Collective Effects in Supernova Neutrinos

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Quick Introduction

Supernova Neutrinos

- A core-collapse supernova happens (CCSN) is a star explosion that happens when a massive star ($M \gtrsim 8M_\odot$) ends its nuclear fuel, collapsing into itself.
- In this process, a large number of neutrinos are emitted ($\sim 10^{53}$ erg) in a time window of about 10 seconds.

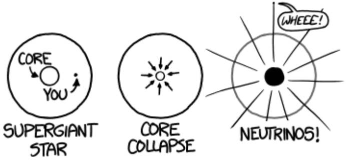


Figure: From https://what-if.xkcd.com/73/

- For supernovae happening in **our galaxy and its neighborhood ($\sim$ some per century)**, their neutrinos can be detected at the Earth, and these neutrinos could bring unique information about the neutrino physics and the supernova mechanism.

Forward Scattering Potentials

- Neutrino-Electron (MSW)

$$H_{\nu e} = \sqrt{2} G_F n_e$$

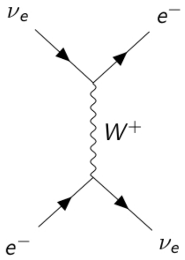


Figure: Forward scattering in electrons.

- Neutrino-Neutrino (Collective Effects) [1]

$$H_{\nu\nu,i} = \sqrt{2} G_F \sum_j (1 - \cos\theta_{ij})(n_{\nu,j}\rho_{\nu,j} - n_{\overline{\nu},j}\rho_{\overline{\nu},j})$$

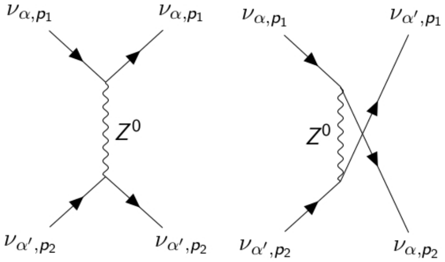


Figure: Forward scattering in neutrinos.

[1] G. Sigl and G. Raffelt, "General kinetic description of relativistic mixed neutrinos", Nucl. Phys. B **406**, 423–451 (1993), P. D. Neto and E. Kemp, "Neutrino–(anti)neutrino forward scattering potential for massive neutrinos at low energies", Mod. Phys. Lett. A **37**, 2250048 (2022)

Evolution Equation

- Considering the forward scattering potentials, the evolution equation for the i-th neutrino in the system becomes

$$i\frac{d}{dt}\rho_i = [\omega H_{vac} + \lambda H_{\nu e} + \mu H_{\nu\nu,i}, \rho_i], \qquad \rho_i \equiv |\psi_{\nu,i}(t)\rangle\langle\psi_{\nu,i}(t)|$$
$$\omega \equiv \frac{\Delta m^2}{2E_i}, \quad \lambda \equiv \sqrt{2}G_F n_e, \quad \mu \equiv \sqrt{2}G_F n_\nu \tag{1}$$

- Although it may appear simple at first glance, **there is no definitive solution to this problem in a supernova environment.**
- The main complications are:
  1. The nonlinear evolution, due to the $\nu - \nu$ interactions;
  2. The angular momenta distribution dependency in the term $\cos\theta_{ij} = \hat{p}_i \cdot \hat{p}_j$;
  3. The complicated geometry of a supernova.

Our Approach to the Problem

Our Approach to the Problem

- In our contribution to understand and solve this problem, we decided to take a **numerical approach**, always trying to compare it with analytical solutions of simpler systems.
- The idea is to **start with a simple model** and increasing its complexity.
- Finally, all the code being developed is made public so that everyone in the community can reproduce the results and maybe help to solve this problem.
- Here, we will present the results of our first models, which consists of an **isotropic neutrino gas** and the **Bulb model with a Single-Angle** approximation.

Polarization Vectors Formalism

- If we consider 2 families of neutrinos $\{\nu_e, \nu_x\}$, all the complex matrices in the evolution equation can be decomposed into the Pauli Matrices, such that the coefficients of expansions will work as components of a vector.

$$H_V = -\tfrac{1}{2}\vec{\sigma}\cdot\vec{B}, \quad H_{\nu e} = -\tfrac{1}{2}\vec{\sigma}\cdot\vec{L}, \quad \rho = \tfrac{1}{2}\mathbf{1} + \tfrac{1}{2}\vec{\sigma}\cdot\vec{P}, \quad \vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3) \tag{2}$$

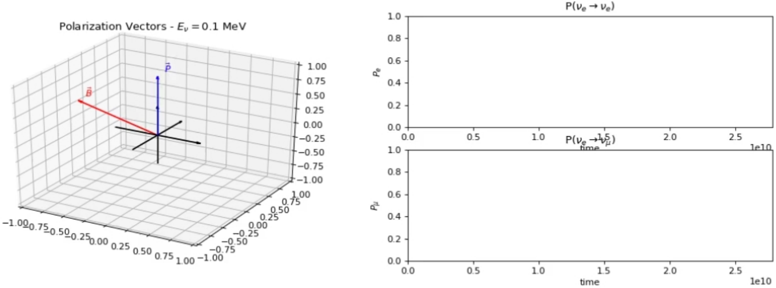
$$\tfrac{d}{dt}\vec{P}_{\nu_i} = \vec{P}_{\nu_i} \times \left[\omega\vec{B} + \lambda\vec{L} + \mu\textstyle\sum_j(1-\cos\theta_{ij})(\vec{P}_{\nu,j} - \vec{P}_{\overline{\nu},j})\right] \tag{3}$$

- Here, the polarization vector $\vec{P}$ has information about the neutrino state, with its third component given the flavor content (ν_e or ν_x) in the flavor basis (P_3^F).

$$P_\beta(t) = \begin{cases} P_e(t) = \frac{1}{2}\left[1 + P_3^F(t)\right] \\ P_x(t) = \frac{1}{2}\left[1 - P_3^F(t)\right] \end{cases} \tag{4}$$

Example - Vacuum Oscillations ($\lambda = 0, \mu = 0$)

$$\frac{d}{dt}\vec{P}_{\nu_i} = \vec{P}_{\nu_i} \times \omega \vec{B} \quad (5)$$



Vacuum oscillation of a neutrino created as $|\nu(0)\rangle = |\nu_e\rangle$

Isotropic and Mono-energetic Neutrino Gas

Isotropic and Mono-energetic Neutrino Gas

- As a first approach in trying to solve neutrino evolution in a high-density neutrino environment, we consider a **mono-energetic and isotropic** ($\langle \cos\theta_{ij} \rangle = 0$) neutrino gas composed of electron neutrinos and antineutrinos $\{\nu_e, \overline{\nu}_e\}$.
- For simplicity, let us consider no matter potential ($\lambda = 0$), so that

$$\frac{d}{dt}\vec{P}_{\nu_i} = \vec{P}_{\nu_i} \times \left[\omega\vec{B} + +\mu(\vec{P}_\nu - \vec{P}_{\overline{\nu}})\right] \tag{6}$$

$$\frac{d}{dt}\vec{P}_{\overline{\nu}_i} = \vec{P}_{\overline{\nu}_i} \times \left[-\omega\vec{B} + \mu(\vec{P}_\nu - \vec{P}_{\overline{\nu}})\right] \tag{7}$$

- Here, $\vec{P}_\nu \equiv \sum_i \vec{P}_{\nu_i}$ and $\vec{P}_{\overline{\nu}} \equiv \sum_i \vec{P}_{\overline{\nu}_i}$ represent the entire ensemble of neutrinos and antineutrinos, respectively.
- Here, we consider $\Delta m^2_{31} = 2.5 \times 10^{-3} eV^2$ and $\sin^2\theta_{13} = 2.1 \times 10^{-2}$ as our vacuum mixing parameters.

The Pendulum Analogy

- It remarkable that the equations for this system is **equivalent to a pendulum**.
- To see this, let us define the vectors $\vec{D} = \vec{P}_\nu - \vec{P}_{\overline{\nu}}$ (difference), $\vec{S} = \vec{P}_\nu + \vec{P}_{\overline{\nu}}$ (sum) and $\vec{Q} = \vec{S} - \frac{\omega}{\mu}\vec{B}$, so that $\vec{Q} \approx \vec{S}$ for $\mu \gg \omega$ and we can rewrite the equations as[2]:

Neutrino Equations

$$\dot{\vec{Q}} = \mu\vec{D} \times \vec{Q} \tag{8a}$$

$$\dot{\vec{D}} = \omega\vec{Q} \times \vec{B} \tag{8b}$$

Pendulum Equations

$$I\dot{\vec{r}} = \vec{L} \times \vec{r} \tag{9a}$$

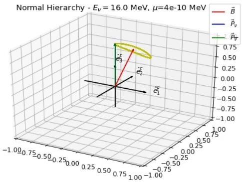
$$\dot{\vec{L}} = \vec{\tau} = \vec{r} \times \vec{F} \tag{9b}$$

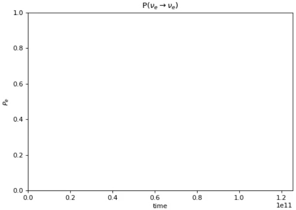
- The neutrino system works as a pendulum attracted by a **force field** $\vec{F} = \omega\vec{B}$, with **angular momentum** $\vec{L} = \vec{D}$, **length** $\vec{r} = \vec{Q}$, and **moment of inertia** $I = m|\vec{r}|^2 = \mu^{-1}$

[2] S. Hannestad et al., "Self-induced conversion in dense neutrino gases: Pendulum in flavour space", Phys. Rev. D **74**, [Erratum: Phys.Rev.D 76, 029901 (2007)], 105010 (2006).

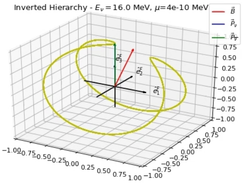
1^{st} Scenario - Symmetric and Constant μ - Numerical Results

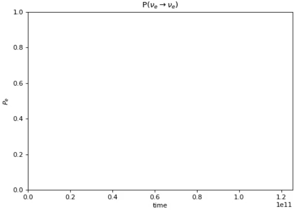
- **Normal Hierarchy ($\Delta m^2 > 0$):** The system is attracted by $\vec{F} = |\omega|\vec{B}$





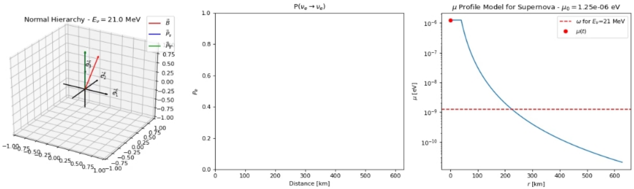
- **Inverted Hierarchy ($\Delta m^2 < 0$):** The system is attracted by $\vec{F} = -|\omega|\vec{B}$



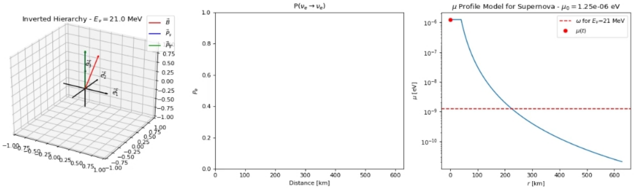


2^{nd} Scenario - Symmetric and Decreasing μ - Numerical Results

- **Normal Hierarchy:** As $I = \mu^{-1}$ increases, the oscillation is damped towards $\vec{F} = |\omega|\vec{B}$.

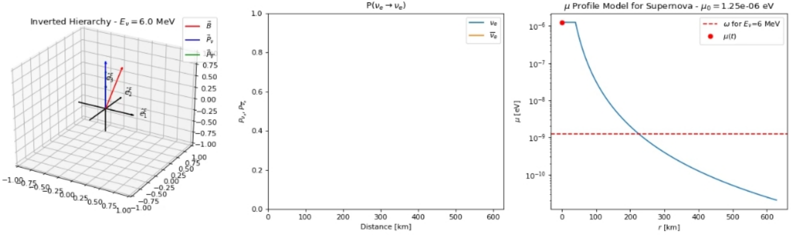


- **Inverted Hierarchy:** As $I = \mu^{-1}$ increase, the oscillation is damped towards $\vec{F} = -|\omega|\vec{B}$



3rd Scenario - Asymmetric and Decreasing μ - Numerical Results

- From the equations of motion, $\vec{D} \cdot \vec{B}$ is conserved, which means that **the initial difference of eigenstates is conserved**. If the mixing angle is small $\theta << 1$, which is the case for the supernova environment due to the high density of matter, this is equivalent to **conservation of net lepton number**.[3]
- **Inverted Hierarchy:** Asymmetry of $N_{\overline{\nu}_e}/N_{\nu_e} = 0.8$.



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[3] A. Mirizzi et al., "Supernova Neutrinos: Production, Oscillations and Detection", Riv. Nuovo Cim. **39**, 1–112 (2016).

Isotropic Neutrino Gas with Spectral Distribution

Isotropic Neutrino Gas with Spectral Distribution

- With a spectral distribution, we can define the polarization vector $\vec{P}_{\nu,\vec{p}_1}$ for each momentum (or momentum interval). Defining $\vec{D} = \sum_{\vec{p}_2} \vec{D}_{\vec{p}_2} = \sum_{\vec{p}_2}(\vec{P}_{\nu,\vec{p}_2} - \vec{P}_{\overline{\nu},\vec{p}_2})$ we may write[4]:

$$\dot{\vec{P}}_{\nu,\vec{p}_1} = \vec{P}_{\nu,\vec{p}_1} \times \left[\omega_{\vec{p}_1}\vec{B} + \mu\vec{D}\right] \quad (10a)$$

$$\dot{\vec{P}}_{\overline{\nu},\vec{p}_1} = \vec{P}_{\overline{\nu},\vec{p}_1} \times \left[-\omega_{\vec{p}_1}\vec{B} + \mu\vec{D}\right] \quad (10b)$$

$$\dot{\vec{S}}_{\vec{p}_1} = \omega\vec{D}_{\vec{p}_1} \times \vec{B} + \mu\vec{D} \times \vec{S}_{\vec{p}_1} \quad (11a)$$

$$\dot{\vec{D}}_{\vec{p}_1} = \omega\vec{S}_{\vec{p}_1} \times \vec{B} \quad (11b)$$

- If $\mu \gg \omega_{\vec{p}_1}$ for all modes, all $\vec{S}_{\vec{p}_1}$ evolve in a similar way, resulting in an evolution similar to the mono-energetic case for each mode.

[4] S. Hannestad et al., "Self-induced conversion in dense neutrino gases: Pendulum in flavour space", Phys. Rev. D **74**, [Erratum: Phys.Rev.D 76, 029901 (2007)], 105010 (2006).

Isotropic Neutrino Gas with Spectral Distribution - Spectral Split

- When considering a decreasing μ, as in a supernova, due to the **net lepton number conservation**, only a fraction of the spectrum can convert its flavor, as given by the following equation:

$$\int_{E_{split}}^{\infty} dE \left[\phi_{\nu_e}(E) - \phi_{\nu_x}(E)\right] = \int_{0}^{\infty} dE \left[\phi_{\overline{\nu}_e}(E) - \phi_{\overline{\nu}_x}(E)\right], \tag{12}$$

- This leads to the phenomenon of **spectral split**, in which there is conversion above certain energy E_{split}, but not bellow it.

Isotropic Neutrino Gas with Spectral Distribution - Numerical Results

- Considering a supernova spectrum, we had the following results:

Normal Hierarchy

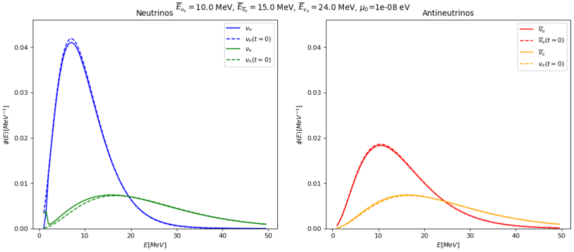

Figure: Initial and final spectrum for the isotropic neutrino gas (NH).

Inverted Hierarchy

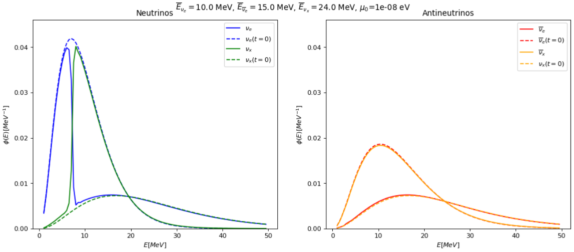

Figure: Initial and final spectrum for the isotropic neutrino gas (IH).

Bulb Model Single Angle Approximation

Bulb Model

- The Bulb model consists of considering that the neutrinos of a supernova are emitted from the surface of a sphere, as light from a bulb.

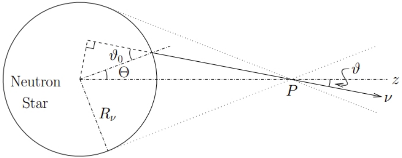

Figure: Bulb model of emission.[5]

- In this case, there is an **azimuthal symmetry** around the z axis, so that the neutrino-neutrino potential can be integrated in this angle.

$$H_{\nu\nu} = \sqrt{2}G_F 2\pi \sum_\alpha \int (1 - \cos\vartheta \cos\vartheta')[\rho_\alpha(p', \vartheta) - \overline{\rho}_\alpha(p', \vartheta)] dp' d\cos\vartheta' \qquad (13)$$

[5] H. Duan et al., "Simulation of Coherent Non-Linear Neutrino Flavor Transformation in the Supernova Environment. 1. Correlated Neutrino Trajectories", Phys. Rev. D **74**, 105014 (2006)

Bulb Model - Single-Angle Approximation

- A further approximation can be made by considering that a single angle ϑ is a good representative of all the other possible trajectories.

$$H_{\nu\nu} = \sqrt{2} G_F 2\pi D(r) \sum_{\alpha} \int [\rho_\alpha(p') - \overline{\rho}_\alpha(p')] dp' \tag{14}$$

- In this case, **the potential is identical to the isotropic case**, with the following geometric factor for $\vartheta = 0$:

$$D(r) = \frac{1}{2}(1 - \cos\vartheta_{\mathrm{max}})^2 = \frac{1}{2}\left[1 - \sqrt{1 - \left(\frac{R_\nu}{r}\right)^2}\right]^2 \tag{15}$$

Bulb Model - Single-Angle Approximation - Numerical Results

- As expected, we have the same results of the isotropic scenario, with the phenomenon of spectral split.

Inverted Hierarchy

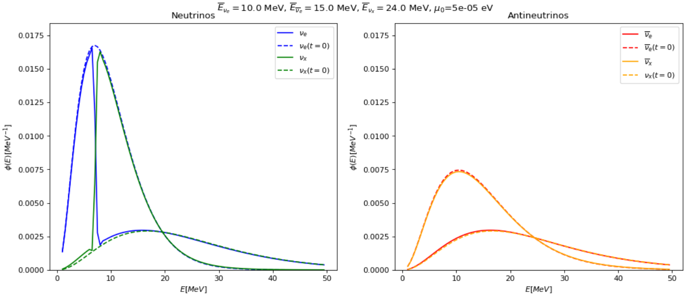

Figure: Initial and final spectrum for the Bulb model with the Single-Angle approximation (IH).

Source Code

Open-Source Code

- When working with **neutrino collective effects**, it is **difficult (almost impossible) to find papers with open-source code** to verify and **reproduce the results**.
- With that in mind, our code was made available and can be found in the following repository[6]: `https://github.com/pedrodedin/Neutrino-Collective-Effects.git`
- We hope that this may help newcomers to understand and reproduce our results without the need to ”reinvent the well“.

[6] We have a paper under constriction in which we describe the physics and the code implementation.

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- In this context, we intend to improve our numerical code to consider more supernova-like systems, while trying to find analytical solutions or interpretations in the formalism of polarization vectors.
- Among these improvements, we may cite:
  - Multi-anlge and non-uniform emission in the Bulb model.
  - Models with electron lepton number crossing in the angular distribution, which can lead to the phenomena of fast oscillations.

Thank you!

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References I

[1] G. Sigl and G. Raffelt, "General kinetic description of relativistic mixed neutrinos", Nucl. Phys. B **406**, 423–451 (1993).

[2] P. D. Neto and E. Kemp, "Neutrino–(anti)neutrino forward scattering potential for massive neutrinos at low energies", Mod. Phys. Lett. A **37**, 2250048 (2022).

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[4] A. Mirizzi, I. Tamborra, H.-T. Janka, N. Saviano, K. Scholberg, R. Bollig, L. Hudepohl, and S. Chakraborty, "Supernova Neutrinos: Production, Oscillations and Detection", Riv. Nuovo Cim. **39**, 1–112 (2016).

[5] H. Duan, G. M. Fuller, J. Carlson, and Y.-Z. Qian, "Simulation of Coherent Non-Linear Neutrino Flavor Transformation in the Supernova Environment. 1. Correlated Neutrino Trajectories", Phys. Rev. D **74**, 105014 (2006).