Dynamical mass generation for a massless minimal scalar with $V(\phi) = \frac{\lambda\phi^4}{4!} + \frac{\beta\phi^3}{3!}$ in de Sitter spacetime

(S. Bhattacharya and N. Joshi, Manuscript under preparation)

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Motivation

- Our interest $\rightarrow$ to study the physics of the very early universe. Corresponding study involves cosmic inflation (a theory of exponential expansion of spacetime in the early universe), decoherence and entanglement in open quantum systems.
- Such study might provide us insight about the initial state as well as the geometry of the early inflationary universe.
- It might provide a clear mechanism on how inflation ended into radiation dominated era and a small value of cosmological constant is reached.
- There can be various approaches towards inflationary field theory such as stochastic & field theoretic.

Introduction & overview

- In this talk $\rightarrow$ stochastic approach to inflation for a massless minimally coupled quantum scalar field with an asymmetric (quartic plus cubic) self interaction, the generation of dynamical mass and the comparison with field theoretic results.
- Potential choice $\rightarrow$ why this potential $V(\phi) = \frac{\lambda\phi^4}{4!} + \frac{\beta\phi^3}{3!}$?
- Stochastic method $\rightarrow$ where we split the quantum fields into long and short wavelength modes and viewing the long wavelength as classical objects evolving stochastically in an environment provided by quantum fluctuations of shorter wavelength. (A. A. Starobinsky and J. Yokoyama, Phys. Rev. D **50**, 6357-6368 (1994).
- Langevin equation $\rightarrow$ a stochastic differential equation that experiences a particular type of random force.

Introduction & overview

- Fokker-Planck equation $\rightarrow$ is a partial differential equation describing the time evolution of the probability density distribution function.
- Dynamical mass $\rightarrow$ means the mass that's get generated due to the self interaction or due to the radiative process even though originally field was massless. Dynamical mass contribution comes from loops which makes the particle massive.
- Backreaction $\rightarrow$ is a shift in the cosmological constant Λ due to the cosmological perturbation.

Potential choice

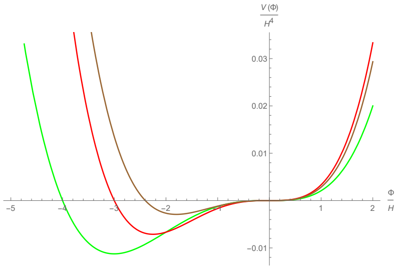

Figure: The Green, red and brown curves correspond to λ and β/H values : (0.01, 0.01), (0.02, 0.015), and (0.02, 0.012) respectively.

- One can expect that the late time $\langle 0|V(\phi)|0\rangle$ may be negative. [1]
- For $\lambda = 0$, the potential becomes unbounded from below and system will roll away to $-\infty$ & for $\beta = 0$, the $\langle V(\phi)\rangle$ will be positive but $\langle\phi\rangle$ contribution will be vanishing.

[1] (S. Bhattacharya, 2202.01593).

Stochastic approach to inflation

- For the interacting potential $V(\phi)$, $\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + V'(\phi) = 0$, we have the following equation for the coarse-grained part $\bar{\phi}$

  $$\dot{\bar{\phi}}(\vec{x}, t) = -\frac{1}{3H} V'(\bar{\phi}) + f(\vec{x}, t)$$

- Above equation is the Langevin equation for the stochastic quantity $\dot{\bar{\phi}}$ with a stochastic noise term $f(\vec{x}, t)$, whose correlation function given as

  $$\langle f_\phi(\vec{x}, t) f_\phi(\vec{x}', t') \rangle = \frac{H^3}{4\pi^2} \delta(t - t')$$

- For the long-wavelength part of $\phi(\vec{x}, t)$ one can write a probability distribution $\rho(\bar{\phi}(\vec{x}, t) = \phi)$ that satisfies a Fokker-Planck equation

  $$\frac{\partial \rho}{\partial t} = \frac{H^3}{8\pi^2} \frac{\partial^2 \rho}{\partial \phi^2} + \frac{1}{3H} \frac{\partial}{\partial \phi} \left(\frac{\partial V}{\partial \phi} \rho(t, \phi) \right)$$

- The general solution of the Fokker-Planck equation is,

  $$\rho(\phi, t) = \exp(-\nu(\phi)) \sum_{n=0}^{\infty} a_n \Phi_n(\phi) e^{-\Lambda_n(t - t_0)}$$

Stochastic approach to inflation

- $\Phi_n(\phi)$ is the eigenfunctions of the Schrodinger-type equation and $\nu(\phi) = \frac{4\pi^2}{3H^4} V(\phi)$
- At late times we have the following static equilibrium solution $\rho_{\rm eq}(\phi) = N^{-1} \exp\left(-\frac{8\pi^2}{3H^4} V(\phi)\right)$
- where N is the normalization fixed by the condition $\int_{-\infty}^{\infty} \rho_{\rm eq}(\phi)\, d\phi = 1$
- If N is finite, then Φ exists, but if it diverges, then there no such equilibrium solution.
- The expectation value of any function $\mathcal{X}$ can be computed as $\langle \mathcal{X}(\phi) \rangle = \int_{-\infty}^{\infty} \mathcal{X}(\phi) \rho_{\rm eq}(\phi) d\phi$

Plots for $\langle \phi \rangle$

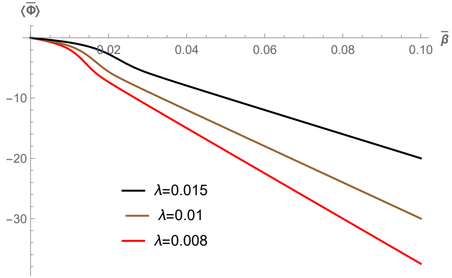

Figure: A 2-D plot for the expectation value of $\overline{\phi} = \frac{\phi}{H}$ vs. $\overline{\beta} = \frac{\beta}{H}$ for three different value of λ.

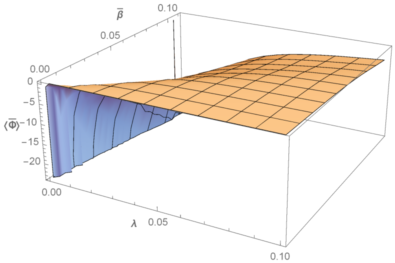

Figure: A 3-D plot for the expectation value of $\overline{\phi} = \frac{\phi}{H}$ vs. λ and $\overline{\beta} = \frac{\beta}{H}$.

Plots for $\langle \phi^2 \rangle$

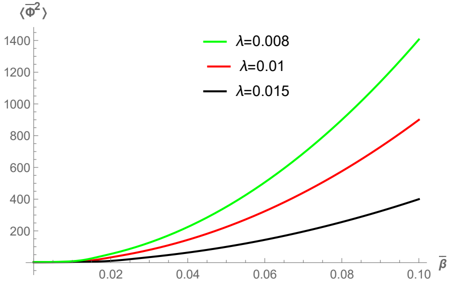

Figure: A 2-D plot for the expectation value of $\overline{\phi}^2 = \frac{\phi^2}{H^2}$ vs. $\overline{\beta} = \frac{\beta}{H}$ for three different value of λ.

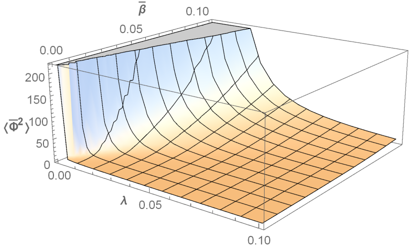

Figure: A 3-D plot for the expectation value of $\overline{\phi}^2 = \frac{\phi^2}{H^2}$ vs. λ and $\overline{\beta} = \frac{\beta}{H}$.

Plots for $\langle V(\phi) \rangle$

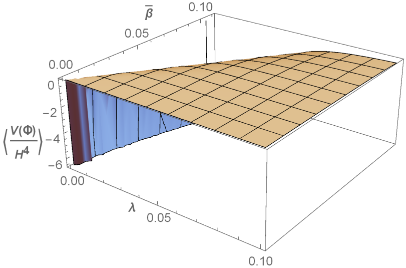

Figure: A 3-D plot for the expectation value of $\frac{V(\phi)}{H^4}$ vs. λ and $\overline{\beta} = \frac{\beta}{H}$.

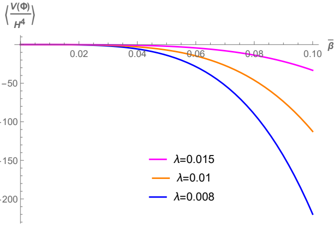

Figure: A 2-D plot for the expectation value of $\frac{V(\phi)}{H^4}$ vs. $\overline{\beta} = \frac{\beta}{H}$ for three different value of λ.

Field theoretic result for $\langle \phi \rangle$

- The final form of $\langle \phi \rangle$ after the renormalisation & resummation look like, where $\overline{\beta} = \frac{\beta}{H}$ & $\overline{\phi} = \frac{\phi}{H}$ are dimensionless [2]

$$\langle \bar{\phi} \rangle = -\frac{0.4781\bar{\beta}}{\lambda}$$

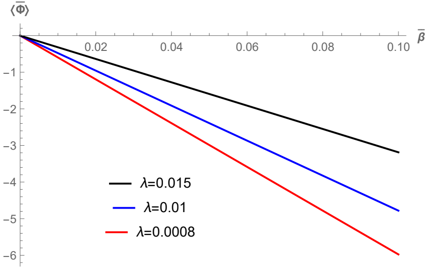

Figure: A 2-D plot for the expectation value of $\overline{\phi} = \frac{\phi}{H}$ vs. $\overline{\beta} = \frac{\beta}{H}$ for three different value of λ upto $\mathcal{O}(\beta)$ & $\mathcal{O}(\lambda\beta)$.

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[2] (S. Bhattacharya, 2202.01593).

Diagrams

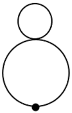

Figure: $\mathcal{O}(\lambda)$ correction to the two-point correlation function in the coincident limit. The dot denotes the spacetime point x, where the two external propagators are identified.

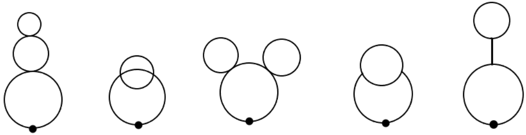

Figure: $\mathcal{O}(\lambda^2)$ correction to the two-point correlation function in the coincident limit (the first three). The last two represent the same for one loop at $\mathcal{O}(\beta^2)$.

Diagrams

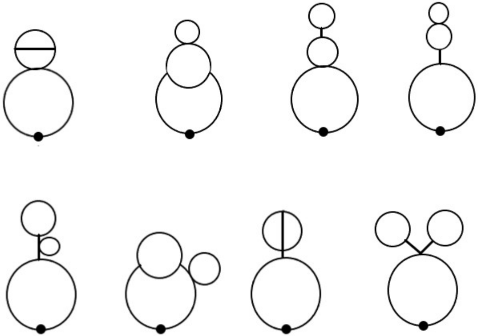

Figure: $\mathcal{O}(\lambda\beta^2)$ correction to the two point correlation in the coincidence limit.

Field theoretic result for $\langle \phi^2 \rangle$

- The perturbative expansion for the $\langle \bar{\phi}^2 \rangle$ is, where $\overline{\beta} = \frac{\beta}{H}$ & $\overline{\phi}^2 = \frac{\phi^2}{H^2}$ are dimensionless.

$$\langle \overline{\phi}^2 \rangle_{\rm local} = \frac{1}{4\pi^2} \ln a - \frac{(\lambda - \frac{\overline{\beta}^2}{2})}{2^4 \times 9\pi^4} \ln^3 a + \frac{\lambda^2}{2^6 \times 27\pi^6} \frac{\ln^5 a}{5}$$

- Following A. Y. Kamenshchik and T. Vardanyan, Phys. Rev. D102, no.6, 065010 (2020), resummed equation look like,

$$\frac{df}{d\mathcal{N}} := A_1 - 3\frac{A_2}{A_1{}^2}\left(\lambda - \frac{\overline{\beta}^2}{2}\right) f^2 + \lambda^2 \left(\frac{5A_3}{A_4{}^4} - \frac{6{A_2}^2}{{A_1}^5}\right) f^4$$

- Where we have

$A_1 = \frac{1}{4\pi^2}$, $A_2 = \frac{1}{2^4 \times 9\pi^4}$, $A_3 = \frac{1}{2^6 \times 135\pi^6}$,

$\mathcal{N}$ =ln a & $\langle \overline{\phi}^2 \rangle$ abbreviated as f.

Field theoretic result for $\langle \phi^2 \rangle$

- We solve above resummed equation numerically to find the plot for $\langle \phi^2 \rangle$.

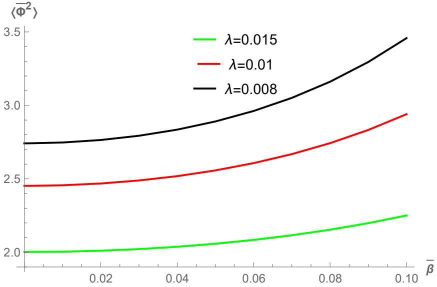

Figure: A 2-D plot for the expectation value of $\overline{\phi}^2 = \frac{\phi^2}{H^2}$ vs. $\overline{\beta} = \frac{\beta}{H}$ for three different value of λ upto $\mathcal{O}(\beta^2)$ and $\mathcal{O}(\lambda^2)$.

Dynamical mass generation

- In d dimension expression of dynamical mass can be written as (R. L. Davis, Phys. Rev. D **45**, 2155 (1992))

  $$m^2_{dyn} = \frac{\Gamma(\frac{d+1}{2}) H^d}{2\pi^{\frac{d+1}{2}} \langle \phi^2 \rangle}$$

- Comparison $\rightarrow$ For λ and β/H values: (0.008; 0.03); (0.01; 0.03); and (0.015; 0.03), stochastic $\frac{m^2_{dyn}}{H^2}$ vs. field theoretic $\frac{m^2_{dyn}}{H^2}$ values are (0.0003; 0.13618); (0.00047; 0.1532); and (0.0011; 0.1879) respectively.

Approximate shift in Λ

- The backreaction to Λ in the Einstein equation will be,
  $$\Lambda \rightarrow \Lambda(1 + 16\pi G\gamma\langle\phi\rangle) \approx \Lambda\left(1 - 1.5 \times 10^5 \times \frac{L_P^2}{L_C L_\gamma}\right)$$
  where $H^{-1} = L_C$ is the length scale of the cosmological event horizon, whereas $L_\gamma = \gamma^{-1}$ is the length scale associated with γ (γ is a constant of length dimension -1) and L_p is the planck length.
- Hence the approximate shift in the inflationary Λ is given by,
  $$\Lambda\left(1 - 10^{-1} \times \frac{L_C}{L_\gamma}\right)$$
  for $\bar{\beta}/\lambda \sim \mathcal{O}(10^2)$
- Similarly the approximate shift in the inflationary Λ due to the potential $\langle V(\phi)\rangle$ is given by, for $\bar{\beta}/\lambda \sim \mathcal{O}(10^2)$
  $$\Lambda\left(1 - 10^{-2} \times \frac{L_C}{L_\gamma}\right)$$

Discussion

- We have studied the stochastic vs. field theoretic approach to inflation for a massless minimally coupled quantum scalar field with $V(\phi) = \frac{\lambda\phi^4}{4!} + \frac{\beta\phi^3}{3!}$and the generation of dynamical mass.
- The nature of plots for $\langle\phi\rangle$are the same for both approaches but the negative value of $\langle\phi\rangle$ stochastically is greater than field theoretic result.
- Upto $\mathcal{O}(\beta^2)$ & $\mathcal{O}(\lambda^2)$, nature of plots for $\langle\phi^2\rangle$ is similar for both approaches but the numeric value is higher stochastically as we increase β & due to this there is difference in dynamical mass also.
- A shift in Λ is of $\mathcal{O}(10^{-1})$ owing to $\langle\phi\rangle$ and $\mathcal{O}(10^{-2})$ owing to $\langle V(\phi)\rangle$.
- We shall be computing power spectra, spectral coefficient & spectral index using stochastic technique.(S. Bhattacharya, Sudesh, N. Joshi, work in progress).

Thank You