Modeling the Shapes of Dark Matter Halos

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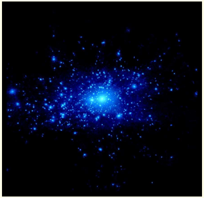
Outline

1. Motivation and Goals
2. The Model
3. Comparison with Simulations
4. Basic Highlights

MAIN REFERENCE

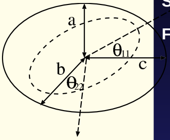
- **G. Rossi**, R. Sheth & G. Tormen (2011), MNRAS (arXiv:1010.2839)

Motivation and Goals



Motivation

- Halos are not spherical!
- Ellipsoidal description improves density profiles
- Implications for weak and strong lensing



Goals

- Statistical study of DM halo morphology
- Link between ellipsoidal shape and primeval principles (gravitational instability)
- Provide scheme for DM halo evolution ($a_{\rm i}, b_{\rm i}, c_{\rm i} \rightarrow a_{\rm f}, b_{\rm f}, c_{\rm f}$)

Shape Parameters

- **Ellipticity**

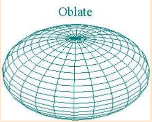
$$e = \frac{\lambda_1 - \lambda_3}{2\delta}$$

- **Prolateness**

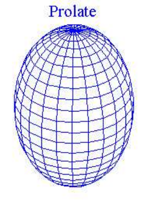
$$p = \frac{\lambda_1 + \lambda_3 - 2\lambda_2}{2\delta}$$

"Prolate" object $\rightarrow 0 \geq p \geq -e$
"Oblate" object $\rightarrow 0 \leq p \leq e$
Sphere $\rightarrow e = 0, p = 0$

- $a \simeq b \gg c$: Oblate object (disk-shaped) $\rightarrow$ pancake



- $b \simeq c \ll a$: Prolate object (cigar-shaped) $\rightarrow$ filament



Essence of the Model

Model made of two independent parts

1. Mapping $(e, p) \rightarrow (b/a, c/a)$ for and initial patch of given R and δ $\rightarrow$ evolution
2. Correct assignment $(e, p) \rightarrow$ halo mass $\rightarrow$ initial conditions

- **First Part** $\rightarrow$ Bond & Myers (1996), improved method
- **Second Part** $\rightarrow$ Sheth & Tormen (2002), improved method

Ellipsoidal evolution (1)

Equations of motion: ellipsoid

$$\frac{d^2A_k}{dt^2} = -4\pi G\bar{\rho}A_k\left[\frac{1+\delta}{3} + \frac{b'_k}{2}\delta + \lambda'_{ext,k}\right]$$

$$= -\frac{G}{A_k^2}\frac{4\pi A_k^3\bar{\rho}(1+\delta)}{3} - 4\pi G\bar{\rho}A_k\left[\frac{b'_k}{2}\delta + \lambda'_{ext,k}\right]$$

$$= -\frac{GM}{A_k^2} - 4\pi G\bar{\rho}A_k\left[\frac{b'_k}{2}\delta + \lambda'_{ext,k}\right]$$

Ellipsoidal evolution (2)

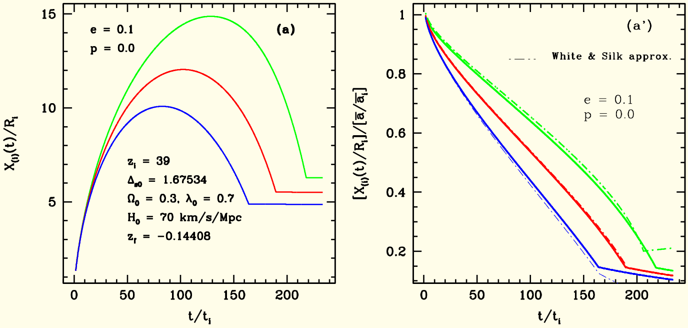
The complication arises from extra terms due to the ellipsoidal potential in a known external tidal field:

$$\Phi = -\pi G \sum_{k=1}^{3} \left[b_k \rho + \left(\frac{2}{3} - b_k \right) \bar{\rho} \right] A_k^2$$

Details

- $b'_k = b_k - 2/3$
- $b_k = A_1(t) A_2(t) A_3(t) \int_0^\infty \frac{d\tau}{[A_i^2(t)+\tau] \prod_{j=1}^{3} [A_j^2(t)+\tau]^{1/2}}$
- $\lambda'_{ext,k} = \frac{D(t)}{D_i(t)} [\lambda_{ext,k}(t_i) - \delta_i/3]$
- D is the linear theory growth mode of perturbations
- $\lambda_{ext,k}(t_i)$ are the initial eigenvalues of the strain tensor
- Extra term needed for Λ-cosmologies

Ellipsoidal evolution (3)



White & Silk approximation: extension

Nelect the time dependence of the $b_k's$ in the equation of motion and replace the quantity with a “spherical average”

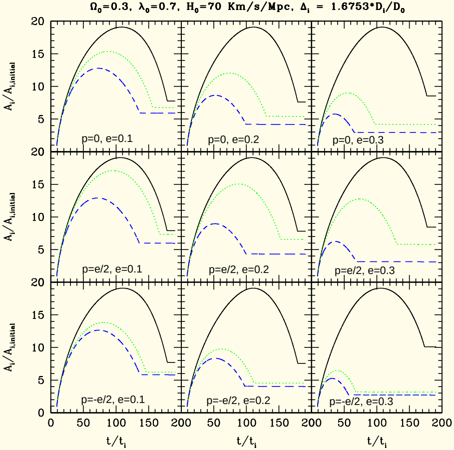
An analytic approximation

$$A_k(t) \simeq \frac{a(t)}{a(t_i)}\left\{A_k(t_i)\left[1-\frac{D(t)}{D(t_i)}\lambda_k(t_i)\right]-A_h(t_i)\left[1-\frac{D}{D_i}\frac{\delta_i}{3}-\frac{a_e(t)}{a(t)}\right]\right\}$$

where

- $A_h(t_i) = 3/\sum_k A_k(t_i)^{-1}$
- $a_e(t)$ = expansion factor of a universe with initial density contrast $\delta_i = \sum_k \lambda_k(t_i)$

SHAPE PARAMETERS: MEANING

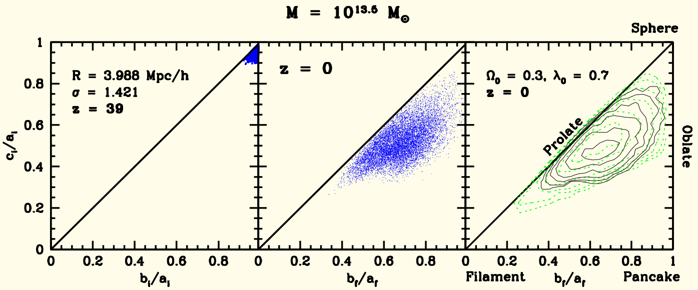


Prolate objects $\rightarrow$ 2 short, 1 long $\rightarrow$ *p* negative (filaments)
Oblate objects $\rightarrow$ 1 short, 2 long $\rightarrow$ *p* positive (pancakes)

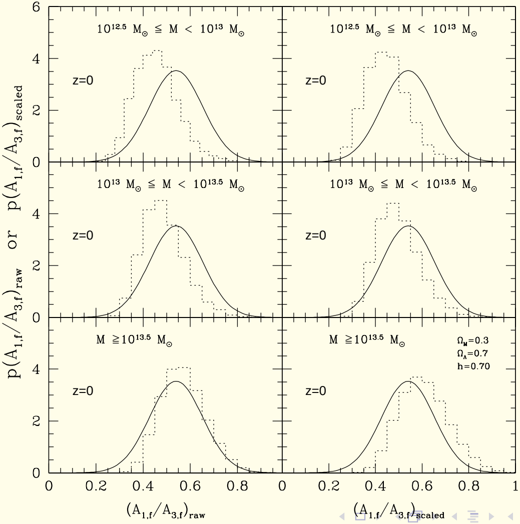
At *p* fixed, if *e* $\uparrow$ then objects are more elongated (filament-like)

Motivations
The Model
Comparison with Simulations
Future Prospects

Model in a Nutshell



COMPARISON WITH SIMULATIONS



Motivations
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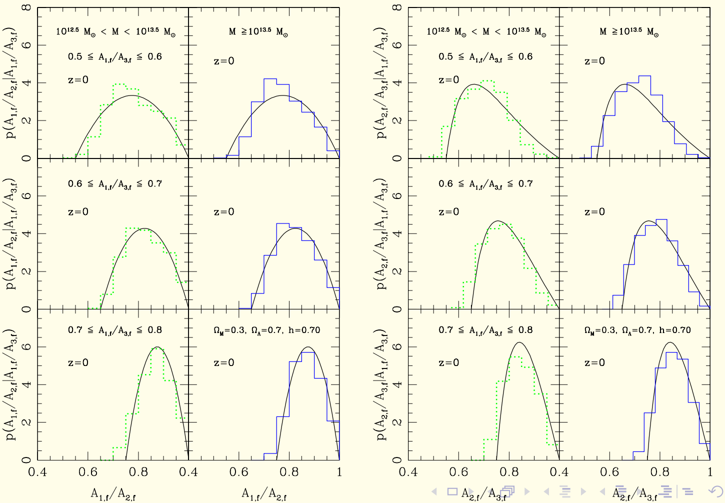
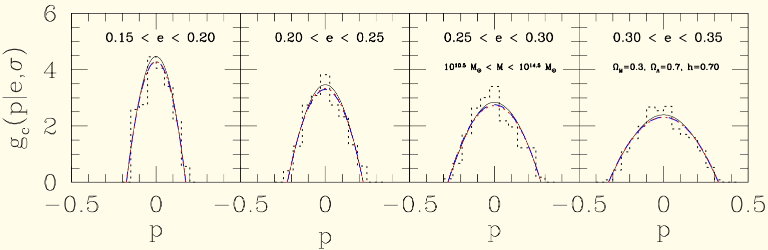
Comparison with Simulations



Universal shape function



Axis-ratio distributions at later times

$$p(A_{12}|A_{13},\delta,\sigma) = \frac{3}{2(1-A_{13})}\left[1 - \frac{(2A_{12}-1-A_{13})^2}{(1-A_{13})^2}\right] \times \exp\left\{-\frac{5}{8\sigma^2}(2A_{12}-1-A_{13})^2\right\}$$

HALO SHAPES: RATIONALE

MOTIVATION

- Theoretical model for dark halo shapes

ACHIEVEMENTS/RESULTS

- Satisfactory match with simulations only around M_*
- Good agreement for conditional distributions
- Analytic insights (i.e. WS approximation, explanations of fitting formulae, etc. ...)

RELEVANCE & ONGOING WORK

- Investigate discrepancies model vs simulations $\rightarrow$ HORIZON RUN simulation @ KIAS
- Numerous theoretical studies up-to-date (i.e. Robertson et al. 2009; Maggiore & Riotto 2010 ...)
- Relevance for EUCLID and WL $\rightarrow$ accurate shapes of galaxies

Kamsahamnida!