V+gamma and V+Jets Production at Hadron Colliders

Theory Overview

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LISHEP, Rio de Janeiro, Brazil - July 2011

OUTLINE – RELEVANT TOPICS

V+gamma

Radiation Zeros: A story of more than three decades

QCD and Electroweak Corrections

Pair Production of Vector Bosons

V+Jets

Large multiplicity processes at NLO: $W+ n$ Jets ($n = 0,1,2,3,4$)

Weak Vector Boson Polarization at the LHC

Showering at NLO

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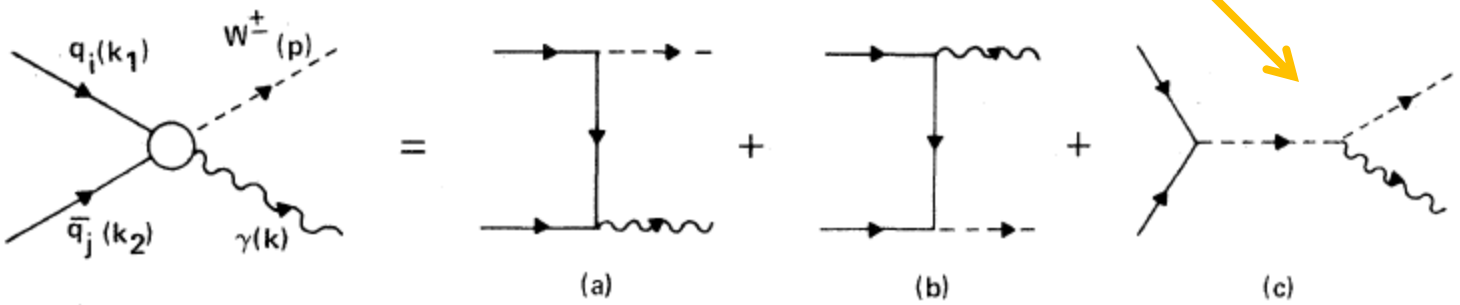
TESTING GAUGE STRUCTURE IN THE SM

$$q_i(k_1)\bar{q}_j(k_2) \rightarrow W^{\pm}(p)\gamma(k)$$

This process allows to measure the trilinear WWγ coupling

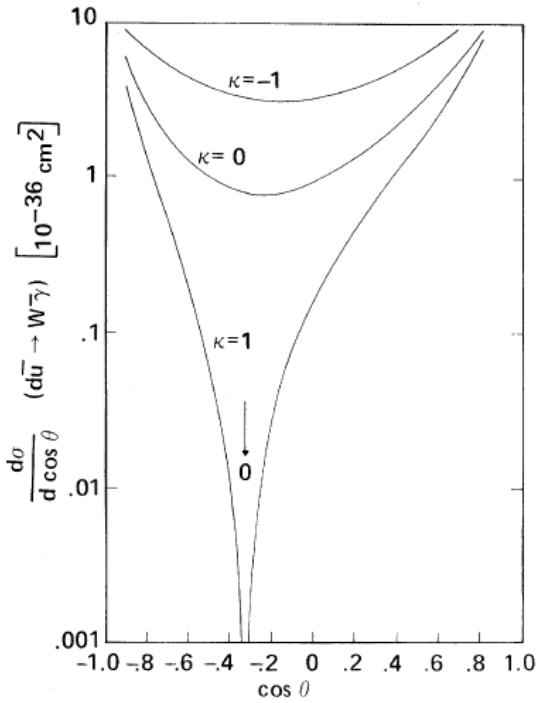


Mikaelian, Samuel, Sahdev; PRL 1979

First Born Level studies for measuring the *W* magnetic moment at hadron machine

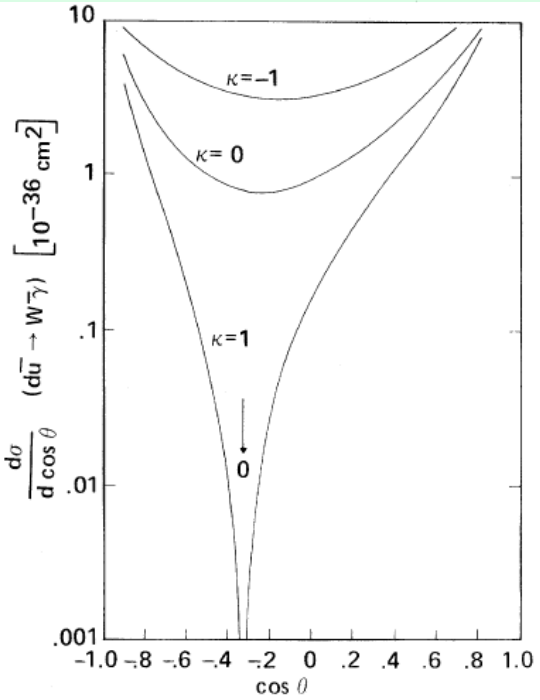
$$\mu_W = (e/2M_W)(1+\kappa)$$

In the SM κ = 1; κ ≠ 1 would be an aTGC



A CURIOUS RADIATION ZERO

Mikaelian, Samuel, Sahdev; PRL 1979



Θ is the angle between W^- and d

For *cos* θ = - 1/3 ; the diff cross section vanishes!

Goebel, Halzen, Levielle; PRD 1981

Actually the amplitude vanishes, due to factorization properties!

Indeed they prove that by general properties (mom conservation, on-shellness, charge conservation) 4-point gauge amplitudes can be arranged in forms like:

$$A = (Q_1 u - Q_2 t)\left[\frac{(p_2 \cdot \epsilon)t - (p_1 \cdot \epsilon)u}{stu}\right]$$

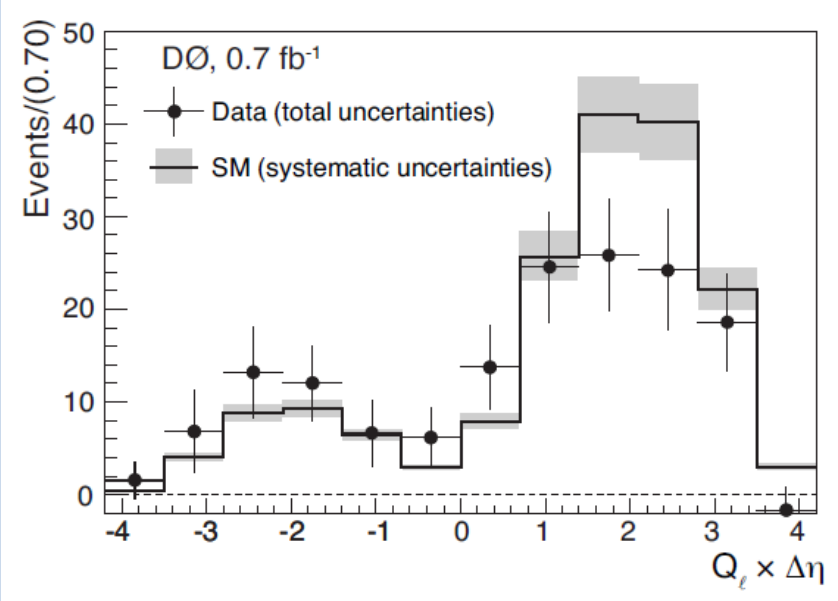
!!!

Bern, Carrasco, Johansson ; arXiv:0805.3993

Almost 3 decades later, the “spatial generalized Jacobi identity” used by GHL in their study, would be generalized to higher point amplitudes within the so called BCJ identities: Useful tool for gauge and gravity amplitudes!

RADIATION AMPLITUDE ZERO MEASURED!

At the hadron level the RAZ shows as a dip in the (η_e-η_γ) distribution. QCD corrections reduce slightly its size. Possible aTGCs basically wash it out.



D0 (**arXiv:0803.0030**) has made a dedicated RAZ study with a 0.7 fb^{-1} data set

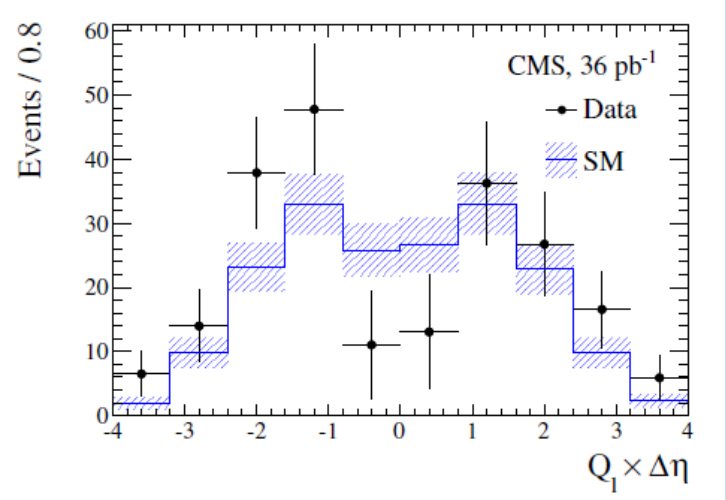
They get a 2.6σ significance for its measurement

Also constrained (CP-conserving) aTGCs to be in the ranges (consistent with the SM):

$$0.49 < \kappa_\gamma < 1.51$$

$$-0.12 < \lambda_\gamma < 0.13$$

OTHER RECENT Wγ AND Zγ MEASUREMENTS



CMS (**arXiv:1105.2758**) has made Wγ and Zγ measurement with a 36 pb^{-1} data set

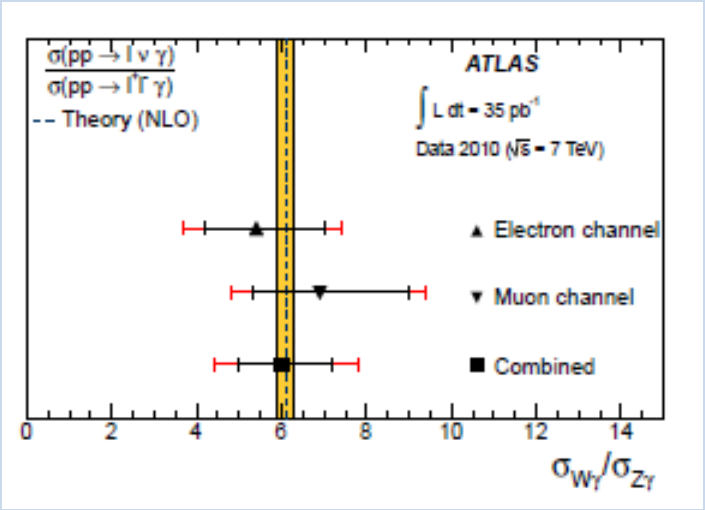
They see agreement with the SM prediction of a RAZ

Constrained (CP-conserving) aTGCs (WWγ, ZZγ and Zγγ)

ATLAS (**arXiv:1106.1592**) has made Wγ and Zγ measurement with a 35 pb^{-1} data set

Made a dedicated study of total and diff cross sections

Discusses Wγ / Zγ ratios. Don't show RAZ or aTGCs studies.



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QUANTUM CORRECTIONS TO Wγ and Zγ PRODUCTION

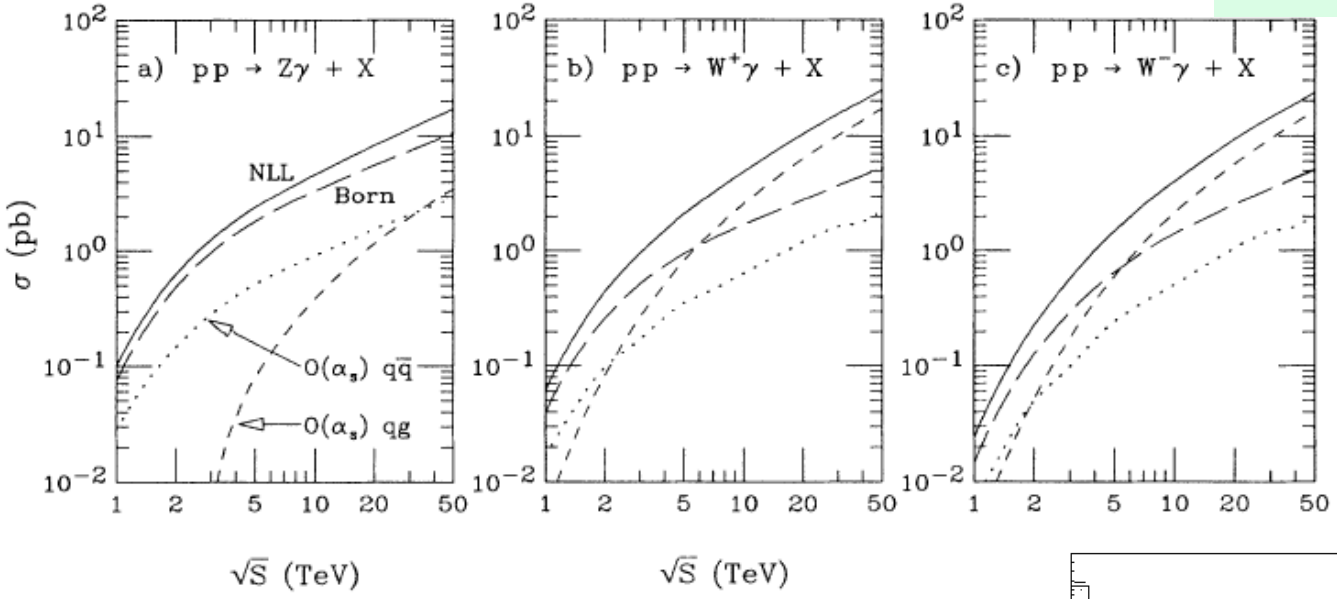
Original work on the impact of QCD corrections in Wγ production was performed by Smith, Thomas and van Neerven in the late 80's	Smith, Thomas, van Neerven; Z.phys.C 1989
Ohnemus also studied Wγ and added QCD corrections to Zγ production	Ohnemus; PRD 1991
Studies of QCD corrections for general TGCs	Baur, Han, Ohnemus; PRD 1993
Fully differential studies at NLO	de Florian, Signer ; hep-ph/0002138
Fully differential (partial) Electroweak corrections	Hollik, Meier ; hep-ph/0402281 Accomando, Denner, Meier ; hep-ph/0509234
Very recent update on general Vector Boson Pair production (including γ radiation from leptons)	Campbell, Ellis, Williams; arXiv:1105.0020

INCLUDED INTO MCFM (v6.0): A PARTON LEVEL NLO MONTECARLO PROGRAM

TOOLS (1/3)

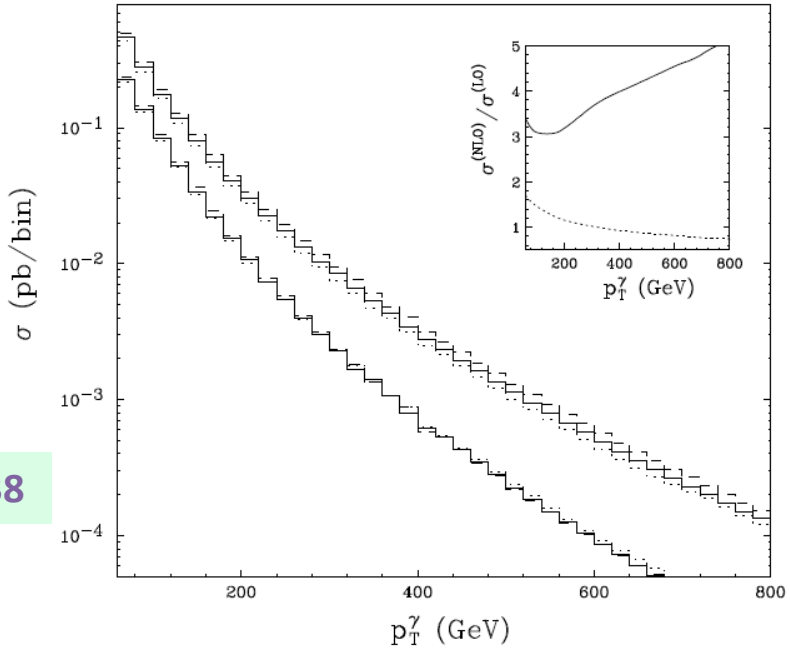
QCD CORRECTIONS: BRIEF RECOUNT

Ohnemus; PRD 1991



Corrections large, specially in Wγ with increased energy. Similar to Drell Yang effects

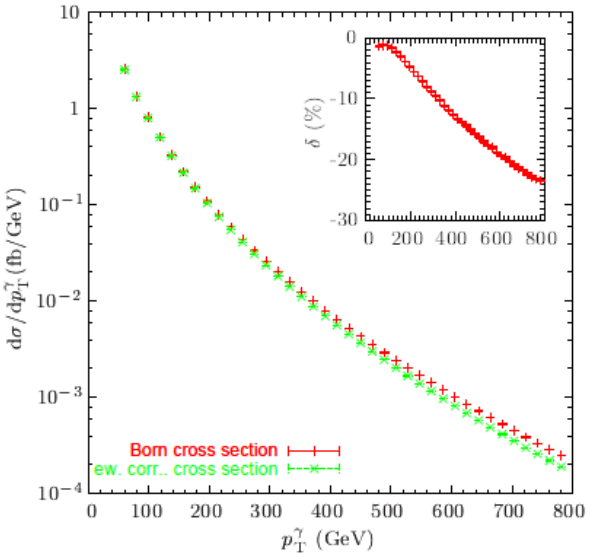
Considerable dependence of the corrections over PS (particularly over γ pT)

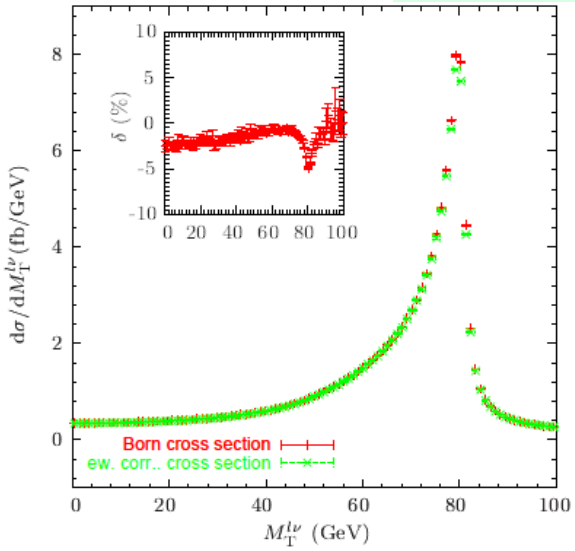


de Florian, Signer ; hep-ph/0002138

ELECTROWEAK CORRECTIONS: BRIEF RECOUNT

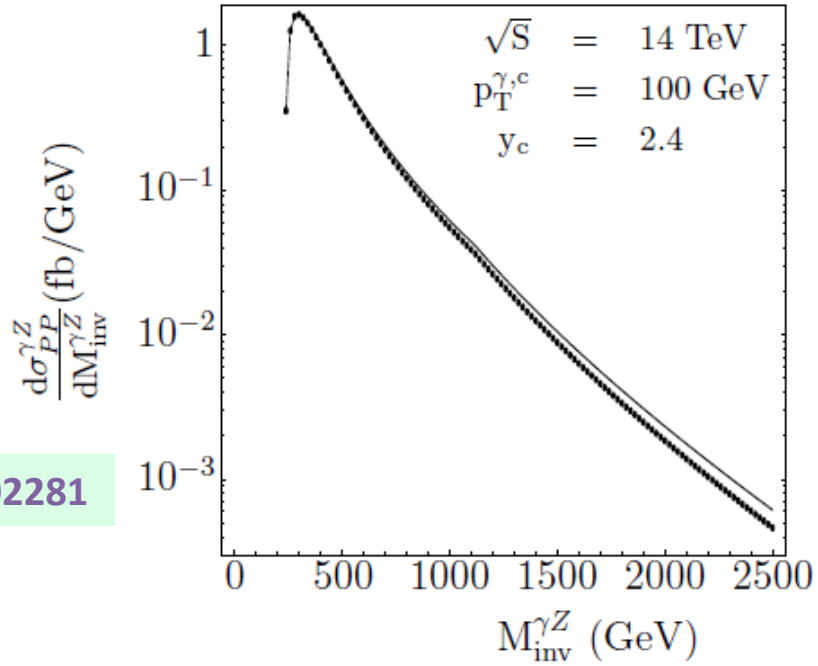
Accomando, Denner, Meier ; hep-ph/0509234





Corrections moderate in Wγ ; in the order of -5% ; tend to increase when testing larger partonic CM energies

Similar observations for Zγ production.



Hollik, Meier ; hep-ph/0402281

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TOOLS (1/3): NLO PARTON LEVEL MONTECARLOS

- Importance of computing quantum correction is clear (better modeling of underlying theories; decrease sensitivity to unphysical scales; good shapes of distributions; etc)
- Often involves cumbersome calculations: So theorist tend to be "busy" with details
- Theory publications can never cover all interesting kinematic scenarios and observables
- So it is important that theorists hand out ***TOOLS*** that allow experimental collaborations to readily study signals and backgrounds at the best available precision

Campbell, Ellis, Williams; arXiv:1105.0020

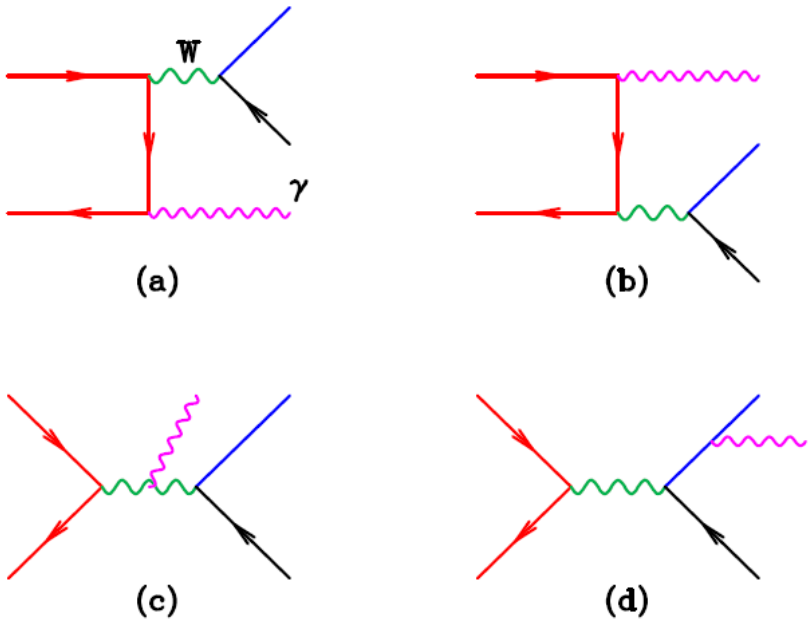
An example: VV PRODUCTION@NLO WITH MCFM (v6.0)

- Latest MCFM release completed the set of all calculations of VV production at NLO QCD
- This includes γγ, Wγ, Zγ, WW, WZ and ZZ production!
- Also allows for studies of different photon isolation schemes

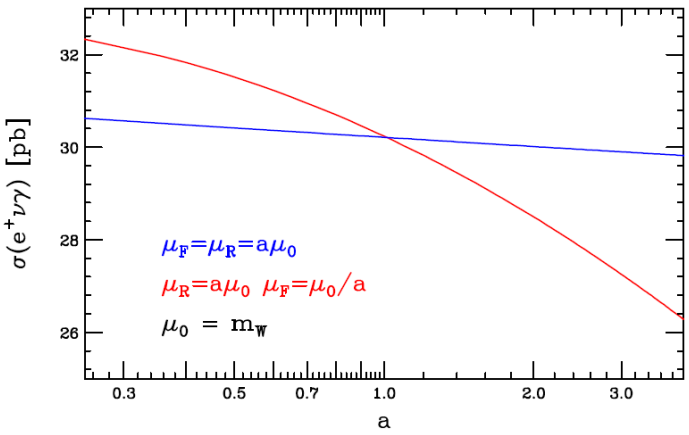
TOOLS (1/3): Wγ Production At The LHC

Campbell, Ellis, Williams; arXiv:1105.0020

Full set of tree level diagrams for $W(\to \ell\nu)\gamma$ production



And we get the typical theory plot:

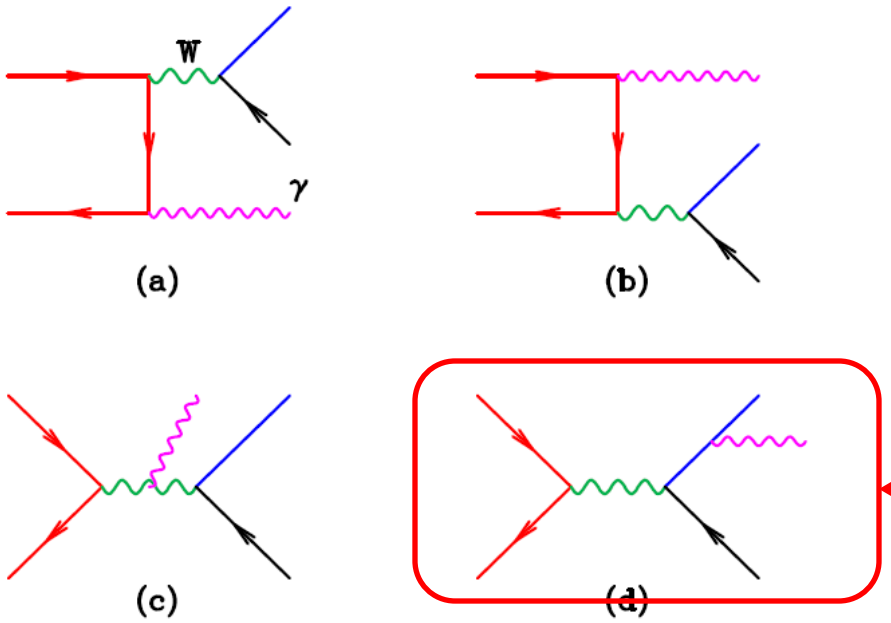


But we might be interested into learn much more than just scale dependence!!!

TOOLS (1/3): Wγ Production At The LHC

Campbell, Ellis, Williams; arXiv:1105.0020

Full set of tree level diagrams for $W(\to \ell\nu)\gamma$ production



For example: which sort of impact has in the quantum corrections the inclusion of (photon) "FSR" associated with this tree level diagram ???

CEW show a study like this and obtain the following:

TOOLS (1/3): Wγ Production At The LHC

Campbell, Ellis, Williams; arXiv:1105.0020

APPLY THESE DIFFERENT SET OF CUTS

$$\text{Basic Photon} : p_T^\gamma > 10 \text{ GeV},\ |\eta_\gamma| < 5,\ R_{\ell\gamma} > 0.7,\ R_0 = 0.4,\ E_T^{max} = 3 \text{ GeV}.$$

$$M_T \text{ cut} : \text{Basic Photon} + M_T > 90 \text{ GeV}.$$

$$\text{Lepton cuts} : M_T \text{ cut} + E_T^{\text{miss}} > 25 \text{ GeV},\ p_T^\ell > 20 \text{ GeV},\ |\eta_\ell| < 2.5.$$

AND LOOK AT THE TOTAL RATES:

Decay	Cuts	$\sigma^{LO}(e^+\nu\gamma)$	$\sigma^{NLO}(e^+\nu\gamma)$
No FSR	Basic γ	4.88	8.74
	M_T cut	1.99	3.78
	Lepton cuts	1.49	2.73
Full	Basic γ	23.0	30.1
	M_T cut	2.12	3.94
	Lepton cuts	1.58	2.85

TOOLS (1/3): Wγ Production At The LHC

- Many similar questions can be answered in a similar way DIRECTLY by the user of MCFM.
- Notice that MCFM contains many other interesting processes, including V+*n* Jets (*n* = 0,1,2).
- This approach is really efficient as long as one can keep computer needs moderate; i.e. running with few variations of inputs (couplings, PDFs, jet algs, etc) and for (relatively) low multiplicity studies.

Depending on the needs, exists several similar programs: like DIPHOX; NLOJet++; JetPHOX; VBFNLO ; etc, etc…

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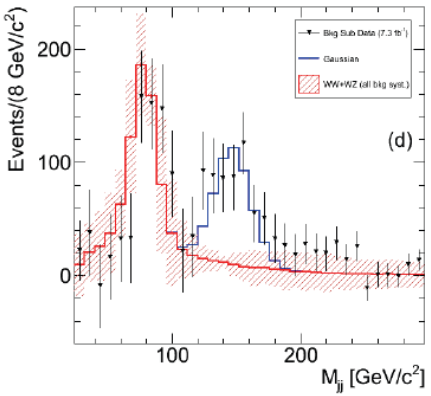
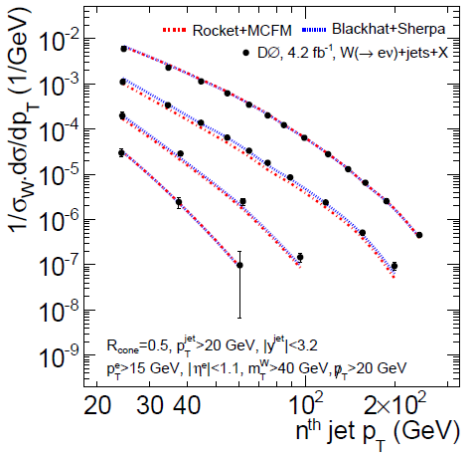
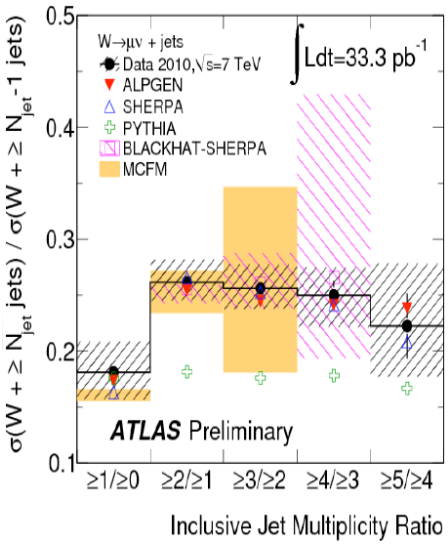
V+JETS: A IMPORTANT SIGNAL



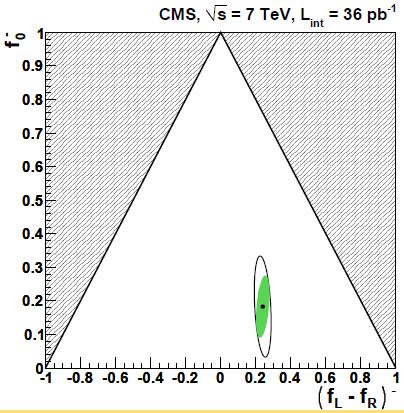
CDF Bump



D0 W+Jets Measurement
arXiv:1106.1457



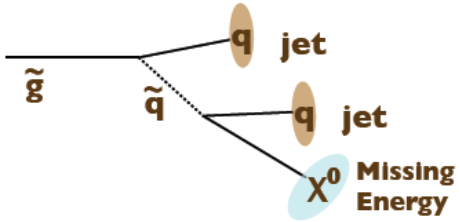
Atlas Jet Ratios



CMS Pol Measurement
arXiv:1104.3829

V + Jets at NLO for SUSY Searches

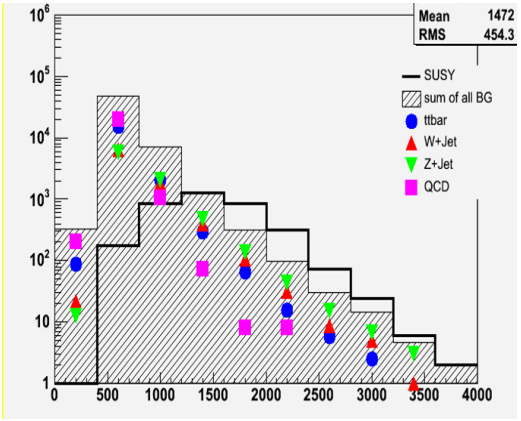
Gluino pair production with:



Mangano [arXiv:0809.1567]

$W^+ + 3$ jet followed by: $W^+ \to \bar{\tau}\nu_\tau \to \bar{\nu}_\tau\nu_\tau + \text{hadrons}$

Z + 4 jet followed by: $Z \to \nu\bar{\nu}$



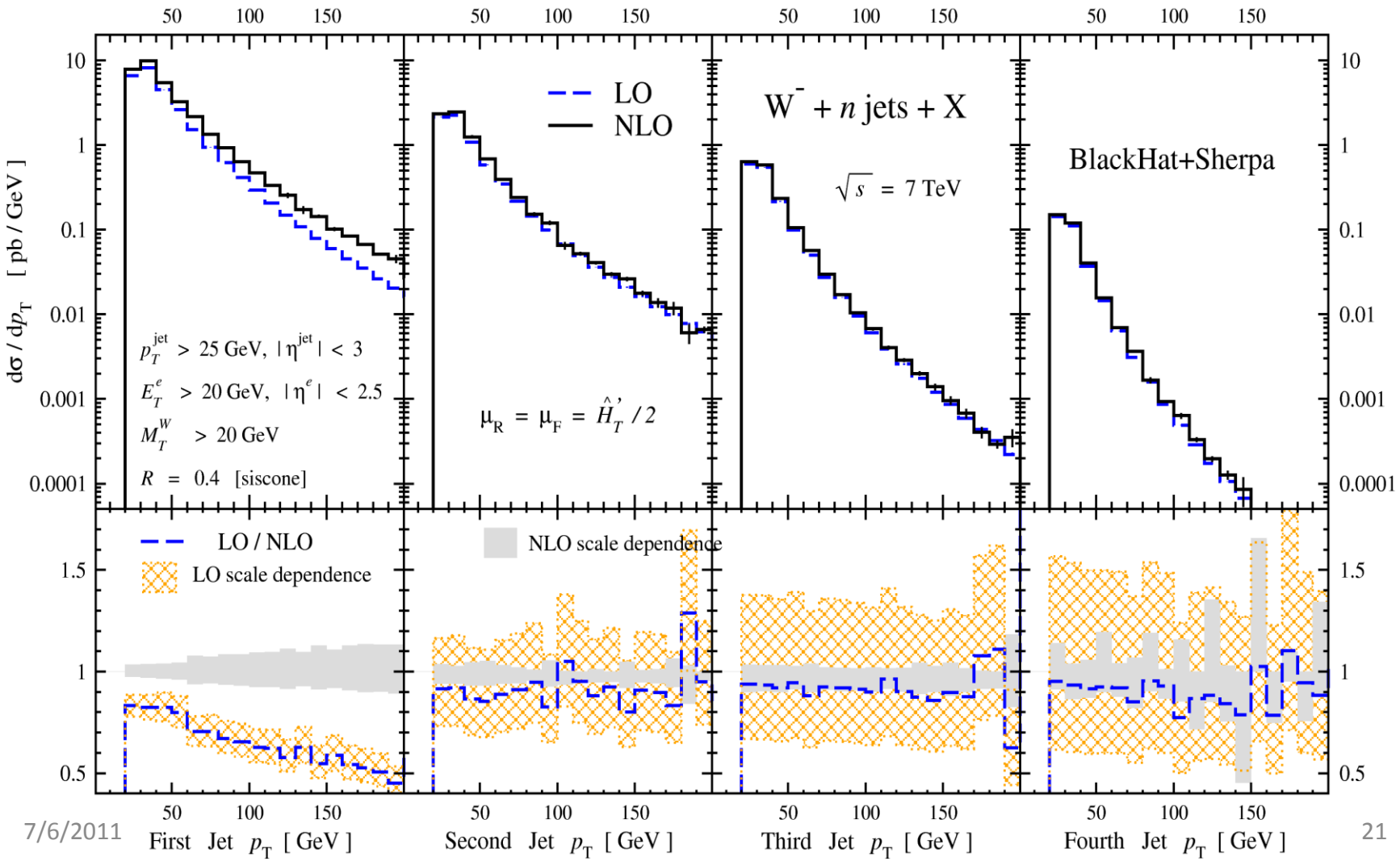
ATLAS simulation for missing ET + Jets signal

We need: pp → W + 1,2,3,4-jets

Berger, Bern, Dixon, FFC, Forde, Gleisberg, Ita, Kosower, Maitre

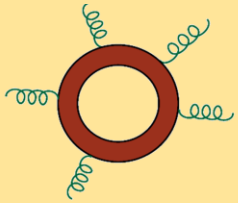
W^- + n-jet + X softest jet p_T

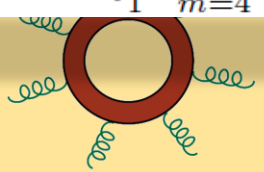
[BlackHat, arXiv:1009.2338]

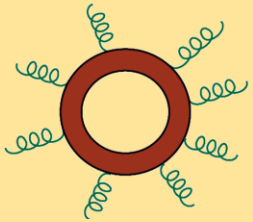


New technology to deal with the Gluon Mess!

# of jets	# 1-loop Feynman diagrams
3	810
4	
5	168,925
6	3,017,490







[Bern, Dixon, Dunbar, Kosower **hep-ph/9409265**]

$$\frac{c_\Gamma(\mu^2)^\epsilon A_{1\,2}^{\text{tree}}(1,\ldots,n)}{2}\left\{\left(\mathrm{K}_0(t_2^{[2]})+\mathrm{K}_0(t_n^{[2]})\right)\right.$$
$$\left.-\frac{1}{t_1^{[2]}}\sum_{m=4}^{n-1}\frac{\mathrm{L}_0\left(-t_2^{[m-2]}/(-t_2^{[m-1]})\right)}{t_2^{[m-1]}}\left(\mathrm{tr}_+[\not{k}_1\not{k}_2\not{k}_m\not{q}_{m,1}]-\mathrm{tr}_+[\not{k}_1\not{k}_2\not{q}_{m,1}\not{k}_m]\right)\right\}$$

looks like magic…

1-loop Amplitudes from Unitarity

See [Bern, Dixon, Dunbar, Kosower hep-ph/9212308]

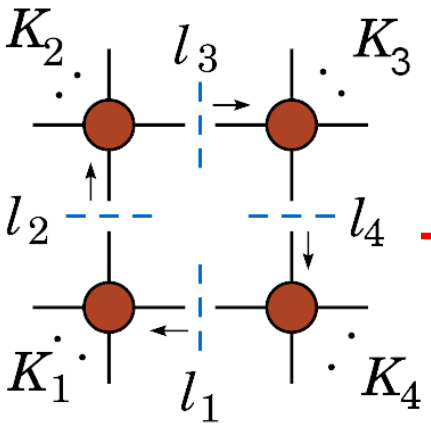
Rational in momenta

Logarithmic in momenta

Process dependent; rational in momenta

$$A = R + C$$

$$C = \sum_i b_i \; + \sum_i c_i \; + \sum_i d_i$$

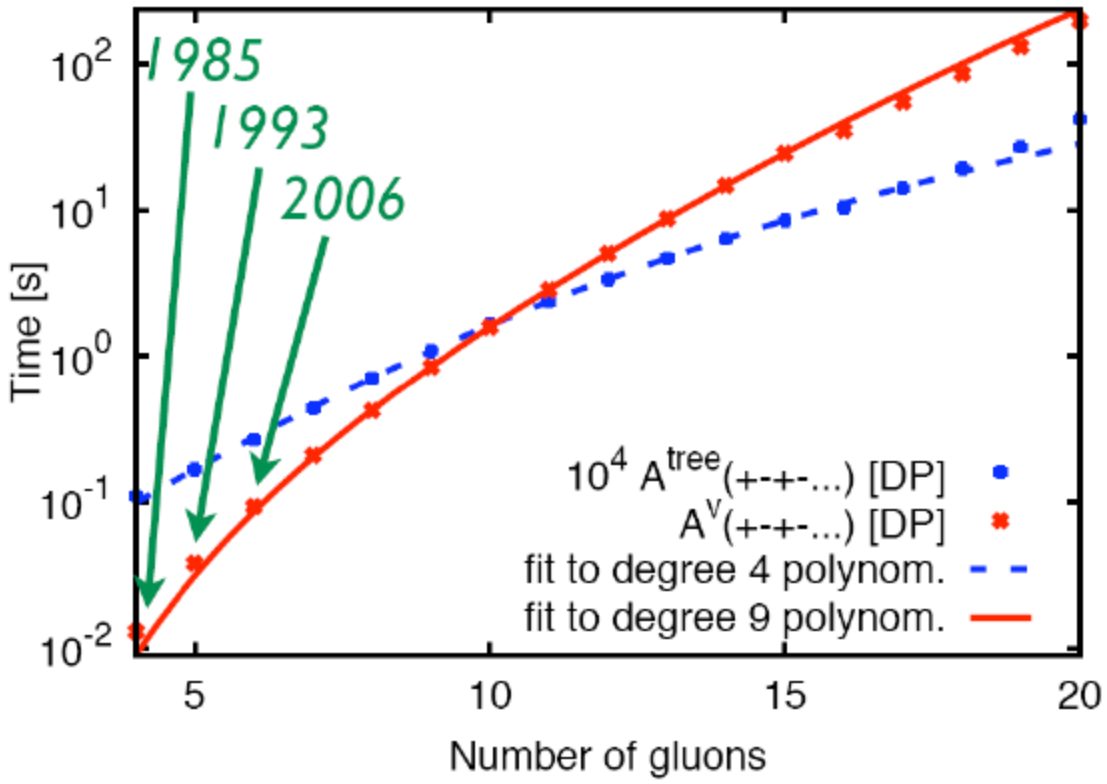


[Britto, Cachazo, Feng hep-th/0412103]

$$b_i = A^{\text{tree}}_{(1)} A^{\text{tree}}_{(2)} A^{\text{tree}}_{(3)} A^{\text{tree}}_{(4)}$$

And then one can extract all coefficients! [Ossola, Papadopoulos, Pittau hep-ph/0609007] [Ellis, Giele, Kunszt arXiv:0708.2398] [Forde arXiv:0704.1835]

A Powerful Technique!



[Giele, Zanderighi arXiv:0806.2152]

BUT STILL VERY COMPUTER INTENSIVE

[BlackHat + Sherpa]

NTUPLES: STORE THE MORE INFORMATION YOU CAN IN YOUR COMPUTATION!

TOOLS (2/3)

TOOLS (2/3): NLO NTUPLES BlackHat+Sherpa

- (Large multiplicity) NLO predictions are CPU expensive
- While generating events for a NLO computation, save in files:
  - Parton information (momenta, flavor)
  - Weight
  - Factorisation and renormalisation scales
  - Additional information for scale and pdf change
- These files, "the ntuples", would be the main results from a theory computation: We can share these files with other theorist and experimentalists

TOOLS (2/3): NLO NTUPLES BlackHat+Sherpa

- Advantages:
  - No need for the end user to run a complicated NLO setup
  - Can produce many plots from the same run
  - Can change scales/pdf
  - Share parts of the computation
- Disadvantages:
  - Large files

Wm2j 7TeV

part	N of files	total events	size of a file	disk/Mevent	total disk usage
born	8	40M	709M	140M	5.7G
bornLO	8	40M	695M	140M	5.6G
real	300	750M	2.8G	1136M	840G
vsub	20	200M	2.7G	270M	54G
loop	100	100M	177M	177M	17.7G
total	436	1130M	----	----	923G

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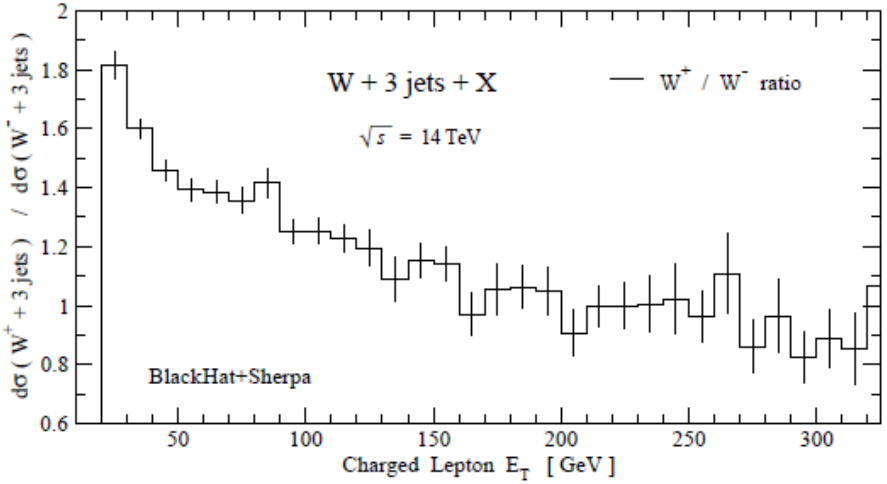
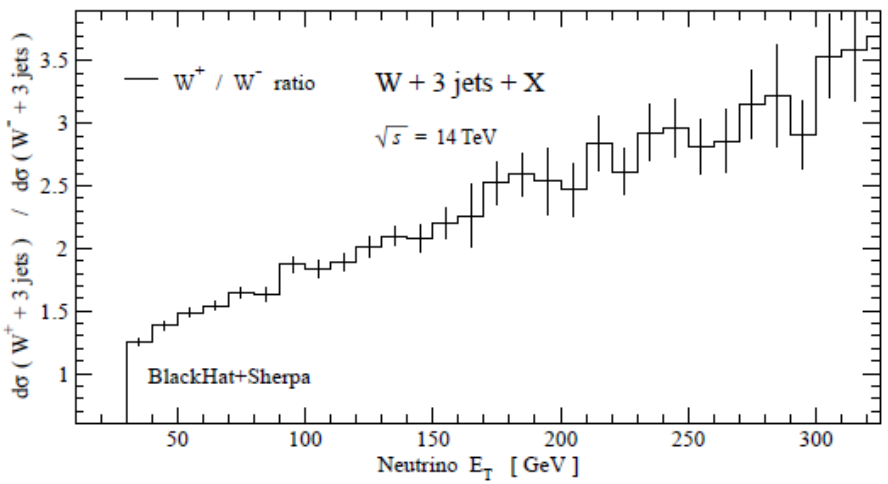
Leptonic E_T in W + 3 jets at LHC

[Berger, et al arXiv:0907.1984]





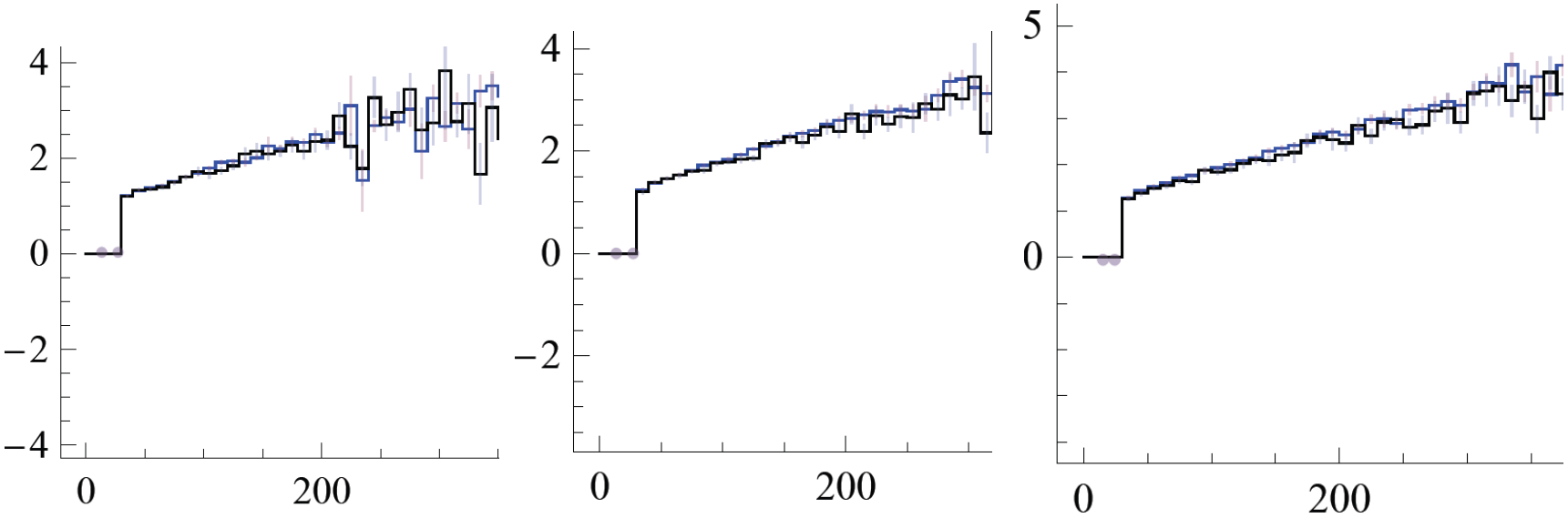
W^+/W^- transverse lepton ratios trace a remarkably large and stable left-handed W polarization at large $p_T(W)$
– independent of number of jets
– will be useful to separate W + n jets from top, maybe also from new physics

BlackHat: **[arXiv:1103.5445]**

$W^{+/-}$ + n jets: Neutrino E_T

NLO LO

BlackHat: [arXiv:1103.5445]



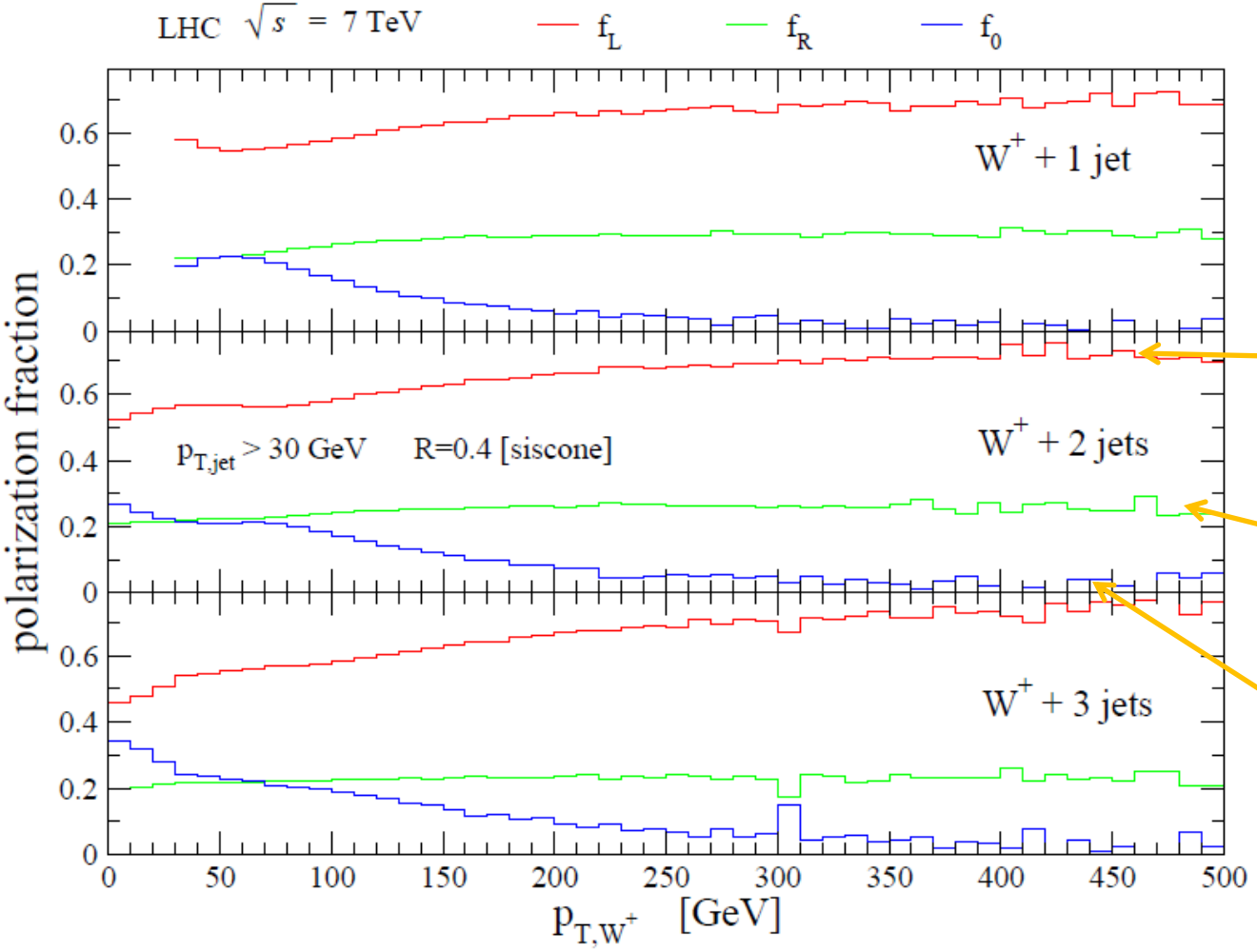
1 jet 2 jets 3 jets

Effect independent of multiplicity! Almost no difference from NLO and LO!

Similarly for charged lepton E_T

Actual W polarization

BlackHat: [arXiv:1103.5445]



Left handed component

Right handed component

Scalar component

Top quark pairs very different

BlackHat: **[arXiv:1103.5445]**

Main production channels are CP invariant:

$$gg \to t\bar{t} \qquad q\bar{q} \to t\bar{t}$$

Semi-leptonic decay involves (partially) left-handed W^+

$$t\bar{t} \to bW^+\bar{b}W^- \to b\,e^+\nu\,\bar{b}jj$$

But conjugate decay involves (same degree) right-handed W^-

$$t\bar{t} \to bW^+\bar{b}W^- \to bjj\,\bar{b}\,e^-\bar{\nu}$$

→ electron and positron have almost identical p_T distributions

→ A nice handle on separating *W*+jets from top

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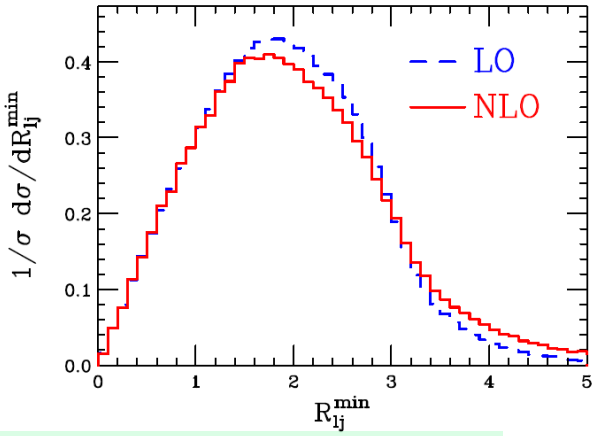
NEED TO GO BEYOND PARTON LEVEL NLO

- Although parton level NLO QCD correction are a necessity, certainly they have a limited reach
- Experimental data are often "corrected" to parton level, unfolding non-perturbative effects (hadronization, underlying event) and shower effects
- This is far from optimal: *Theory should get close to data, **not** the other way around!*
- Algorithms to have consistent NLO showers are needed

TOOLS (3/3): NLO SHOWER ALGORITHMS

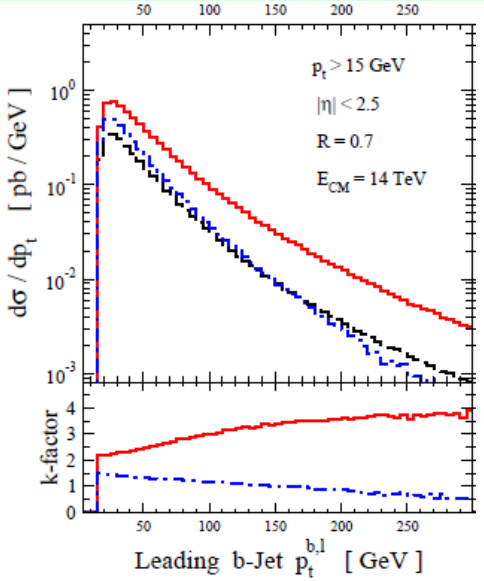
- Great advances over the last years on automation of consistent NLO showers: in particular within the MC@NLO program and with the POWHEG method
- Several automated codes exist in the market: The POWHEG Box, POWHEG in SHERPA, aMC@NLO, etc
- More and more processes included within these frameworks

An example: $Vb\bar{b}$ PRODUCTION



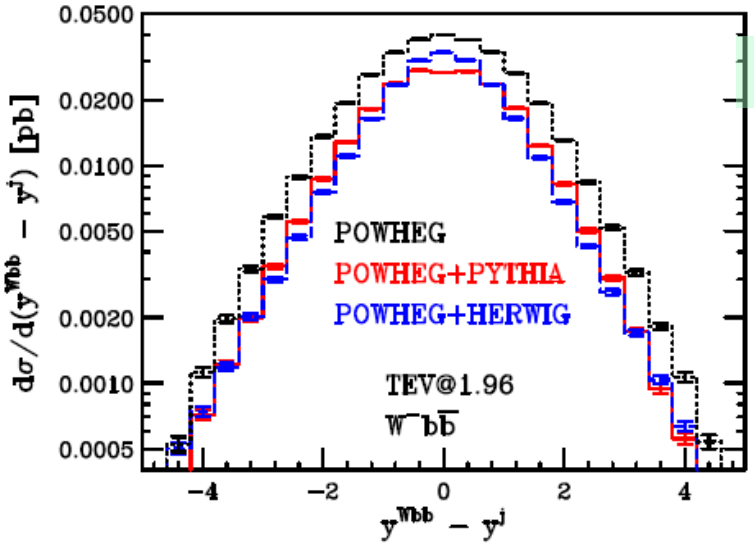
Badger, Campbell, Ellis; arXiv:1011.6647

FFC, Reina, Wackeroth; arXiv:0906.1923

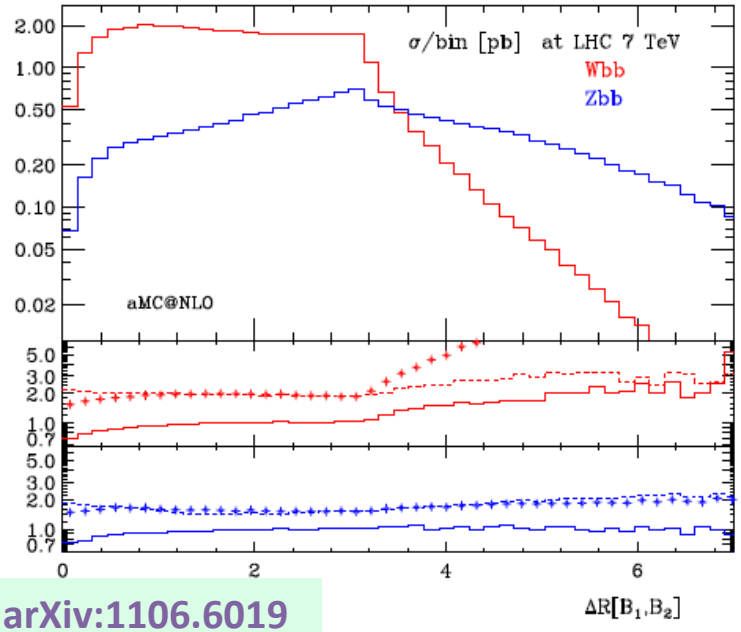


TOOLS (3/3): An Example: $Vb\bar{b}$ Production

An NLO shower is particularly desirable for this process!



Reina, Oleari; arXiv:1105.4488



Frederix, Frixione, Hirschi, Maltoni,Pittau, Torrielli; arXiv:1106.6019

Closing Remarks

- Weak vector boson production in association with a photon or with jets are very important ways in the **exploration of the validity of the SM**
- The relevance of developing **TOOLS** useful for experimentalists
  - (1/3) Parton Level NLO Montecarlos
  - (2/3) Ntuples for complex high multiplicity calculations
  - (3/3) Consistent NLO (QCD) showering program
- Theory community have been busy keeping up with the challenges that the LHC presents
- Hope for more Theorist/Experimentalist collaborations!!!

BACKUP SLIDES…

Numerical Stability

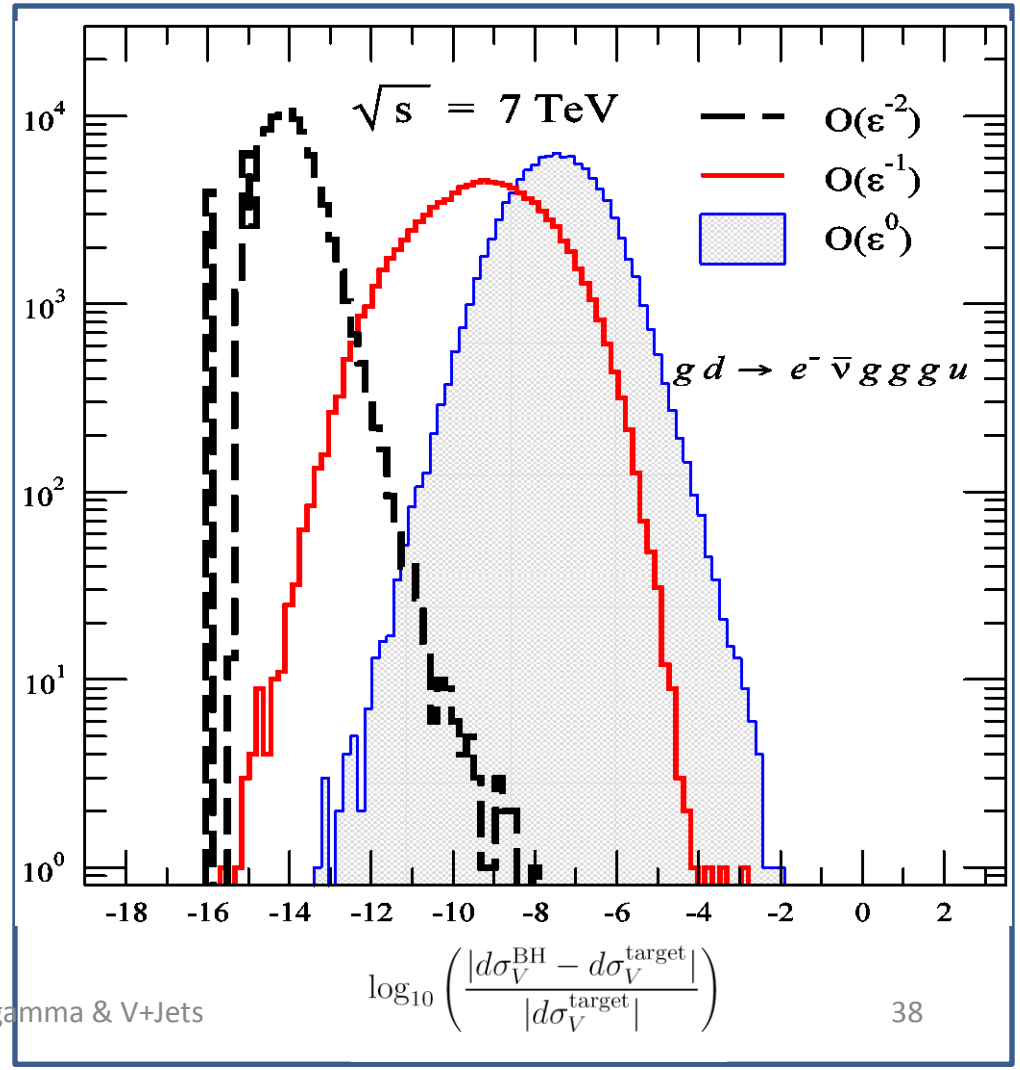
Good numerical control over matrix elements

- Virtual contribution
- gd → e^- $\bar{\nu}$ g g g u
- Test over physical phase space (10^5 PS points)

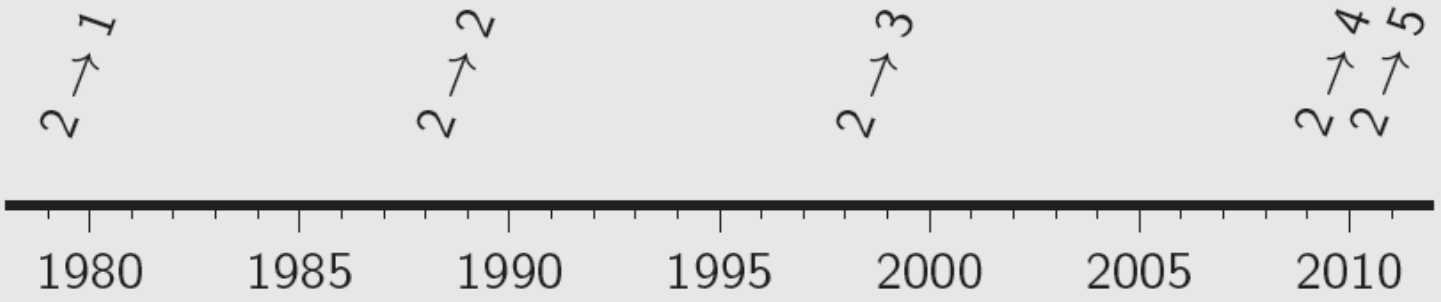
Needed precision checks & rescue system [BlackHat:0803.4180]

- Locally compute with more digits when needed (double, quadruple,…)

BlackHat 1005.3728



The NLO revolution

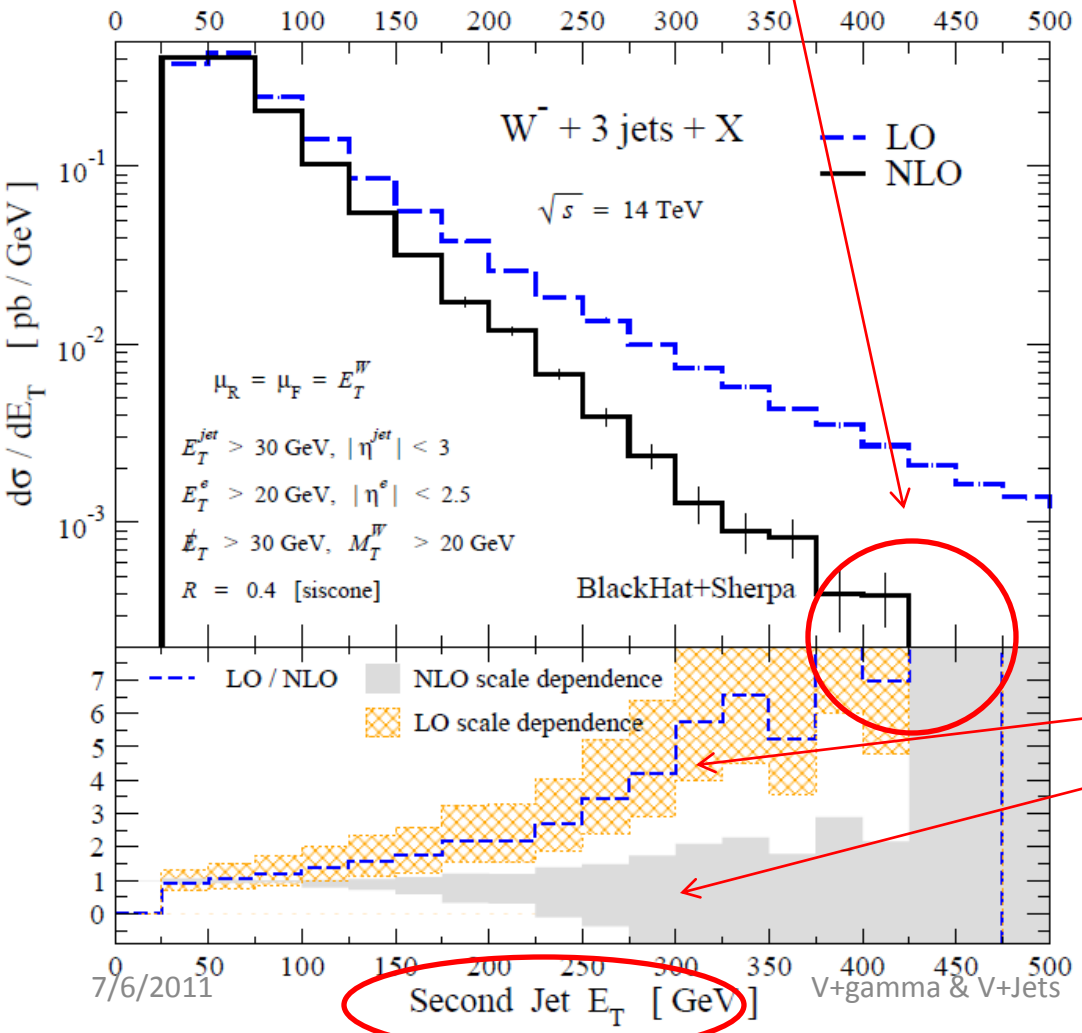


2009: NLO $W+3j$ [Rocket: Ellis, Melnikov & Zanderighi] [unitarity]
2009: NLO $W+3j$ [BlackHat: Berger et al] [unitarity]
2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al] [traditional]
2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al] [unitarity]
2009: NLO $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth et al] [traditional]
2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al] [unitarity]
2010: NLO $Z+3j$ [BlackHat: Berger et al] [unitarity]
2010: NLO $W+4j$ [BlackHat: Berger et al] [unitarity]

Look Out for Scales at the LHC

The renormalization scale common at Tevatron:
Turns out to be a bad choice at LHC.

$$\mu = E_T^W \equiv \sqrt{M_W^2 + p_T^2(W)}$$



Complicated processes have many scales.

LHC has a much greater dynamic range than Tevatron; M_W not characteristic scale.

Other signs of bad scale choice:

- Negative cross section.
- Large LO/NLO ratio.
- Rapid growth of scale bands with ET.

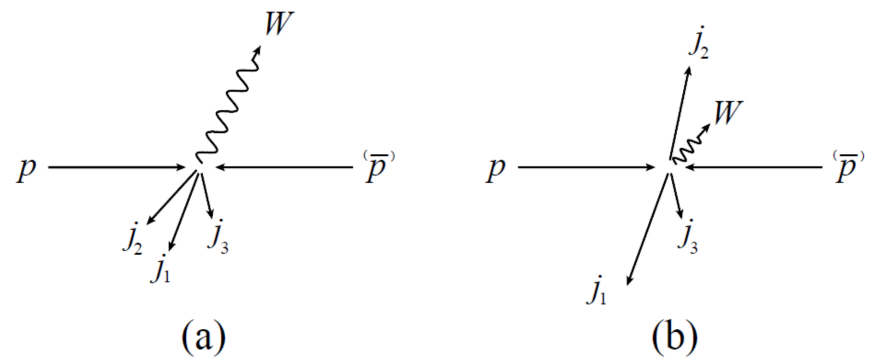
The Trouble with E_T^W

See also: Mangano, Parke '90; Frixione '93; Arnold, Reno '89; Baur, Han, Ohnemus (9507336); Bozzi Jager, Oleari, Zeppenfeld (0701150)

Consider these 2 configurations:

- For (a) $\mu = E_T^W \equiv \sqrt{M_W^2 + p_T^2(W)}$ physical scale of interactions.

- For (b) E_T^W may be low and under-estimating the physical scale.



Looking at large E_T for the 2nd jet forces configuration (b).

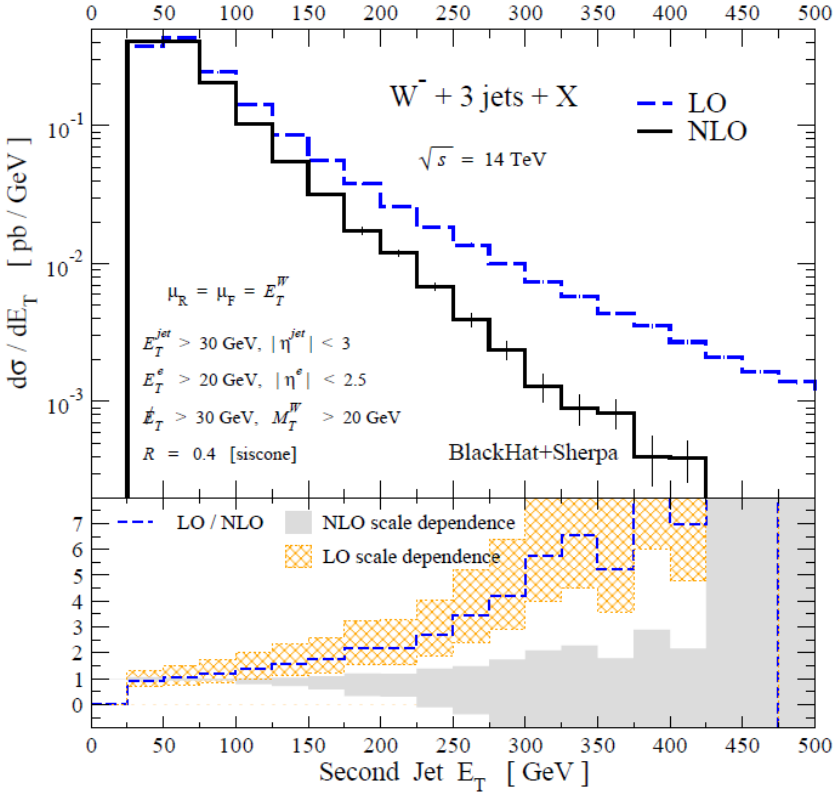
- The total (partonic) transverse energy is a better variable; gets large for both (a) and (b).

$$\hat{H}_T = \sum_p E_T^p + E_T^e + E_T^\nu$$

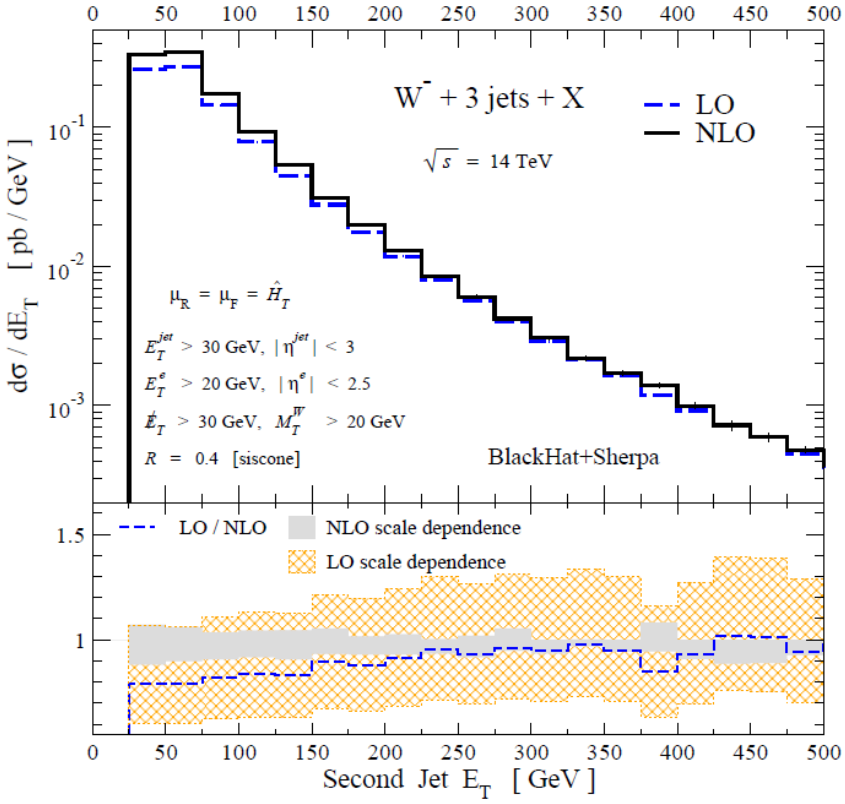
- Other reasonable scales are for example *invariant mass of the n jets* [Bauer, Lange arXiv:0905.4739] or *local scales* (at LO) inspired in CKKW reweighting [Melnikov, Zanderighi arXiv:0910.3671]

Compare Two Scale Choices

$\mu = E_T^W$



$\mu = \hat{H}_T$

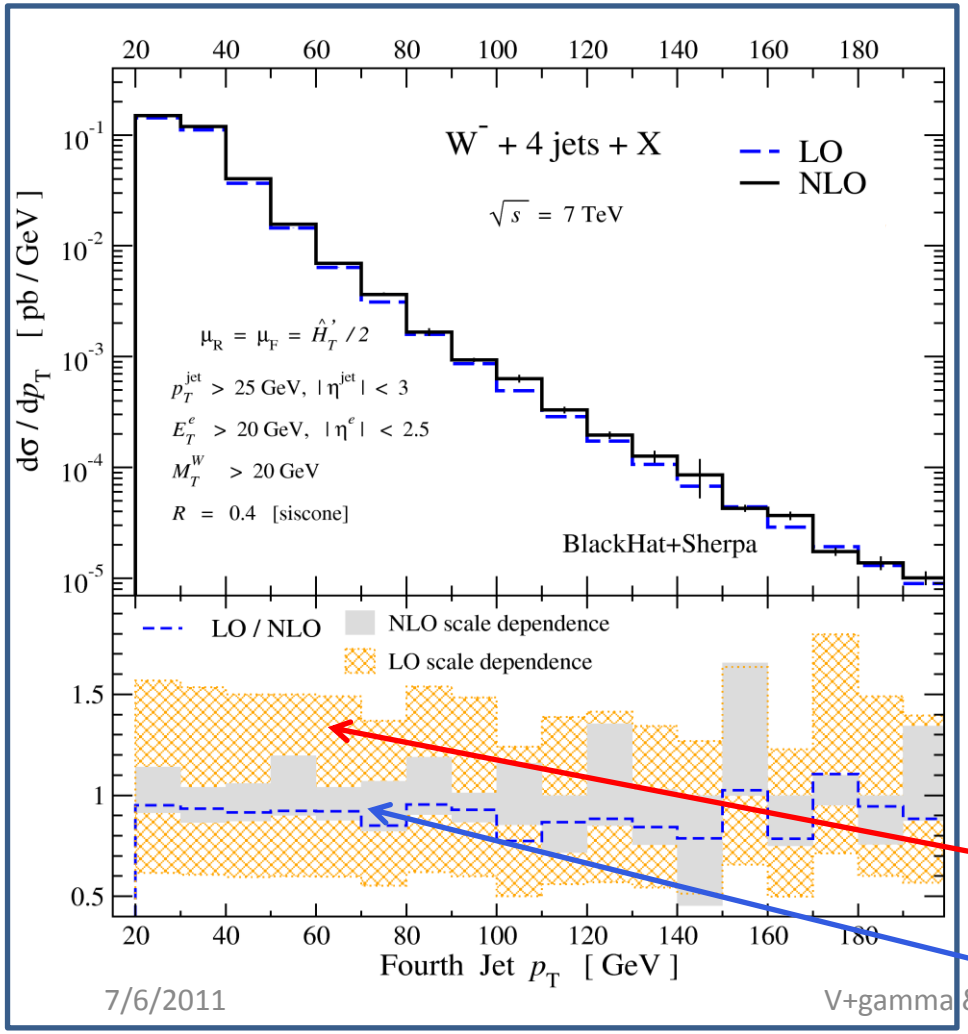


Message: Do not use $\mu = E_T^W$

- LO/NLO ratio sensible.
- NLO scale dependence very good.

pp → W+4 jets @ the LHC

[BlackHat, arXiv:1009.2338]



- First ever 2→5 NLO
- Used in recent ATLAS $t\bar{t}$ results measurement
- Leading color virtual. Will add in subleading color soon (~3% effect).

LO uncertainty > 50%

NLO uncertainty ~ 20%

pp → W+4 jets @ the LHC

[BlackHat, arXiv:1009.2338]

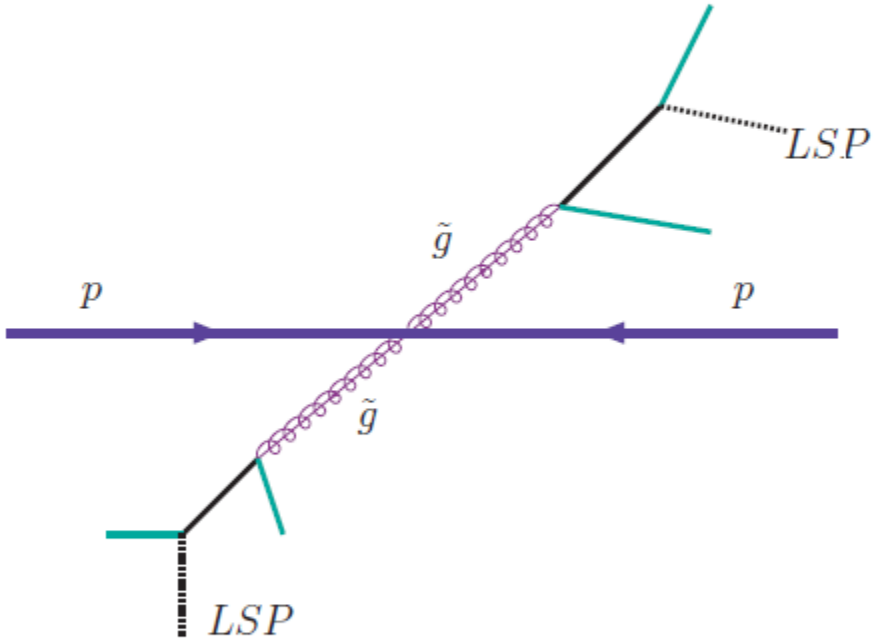
- First ever 2→5 NLO

no. jets	W^- LO	W^- NLO	W^+/W^- LO	W^+/W^- NLO	$W^- n/(n-1)$ LO	$W^- n/(n-1)$ NLO
0	$1614.0(0.5)^{+208.5}_{-235.2}$	$2077(2)^{+40}_{-31}$	1.656(0.001)	1.580(0.004)	—	—
1	$264.4(0.2)^{+22.6}_{-21.4}$	$331(1)^{+15}_{-12}$	1.507(0.002)	1.498(0.009)	$0.1638(0.0001)^{+0.044}_{-0.031}$	0.159(0.001)
2	$73.14(0.09)^{+20.81}_{-14.92}$	$78.1(0.5)^{+1.5}_{-4.1}$	1.596(0.003)	1.57(0.02)	$0.2766(0.0004)^{+0.051}_{-0.037}$	0.236(0.002)
3	$17.22(0.03)^{+8.07}_{-4.95}$	$16.9(0.1)^{+0.2}_{-1.3}$	1.694(0.005)	1.66(0.02)	$0.2354(0.0005)^{+0.034}_{-0.025}$	0.216(0.002)
4	$3.81(0.01)^{+2.44}_{-1.34}$	$3.55(0.04)^{+0.08}_{-0.30}$	1.812(0.001)	1.73(0.03)	$0.2212(0.0004)^{+0.026}_{-0.020}$	0.210(0.003)

SUSY searches

- Gluinos/squarks are pair produced
- Generic signature is MET + jets

Typical SUSY event:



- How can SM mimic this?
  - $W \rightarrow l^{\pm}\nu$ with undetected lepton
  - QCD with mismeasured jet
  - $Z \rightarrow \nu\overline{\nu}$ Irreducible background - subject of this talk

Data Driven Background Estimation

Bern, Diana, Dixon, FFC, Hoche, Ita, Kosower, Maitre, Ozeren; arXiv:1106.1423

- CMS uses photons to measure Z (Incandela's Group)

[CMS PAS SUS-08-002]

[CMS PAS SUS-10-005]

theory input

$$\sigma(pp \to Z(\to \nu\bar{\nu})) = \sigma(pp \to \gamma) \times R_{Z/\gamma}$$

SM irr. BG

measure this

- Can also use $Z \to \mu\bar{\mu}$, but γ has better statistics
- So what is the conversion factor R?

Photon Isolation a la Frixione [hep-ph/9801442]

- In pQCD, have to be careful to preserve Infra-Red Safety
- Can't veto QCD radiation arbitrarily!
- Frixione: "here is a way to remove frag. photons in an IR safe way"

$$\sum_i E_{iT}\theta(\delta - R_{i\gamma}) \leq H(\delta)$$

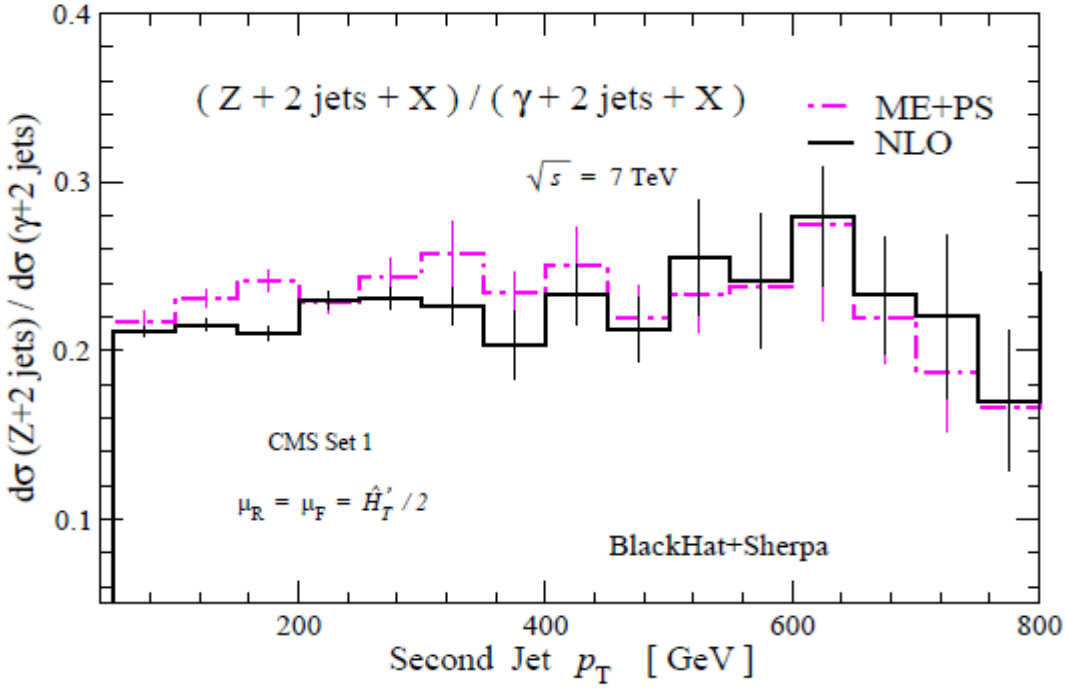
$$H(\delta) = E_T^{\gamma}\epsilon\left(\frac{1-\cos\delta}{1-\cos\delta_0}\right)^n$$

- Important: $H(\delta) \to 0$ as $\delta \to 0$
- We choose $\epsilon = 0.025$, $\delta_0 = 0.3, n = 2$

Preliminary Results Z/γ ratio

Plot of Z/γ ratio

Bern, Diana, Dixon, FFC, Hoche, Ita, Kosower, Maitre, Ozeren; arXiv:1106.142



- Ratio roughly constant across phase space
- Good agreement between NLO / MEPS
- Take difference as error estimate, as scale variation largely cancels in ratio

process	LO	ME+PS	NLO
$\gamma + 2j$	$2.220^{+0.762}_{-0.526}$	2.110	$2.609^{+0.159}_{-0.241}$
$Z + 2j$	$0.521^{+0.180}_{-0.124}$	0.478	$0.560^{+0.012}_{-0.043}$
ratio	0.235	0.226	0.214