How Special are Black Holes?: Black Hole-Saturon Correspondence

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See talk by M. Zantedeschi

How Special Are Black Holes?

1. Their entropy satisfies the *area law*:

$$S \sim \frac{Area}{G_N} \sim \frac{Area}{M_p^{-2}}$$

2. They exhibit a (semiclassical) *information horizon.*

3. Decay rate is thermal and they have *temperature*

$$T \sim \frac{1}{R}$$

4. Time-scale required for beginning of the *information retrieval* is

$$t_{min} = SR \sim \frac{R^3}{M_p^{-2}} \sim \frac{Volume}{M_p^{-2}}$$

- S. W. Hawking, Commun. Math. Phys. 43, 199 (1975).
- D. N. Page, Phys. Rev. Lett. 71, 3743 (1993).

How Special Are Black Holes?

- Does the *area-law* entropy bound extend beyond gravity?
  - What is its underlying meaning?

$$S \leqslant \frac{Area}{G_{Gold}}$$

The entropy bound is imposed by unitarity

- G. Dvali, JHEP03(2021) 126, arXiv:2003.05546.
- G. Dvali, PTRS A, arXiv:2107.10616

Saturons

1. Their entropy satisfies the *area law*:

$$S \sim \frac{Area}{G_{Gold}} \sim \frac{Area}{f^{-2}}$$

2. They exhibit a (semiclassical) *information horizon.*

3. Decay rate is thermal and they have *temperature*

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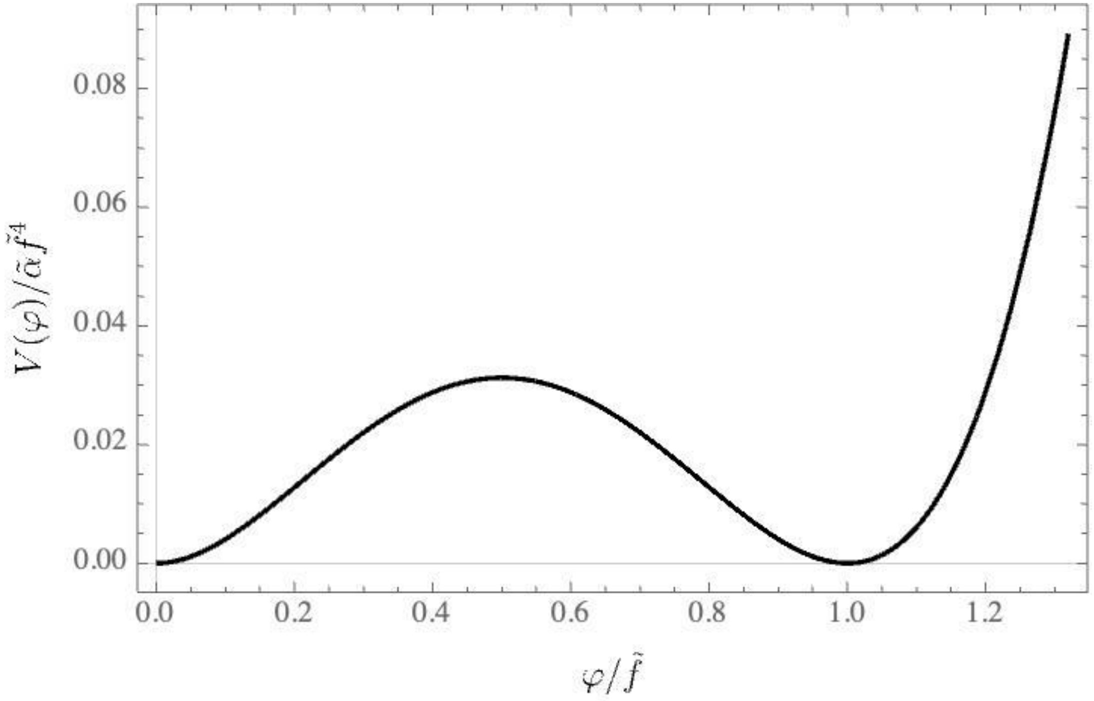
$G_{Gold} \equiv f^{-2}$ Goldstone Coupling

Model of a Saturon as a Vacuum Bubble

- $d = 4$
- ϕ in the *adjoint rep.* of $SU(N)$
- $N \gg 1$

$$\mathcal{L} = \frac{1}{2}\,\mathrm{tr}\left[(\partial_\mu \phi)(\partial^\mu \phi)\right] - V[\phi]$$
$$V[\phi] = \frac{\alpha}{2}\mathrm{tr}\left[\left(f\phi - \phi^2 + \frac{I}{N}\mathrm{tr}\,[\phi^2]\right)^2\right]$$

- *Unitarity requires:* $\alpha \leq \frac{1}{N}$



$$\varphi^2 \equiv \mathrm{tr}[\phi^2]$$
$$V(\varphi) \sim \frac{\alpha}{2}\varphi^2(f - \varphi)^2$$

Model of a Saturon as a Vacuum Bubble

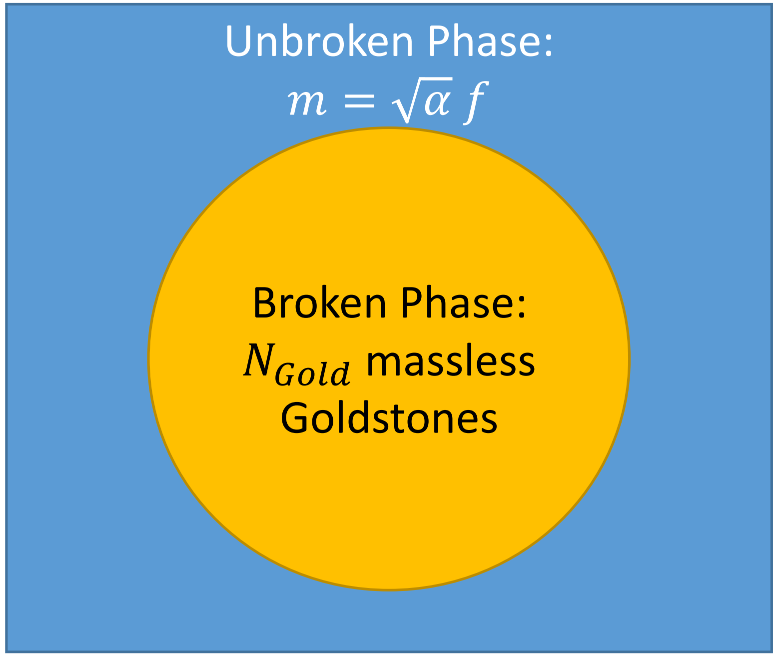
Vacuum Bubbles:

$$\Phi_{\mathrm{D}} = \frac{\varphi(r)}{\sqrt{N(N-1)}} \mathrm{diag}\left((N-1), -1, \dots, -1\right)$$

$$\phi = U^{\dagger} \Phi_{\mathrm{D}} U$$

- $U = \exp[-i\theta T]$,
- T corresponds to the broken generators

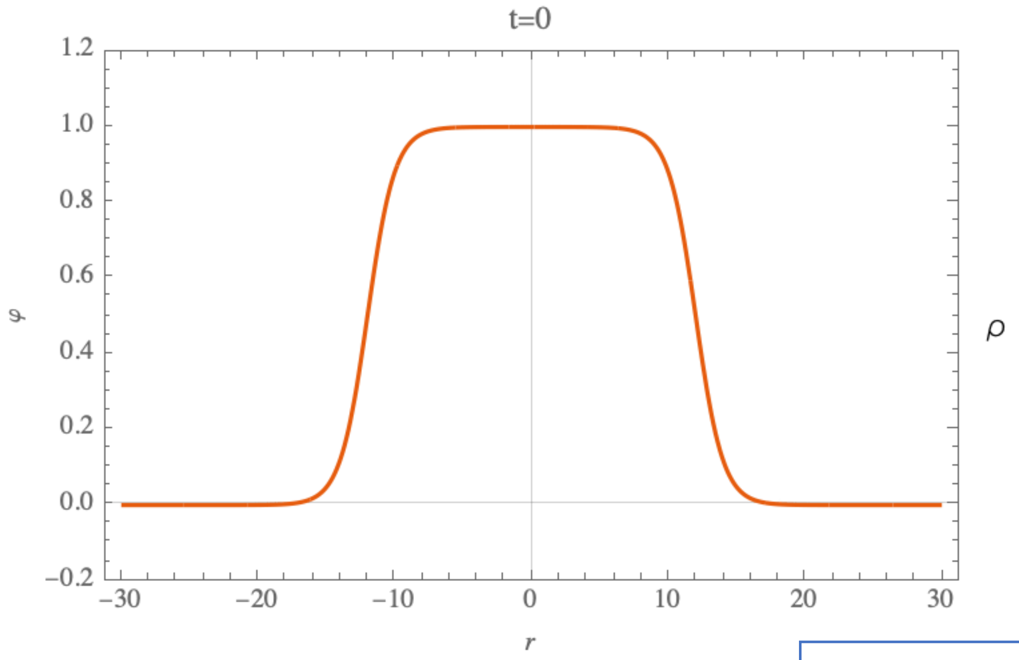
$$\theta = \omega t$$



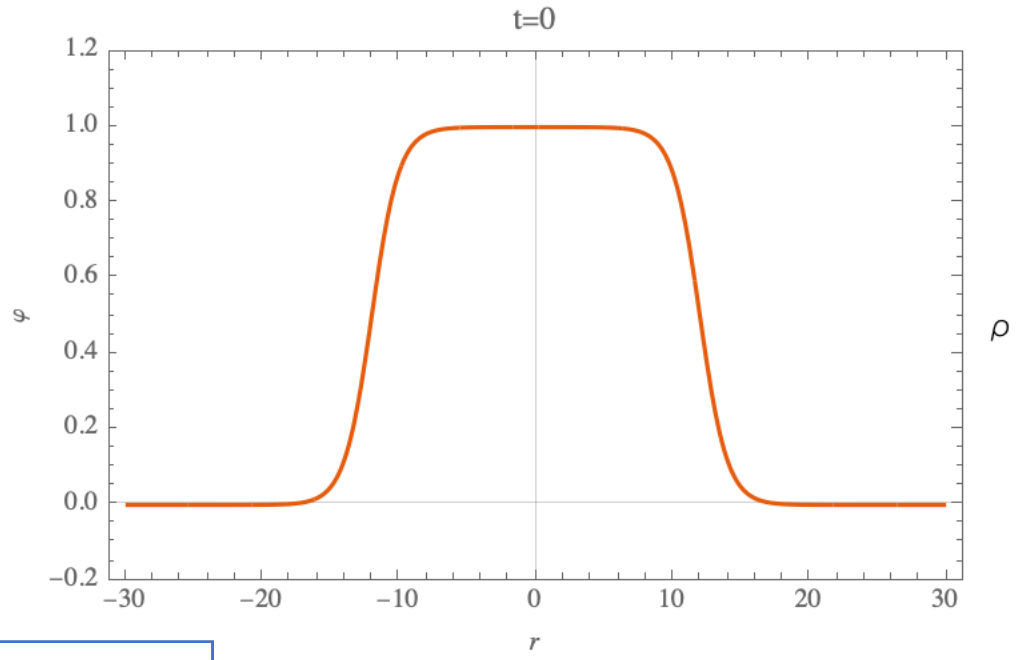
$$SU(N) \rightarrow SU(N-1) \times U(1),$$

Vacuum Bubbles Stabilization:

$\dot{\boldsymbol{\theta}} = \mathbf{0}$



$\dot{\boldsymbol{\theta}} = \boldsymbol{\omega}$



Thin wall approximation for:
$\omega \approx 0.24\, m,$
$R_\omega \sim \frac{m}{\omega^2} \approx \frac{12}{m}$

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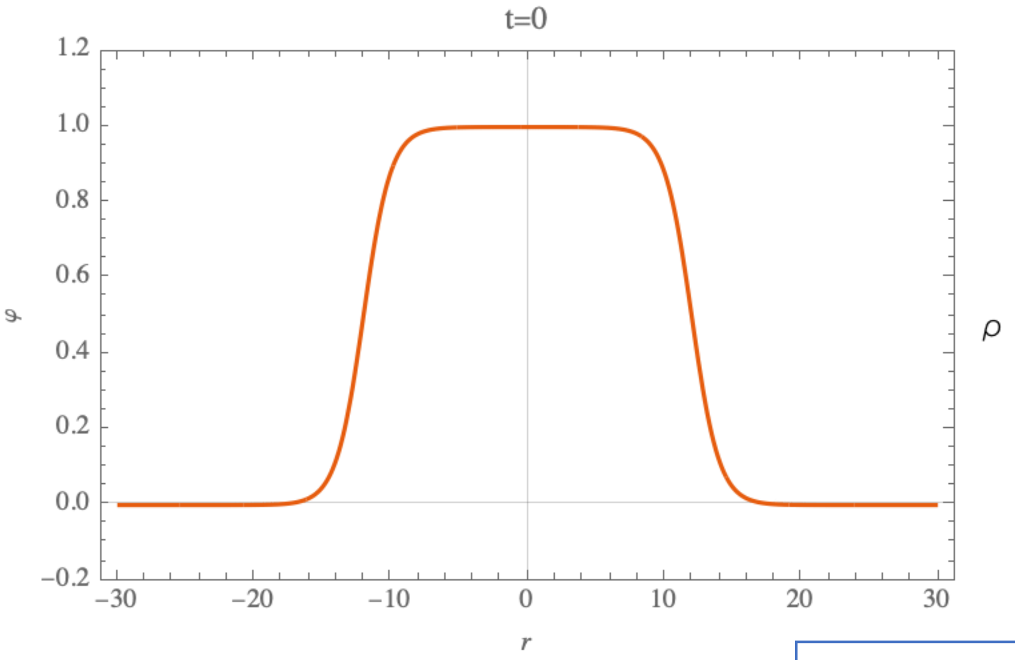

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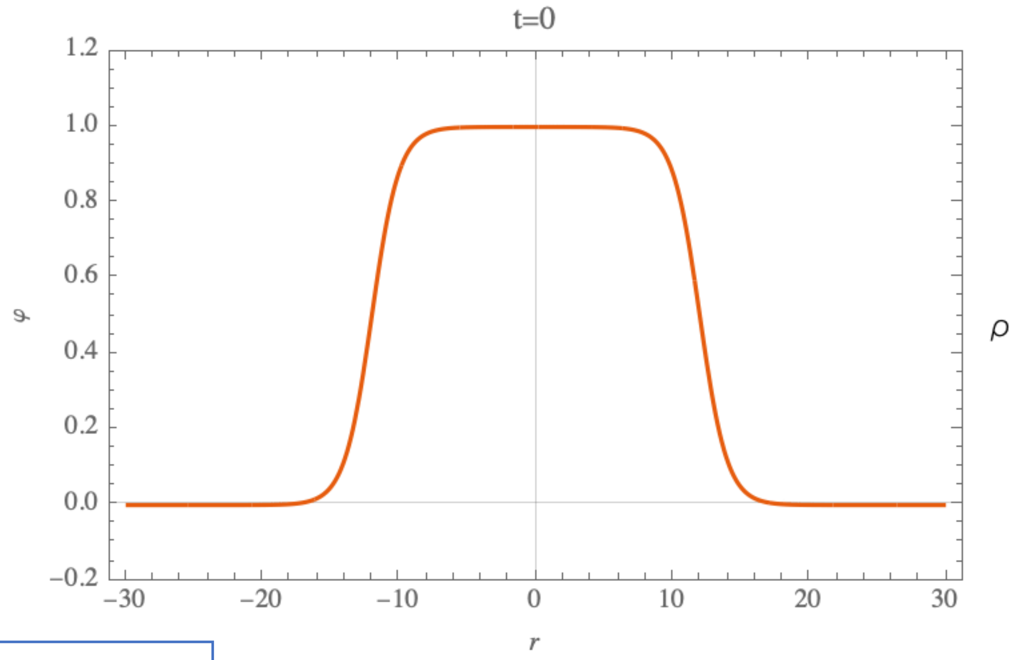

Thin wall approximation for:

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Vacuum Bubbles Microstates

- $N_G = E_{int}/\omega$ is the total mean *occupation number*.
- $N_{Gold} = 2(N-1)$ Goldstone modes (flavors).
- N_G can be arbitrarily *redistributed* among the N_{Gold} modes.

$$\sum_{a=1}^{N_{Gold}} n^a = N_G$$

- Each sequence represents a *memory pattern*

$$|Pattern\rangle = |\, n^1, n^2, \dots \rangle$$

- The number of degenerate micro-states, n_{st}, is the number of *patterns* satisfying the *constraint* above, and the ***entropy*** is

$$S = \ln n_{st} \approx 2N \ln\left[\left(1+\frac{2N}{N_G}\right)^{\frac{N_G}{2N}}\left(1+\frac{N_G}{2N}\right)\right]$$

Vacuum Bubbles Stabilization: Quantum picture of classical stability

- *A stationary bubble is obtained thanks to the excitations of the Goldstone mode(s).*
- The bubble is stable because of two factors:
  1) The fact that the Goldstone $SU(N)$ charge is conserved; and
  2) The fact that the same amount of charge in the exterior vacuum would cost higher energy.

- G. Dvali, JHEP03 (2021) 126, arXiv:2003.05546.
- G. Dvali, arXiv:1810.02336.
- G. Dvali, L. Eisemann, M. Michel, and S. Zell, JCAP03 (2019) 010, arXiv:1812.08749.

Saturon as a Vacuum Bubble

- Thick wall Bubbles correspond to Saturons

$$\omega \sim m$$

$$m^{-1} \sim R$$

$$N_G \sim \frac{1}{\alpha} \sim N \sim N_{Gold}$$

$$\mathrm{S} \sim S_{max} \sim \frac{1}{\alpha} \sim E_{\mathrm{Bubble}} R$$

- J. D. Bekenstein Phys. Rev. D23no. 2 (1981), 287-298.47

Information Horizon

Saturons in semiclassical limit

Semi-classical Limit

- The limit in which the classical bubble solution experiences no back reaction from quantum fluctuations

$$\alpha \to 0, \qquad R = \text{finite}, \qquad \omega = \text{finite}, \qquad \alpha N = \text{finite}$$

- Simultaneously

$$f \to \infty, \qquad m = \text{finite}, \qquad N \to \infty$$

- In this limit, saturons possess a strict *information horizon*.
- *Recall: For BH* $f \sim M_p$

Semi-classical Limit

- In Semi-classical limit, the effective coupling of a Goldstone mode of frequency ε

$$\alpha_G = \frac{\varepsilon^2}{f^2} \rightarrow 0$$

- **Even at finite $\boldsymbol{f}$**, and $\varepsilon \ll m$, the mode cannot propagate outside the bubble:
  1. The energy ε is such propagation is impossible due to the finite energy gap.

  2. $\varepsilon \ll m$, the perturbation energy can exceed the mass gap at the expense of a large occupation number n_ε. But the process

     $$n_\varepsilon \rightarrow 1,$$

     is exponentially suppressed by a factor $\mathrm{e}^{-n_\varepsilon}$

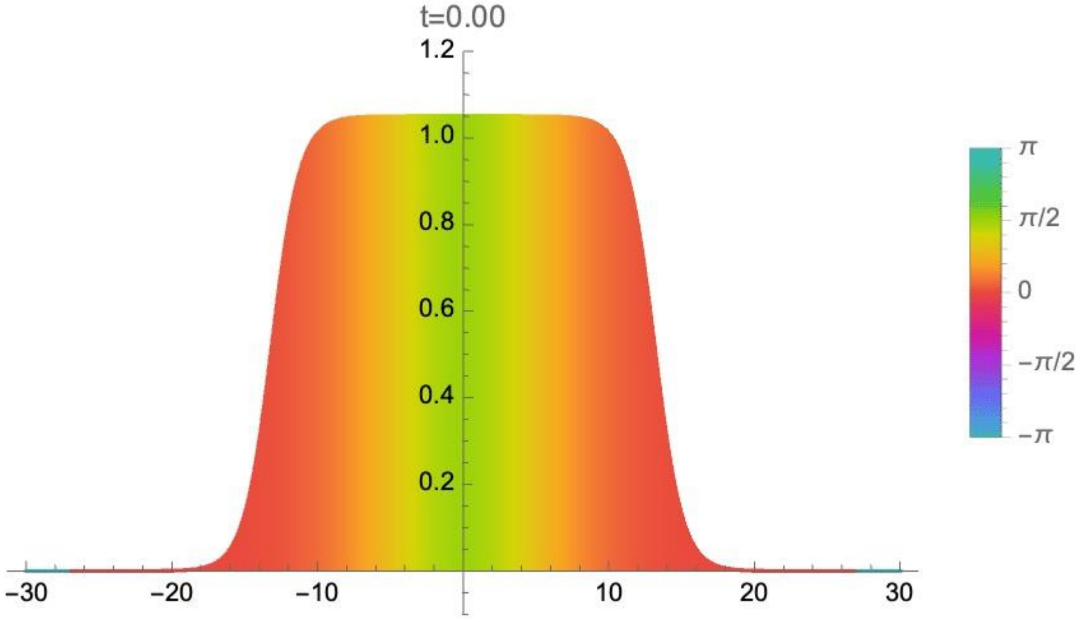
- G. Dvali, PTRS A, arXiv:2107.10616

Goldstone Horizon: An Example

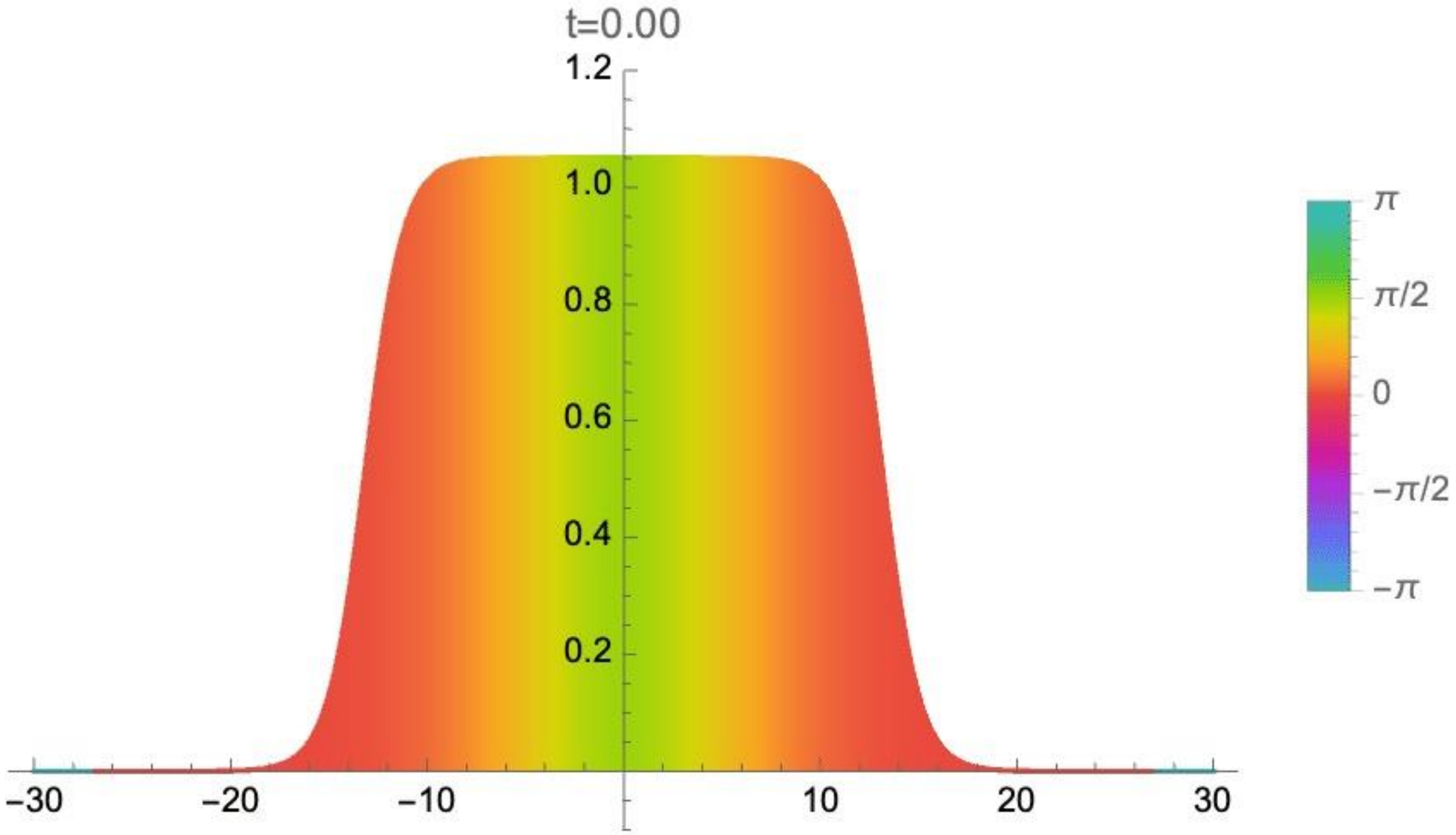
- Lets consider a perturbation on a stable vacuum bubble, ϕ_{VB},

$$\phi\Big|_{t=0} = P^{\dagger}(r)\phi_{VB}(r)P(r)$$

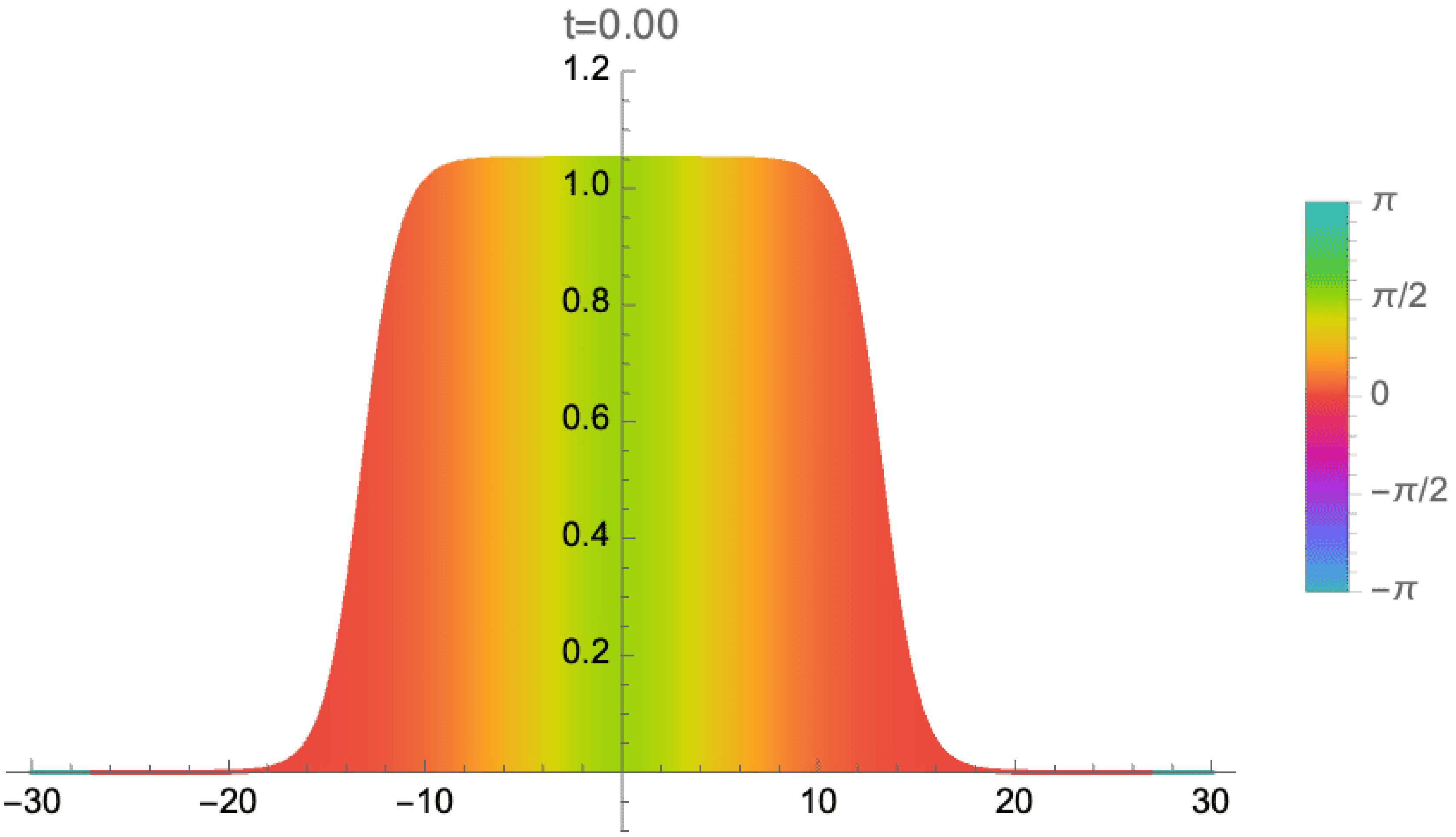
$$P(r) = \exp\left[\frac{i\pi}{2}\mathrm{e}^{-\frac{r^2}{2r_0^2}}T\right]$$



Goldstone Horizon: An Example



Goldstone Horizon: An Example



Correspondence to Black Holes

Correspondence to Black Holes

Saturons

- $S = (fR)^2 = \alpha^{-1}$
- $T = R^{-1}$
- $t_{\min} = R^3\, f^2 = SR$
- Information/Goldstone Horizon

Black Holes

- $S = (M_P R)^2$
- $T = R^{-1}$
- $t_{\min} = R^3\, M_P^2 = SR$
- Information Horizon

Conclusions and outlook

- **Black Holes are certainly special but not unique.** They belong to a larger class of objects: Saturons, which saturate the entropy bound.
- We have shown an explicit example of a Saturon as a Vacuum Bubble.
- Vacuum Bubbles exhibit a **goldstone horizon**, analog to the information horizon of saturons.

- G. Dvali, JHEP03(2021) 126, arXiv:2003.05546, arXiv:2107.10616
- G. Dvali and O. Sakhelashvili, PRD 105 (2022) 6, 065014arXiv:2111.03620.
- G. Dvali, R. Venugopalan, PRD 105 (2022) 5, 056026

Conclusions and outlook

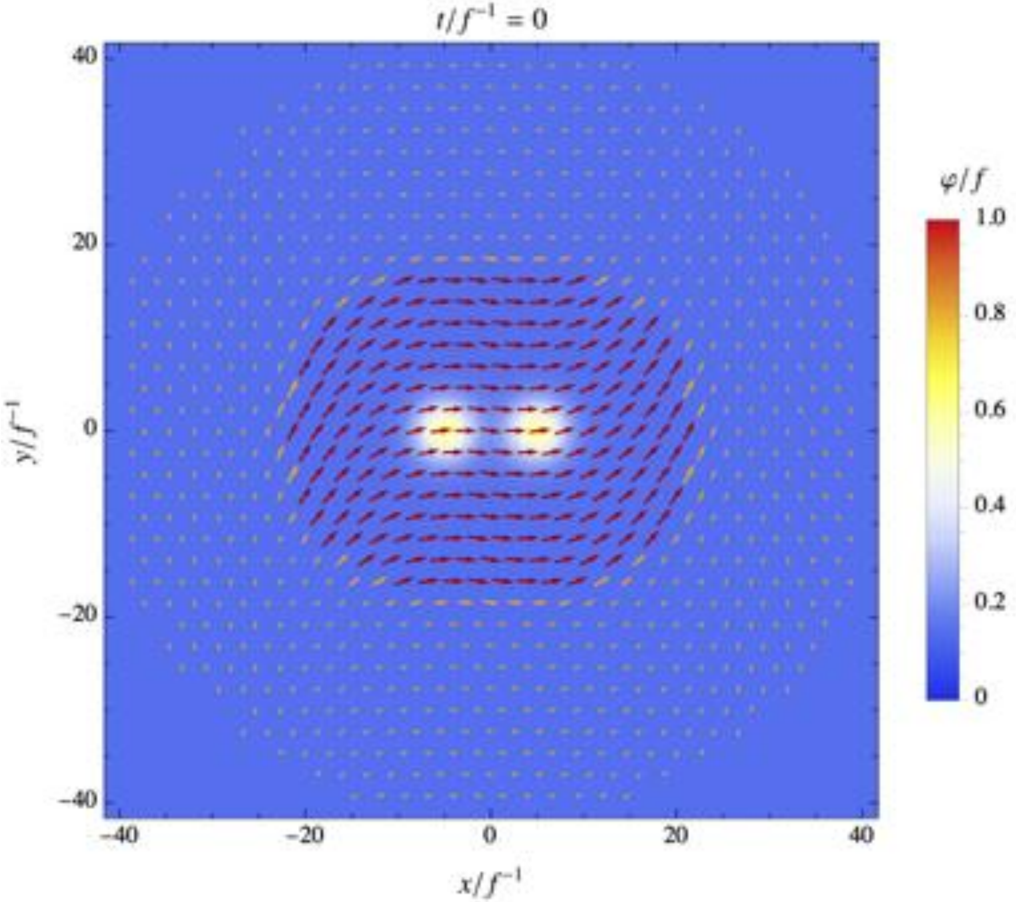
- Departures from semi-classical behavior can become observable for BH that are relatively old and close to their half-decay time.

- The light *Primordial Black Holes*, provided they exist, can be within a potentially interesting window.

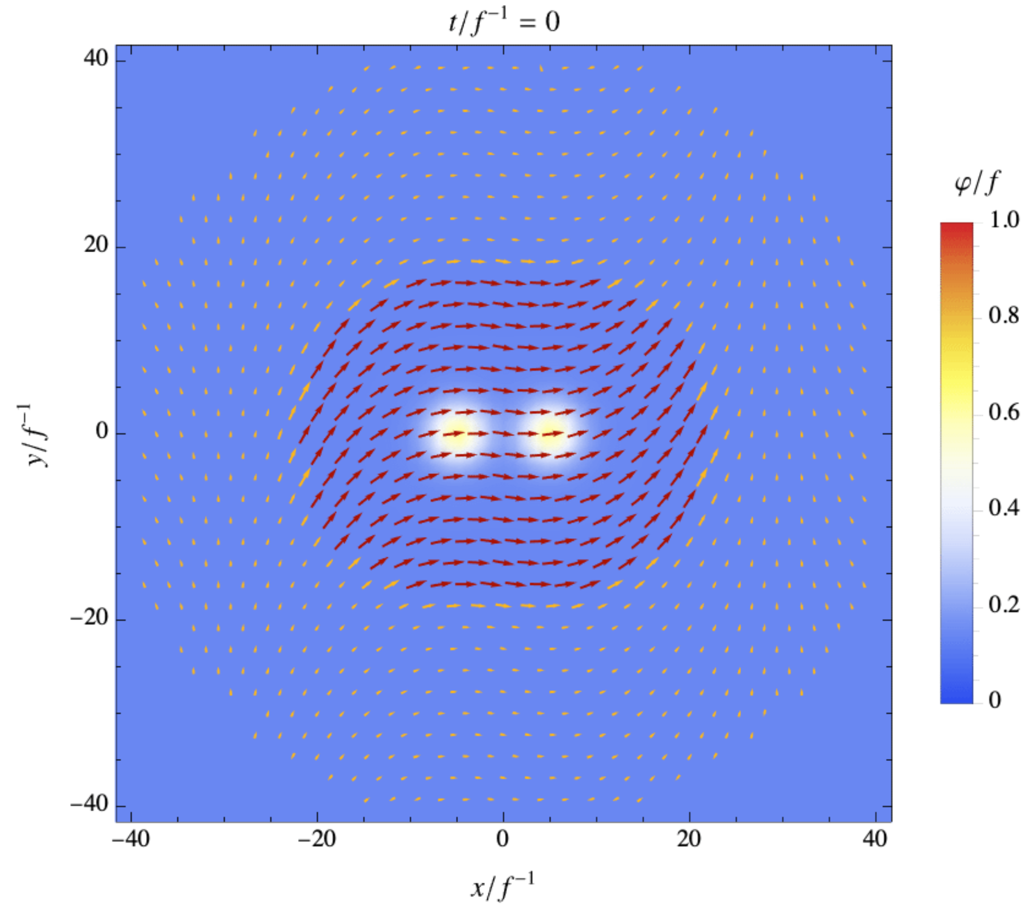
- Possible observational consequences for rotating black holes.

- G. Dvali, F. Kühnel and M. Zantedeschi, hep-th/2112.08354.

Outlook



Outlook

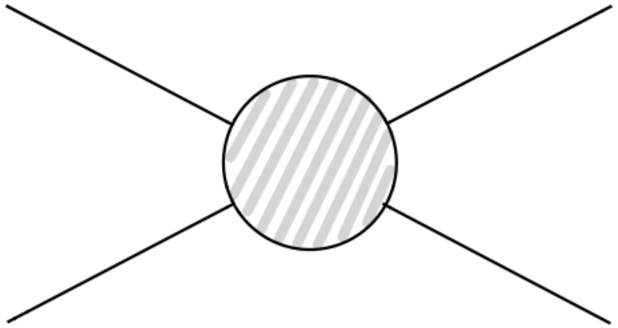


Thank you

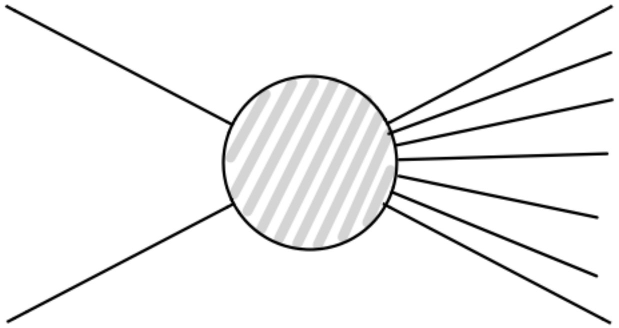
Entropy Bound

Entropy Bound Imposed by Unitarity

Cross-section:
$\boldsymbol{\sigma_{2\to 2} \sim \alpha}$



Cross–section:
$\boldsymbol{\sigma_{2\to N}\, n_{st} \sim \alpha^N N!\, n_{st}}$



The *bound* reads

$$S \leqslant \frac{1}{\alpha} \sim N$$

For self-sustained objects of size R, $\alpha = \alpha(q)$ is as an *effective running coupling* evaluated at the scale $q \sim 1/R$, and $\alpha N \sim 1$

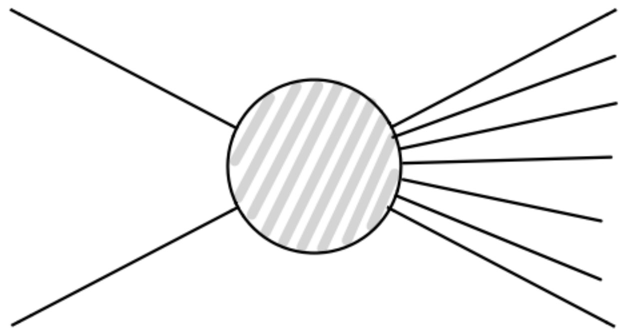
G. Dvali, JHEP03(2021) 126, arXiv:2003.05546.
G. Dvali, PTRS A, arXiv:2107.10616

Saturation of Unitarity

If $N \sim \frac{1}{\alpha} \gg 1$

$$\boldsymbol{\sigma_{2\to N}\, n_{st} \sim \alpha^N N!\, n_{st}}$$

$$\sigma_{2\to N}\, n_{st} \sim \alpha^N \left(\frac{1}{\alpha}\right)^N e^{-\frac{1}{\alpha}}\, e^S$$



- Exponential suppression of high occupancy state (*classical lumps*), unless

$$S \sim \frac{1}{\alpha} \sim N$$

Entropy Bound Imposed by Unitarity

- E.g. consider bound states of Goldstone bosons of de Broglie wavelength R

$$\alpha = \frac{q^2}{f^2} = \frac{1}{(fR)^2}$$

f is the canonically normalized *Goldstone decay constant*.

- Thus

$$S \leqslant \frac{1}{\alpha} = \frac{Area}{G_{Gold}}$$

- $Area \sim R^2$
- $G_{Gold} \equiv f^2$ is the Goldstone coupling

Saturons

- We refer to the objects saturating the entropy bounds as ***Saturons***.
- Different saturons are discussed in [1-3]. These include:
  - ***Black Holes***,
  - ***Vacuum Bubbles.***
  - *Classical lumps, Instantons, Monopoles…*
  - *Color Glass Condensates*
- *Area law*:

$$S_{\mathrm{Max}} = \frac{1}{\alpha} = \frac{Area}{f^{-2}}$$

- *Temperature:*

$$T \sim \frac{1}{R},$$

- *Information retrieval*

$$t_{min} = \frac{Volume}{G_{Gold}} = \frac{R}{\alpha} = S_{\mathrm{max}}$$

- *Information horizon.*

1. G. Dvali, JHEP03(2021) 126, arXiv:2003.05546, arXiv:2107.10616
2. G. Dvali and O. Sakhelashvili, PRD 105 (2022) 6, 065014arXiv:2111.03620.
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Vacuum Bubbles Microstates

Towards Entropy Saturation

Vacuum Bubbles Stabilization: Quantum picture of classical stability

$$E_{\text{Bubble}} = E_{\text{int}} + E_{\text{wall}} = \frac{\omega}{\alpha}\frac{m^5}{\omega^5}\left(\frac{40\pi}{81}\right),$$

- In terms of the *occupation numbers* of the corresponding quanta, the energies are:

$$E_{\text{int}}=\omega N_G, \quad \text{where,} \quad N_G \equiv \frac{1}{\alpha}\frac{m^5}{\omega^5}\left(\frac{16\pi}{81}\right),$$

$$E_{\text{wall}}=m N_\varphi, \quad \text{where,} \quad N_\varphi \equiv \frac{1}{\alpha}\frac{m^4}{\omega^4}\left(\frac{8\pi}{27}\right).$$

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- G. Dvali, Entropy Bound and Unitarity of Scattering Amplitudes, JHEP03 (2021) 126, arXiv:2003.05546.
- G. Dvali, A Microscopic Model of Holography: Survival by the Burden of Memory, arXiv:1810.02336.
- G. Dvali, L. Eisemann, M. Michel, and S. Zell, Universe’s Primordial Quantum Memories, JCAP03 (2019) 010, arXiv:1812.08749.

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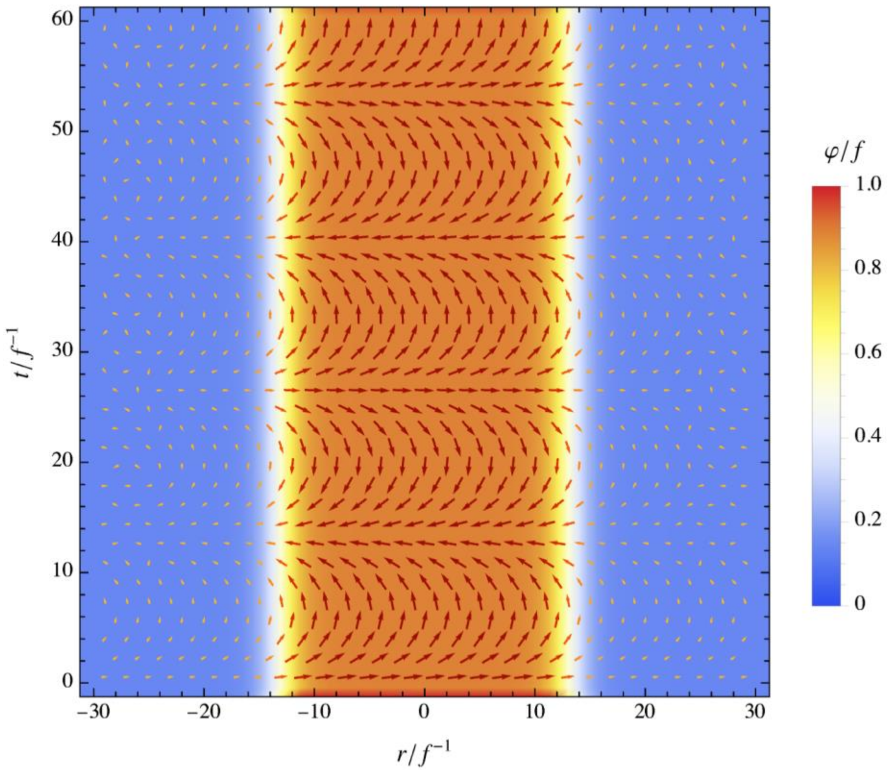
- **Memory Burden Effect**
  - Large amount of Memory patterns
  - Stored quantum information.
  - Slowdown of the system’s evolution
- **Vaccum Bubbles Stabilization**
  - Large amount of Bubble micro-states
  - Excitations of the Goldstone modes
  - Slowdown of bubble's decay

Vacuum Bubbles Stabilization

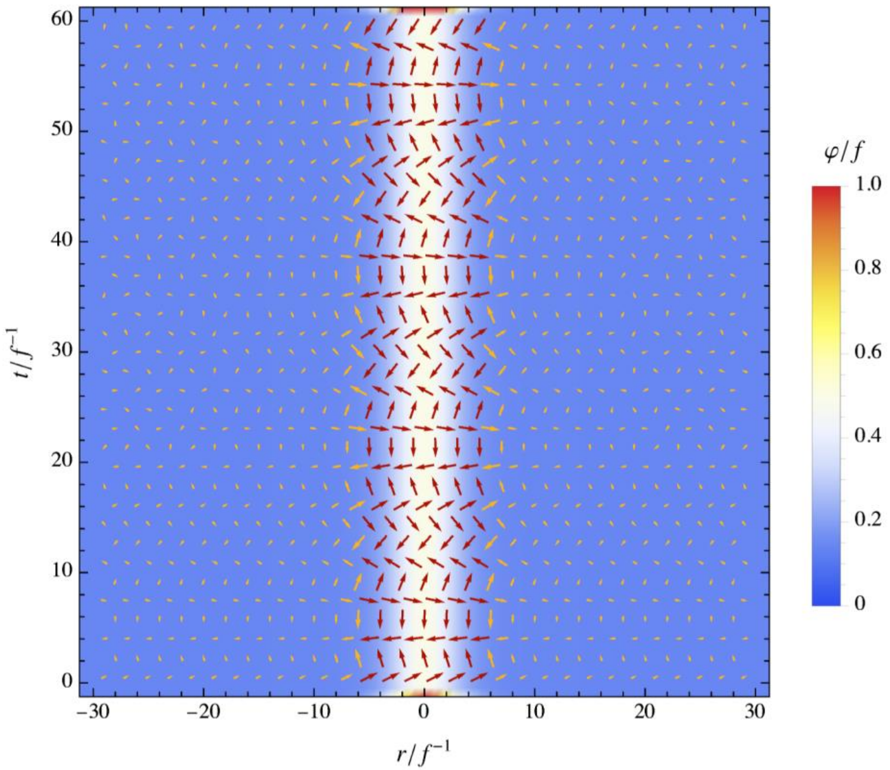
Large Bubbles

$R \sim 12m^{-1}$



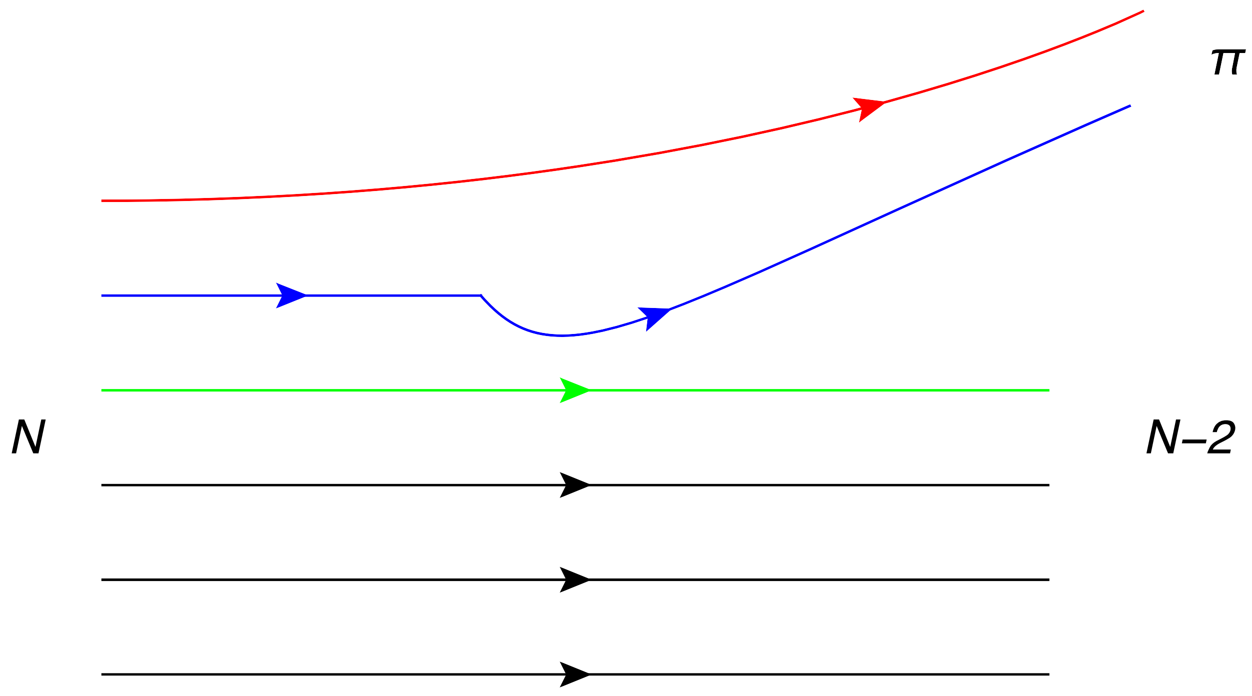
Small Bubbles

$R \sim 1.02m^{-1}$

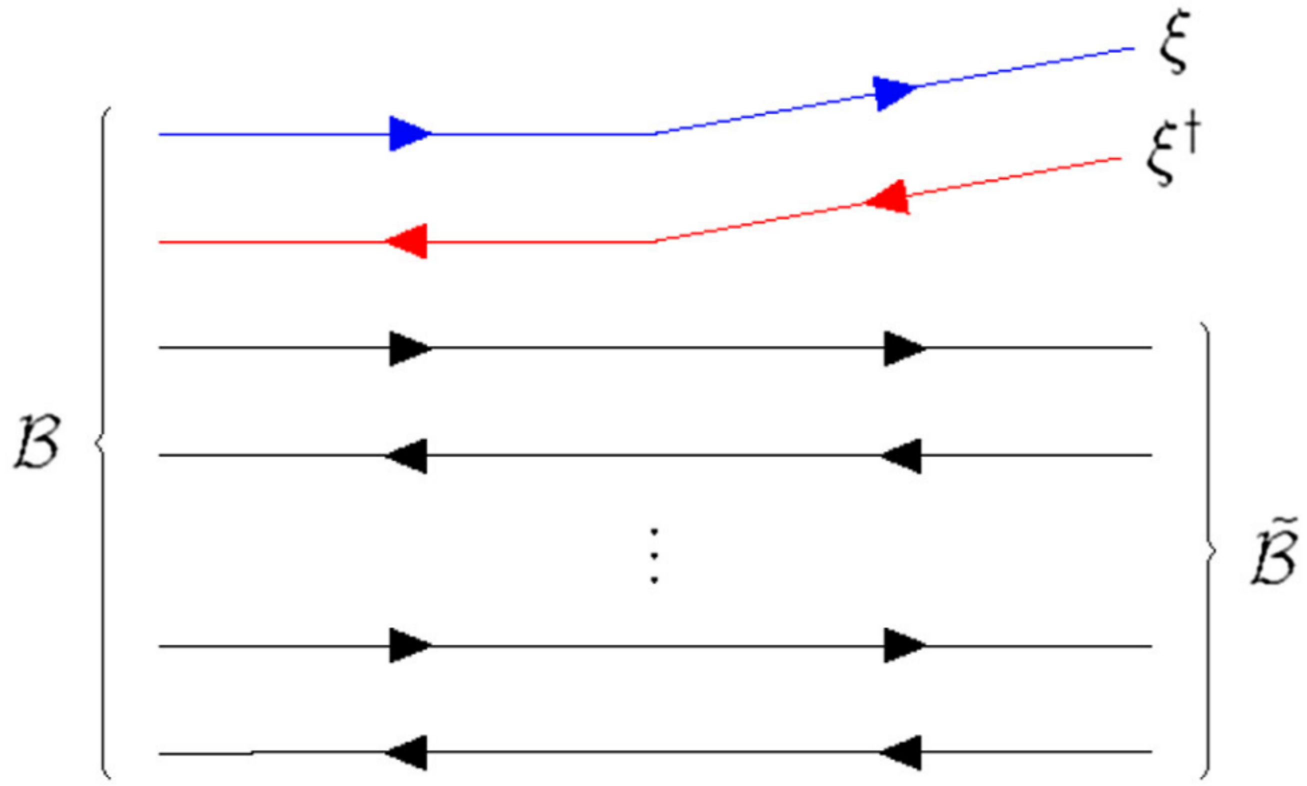


Hawking Evaporation

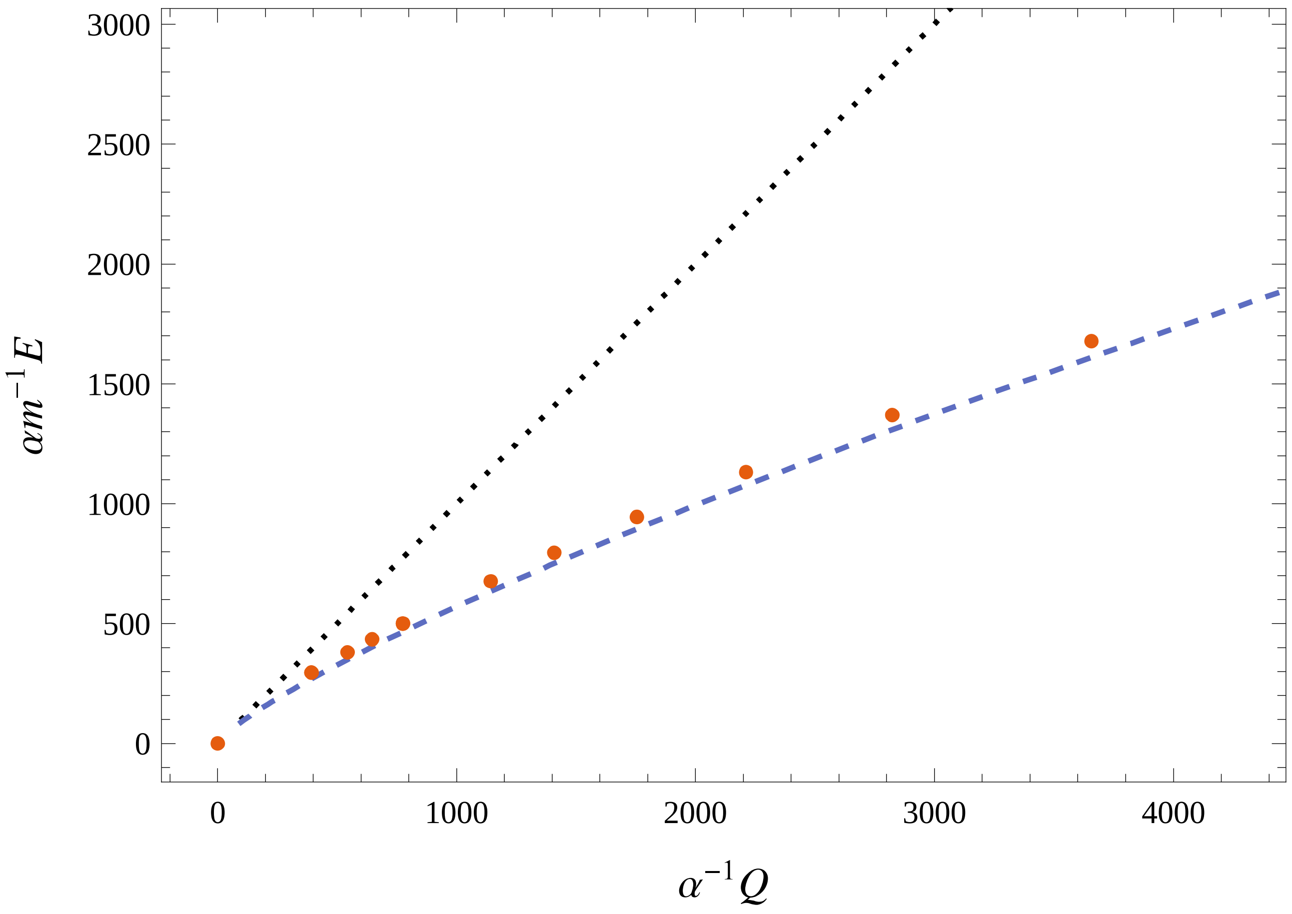
Hawking Evaporation: Temperature



Hawking Evaporation: Temperature

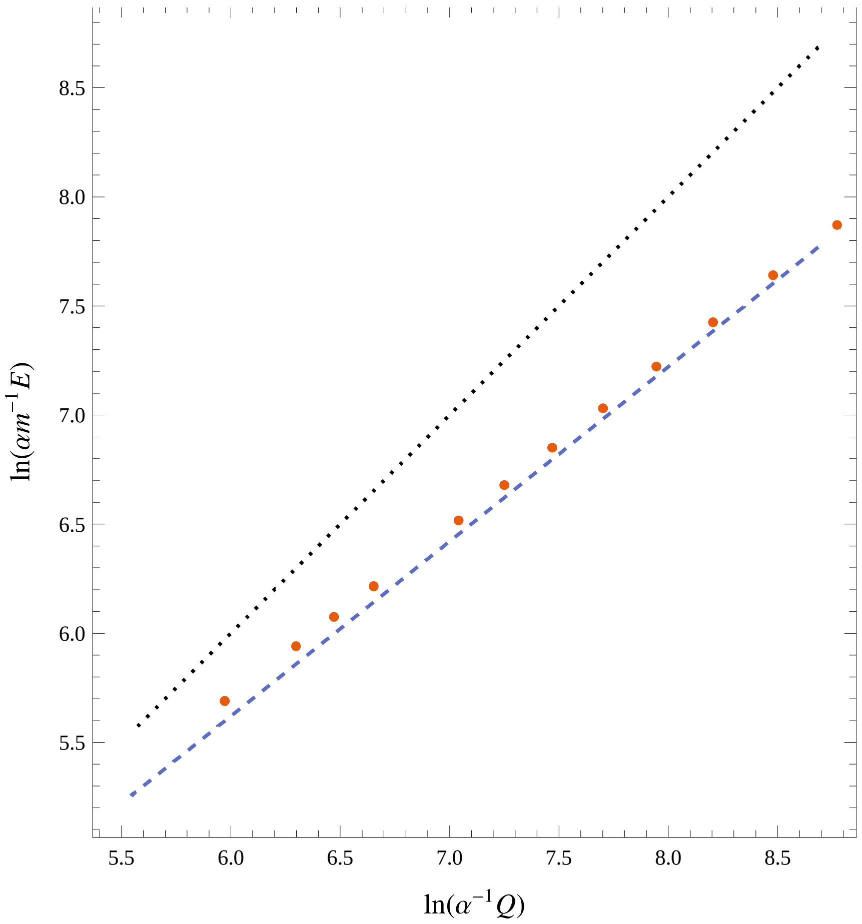


Spectrum



3000
2500
2000
1500
1000
500
0
0
1000
2000
3000
4000
$\alpha m^{-1} E$
$\alpha^{-1} Q$

Spectrum



1+1 Dim