

**PASCOS**

PARTICLES  
STRINGS  
COSMOLOGY

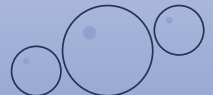
25-29 JULY 2022 - MPIK - HEIDELBERG - GERMANY

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# Phase transitions in the early universe: quantum corrections

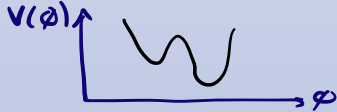
**Mariana Carrillo González**

Based on: arXiv:[2204.03480](https://arxiv.org/abs/2204.03480) with J.E. Camargo Molina and A. Rajantie

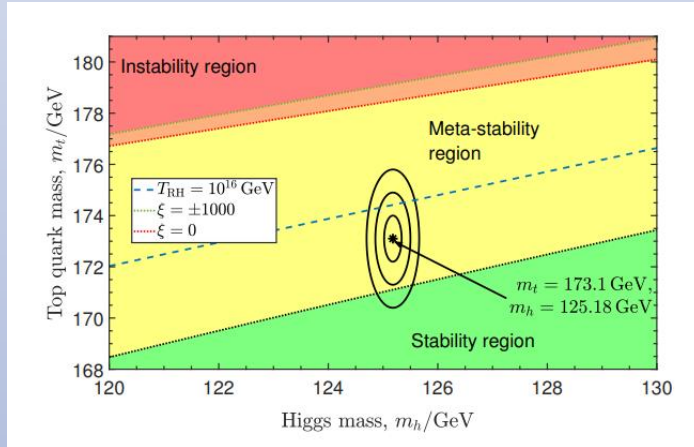


# Phase transitions → Bubble nucleation

- Arise in the presence of metastable vacuum

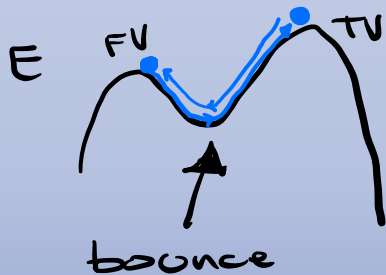
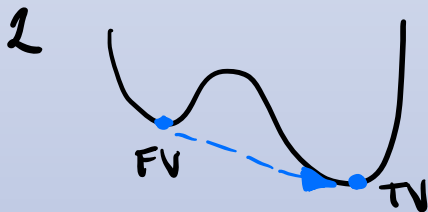


- Spontaneous symmetry breaking
- Beyond Standard Model Physics
- Current measurements: SM Higgs metastable state



Markkanen, Rajantie, Stopyra; 2018

## Decay rates in flat space



$$\Gamma = -2 \text{Im} E = 2 \text{Im} \left( \lim_{T \rightarrow \infty} \frac{\ln Z}{T} \right)$$

Saddle-point approximation:

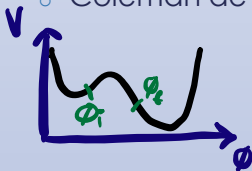
$$\frac{\Gamma}{\gamma} = \left( \frac{B}{2\pi} \right)^2 \left| \frac{\det' S_b''}{\det S_{fv}''} \right|^{-\frac{1}{2}} e^{-B}$$

$$B \equiv S_b - S_{fv}$$

# Decay rates in de Sitter

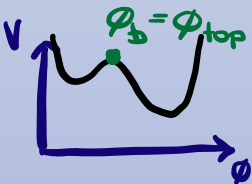
## Saddle-point approximation

- Coleman de Lucia



Similar to flat space bounce  
dif. boundary conditions

- Hawking-Moss



No flat space analog.  
(thermal transition)

## Stochastic Formalism <sup>Starobinski, Yokoyama</sup>

- Describes dynamics of long-wavelength modes accounting for short-wavelength modes as noise through Fokker-Planck eq:

$$\frac{\partial P}{\partial t} = \frac{3H^3}{8\pi^2} \left( \partial_\phi^2 - \frac{4\pi^2}{3H^4} (V(\phi)^2 - V'(\phi)) \right) P(t; \phi)$$

$V(\phi)$  potential Probability distribution of  $\phi$  at time  $t$

- The lowest non-zero eigenvalue gives the decay rate.

## Early Universe: large curvature

- Dominated by Hawking-Moss
  - one negative eigenvalue for:

$$H > \sqrt{|V''_{\text{top}}|} / 2$$

- Assume fixed dS background:

$$V_{\text{top}} - V_{\text{tv}} \ll V_{\text{tv}}$$

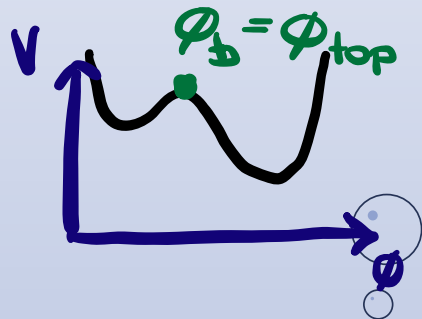
- At smaller curvatures, expect transition to Coleman de Lucia dominated decay rates.
- Akin to thermal transition

$$T = \frac{H}{2\pi}$$

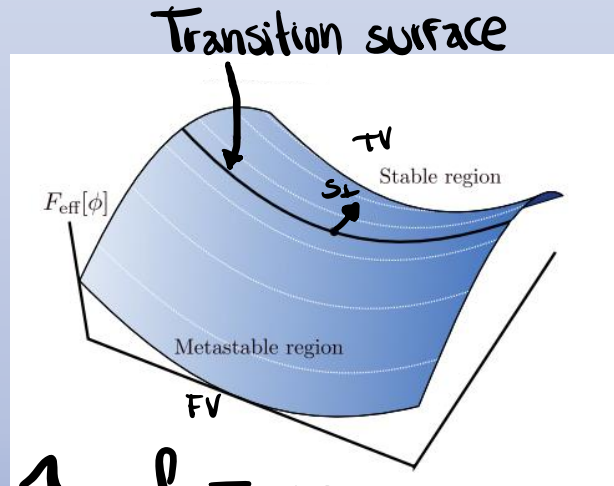
$$\Gamma \sim e^{-B}$$

$$B = S_{\text{HM}} - S_{\text{FV}} = \frac{8\pi^2 \Delta V}{3H^4}$$

$$B = \beta \Delta E = \left( \frac{2\pi}{H} \right) \left( \frac{4\pi}{3} \frac{1}{H^3} \right) \Delta V$$



# Thermal interpretation and semi-classical approximation



$$\Gamma = \int_{TS} \mathbf{J} \cdot d\mathbf{S}_\perp$$

- At high temperature (curvature) physics is driven by long-wavelength modes:
  - Semi-classical approximation
- Dynamics governed by Langevin eq.

$$(\partial_t^2 - \bar{\nabla}^2)\phi + \frac{\partial V}{\partial \phi} + \beta H \dot{\phi} = \xi \leftarrow \begin{array}{l} \text{noise} \\ \text{encoding} \\ \text{thermal} \\ \text{fluctuations} \end{array}$$

- Use flux-over-population method
  - Escape rate (Kramers; 1940 Langer; 1969)
  - Thermal Field theory (Berrera, Mabillard, Mintz, Ramos; 2019, Gould, Hirvonen; 2021)

# Hawking-Moss decay rate at 1-loop

$$\Gamma = \int_{T_S} J \cdot dS_{\perp} = \frac{1}{Z_{FV}} \int D\pi D\phi e^{-S[\phi]} \sigma \delta(c_-)$$

growth rate of  $\phi_- \rightarrow k$   
 S.P.  $\frac{k}{2\pi} \left| \frac{\det S''_{\text{top}}}{\det S''_{FV}} \right|^{-1/2} e^{-B}$

deviation from thermal eq.  $\rightarrow$   
 negative eigenmode coef.  $\rightarrow$

Match to S.F.  $\rightarrow$   $T^2 \gg V''_{\text{top}}$

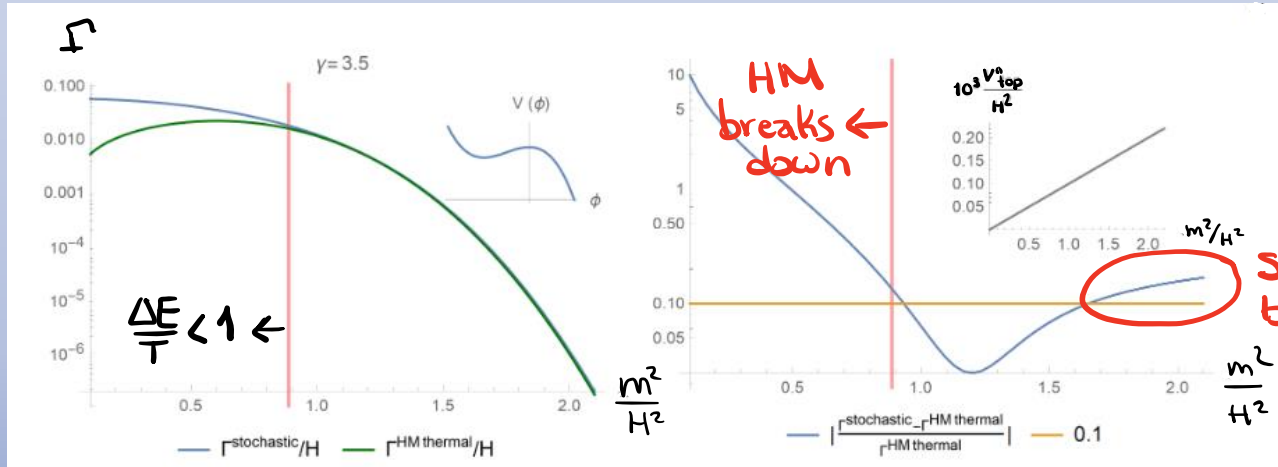
$$\frac{|V''_{FV} V''_{\text{top}}|^{1/2}}{2\pi} e^{-\frac{8\pi^2}{3H^4} \Delta U_{1\text{-loop}}}$$

Constraint effective potential

# Comparison: HM thermal vs Stochastic F.

$$U(\phi) = 3H^2 M_{pl}^2 - \frac{1}{2} m^2 \phi^2 - \frac{\delta}{3!} H \phi^3$$


1 loop







# Conclusions

- At large Hubble rates, **thermal** interpretation of **Hawking-Moss** instanton allows us to compute **higher order corrections**.
  - **Matching** when saddle-point approx. is valid tells us that **quantum corrections** can be captured by **stochastic formalism** using **constraint effective potential**.
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## Complementary approaches:

### Hawking-Moss

Breaks down when perturbation theory is not valid and thermal fluctuations are large

### Stochastic Formalism

Breaks down for large masses due to overdamped approximation.

