains unchanged

LQC in anisotropic Universe

In diagonal Bianchi I model LQC changes c_i to

$$c_i \to \sin\left(\bar{\mu}_i c_i\right), \tag{9}$$

where $\bar{\mu}_i$ are some generalisations of $\bar{\mu}$. Two ways of doing that:

The old $\bar{\mu}_i$*scheme*

Szulc '08: $\bar{\mu}_i = \sqrt{\Delta/p_i} \Rightarrow$ problems with scaling

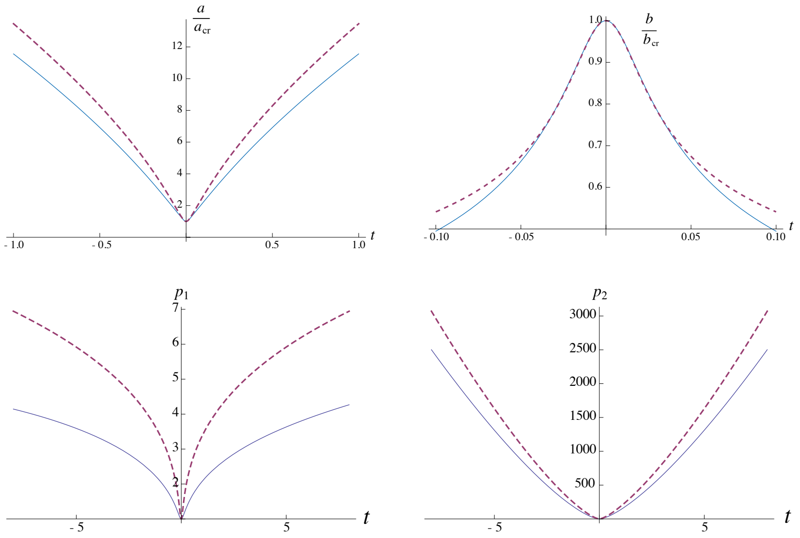
The new $\bar{\mu}_i$ scheme

Ashtekar, Wilson-Ewing '09: $\bar{\mu}_1 = \sqrt{\Delta/p_2}, \quad \bar{\mu}_2 = \sqrt{\Delta p_2}/p_1$

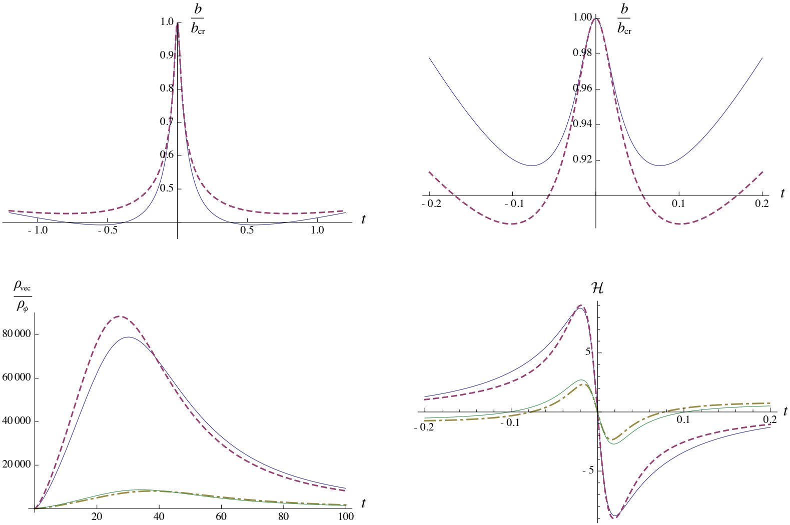
In both schemes p_i and $\mathcal{C}_{mat}$ are not changed by LQC

And now... The most important thing

LQC with massless vector field



LQC with massive vector and scalar



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- Isotropic low energy limit for massive vectors
- In LQC with vector fields $b(t)$ shrink for a while, but...
- everything is fine since areas always grow