Energy distribution and equation of state after inflation

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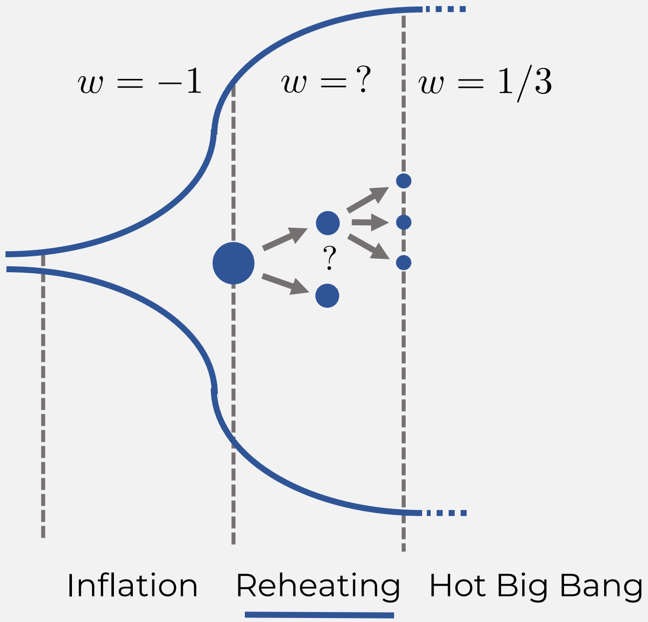
(arXiv: 2005.07563/2112.11280, S. Antusch, D. Figueroa , K. Marschall, F. Torrentí)

PASCOS 2022

27th International Symposium on Particles, Strings and Cosmology

Universität Basel

The Primordial Dark Age



Why should we study the phase of reheating/preheating?

- Energy Transfer
- Expansion History (i.e. equation of state)
  - ➔ Number of e-folds N_k of inflation ($N_k = 55 \pm 5$)
  - ➔ Accurate predictions for CMB observables n_s and r

Inflationary Potential and Reheating Model

$$V(\phi, X) = \frac{1}{p}\Lambda^4 \tanh^p\left(\frac{|\phi|}{M}\right) + \frac{1}{2}g^2\phi^2X^2$$

Inflationary Potential and Reheating Model

inflaton

daughter field

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inflation

(p)reheating

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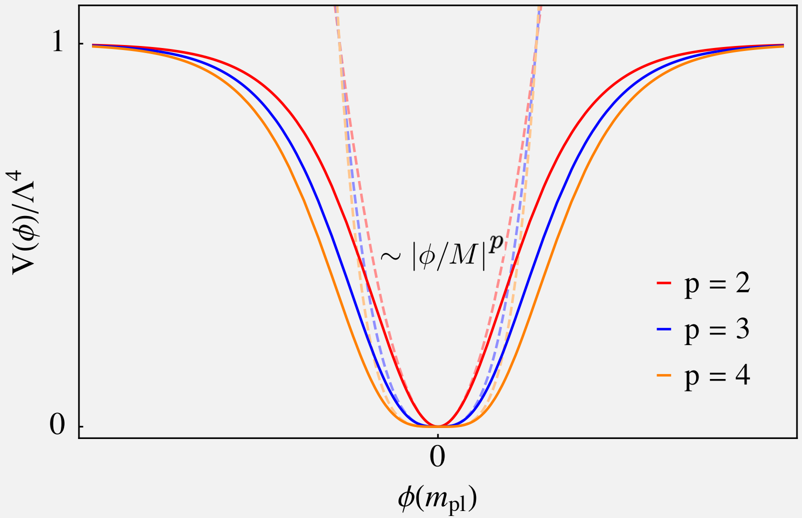
$$V(\phi) \simeq \begin{cases} \Lambda^4 & |\phi| \gg M \\ \dfrac{1}{p}\dfrac{\Lambda^4}{M^p}|\phi|^p & |\phi| \ll M \end{cases}$$



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We consider $M > 2m_{\rm pl}$, which is consistent with CMB observations

→ ϕ oscillates in positive curved region of the potential

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- Homogeneous phase
- Resonance phase
- Non-linear phase

Reheating: Homogeneous Evolution

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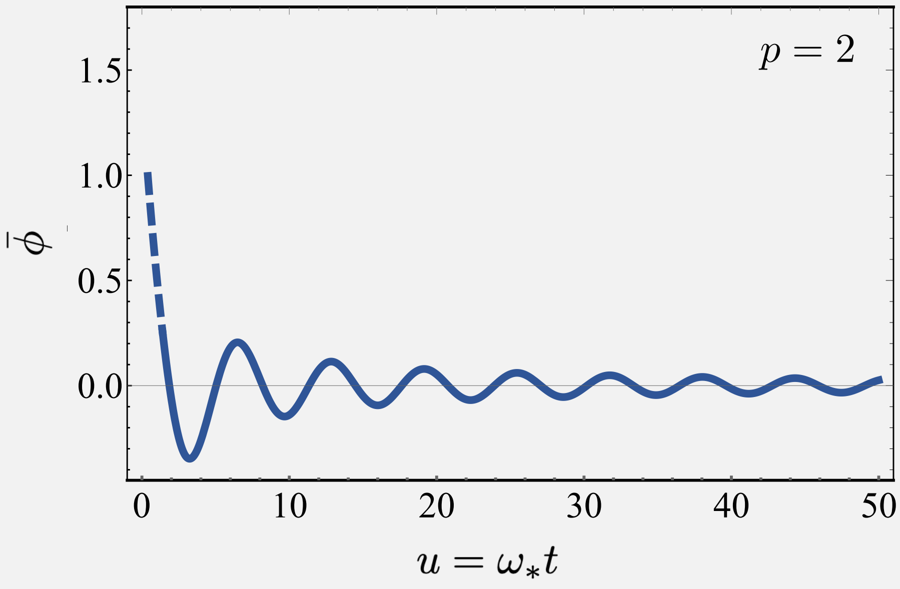
EoM of the homogeneous inflaton condensate $\bar{\phi}$:

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + \partial_{\bar{\phi}}V = 0$$



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‘inflation’ (p)reheating

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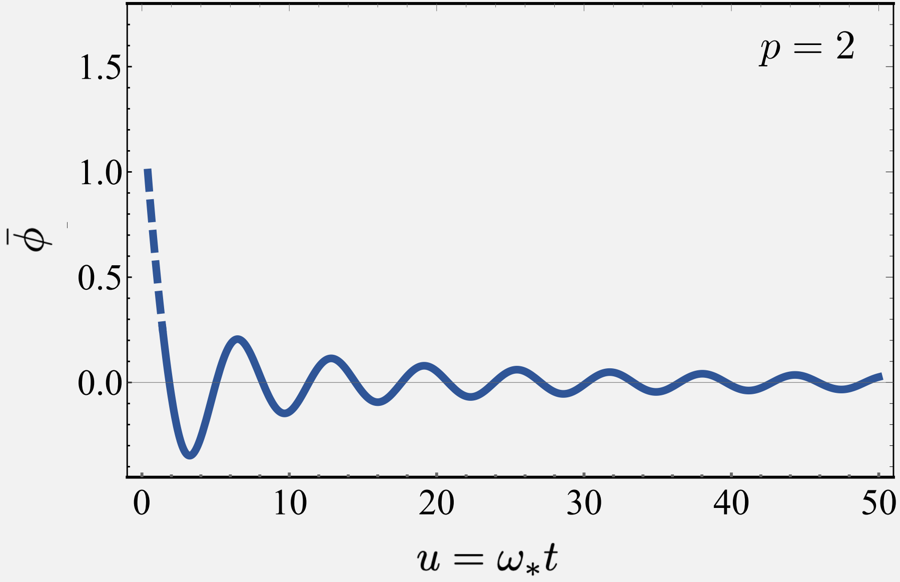

amplitude: $\bar{\phi} \propto a^{-\frac{6}{p+2}}$

oscillation frequency: $\omega^2 = \omega_*^2 a^{\frac{6p-12}{p+2}}$ $\qquad \omega_*^2 \equiv \frac{\Lambda^4}{M^p}\phi_*^{p-2}$

initial amplitude

Resonance Phase: Parametric Resonance

Linearized mode equations for χ:

$$\delta\chi_k'' + (\tilde{\kappa}^2 + \tilde{q}\bar{\varphi}^2)\delta\chi_k \simeq 0$$

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Redefinition of *field* and *space-time variables*:

$$\varphi \equiv \frac{\phi}{\phi_*} a^{\frac{6}{p+2}} \qquad \chi \equiv \frac{X}{\phi_*} a^{\frac{6}{p+2}}$$

$$t \to u \equiv \int_{t_*}^{t} \omega_* a(t')^{\frac{3(2-p)}{p+2}} dt'$$

$$\vec{x} \to \vec{y} \equiv \omega_* \vec{x} \qquad \tilde{\kappa} = \frac{k}{\omega_*} a^{\frac{2(p-4)}{p+2}}$$

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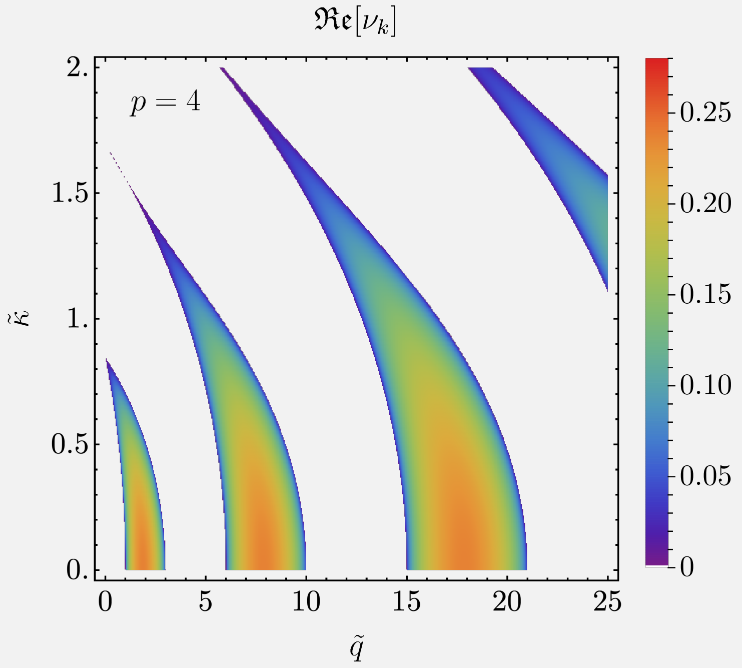

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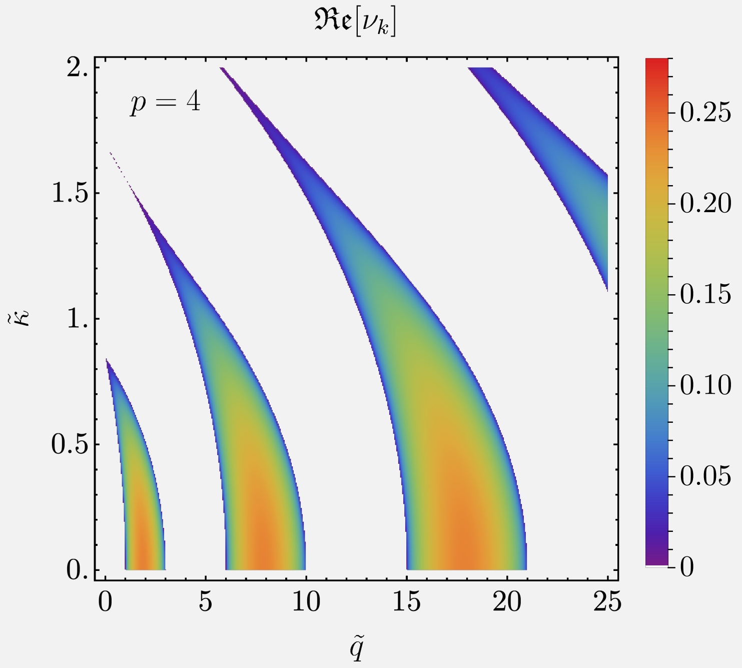
Floquet Index



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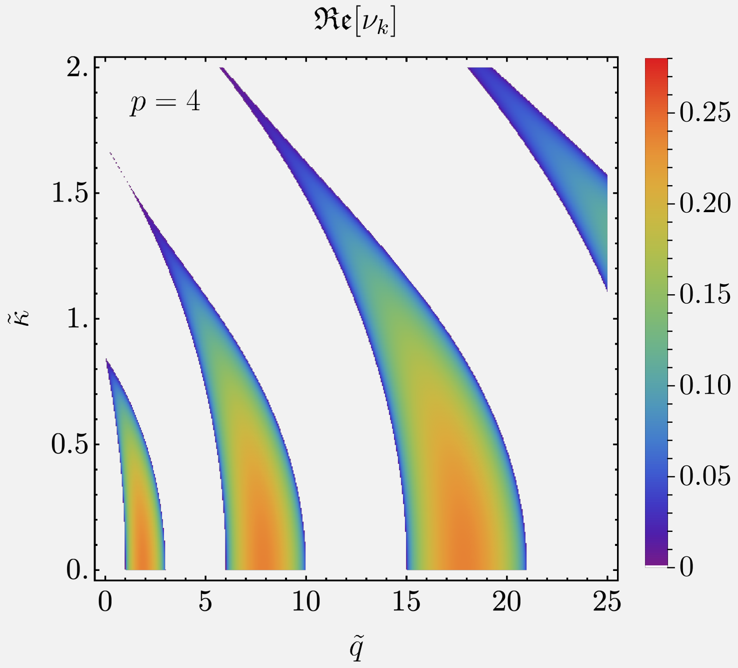
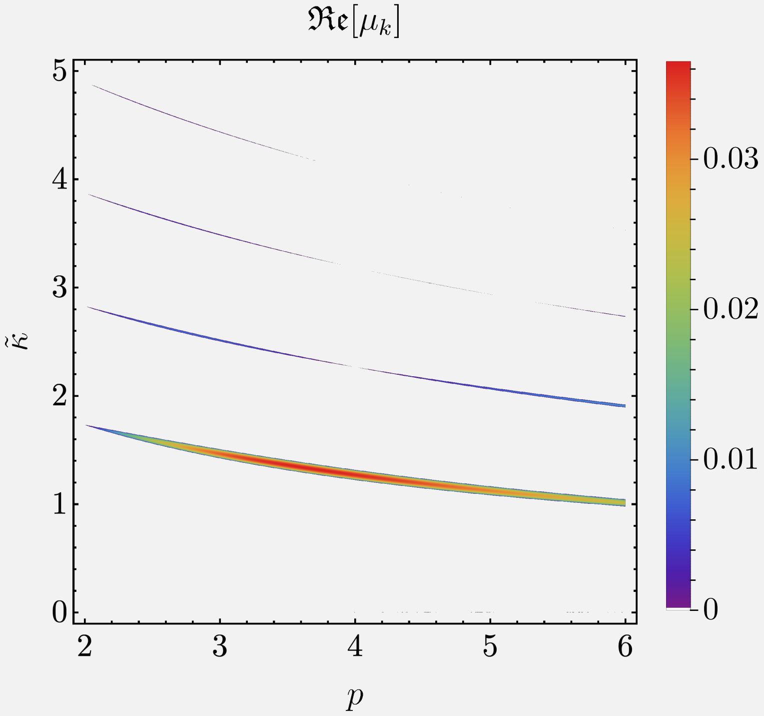


$\tilde{q} \lesssim 1$: narrow resonance

$\tilde{q} \gtrsim 1$: broad resonance

Resonance Phase: Self-Resonance



→ only for $p > 2$

Linearized mode equations for φ:

$$\delta\varphi_k'' + (\tilde{\kappa}^2 + (p-1)|\bar{\varphi}|^{p-2})\delta\varphi_k \simeq 0$$

→ $$|\delta\varphi_k|^2 \propto e^{2\mu_k(p)u}$$

usually $\nu_k^{\max} > \mu_k^{\max}$:

→ daughter field is excited much faster

Lattice Simulations

Dynamics become non-linear at $u_{\rm br} \sim 100$ → *lattice simulations*

- *CosmoLattice (arXiv: 2102.01031)*
  → Solves EoM on an expanding lattice:

$$\ddot{f} - a^{-2}\nabla^2_{\vec{x}} f + 3H\dot{f} + \partial_f V = 0$$

- Simulations done in $(2+1)$ dimensions and with $N = 128 - 512$ grid points
  → results almost identical to the ones in $(3+1)$ dimensions!

Non-Linear Phase: Lattice Simulations

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Focus on the following *volume* and *oscillation averaged* quantities:

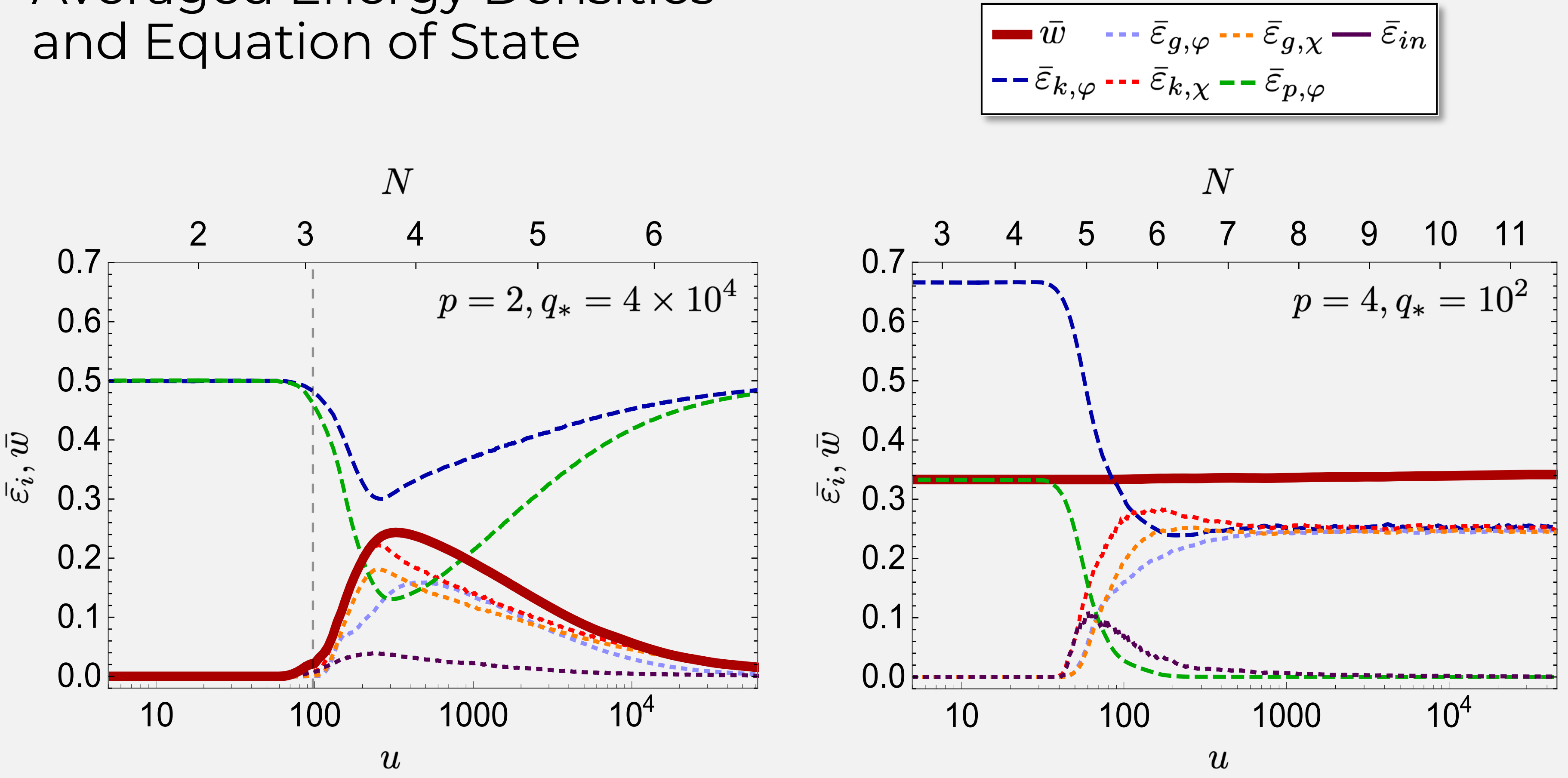
Energy density ratios: $\bar{\varepsilon} \equiv \langle E_i \rangle_{V,T} / \langle E_{\rm tot} \rangle_{V,T}$

$$E_{\rm tot} = \underbrace{E_{k,\varphi} + E_{k,\chi}}_{\text{kinetic}} + \underbrace{E_{g,\varphi} + E_{g,\chi}}_{\text{gradient}} + \underbrace{E_{p,\varphi}}_{\text{potential}} + \underbrace{E_{\rm int}}_{\text{interaction}}$$

Equation of state: $\bar{w}$

$$w = \left(E_{k,\varphi} + E_{k,\chi} - \frac{1}{3}E_{g,\varphi} - \frac{1}{3}E_{g,\chi} - E_{p,\varphi} - E_{\rm int} \right) / E_{\rm tot}$$

Averaged Energy Densities and Equation of State



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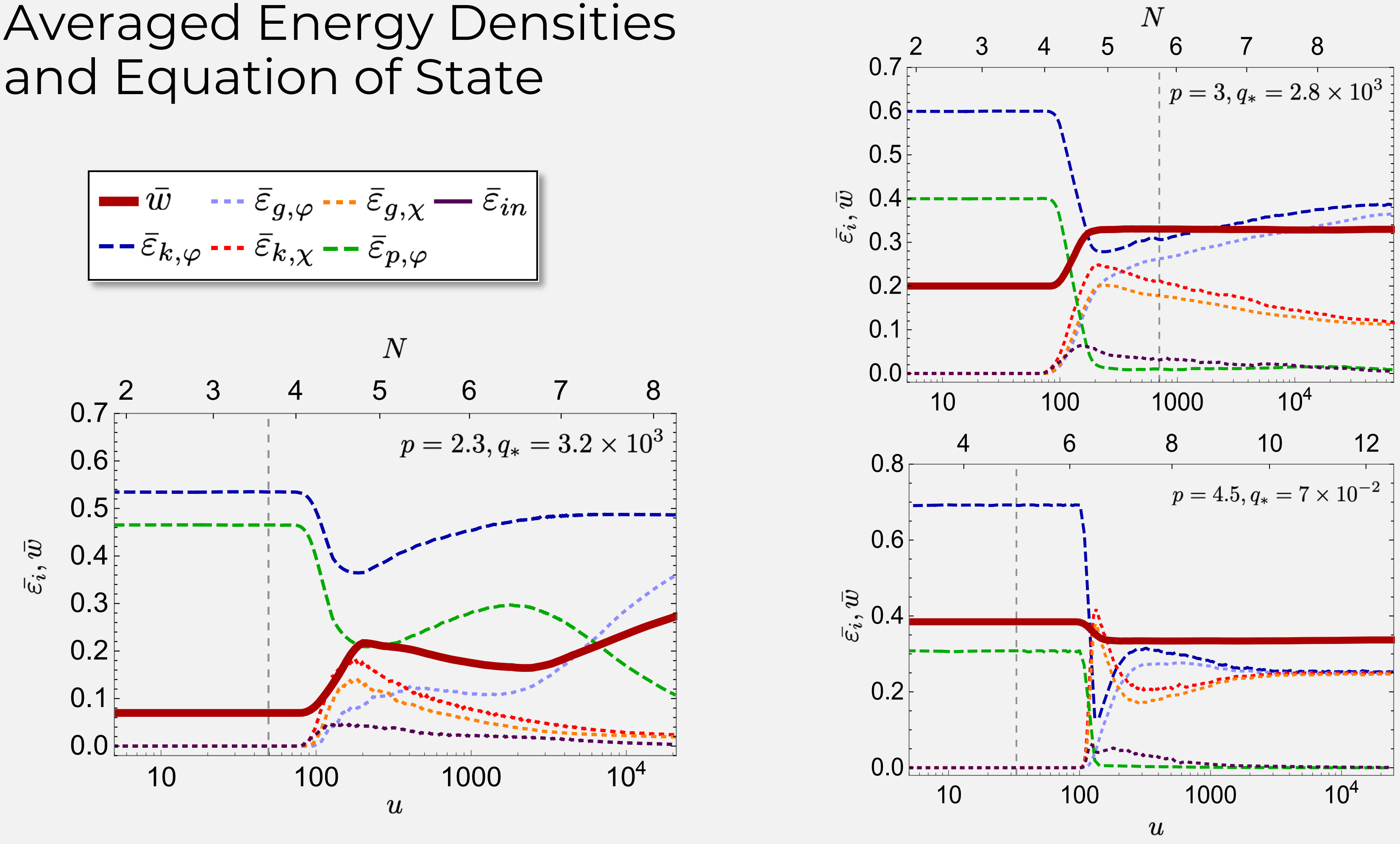
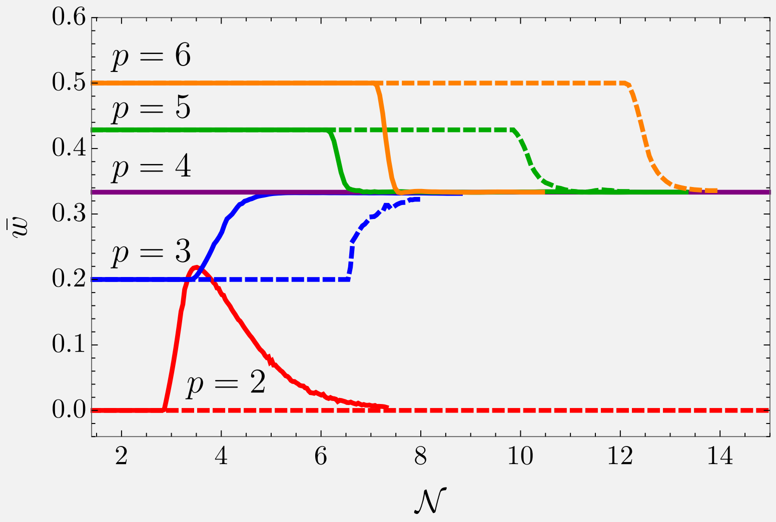


Determine exact Number of e-folds



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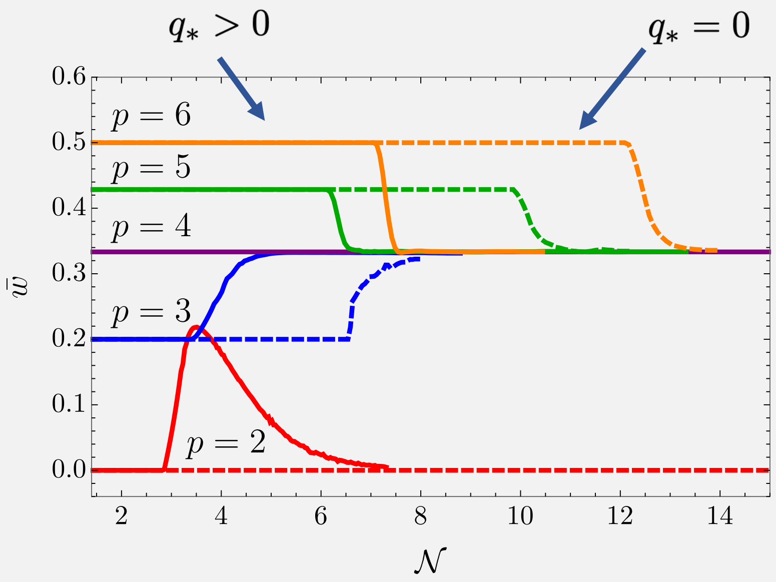

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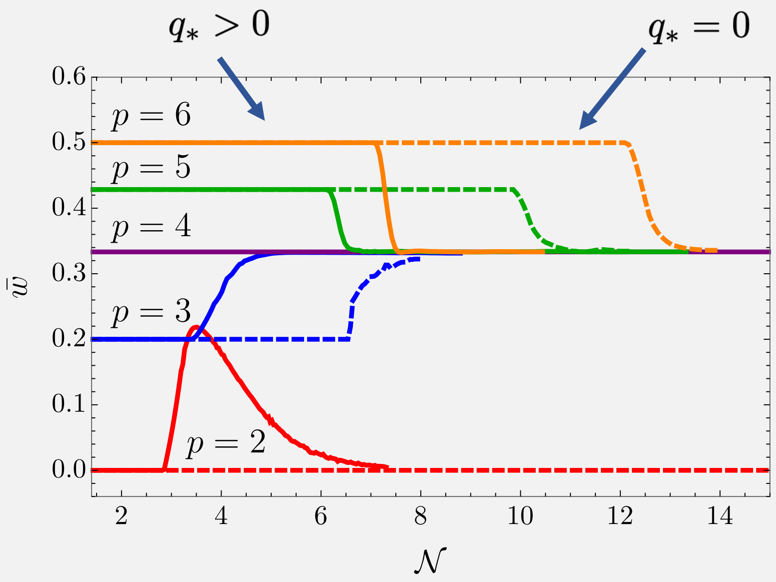

Assuming RD until BBN followed by RD ➔ MD ➔ ΛD phases until today, allows us to determine N_k via:

$$N_k \simeq 67 - \ln\frac{k}{a_0 H_0} + \ln\frac{V_k^{1/2}}{m_{\rm pl}\rho_{\rm end}^{1/4}} + \frac{1-3\bar{w}_{\rm end}^{\rm rd}}{12(1+\bar{w}_{\rm end}^{\rm rd})}\ln\frac{\rho_{\rm rd}}{\rho_{\rm end}}$$

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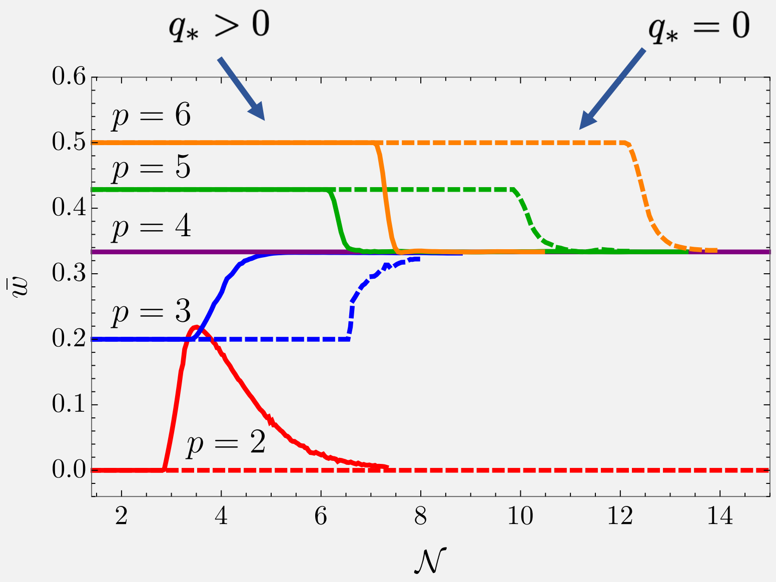

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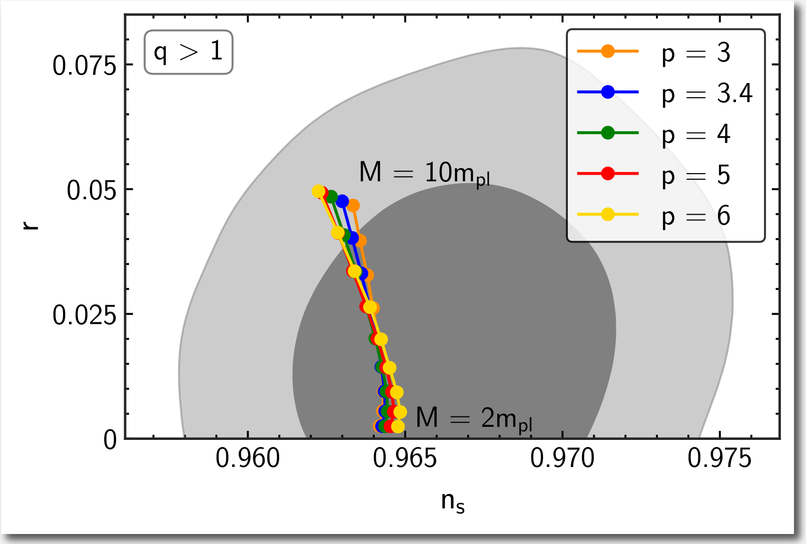
N_k for *pivot scale* $k = 0.05\text{Mpc}^{-1}$:

M \ p	3	4	5	6
$2m_{\rm pl}$	55.5	55.8	56.1	56.5
$10m_{\rm pl}$	56.9	57.4	57.8	58.3

Number of e-folds and Inflationary Observables

With exact N_k, accurate predictions for CMB observables n_s and r can be given:



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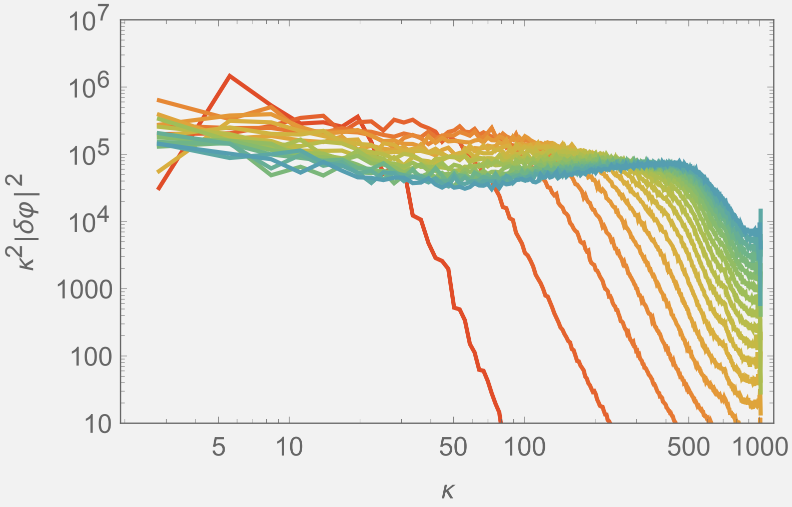
Final energy ratios and EoS

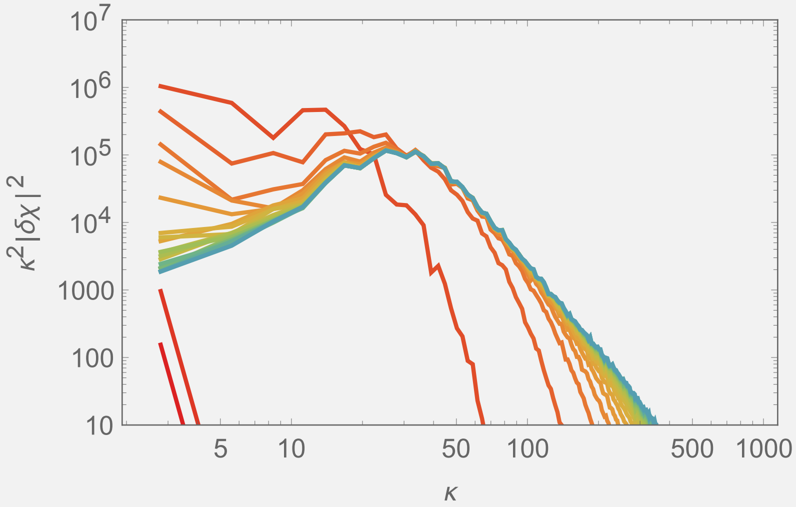
	final EoS:	final energy ratios:
$p = 2:$	$\bar{w} \to 0$	$\bar{\varepsilon}_{k,\varphi} \to 1/2$ $\bar{\varepsilon}_{p,\varphi} \to 1/2$
$2 < p < 4:$	$\bar{w} \to 1/3$	$\bar{\varepsilon}_{k,\varphi} \to 1/2$ $\bar{\varepsilon}_{g,\varphi} \to 1/2$
$p \geq 4:$	$\bar{w} \to 1/3$	$\bar{\varepsilon}_{k,\varphi} \to 1/4 \quad \bar{\varepsilon}_{g,\varphi} \to 1/4$ $\bar{\varepsilon}_{k,\chi} \to 1/4 \quad \bar{\varepsilon}_{g,\chi} \to 1/4$

- Parametric resonance *dominates* over self-resonance, but inflaton fluctuations are produced even *after* backreaction for $p > 2$.
- Final amount of energy transferred to daughter field depends *only* on p.
- Further mechanisms are needed to transfer energy from the inflationary sector to SM.
- For $p > 2$ the system becomes radiation dominated (RD) and one can calculate N_k.

Thank you!

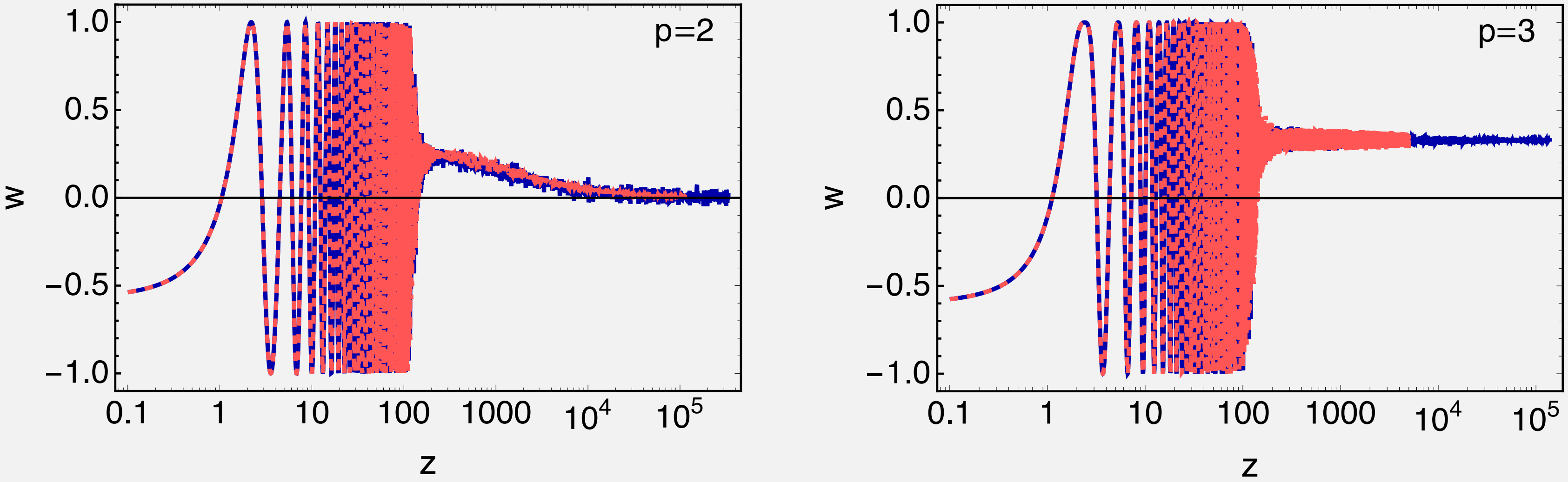
Spectra p=2.3, q=600:





Comparison: (2+1)D and (3+1)D simulations

Equation of state:



(3D: dashed red, 2D: solid blue)

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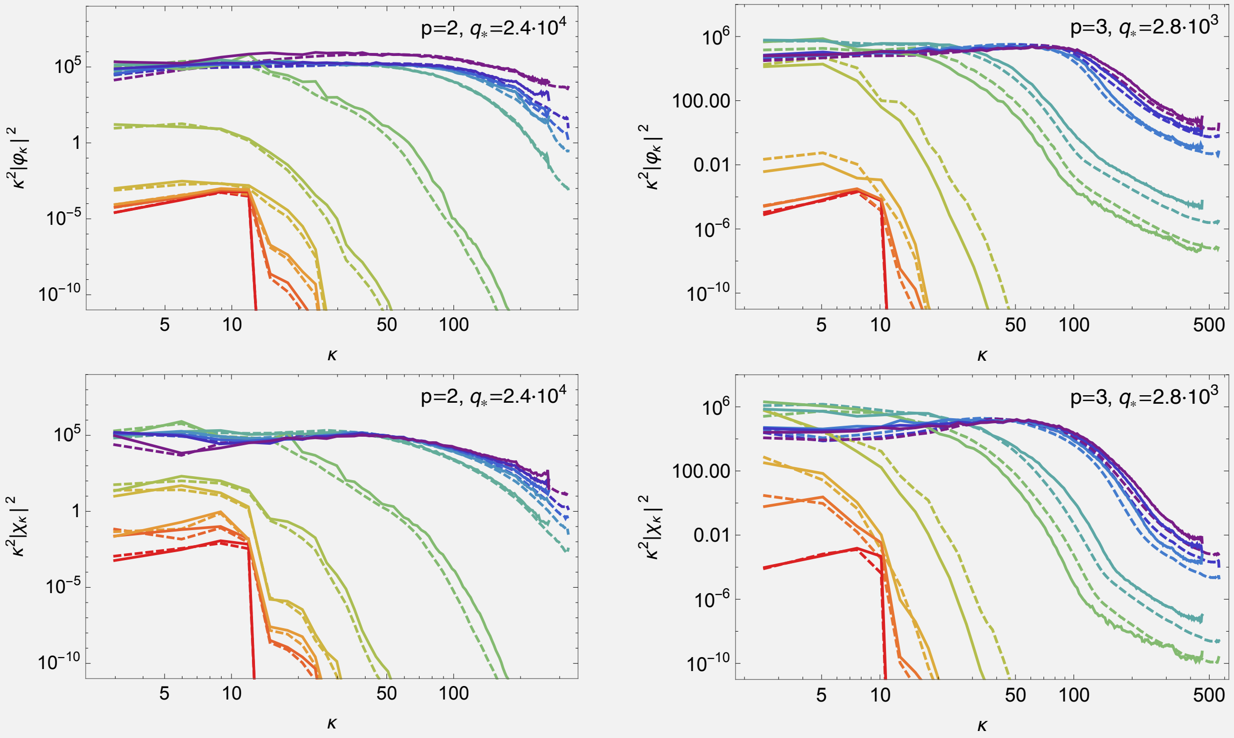
Power Spectra:



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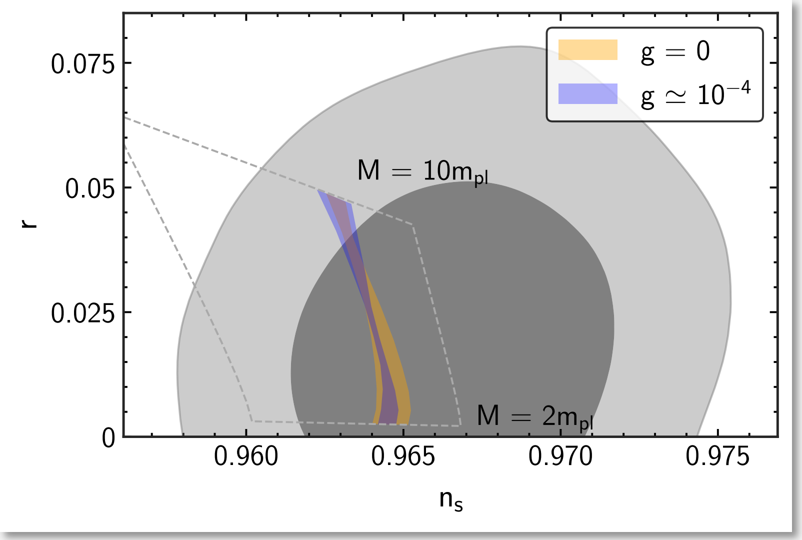

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$V_k(p, M, N_k)$ → determine N_k iteratively

M \ p	3	4	5	6
$2m_{\rm pl}$	55.5	55.8	56.1	56.5
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Homogeneous evolution and field and redefinitions

$$V(\phi, X) = \frac{1}{p}\frac{\Lambda^4}{M^p}|\phi|^p + \frac{1}{2}g^2\phi^2X^2$$

Inflaton performs homogeneous oscillations with frequency:

$$\omega^2 = \omega_*^2 a^{\frac{6p-12}{p+2}} \qquad \omega_*^2 = \frac{\Lambda^4}{M^p}\phi_*^{p-2}$$

Which suggests a redefinition of *field* and *space-time variables*:

$$\varphi \equiv \frac{\phi}{\phi_*}a^{\frac{6}{p+2}} \qquad \vec{x} \to \vec{y} \equiv \omega_* \vec{x}$$

$$\chi \equiv \frac{X}{\phi_*}a^{\frac{6}{p+2}} \qquad t \to z \equiv \int_{t_*}^{t} \omega_*\, a(t')^{\frac{3(2-p)}{p+2}}dt'$$

→

equations of motion:

$$\varphi'' - a^{\frac{-(16-4p)}{2+p}}\nabla_{\vec{y}}^2\varphi + (|\varphi|^{p-2} + q_{\rm res}\chi^2 + \Delta)\varphi = 0$$

$$\chi'' - a^{\frac{-(16-4p)}{2+p}}\nabla_{\vec{y}}^2\chi + (q_{\rm res}\varphi^2 + \Delta)\chi = 0 \qquad (\Delta \sim z^{-2})$$

Resonance parameter $\quad q_{\rm res}(a) \equiv q_* a^{\frac{6(p-4)}{p+2}} \qquad q_* \equiv \frac{g^2\phi_*^2}{\omega_*^2}$

Energy densities and equation of state

In terms of *natural fields* $f = \{\varphi, \chi\}$ the different energy density of the system are given by:

$$E_{\rm k,f} \equiv \frac{1}{2}\left(f' - \frac{6}{p+2}\frac{a'}{a}f\right)^2 \qquad E_{\rm g,f} \equiv \frac{1}{2}a^{\frac{4p-16}{p+2}}|\nabla_{\vec{y}} f|^2$$

$$E_{\rm pot} \equiv \frac{1}{p}\varphi^p \qquad E_{\rm int} \equiv \frac{1}{2}a^{\frac{6p-24}{p+2}} q_* \varphi^2 \chi^2$$

we consider the *energy density ratios:*

$$\varepsilon_i = E_i / E_{\rm tot}$$

and *equation of state:*

$$w = \varepsilon_{\rm k,\varphi} + \varepsilon_{\rm k,\chi} - \frac{1}{3}(\varepsilon_{\rm g,\varphi} + \varepsilon_{\rm g,\chi}) - (\varepsilon_{\rm pot} + \varepsilon_{\rm int})$$

The energy density ratios preserve the following *equipartition identities*

$$\langle \varepsilon_{\rm k,\varphi} \rangle \simeq \langle \varepsilon_{\rm g,\varphi} \rangle + \frac{p}{2}\langle \varepsilon_{\rm pot} \rangle + \langle \varepsilon_{\rm int} \rangle$$

$$\langle \varepsilon_{\rm k,\chi} \rangle \simeq \langle \varepsilon_{\rm g,\chi} \rangle + \langle \varepsilon_{\rm int} \rangle$$

during the phase of homogeneous inflaton oscillations:

$$\langle \varepsilon_{\rm k,\varphi} \rangle \simeq \frac{p}{2}\langle \varepsilon_{\rm pot} \rangle$$

and

$$w_{\rm hom} = \frac{p-2}{p+2}$$