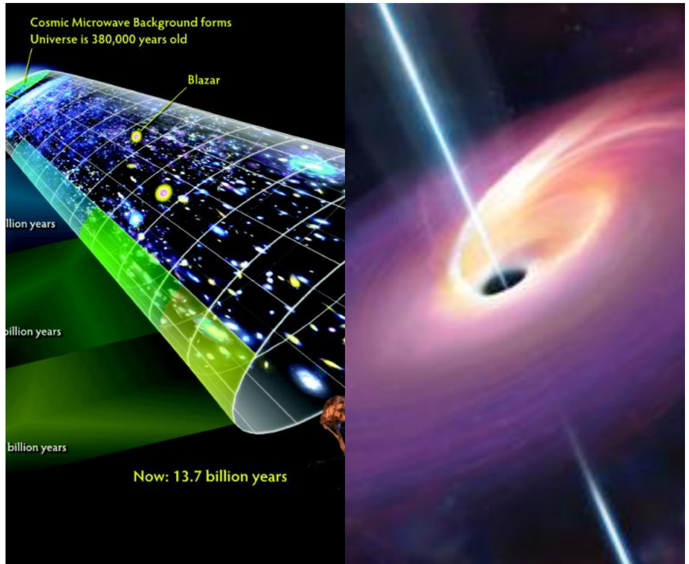

Albert Escrivà

Formation of PBHs during the QCD phase transition

In preparation

with E. Bagui and S. Clesse

Service de Physique Théorique
Université Libre de Bruxelles

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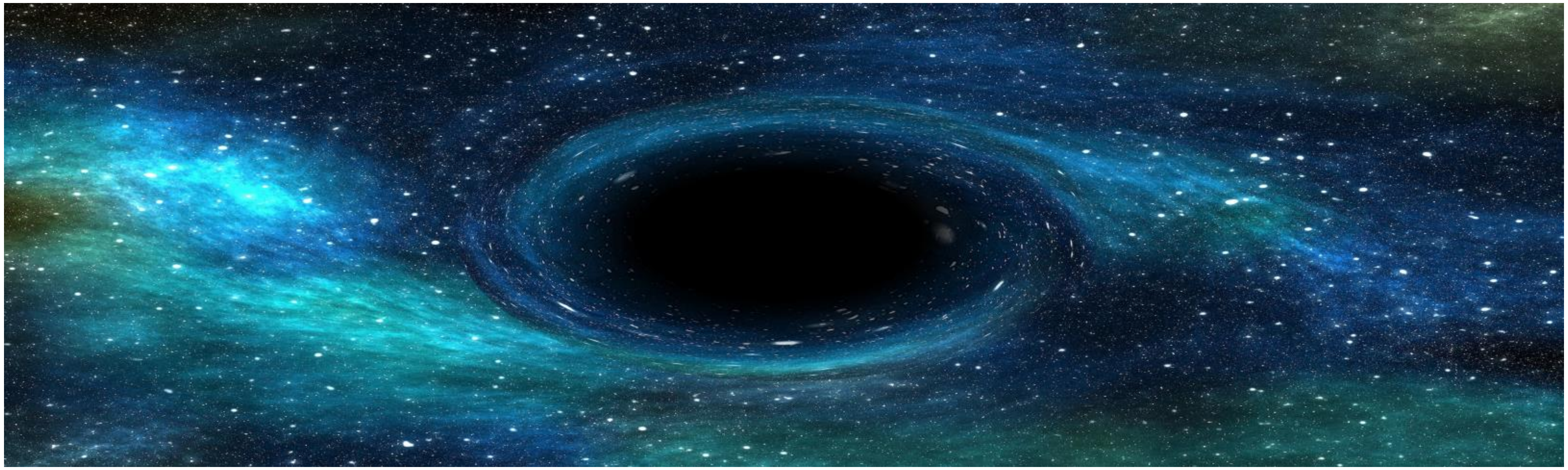
What are PBHs????

<u>Black holes that could have been formed in the very early Universe (no estelar origin)</u>



- First proposal by Novikov & Zel'dovič (1960), but refined later by Carr & Hawking (1974)

- PBHs can have many implications in the history of the Universe, but importantly, they can constitute the dark matter! (or a significant fraction)

Carr & Kuhnel. ArXiv:2006.02838

PBHs from cosmological fluctuations

- A lot of mechanisms for PBH production! (bubbles, strings, domains walls, Q-balls , quark confinement ,etc, etc)

The most considered case: from cosmological fluctuations!

- Some fluctuations generated during inflation could be sufficiently large (**very rare events**) and collapse during **radiation epoch** when they reenter the cosmological horizon

- These very rare fluctuations will have roughly spherical symmetry (spherical peaks)

J. M. Bardeen, J. R. Bond, N. Kaiser and A. S. Szalay 1986

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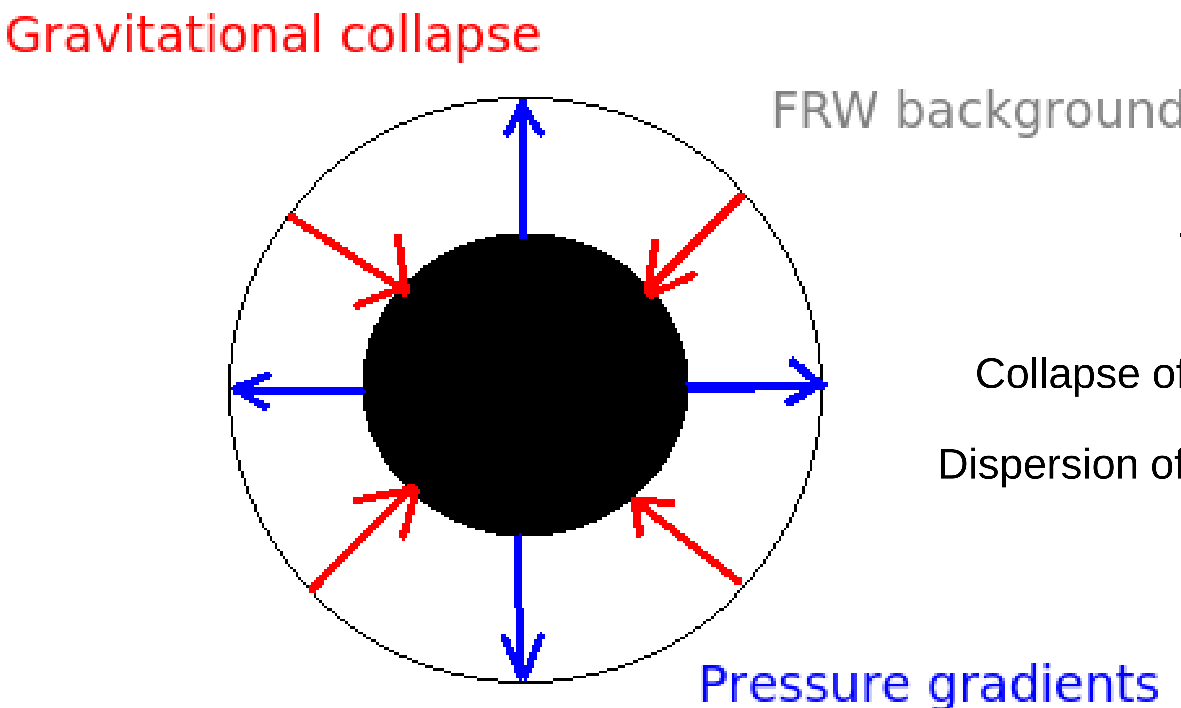
Collapse of fluctuations

<u>Spherical Collapse</u> of cosmological perturbations leading to PBH formation



Threshold → δ_c

Collapse of the perturbation: $\delta > \delta_c$

Dispersion of the perturbation: $\delta < \delta_c$

“amplitude of the perturbation”

$$p = w\rho$$ Perfect fluid

$$w = 1/3$$ (radiation)

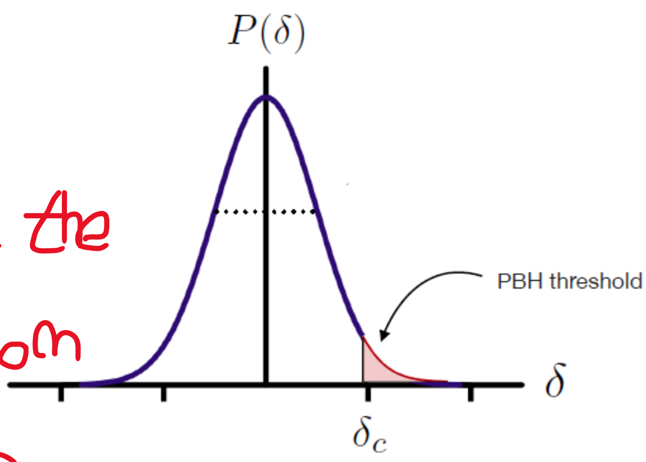
The abundance…

- Statistically, the prob. to have a large fluctuation is exponentially small

<u>**The abundance of PBHs is exponentially sensitive to the threshold**</u> → $$P(\delta) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\delta^2}{2\sigma^2}}$$

→ depends on ω

→ depends on the shape of the fluctuation



$$\beta \equiv \frac{\rho_{\rm PBH}}{\rho_{\rm tot}} = 2\int_{\delta_c}^{\infty} \frac{M_{\rm PBH}}{M_{\rm H}} P(\delta) d\delta \sim {\rm erf}\left(\frac{\delta_c}{\sqrt{2}\sigma}\right)$$

Press–Schechter formalism

The threshold…

- There are some analytical approximations,

Harada,Yoo,Kohri. ArXiv:1309.4201

$$\delta_{HYK} = \frac{3(1+w)}{5+3w}\sin^2\left(\frac{\pi\sqrt{w}}{1+3w}\right) \approx 0.41$$

(Jeans length aproximation), B. Carr 1975

$$\delta_{\rm Carr} = w = 1/3$$

In radiation

The threshold was considered time ago as a constant value, **but actually, it depends on the specific shape of the cosmological fluctuation**

Germani , Musco. ArXiv:1805.04087

Only can be got numerically with accuracy!

Do simulations is essential!!

(ACTUALLY NOT!)

Escrivà, Germani , K. Sheth. ArXiv:2007.05564.

Escrivà, Germani , K. Sheth. Arxiv:1907,13311

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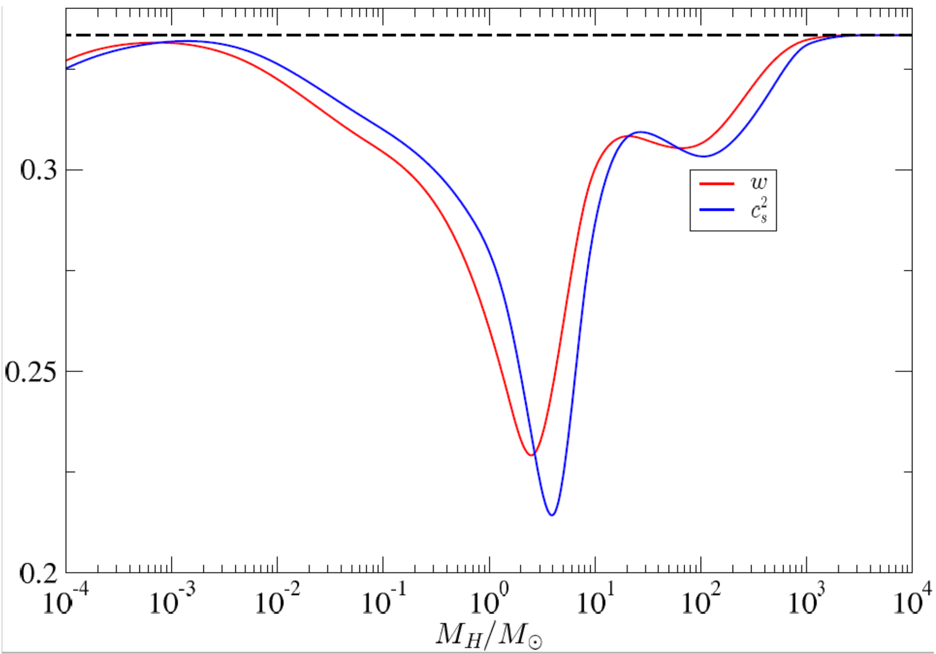
The QCD scenario!

The problem for a constant eq. of state (especially for radiation fluid) is very well studied

BUT

Scenario with a QCD phase transition?

enhances the probability of forming PBH with stellar masses



$$p = w(\rho)\rho$$

From lattice QCD simulations Arxiv: 1606.07494

Never explored numerically, we need to do simulations!

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How we simulate PBH formation?
Solving Misner-Sharp equations

Misner,Sharp. Phys.Rev. 136 (1964)

- The Misner-Sharp equations describes the motion of a relativistic fluid under a curved spacetime (comoving-gauge)

- We consider a perfect fluid with an equation of state $p = w(\rho)\rho$

$$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$$

QCD pase transition

- Spherically symmetric spacetime

$$ds^2 = -A^2(r,t)dt^2 + B^2(r,t)dr^2 + R^2(r,t)d\Omega^2$$

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Misner-Sharp equations

$$\dot{U} = -A\left[\frac{c_s^2}{1+w(\rho)}\frac{\Gamma^2}{\rho}\frac{\rho'}{R'} + \frac{M}{R^2} + 4\pi R w(\rho)\rho\right]$$

$$\dot{\rho} = -A\rho(1+w(\rho))\left(2\frac{U}{R} + \frac{U'}{R'}\right)$$

$$\dot{M} = -4\pi A w(\rho)\rho U R^2$$

$$\dot{R} = AU$$

$$c_s^2 = w(\rho) + \rho\frac{\partial w(\rho)}{\partial \rho}$$

$$M = \int_0^R 4\pi R^2 \rho \, dR$$

$$A' = -A\frac{\rho'}{\rho}\frac{c_s^2}{1+w}$$

$$\Gamma^2 = 1 + U^2 - 2\frac{M}{R}$$

It is problematic now!!

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Numerical procedure

How can we solve this numerical problem???

- **Spectral methods** (other procedures are possible)

Escrivà.ArXiv:1907.13065

→ RK4 for the time derivative

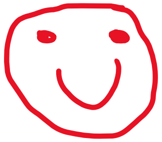
→ Spectral methods for spatial derivative

$$
D_N = \left[\begin{array}{c|c|c}
\frac{2N^2+1}{6} & 2\frac{(-1)^j}{1-x_j} & \frac{1}{2}(-1)^N \\
\hline
-\frac{1}{2}\frac{(-1)^i}{1-x_i} & \begin{array}{ccc} & & \frac{(-1)^{i+j}}{x_i-x_j} \\ & \frac{-x_j}{2(1-x_j^2)} & \\ \frac{(-1)^{i+j}}{x_i-x_j} & & \end{array} & \frac{1}{2}\frac{(-1)^{N+i}}{1+x_i} \\
\hline
-\frac{1}{2}(-1)^N & -2\frac{(-1)^{N+j}}{1+x_j} & -\frac{2N^2+1}{6}
\end{array}\right]
$$

where $x_j = \cos(\frac{j\pi}{N}), j = 0, \ldots, N$

$\vec{w} = D_N \cdot u(\vec{x}) \approx u'(\vec{x})$ with spectral accuracy.
$\vec{w} = {D_N}^2 \cdot u(\vec{x}) \approx u''(\vec{x})$ with spectral accuracy.

Essential when QCD phase transition is considered: solving the lapse equation is straightforward!



Basics on primordial black hole formation

Shibata,Sasaki. ArXiv:gr-qc/9905064

- PBHs could be formed by sufficiently large cosmological perturbations collapsing after re-entering the cosmological horizon. Assuming spherical symmetry, such regions can be described by the following approximate form of the metric at super-horizon scales (gradient expansion approach)

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - K(r)r^2} + r^2 d\Omega^2\right]$$

- The curvature profile K(r) characterize the cosmological perturbation $\frac{\delta\rho}{\rho}$

Polnarev, Musco. ArXiv:gr-qc/0605122

- The criteria of PBH formation is using the compact function

$$\mathcal{C} = 2\frac{M - M_b}{R}$$

$$C(r) = f(\omega)K(r)r^2$$

$$K(r_m) + \frac{r_m}{2}K'(r_m) = 0$$

(at super-horizon scales)

$$\delta_c \equiv \mathcal{C}_c(r_{\text{peak}}) = \mathcal{C}_c(r_m)$$

Condition for PBH formation

- The peak of the compaction function must be greater than a given threshold

$$C_{\rm peak} = C(r_m)$$

$$C_{\rm peak} \geq \delta_c \longrightarrow$$ PBH formation

$$C_{\rm peak} < \delta_c \longrightarrow$$ NO PBH formation

Shibata,Sasaki. ArXiv:gr-qc/9905064

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Curvature profiles

- Consider a set of exponential profiles

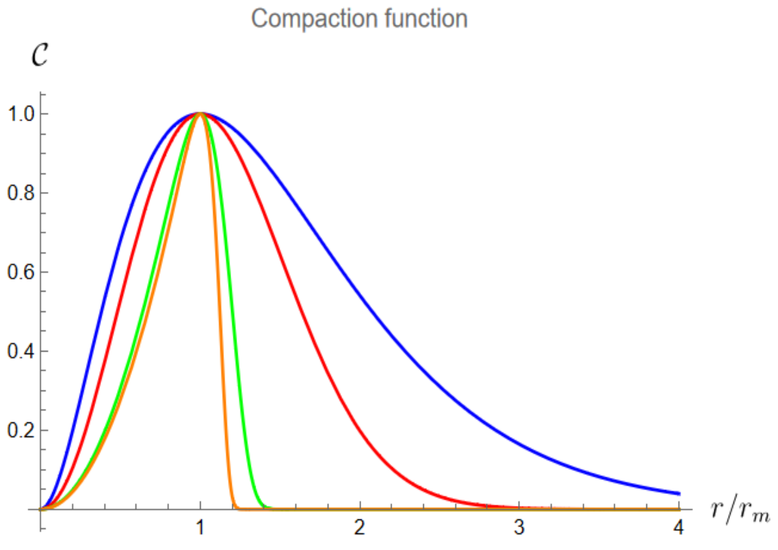
$$K_{\exp}(r) = \frac{\delta_m}{f(w) r_m^2} \; e^{\frac{1}{q}\left(1 - \left(\frac{r}{r_m}\right)^{2q}\right)}$$

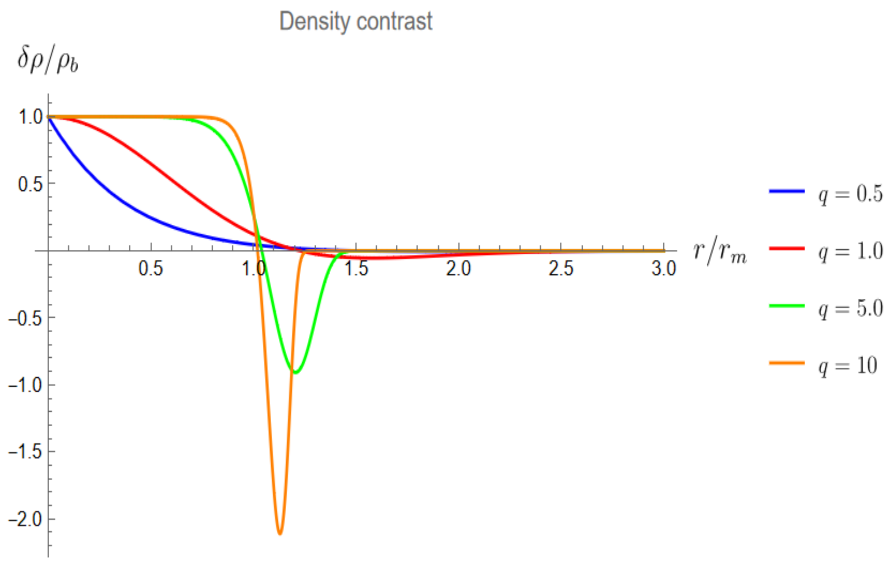
$$\frac{\delta\rho}{\rho_b}(r) = \frac{3(1+w)}{5+3w}\left(\frac{1}{aH}\right)^2 \left[K(r) + \frac{r}{3} K'(r)\right]$$

Gaussian profile: q=1

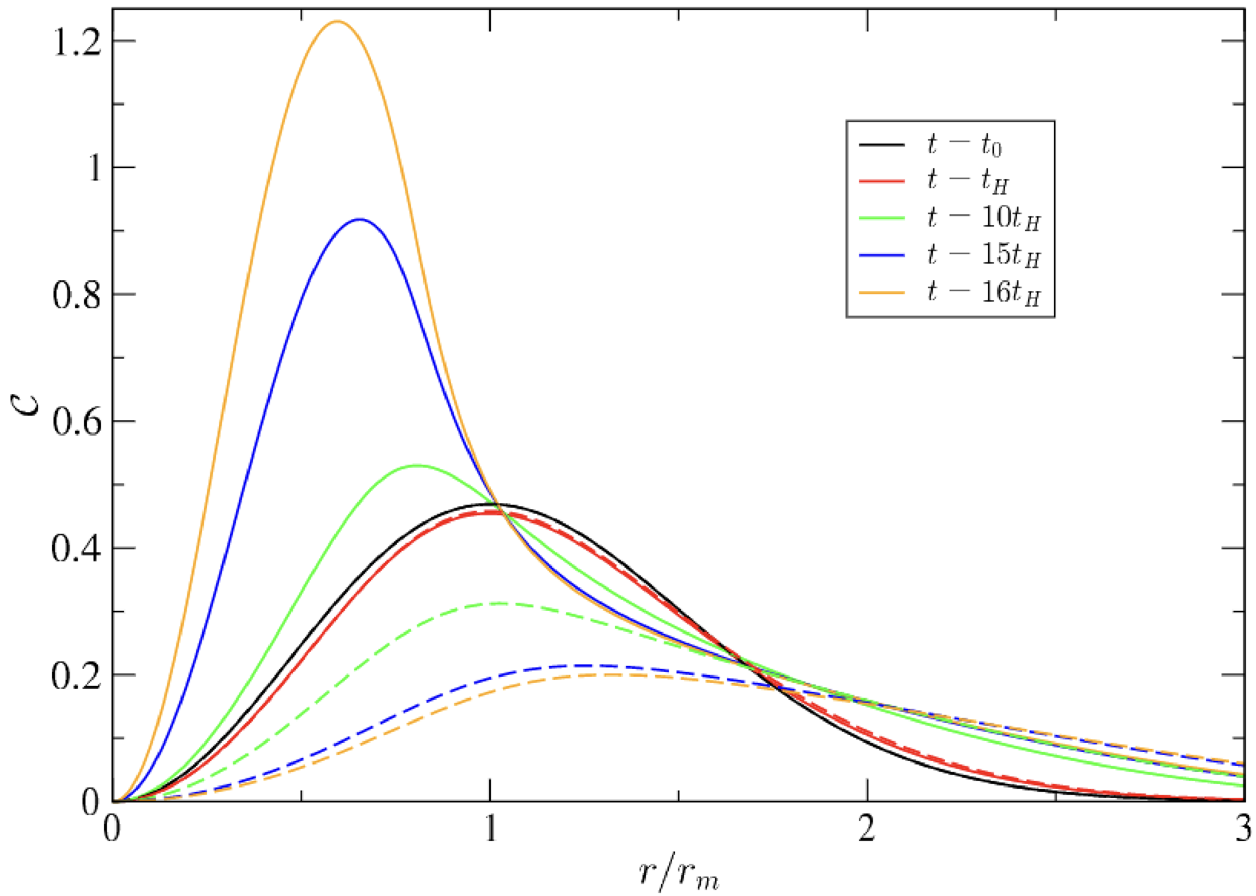
$$q = -\frac{r_m^2 C''(r_m)}{4C(r_m)}$$

Escrivà, Germani , K. Sheth
Arxiv:1907,13311





Example of the simulations: Gaussian profile



$\delta_m \sim 0.47 \quad || \quad M_H(t_H) \sim 2.5 M_\odot$

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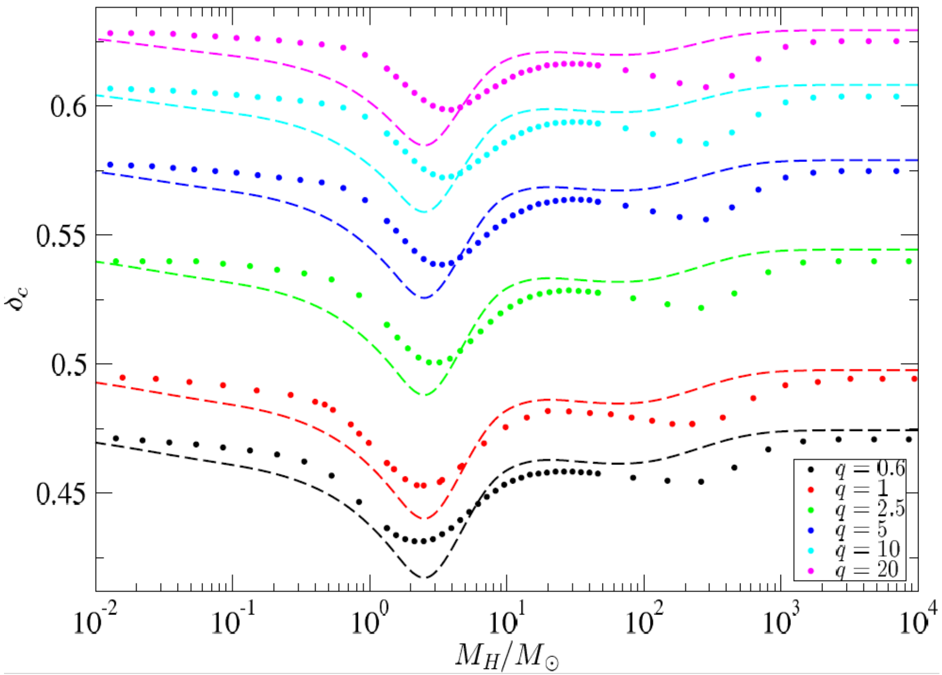
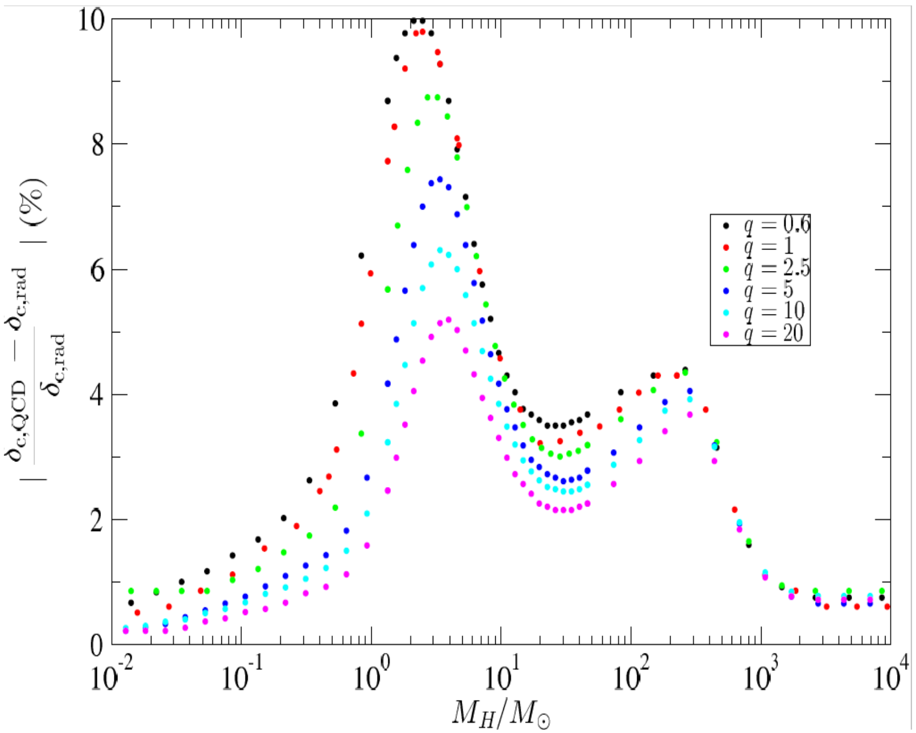
The Thresholds





The minimum threshold is located around: $[1-4]\,\frac{M_H}{M_\odot}$

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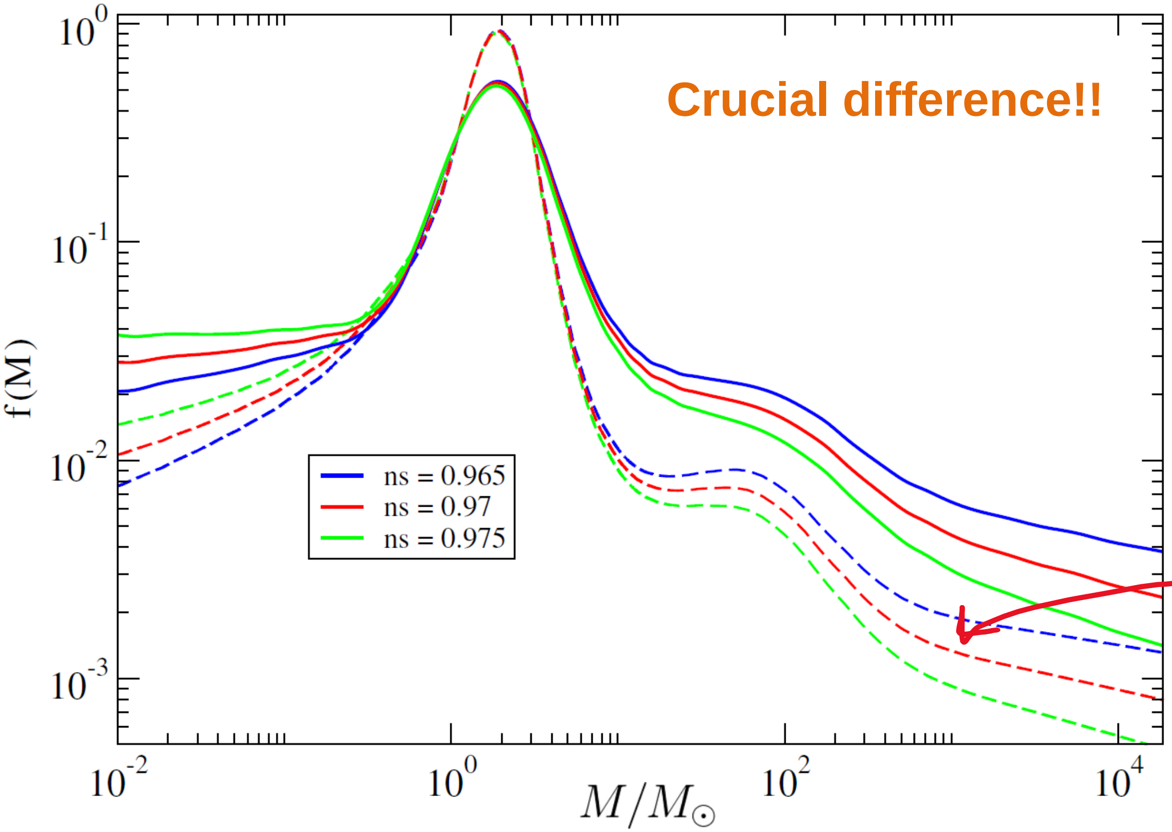
Crucial effect on the mass function (example)



Fraction of PBHs to be the dark matter

$$\beta(M) = \mathrm{erfc}\left(\frac{\delta_{\mathrm{c}}(\mathrm{M})}{\sqrt{2}\delta_{\mathrm{rms}}(\mathrm{M})}\right)$$

Carr , Clesse, Garcia-Bellido, Kuhnel.
Arxiv:1906.08217

Computation of merging rates, comparison with GWTC3 catalog

$$f(M) = \frac{1}{\rho_{\mathrm{DM}}}\frac{d\rho_{\mathrm{PBH}}}{d\ln(M)} \approx \frac{2.4}{f_{\mathrm{PBH}}}\beta(M)\sqrt{\frac{M}{M_{\mathrm{eq}}}}$$

Mass function!

Conclusions

- The QCD phase transition affects the process of PBH formation significantly compared with the radiation fluid

- The thresholds of PBH formation are clearly smaller in comparison with the standard case of radiation

- The QCD transition produce a peak in the mass function around two solar masses

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Thanks for your attention!

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