How special are black holes?

Correspondence with objects saturating unitarity bounds in generic theories

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Based on: Phys. Rev. D **105**, 056013 (2022)
Dvali, OK, Valbuena-Bermúdez

See also talk by: Valbuena-Bermúdez

Outline

Saturons

Saturon: *n*-particle composite classical object with $S = S_{\text{max}}$

Non-perturbative saturation of unitarity by $2 \to n$ particle scattering amplitudes at the point of optimal truncation

$$\boxed{S_{\text{max}} = \frac{1}{\alpha} = \frac{Area}{G_{\text{Gold}}}}$$

α: eff. running coupling evaluated at mom.-transfer scale $1/R$

G_{Gold}: coupling of Goldstone field of a spont. broken symmetry

Dvali: JHEP **03**, 126 (2021) & refs. therein

A saturon model

Theory of a scalar field ϕ in the adjoint rep. of $SU(N)$, $N \geq 3$

ϕ_{α}^{β} is an $N \times N$ traceless Hermitian matrix

$$\mathcal{L} = \frac{1}{2}\mathrm{tr}\left[(\partial_\mu \phi)(\partial^\mu \phi)\right] - V[\phi]$$

$$V[\phi] = \frac{\alpha}{2}\mathrm{tr}\left[\left(f\phi - \phi^2 + \frac{I}{N}\mathrm{tr}\left[\phi^2\right]\right)^2\right]$$

't Hooft coupling: $\lambda_{\mathrm{t}} \equiv \alpha N \lesssim 1$ (non-perturbative bound)

Saturation: $\lambda_{\mathrm{t}} \sim 1$

Dvali: JHEP **03**, 126 (2021)

Vacua

Vacuum equations

$$f\phi_\alpha^\beta - \left(\phi^2\right)_\alpha^\beta + \frac{\delta_\alpha^\beta}{N}\text{tr}\left[\phi^2\right] = 0$$

admit various solutions degenerate in energy, corresponding to vacua with different unbroken symmetries

$$SU(N) \to SU(N-K) \times SU(K) \times U(1)\ , \quad 0 < K < N$$

Consider 2 vacua:
(1) unbroken $SU(N)$ symmetry, $\phi = 0$
(2) $K = 1$

In $K = 1$ vacuum, only the component

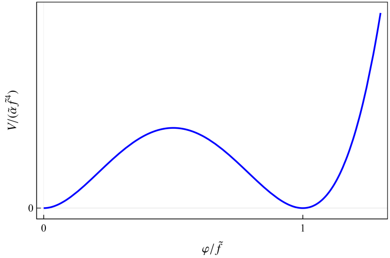
$$\phi_\alpha^\beta = \frac{\varphi(x)}{\sqrt{N(N-1)}} \mathrm{diag}\left[(N-1), -1, \ldots, -1\right]$$

has a non-zero expectation value

$$\langle \phi \rangle \equiv \tilde{f} = f \frac{\sqrt{N(N-1)}}{(N-2)} \xrightarrow{N \to \infty} f$$

$$V[\phi] = \frac{\tilde{\alpha}}{2} \varphi^2 \left(\varphi - \tilde{f}\right)^2 ,$$

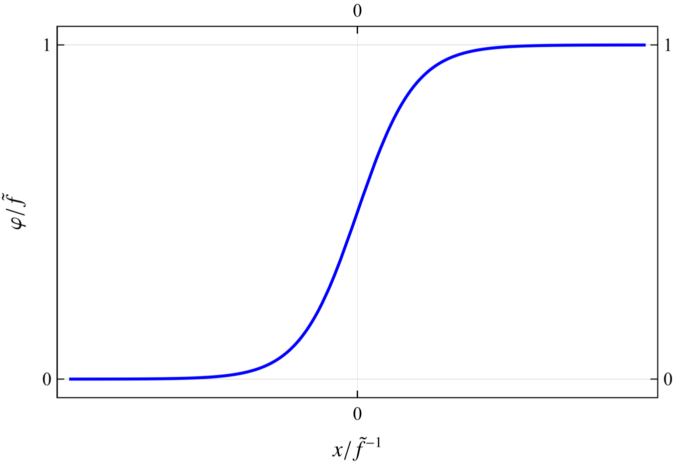
$$\tilde{\alpha} \equiv \alpha \frac{(N-2)^2}{N(N-1)} \xrightarrow{N \to \infty} \alpha$$



$$\text{minimized by } \varphi = \begin{cases} 0, & SU(N);\ \text{unbroken phase} \\ \tilde{f}, & SU(N-1) \times U(1);\ \text{broken phase} \end{cases}$$

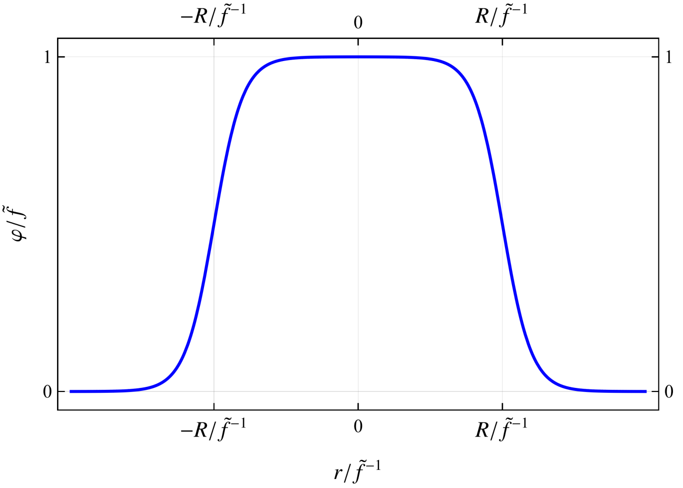
Domain wall

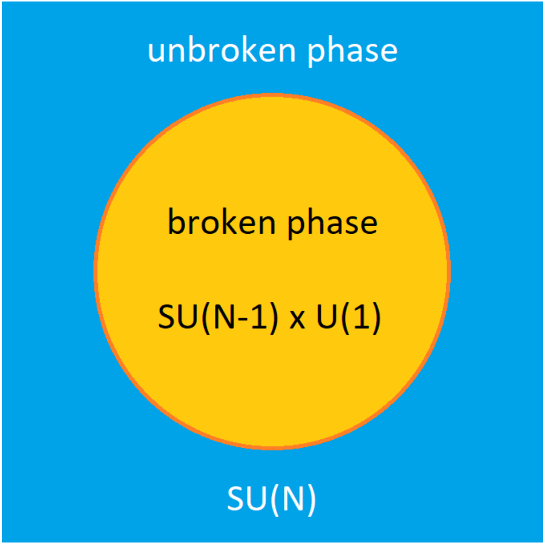
$$\varphi(x) = \frac{\tilde{f}}{2}\left[1 + \tanh\left(\frac{mx}{2}\right)\right] \ , \quad \delta_{\rm w} \sim m^{-1} \ , \quad m \equiv \sqrt{\alpha} f = \sqrt{\tilde{\alpha}} \tilde{f}$$



Bubble

$$\varphi(r) = \frac{\tilde{f}}{2}\left[1 + \tanh\left(\frac{m(R-r)}{2}\right)\right]$$





$SU(N)$ phase: mass gap $m = \sqrt{\alpha} f$
$SU(N-1) \times U(1)$ phase: N_{Gold} massless Goldstones

Goldstone excitations

$$\phi_\alpha^\beta = \left(U^\dagger \Phi U\right)_\alpha^\beta$$

$$\Phi_\alpha^\beta = \frac{\varphi(t,\vec{x})}{\sqrt{N(N-1)}} \mathrm{diag}\left[(N-1), -1, \ldots, -1\right]$$

$$U = \exp\left[-i\theta^a T^a\right] \ , \quad \theta^a = \theta(t,\vec{x})\delta^{a1}$$

$$\Rightarrow \ U = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) & & & \\ -i\sin(\theta/2) & \cos(\theta/2) & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\varphi\right)\left(\partial^\mu\varphi\right) + \frac{N}{4(N-1)}\varphi^2\left(\partial_\mu\theta\right)\left(\partial^\mu\theta\right) - \frac{\tilde{\alpha}}{2}\varphi^2\left(\varphi - \tilde{f}\right)^2$$

Memory burden is a **collective** effect

Re-formulation

$$\Psi \equiv \frac{\varphi}{\sqrt{2}} e^{i\chi/\tilde{f}} \ , \quad \chi \equiv \sqrt{\frac{N}{2(N-1)}} \tilde{f}\theta$$

$$\Rightarrow \mathcal{L} = (\partial_\mu \Psi^*)(\partial^\mu \Psi) - \tilde{\alpha} |\Psi|^2 \left(\sqrt{2} |\Psi| - \tilde{f} \right)^2$$

$$\text{EOM: } \boxed{\Box \Psi + \tilde{\alpha} \Psi \left(\sqrt{2} |\Psi| - \tilde{f} \right) \left(2\sqrt{2} |\Psi| - \tilde{f} \right) = 0}$$

$$\varphi = \sqrt{2} |\Psi| \ , \quad \theta = \sqrt{\frac{2(N-1)}{N}} \mathrm{Arg}(\Psi)$$

Time evolution

Initial conditions:

$$\Psi(t,r)|_{t=0} = \frac{\tilde{f}}{2\sqrt{2}}\left[1 + \tanh\left(\frac{m(R-r)}{2}\right)\right]$$

$$\partial_t \Psi(t,r)|_{t=0} = i\tilde{\omega}\frac{\tilde{f}}{2\sqrt{2}}\left[1 + \tanh\left(\frac{m(R-r)}{2}\right)\right]$$

Or, equivalently:

$$\varphi(t,r)|_{t=0} = \frac{\tilde{f}}{2}\left[1 + \tanh\left(\frac{m(R-r)}{2}\right)\right]$$

$$\dot{\theta} \equiv \partial_t \theta(t,r)|_{t=0} = \sqrt{\frac{2(N-1)}{N}}\tilde{\omega}$$

$$\tilde{\omega} = \sqrt{\frac{N}{2(N-1)}}\omega$$

Stability

Balance: wall tension $\leftrightarrow$ interior pressure due to internal rotation

$$R \gg m^{-1} \quad : \quad E = \frac{2\pi}{3\alpha} m^3 R^2 (1 - \underbrace{\dot{R}^2}_{=0})^{-1/2} + \frac{2\pi}{3\alpha} m^2 \omega^2 R^3$$

$$\dot{Q} = 0 \text{ with } Q = -i \int r^2 \, \mathrm{d}r \, (\Psi^* \partial_t \Psi - \Psi \partial_t \Psi^*) = \frac{2\pi}{3\alpha} m^2 \omega R^3$$

$$\Rightarrow \; E = \frac{2\pi}{3\alpha} m^3 R^2 + \frac{3\alpha Q^2}{2\pi m^2 R^3} \quad \Rightarrow \quad \frac{\mathrm{d}E}{\mathrm{d}R} \overset{!}{=} 0 \; \Leftrightarrow \; \boxed{R = \frac{2m}{3\omega^2}}$$

$$\left.\begin{aligned} E_{\text{int}} &= \omega N_{\text{G}} \;, \quad N_{\text{G}} \equiv \frac{1}{\alpha} \frac{m^5}{\omega^5} \left(\frac{16\pi}{81}\right) \\ E_{\text{wall}} &= m N_{\varphi} \;, \quad N_{\varphi} \equiv \frac{1}{\alpha} \frac{m^4}{\omega^4} \left(\frac{8\pi}{27}\right) \end{aligned}\right\} \Rightarrow \frac{N_{\text{G}}}{N_{\varphi}} = \frac{2m}{3\omega} \gg 1$$

Goldstone $SU(N)$ charge is conserved

Same amount of charge costs more energy in the exterior vacuum

Microscopic picture

SSB: $SU(N) \to SU(N-1) \times U(1)$

$\Rightarrow$ broken vacuum (bubble interior) contains
$N_{\mathrm{Gold}} = N^2 - 1 - [(N-1)^2 - 1 + 1] = 2(N-1) \simeq 2N$
species of massless Goldstones θ^a

$$N_{\mathrm{G}} \simeq \sum_{a=1}^{2N} n^a$$

$$n_{\mathrm{st}} \simeq \binom{N_{\mathrm{G}} + 2N}{2N} \sim \left(1 + \frac{2N}{N_{\mathrm{G}}}\right)^{N_{\mathrm{G}}} \left(1 + \frac{N_{\mathrm{G}}}{2N}\right)^{2N}$$

$$S = \ln(n_{\mathrm{st}}) \simeq 2N \ln\left[(1+\lambda)^{\frac{1}{\lambda}} \left(1 + \frac{1}{\lambda}\right)\right] \text{ with } \lambda \equiv 2N/N_{\mathrm{G}}$$

Entropy of bubbles

$$S = \ln(n_{\mathrm{st}}) \simeq 2N \ln\left[(1+\lambda)^{\frac{1}{\lambda}}\left(1+\frac{1}{\lambda}\right)\right] \text{ with } \lambda = 2N/N_{\mathrm{G}}$$

Thin-wall bubbles: $R \gg \delta_{\mathrm{w}} \sim m^{-1}$
$\Leftrightarrow\ \omega \ll m\ \Leftrightarrow\ N_{\mathrm{G}} \gg \alpha^{-1} \sim N\ \Leftrightarrow\ \lambda \ll 1$
$\Rightarrow$ under-saturated state:
$S \simeq 2N \ln\left(\frac{\mathrm{e}}{\lambda}\right) \sim \frac{1}{\alpha} \ln\left(\frac{m^{10}}{\omega^{10}}\right) \ll S_{\mathrm{max}} \sim \frac{1}{\alpha}\frac{m^6}{\omega^6}$

Thick-wall bubbles: $R \sim \delta_{\mathrm{w}} \sim m^{-1}$
$\Leftrightarrow\ \omega \sim m\ \Leftrightarrow\ N_{\mathrm{G}} \sim \alpha^{-1} \sim N\ \Leftrightarrow\ \lambda \sim 1$
$\Rightarrow$ saturon: $S \sim S_{\mathrm{max}} \sim \frac{1}{\alpha}$

Stabilization via memory burden

Memory burden effect	**Bubble stabilization**
Many memory patterns	Many bubble micro-states
Stored quantum information	Excitations of Goldstone modes
Impediment of system's evolution	Impediment of bubble's decay

Summary

Saturons: $S = S_{\text{max}}$

BHs are not special

Memory burden is an important, collective effect

Outlook:
BH-like properties: G_{Gold}, α, S, T, t_{min}, information horizon
Bubble dynamics, interaction

Dvali, OK, Valbuena-Bermúdez: Phys. Rev. D **105**, 056013 (2022)

Dvali: JHEP **03**, 126 (2021); Phil. Trans. A **380**, 20210071 (2022); Fortschr. Phys. **69**, 2000090 (2021); Fortschr. Phys. **69**, 2000091 (2021)

Dvali, Kühnel, Zantedeschi: [2112.08354]

Dvali, Sakhelashvili: Phys. Rev. D **105**, 065014 (2022)

Dvali, Venugopalan: Phys. Rev. D **105**, 056026 (2022)

Saturons

Saturon: n-particle composite classical object with $S = S_{\text{max}}$

Non-perturbative saturation of unitarity by $2 \to n$ particle scattering amplitudes at the point of optimal truncation

$$2 \to n \text{ cross-section: } \sigma_{2\to n} = \underbrace{c_n}_{\mathcal{O}(1)} n!\ \alpha^n$$

Stop series expansion in α when $\sigma_{2\to n}$ reaches minimum in n: at $n = \alpha^{-1}$, i.e. at $\lambda_c \equiv \alpha n = 1 \ \Rightarrow \ \sigma_{2\to n} = n!\ n^{-n}$

Can only be trusted for $n \leq \alpha^{-1}$

Stirling's approximation: $\ln(n!) = n\ln(n) - n + \Theta\left[\ln(n)\right]$

$$\Rightarrow \ \boxed{\sigma_{2\to n} = \mathrm{e}^{-n} = \mathrm{e}^{-1/\alpha}}$$

Dvali: JHEP **03**, 126 (2021)

Entropy saturation

$$\text{Total cross-section: } \sigma = \sum_{\text{micr. st.}}^{n_{\text{st}}} \sigma_{2\to n} = \underbrace{\sigma_{2\to n}}_{e^{-1/\alpha}} \overbrace{n_{\text{st}}}^{e^{S}} = e^{-\frac{1}{\alpha}+S}$$

Non-perturbative methods[1]: for $n \gg \alpha^{-1}$: $\sigma_{2\to n} \lesssim n!\ n^{-n} \sim e^{-n}$

$$\boxed{S_{\max} = \frac{1}{\alpha} = \frac{Area}{G_{\text{Gold}}}}$$

α: eff. running coupling evaluated at mom.-transfer scale $1/R$

G_{Gold}: coupling of Goldstone field of a spont. broken symmetry

Bonus: Bekenstein bound[2]: $S_{\max} = 2\pi MR$

[1] Dvali: [1804.06154]; JHEP **03**, 126 (2021) & refs. therein
[2] Bekenstein: Phys. Rev. D **23**, 287 (1981)

Enhanced memory capacity

Want: system with high capacity to store information

$$\left[\hat{a}_j, \hat{a}_k^\dagger\right] = \delta_{jk}$$

Memory pattern: micro-state $|n_1, \ldots, n_K\rangle$

$n_{\rm st}$-many micro-states degenerate in energy $\Rightarrow$ entropy $S = \ln(n_{\rm st})$

$$\hat{H} = \qquad \varepsilon \sum_{k=1}^{K} \hat{n}_k$$

Dvali: [1810.02336]

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$$\hat{H} = \left(1 - \frac{\hat{n}_a}{N}\right) \varepsilon \sum_{k=1}^{K} \hat{n}_k$$

Assisted gaplessness: highly occupied master mode $\hat{n}_a$ interacts attractively with memory modes $\hat{n}_k$, lowers their energy gaps

Dvali: [1810.02336]

Memory burden

$$\hat{H} = \left(1 - \frac{\hat{n}_a}{N}\right) \varepsilon \sum_{k=1}^{K} \hat{n}_k + C_b \left(\hat{a}^\dagger \hat{b} + \text{H.c.}\right)$$

Memory burden: stored information stabilizes system

$$|in\rangle = |n_a, n_b, n_1, \ldots, n_K\rangle = |N, 0, n_1, \ldots, n_K\rangle$$

$$\mu = -\frac{1}{N} \varepsilon \sum_{k=1}^{K} \langle \hat{n}_k(t) \rangle$$

$$\langle \hat{n}_a(t) \rangle = N \left[1 - \frac{C_b^2}{C_b^2 + \left(\frac{\mu}{2}\right)^2} \sin^2 \left(\sqrt{C_b^2 + \left(\frac{\mu}{2}\right)^2}\, t\right)\right]$$