Non-Gaussian tails from multifield inflation

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Inflation

- A useful way of thinking about inflation

Quasi de Sitter epoch

Broken time translations

$$ds^2 = -dt^2 + e^{Ht} e^{-2\zeta} d\vec{x}^2$$

- Perturbations around the background are very close to Gaussian

- At CMB scales $\Delta_\zeta^2 \sim 10^{-9}$

- $$\frac{\langle \zeta^n \rangle}{\langle \zeta^2 \rangle^{n/2}} \ll 1$$

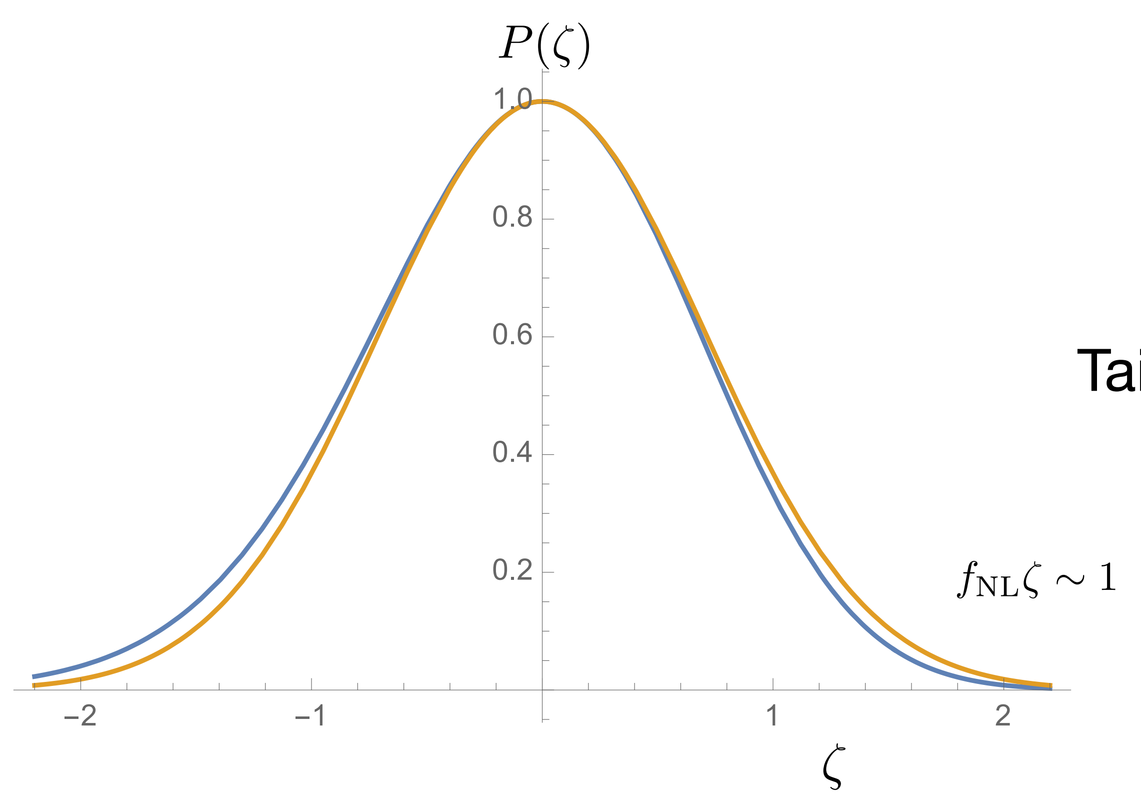
Gaussian approximation

$$P(\zeta) \sim \exp\left[-\frac{\zeta^2}{2\sigma_\zeta^2}\left(1 + f_{\rm NL}\zeta + g_{\rm NL}\zeta^2 + \cdots\right)\right]$$

$$\sigma_\zeta^2 = \int d^3k P_\zeta(k)$$

$$f_{NL}\zeta \sim \frac{\langle\zeta\zeta\zeta\rangle}{\Delta_\zeta^2}\zeta \quad \ll 1$$

$$g_{NL}\zeta \sim \frac{\langle\zeta\zeta\zeta\zeta\rangle}{\Delta_\zeta^3}\zeta^2 \quad \ll 1$$



Tails cannot be computed using perturbation theory

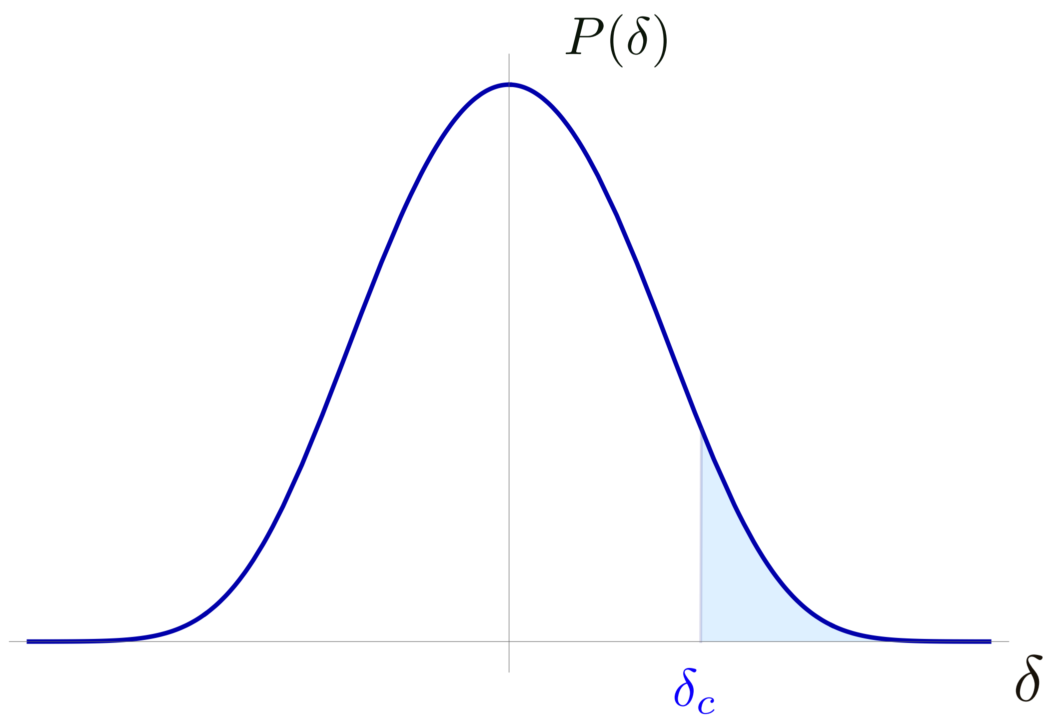
Tails

- Information about tails can be important
    - PBHs,
    - Eternal inflation
    - Galaxy formation

- Tails have been obtained in several contexts
    - EFT of inflation **Celoria *et al* '21**
    - Ultraslow roll **Ezquiaga *et al* '19** **Figueroa *et al* '19** **Pattison *et al* '21**
    - Multified inflation **Panagopoulos and Silverstein '19** **Achucarro, SC *et al* '21**

Primordial black holes

- Black holes formed during radiation (also matter) domination due to large amplitude fluctuations of the curvature field



$$\beta(M_{\rm PBH}) = \frac{\rho(M_{\rm PBH})}{\rho_{\rm tot}} = \int_{\delta_c}^{\infty} P(\delta) d\delta$$

All DM to be formed by PBHs requires

$$\Delta_\zeta^2 \sim 10^{-2} - 10^{-3}$$

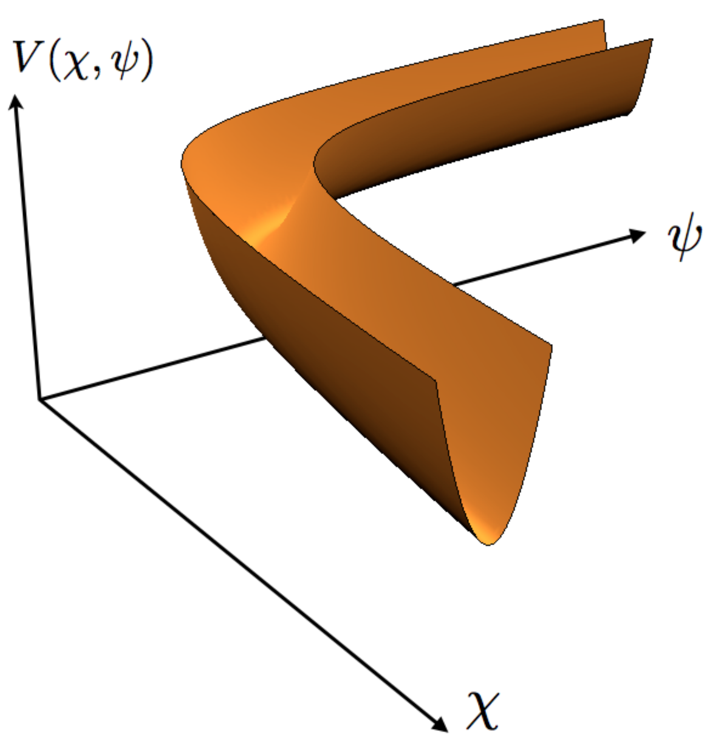
- Formation can be explained by several inflationary mechanisms (USR, multifield, tachyonic instabilities)

Multifield inflation

Multifield inflation

- Considering the simple setup.

$$S = \frac{\mathrm{Mpl}^2}{2}\int d^4x\sqrt{-g}R - \frac{1}{2}\int d^4x\sqrt{-g}\left[\gamma_{ab}(\phi)\partial\phi^a\partial\phi^b + V(\phi)\right]$$



Using background solution $T^a = \frac{\dot{\phi}_0^a}{\dot{\phi}_0}$ N_a

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + T^aV_a = 0$$

Slow roll evolution

$$\epsilon \equiv -\frac{1}{M_{\mathrm{pl}}^2}\frac{\dot{\phi}_0^2}{2H^2} \ll 1, \ldots$$

Gordons et al ’00
Groot Nibelink and Van Tent ‘00

$$\Omega \equiv N^a\nabla_aV/\dot{\phi}_0$$

Measure deviations from geodesic

Perturbations of two fields systems

- We can study perturbations by introducing a Goldstone boson

- Changing to the curvature field

$$S = \frac{1}{2}\int d^4x a^3 \left[f_\zeta^2 \left(\dot{\zeta} - \frac{2\Omega}{f_\zeta}\psi \right)^2 - \frac{f_\zeta}{a^2}(\nabla\zeta)^2 + \dot{\psi}^2 + \frac{1}{a^2}(\nabla\psi)^2 + \mu^2\psi^2 + \dots \right]$$

$\Omega \neq 0$ Deviations from geodesic trajectory

Entropy mass

$$\mu^2 = m^2 + 4\Omega^2$$

$$f_\zeta^2 \equiv 2M_{\rm pl}^2 |\dot{H}|/H^2 = 2\epsilon M_{\rm pl}^2$$

$$f_\zeta^2 \gg H^2$$

Light Entropy mass

- For $\mu^2 \ll H^2$
  - In general interactions imply that curvature perturbations keep growing after leaving the horizon
  - When $\mu^2 = 0$ ultralight field. Curvature perturbations grow until the end of inflation. Suppressed non-Gaussianity.

Achucarro *et al* '16

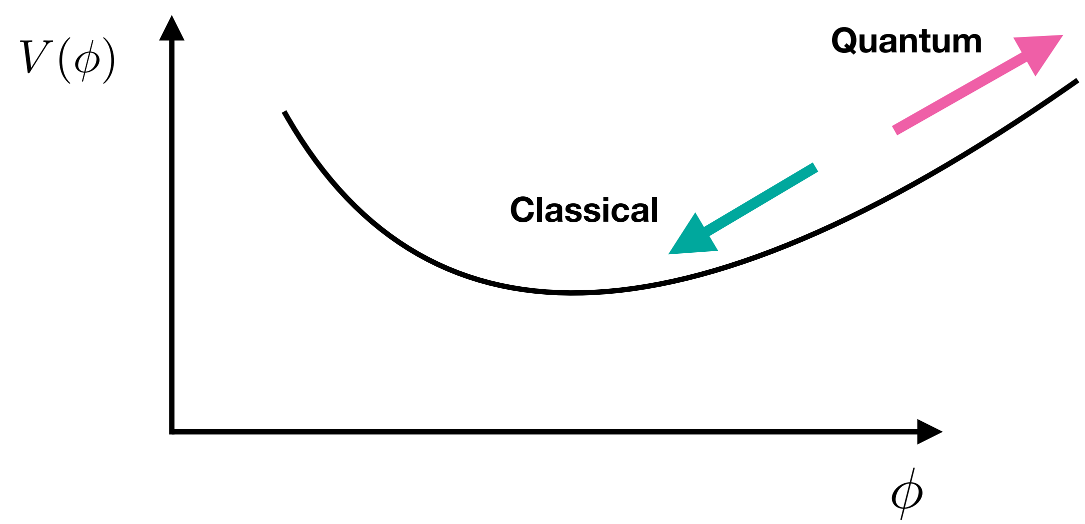
  - If $\mu^2 < 0$ tachyonic instability enhances exponentially the curvature mode

Reneaux-Patel and Turzinky '15
Linde and Kallosh '16
Brown '17
Ballesteros, SC, Santoni '21

PBH formation

Stochastic inflation

Classical vs Quantum



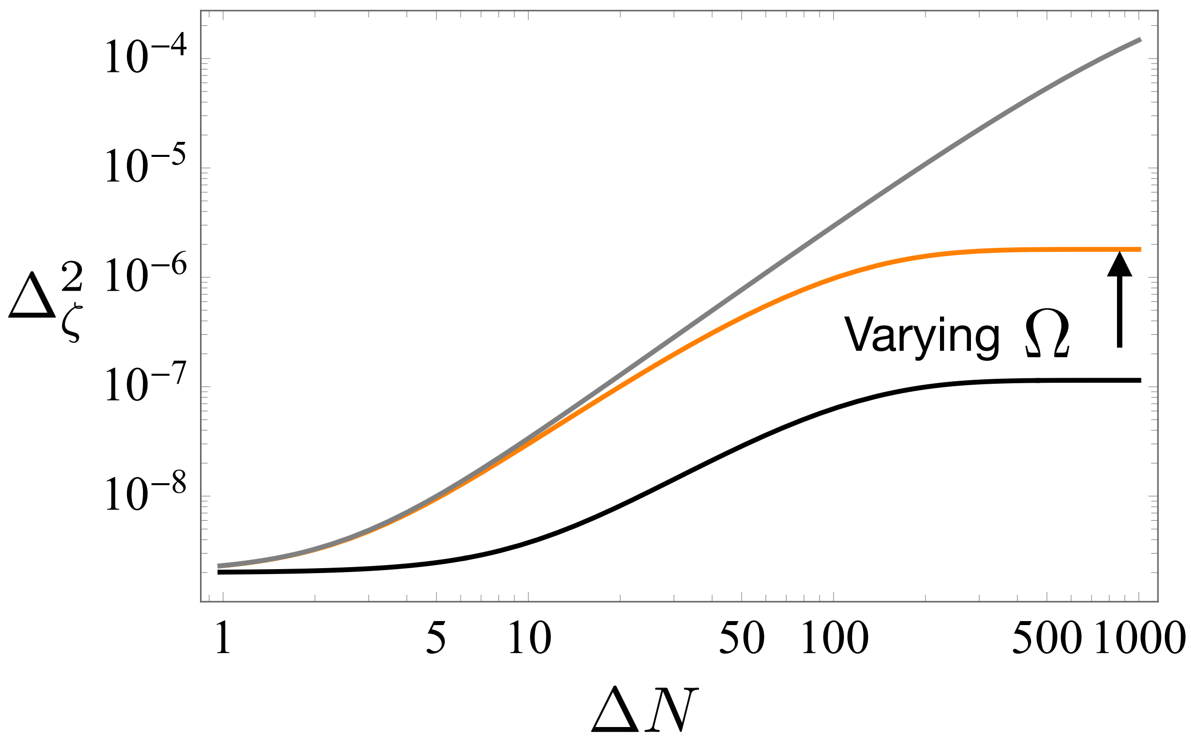
- Fluctuations during inflation can be understood as an stochastic process
- Separate the field $\phi = \phi_{\rm long} + \phi_{\rm short}$

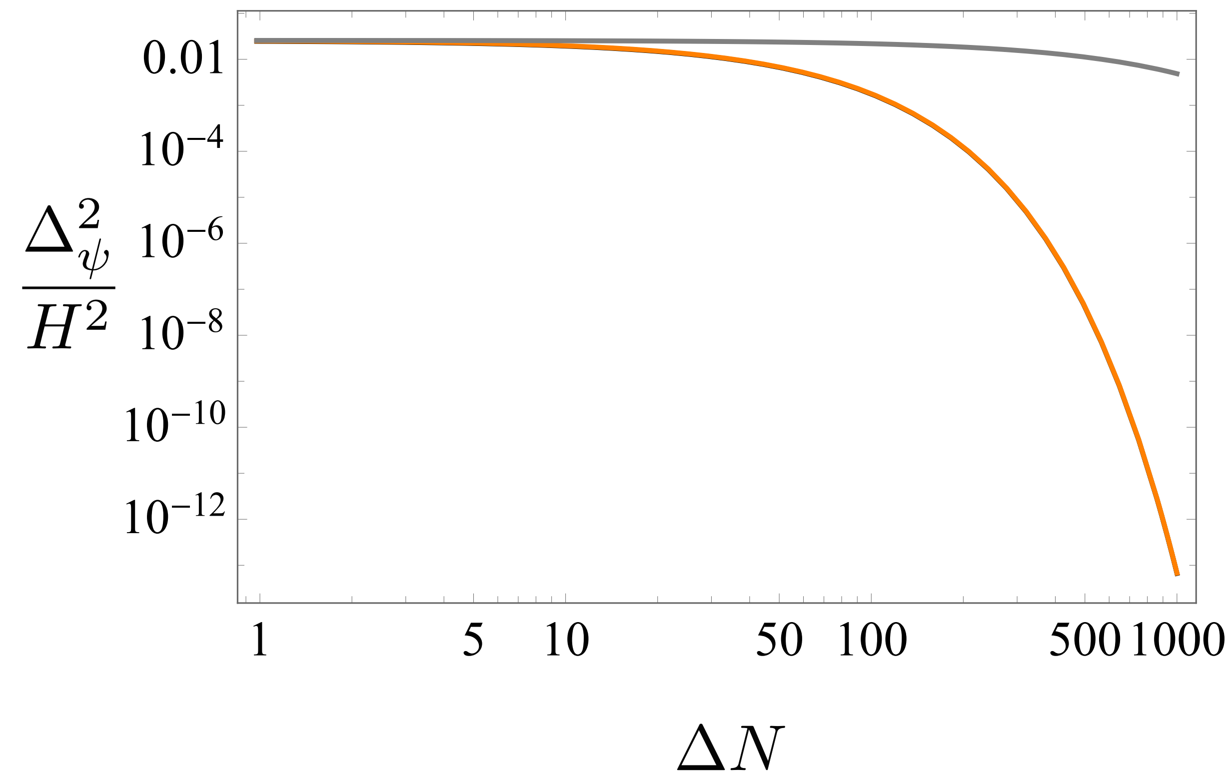
Linde '82
Vilenkin and Ford '82
Starobinsky '82

- Long wavelength dynamics can be understood as a Langevin equation $$\dot{\phi}_{\rm long} = -\frac{1}{3H} V_{,\phi} + \eta_\phi$$
- Equivalently to solve a Fokker-Planck equation for

$$\frac{d}{dt}P(\phi,t) = \frac{1}{3H}\frac{\partial}{\partial\phi}(V'(\phi)P(\phi,t)) + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial\phi^2}P(\phi,t)$$

Light scalar fields





- For single field inflation we find,

$$P(\zeta, t) = \frac{1}{\sqrt{2\pi}\sigma_\zeta} \exp\left(-\frac{1}{2\sigma_\zeta^2}\zeta^2\right)$$

$$\sigma_\zeta^2 = \frac{H^3}{8\epsilon\pi^2}(t - t_0)$$

- For an spectator field

$$\lim_{t\to\infty} P(\psi) \sim \exp\left(-\frac{8\pi^2 V(\psi)}{3H^4}\right)$$

- At large time there is an equilibrium distribution
- Solution is non perturbative in the couplings

Starobinsky and Yokoyama '94

For $V(\phi) = \frac{\lambda}{4}\phi^4$ one finds $\langle\phi^2\rangle \sim \frac{H^2}{\lambda^{1/2}}$

Stochastic dynamics of two fields

- Let's consider the linear case first

$$S = \frac{1}{2}\int d^4x a^3 \left[f_\zeta^2 \left(\dot{\zeta} - \frac{2\Omega}{f_\zeta}\psi \right)^2 - \frac{f_\zeta}{a^2}(\nabla\zeta)^2 + \dot{\psi}^2 + \frac{1}{a^2}(\nabla\psi)^2 + \mu^2\psi^2 + \ldots \right]$$

- Splitting the fields

$$\zeta = \zeta_l + \zeta_s$$

$$\psi = \psi_l + \psi_s$$

- The PDF is,

$$P(\tilde{\zeta}, \psi, t) \sim \exp\left(-\frac{\psi^2}{\sigma_\psi^2} - \frac{1}{2\sigma_\zeta^2 - 2\kappa^2/\sigma_\psi^2}\left(\tilde{\zeta} - \frac{\kappa}{2\sigma_\psi^2}\psi \right)^2 \right)$$

$$\langle\zeta\zeta\rangle \simeq \langle\psi\psi\rangle \frac{4\Omega^2}{f_\zeta^2}(\Delta N)^2$$

Achucarro *et al* '16

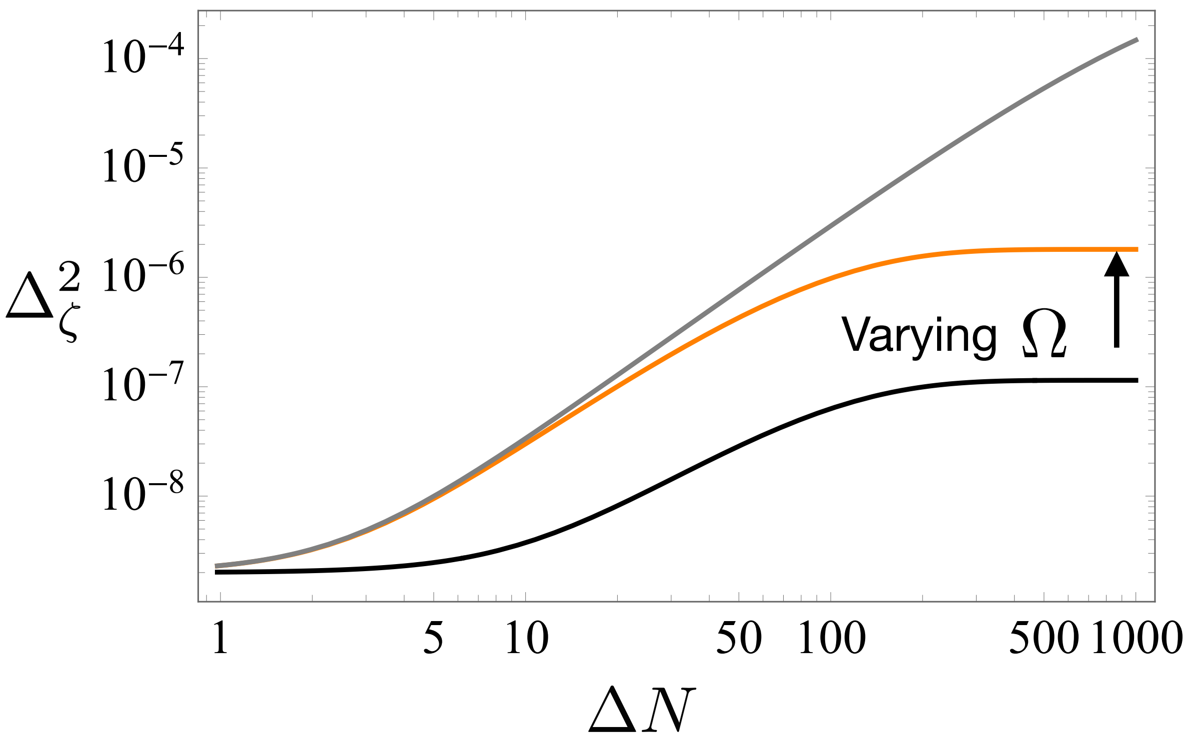
$$\kappa \equiv \frac{2}{3} t_\psi \Delta_\zeta^2 f_\zeta \Omega (1 - e^{-t/t_\psi})$$

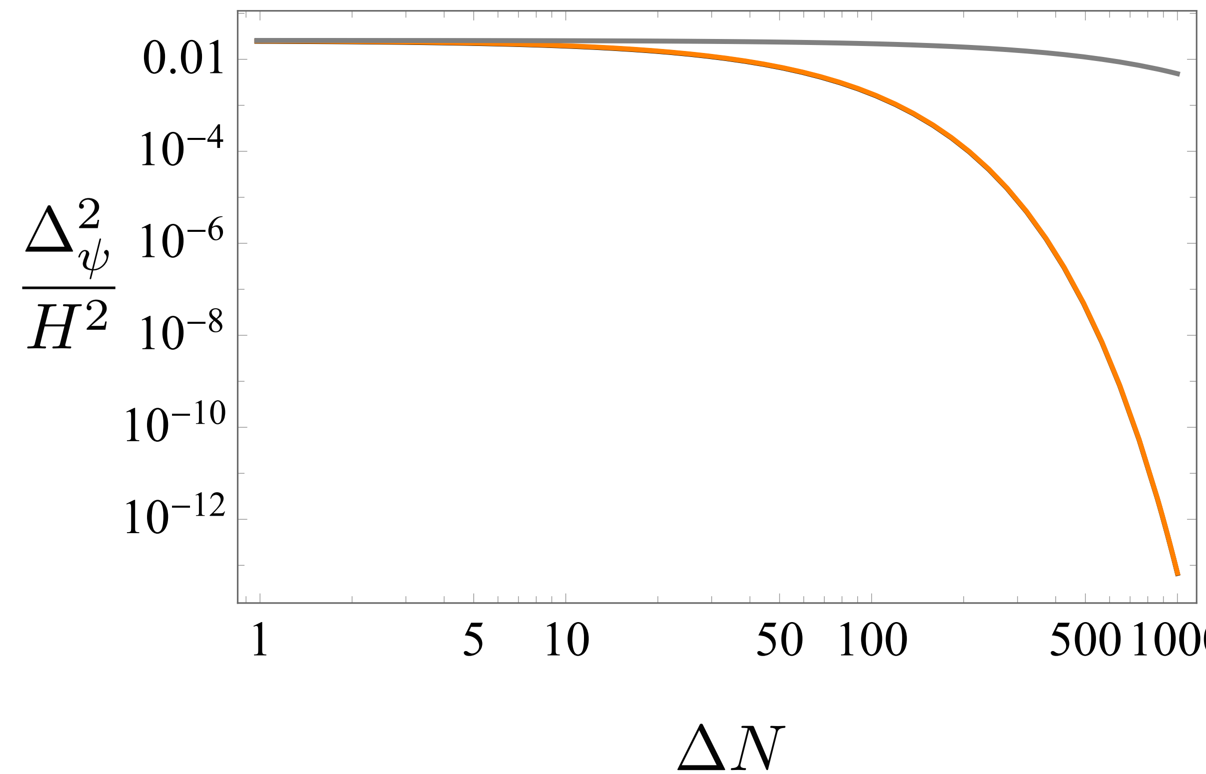
$$\kappa \sim Ht\Omega/f_\zeta$$

to $Ht_\psi\Omega/f_\zeta$

Result matches perturbative computations

Light scalar fields





Fields behave as ultralight until

$$\Delta N \geq \frac{H^2}{\mu^2}$$

$$\sigma_\psi^2 \approx \frac{3H^4}{8\pi^2}\left(1 - e^{-\frac{2\mu^2}{3H^2}\Delta N}\right)$$

Adding non linear terms

- Let us consider the cubic action

$$S = \frac{1}{2}\int d^4x a^3 \left\{ f_\zeta^2 \left(\dot{\zeta} - \frac{2\Omega}{f_\zeta}\psi\right)^2 - f_\zeta^2 \frac{(\nabla\zeta)^2}{a^2} + \dot{\psi}^2 - \frac{(\nabla\psi)^2}{a^2} - \mu^2\psi^2 \right.$$
$$\left. + \frac{6\Omega^2}{H}\psi^2\left(\dot{\zeta} - \frac{2\Omega}{f_\zeta}\psi\right) + \frac{2f_\zeta\Omega}{H}\psi\left(\dot{\zeta} - \frac{2\Omega}{f_\zeta}\psi\right)^2 - \frac{\tilde{\lambda}}{3}\psi^3 + \dots \right\}$$

Derivative interactions $v_\zeta^n \psi^m$

If $\gamma_{ab} = \delta_{ab}$ only two interactions

Interactions with more that two derivatives are turned off after horizon crossing

Adding non linear terms

Langevin equations in phase space

$$\frac{dv_\zeta}{dt} + 3Hv_\zeta + \frac{6\Omega}{f_\zeta} v_\zeta \psi + \frac{18\Omega^2}{f_\zeta^2}\psi^2 = 3H\eta_\zeta,$$

$$3H\dot{\psi} + \mu^2\psi + \underline{2\Omega f_\zeta v_\zeta} - \underline{\frac{2\Omega^2}{H}\psi v_\zeta} - \frac{\Omega f_\zeta}{H} v_\zeta^2 = \eta_\psi$$

We have that

$$\frac{6\Omega^2}{f_\zeta^2}\frac{\psi^2}{Hv_\zeta} \ll 1$$

- At leading order we can consider the first equation to be linear
- Larger effect from $v_\zeta \psi^2$
- Velocity field decays faster than the other variables and can be systematically integrated out from the Fokker Plank equation

$$t_v \sim 1/H \ll t_\psi \sim H/\mu^2$$

Non linear PDF

- Integrating out the velocity field we find the Fokker-Plank equation,

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \psi}\left(\frac{\psi P}{t_\psi} + \frac{D_\psi}{2}\frac{\partial P}{\partial \psi}\right) + \frac{2\Omega}{3}\Delta_\zeta^2(t)\frac{\partial^2}{\partial\psi\partial\zeta}\left(\frac{\Omega}{H}\psi P - f_\zeta P\right) + \frac{H\Delta_\zeta^2(t)}{2}\frac{\partial^2 P}{\partial^2\zeta}$$

We can solve it analytically in some limits

$$P(\zeta,\psi) = \exp\left[-\frac{\psi^2}{2\sigma_\psi^2} - \frac{1}{2\sigma_\zeta^2}\left(\zeta + \frac{2f_\zeta\Omega}{3H}\frac{\sigma_\zeta^2}{\sigma_\psi^2}\psi - \frac{\Omega^2}{3H^2}\frac{\sigma_\zeta^2}{\sigma_\psi^2}\psi^2\right)^2\right] \quad 1 \ll Ht \ll Ht_\psi$$

Non-Gaussian tails

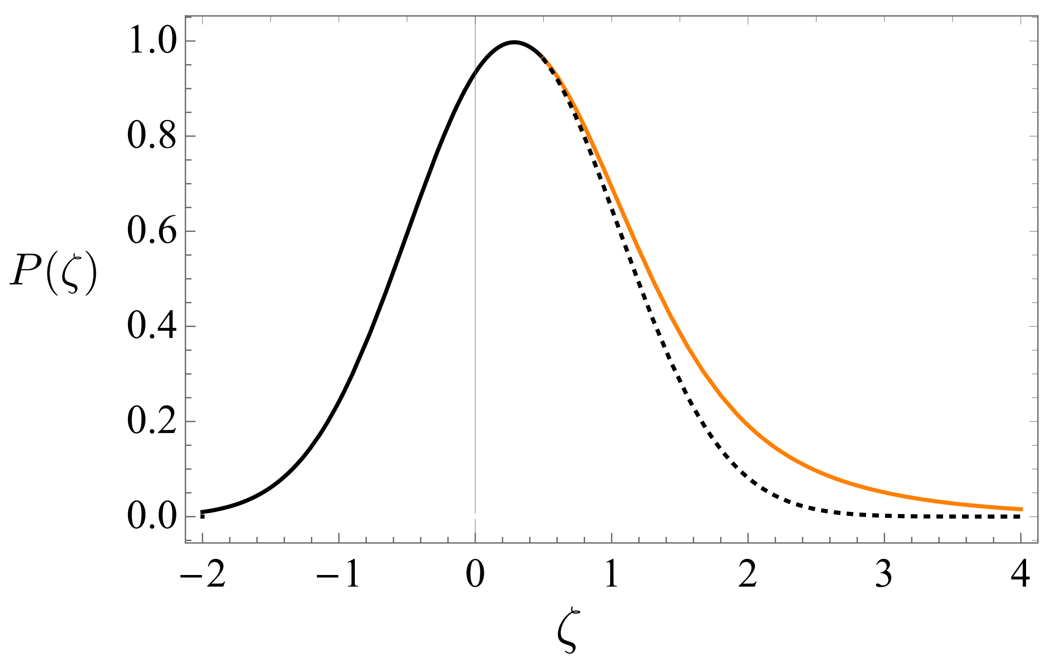
- Let us study the simplified PDF

$$f_{\rm NL}\zeta \sim \frac{\Omega^2}{H^2}\zeta \geq 1$$

$$P(\zeta,\psi) = \exp\left[-\frac{\psi^2}{2\sigma_\psi^2} - \frac{1}{2\sigma_\zeta^2}\left(\zeta - \bar{\kappa}\frac{\psi^2}{2\sigma_\psi^2}\right)^2\right]$$

$$\bar{\kappa} \equiv \frac{2\Omega^2}{3H^2}\sigma_\zeta^2$$

Integrating out ψ



Saddle points at $\psi = 0$

$$\bar{\psi} = \pm\sqrt{\frac{2\sigma_\psi^2}{\bar{\kappa}}}\sqrt{\zeta - \frac{\sigma_\zeta^2}{\bar{\kappa}}}$$

Expanding around $\psi = 0$

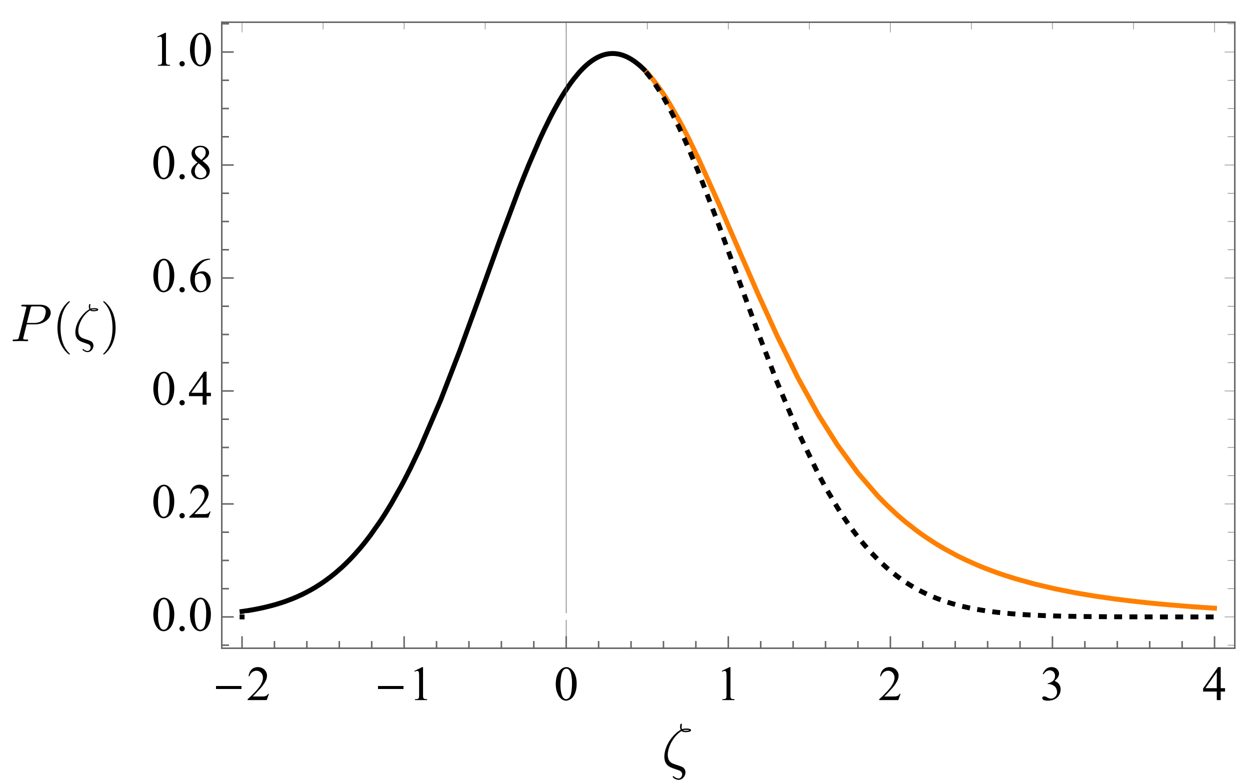
$$P(\zeta) \sim \exp(-\zeta^2/2\sigma_\zeta^2)$$

Non-Gaussian tails

- Let us study the simplified PDF

$$P(\zeta,\psi) = \exp\left[-\frac{\psi^2}{2\sigma_\psi^2} - \frac{1}{2\sigma_\zeta^2}\left(\zeta - \bar{\kappa}\frac{\psi^2}{2\sigma_\psi^2}\right)^2\right]$$

$$\bar{\kappa} \equiv \frac{2\Omega^2}{3H^2}\sigma_\zeta^2$$



For $\zeta \gg \sigma_\zeta^2/\bar{\kappa} \sim 1/f_{\rm NL}$

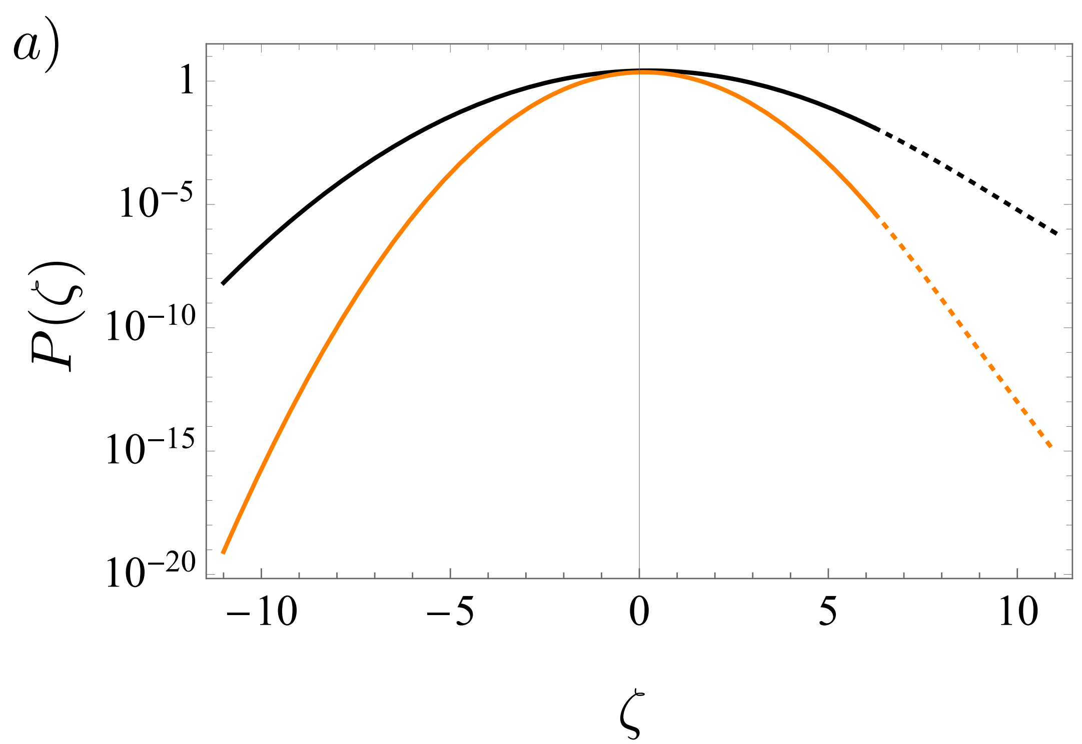
The second saddle point becomes real

$$P(\zeta) \sim \exp\left(-\frac{\zeta}{\bar{\kappa}}\right)$$

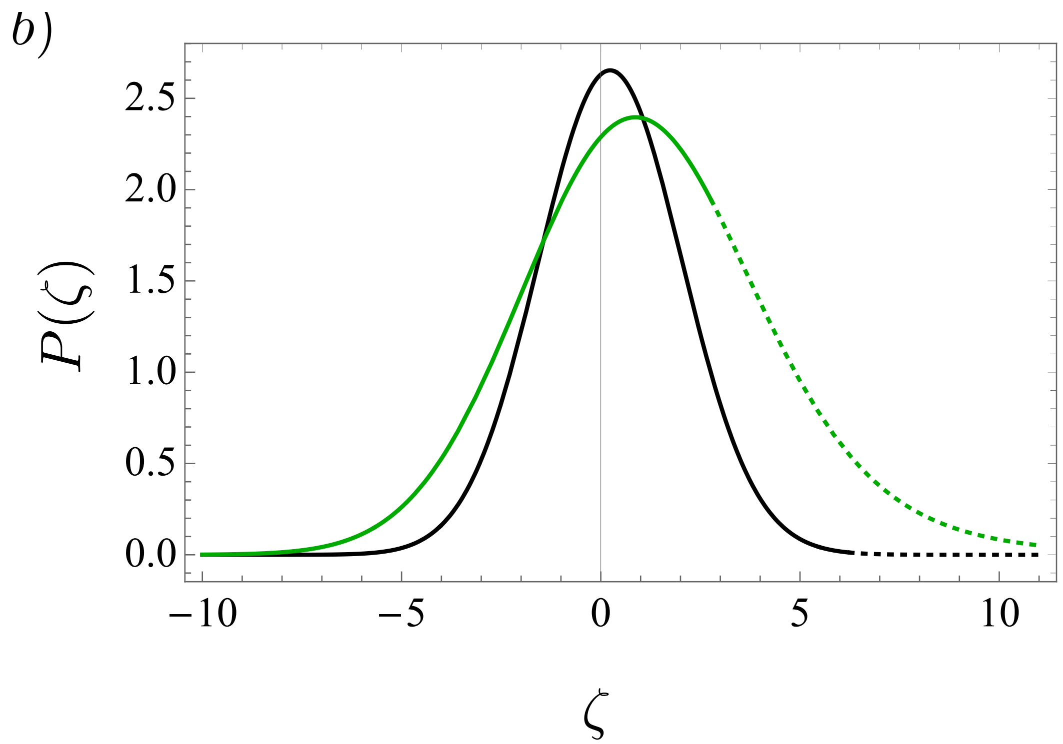
See also Panagopoulos and Silverstein ’19

When is the tail important?

Increasing the variance



Increasing the coupling Ω



Adding other terms

- Non perturbative effects come from the spectator field.
- In general adding higher order corrections will modify the tail making it heavier
- It will also be modifications to the variance for large ζ
- The coupling parameters $\kappa,\ \bar{\kappa}$ are quasi constant until the end of inflation

General potential

- We can study a general $V(\psi)$

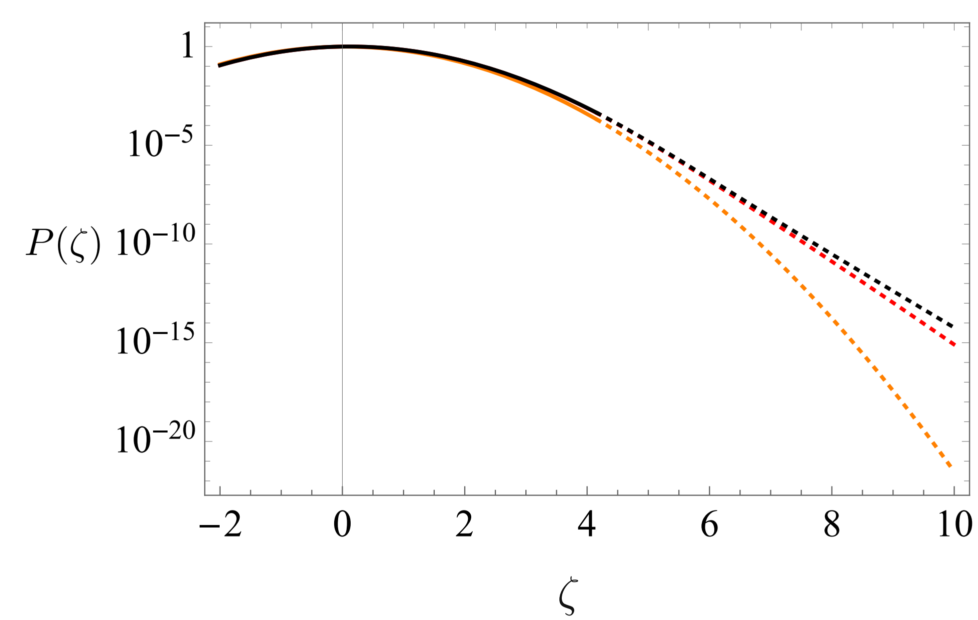
$$P(\zeta,\psi) \sim \exp\left[-\frac{8\pi^2 V(\psi)}{3H^2} - \frac{1}{2\sigma_\zeta^2}\left(\zeta - \frac{\bar{\kappa}'}{2\sigma_{0\psi}^2}\psi^2\right)^2\right]$$

Example: If we add a quartic term

For $\zeta \gg \sigma_\zeta^2/\bar{\kappa}$

$$P(\zeta) \sim \exp\left(-\frac{\zeta^2}{\frac{H\bar{\kappa}^2}{3t_\psi \lambda \sigma_\zeta \sigma_\psi^2} + 2\sigma_\zeta^2} - \frac{\zeta}{\bar{\kappa} + \frac{6t_\psi \lambda \sigma_\psi^2 \sigma_\zeta^2}{H\bar{\kappa}}}\right)$$

$$\frac{f_{\rm NL}^{(\lambda)}}{f_{\rm NL}} \sim \frac{\lambda}{\Omega^2}\sigma_\psi^2$$

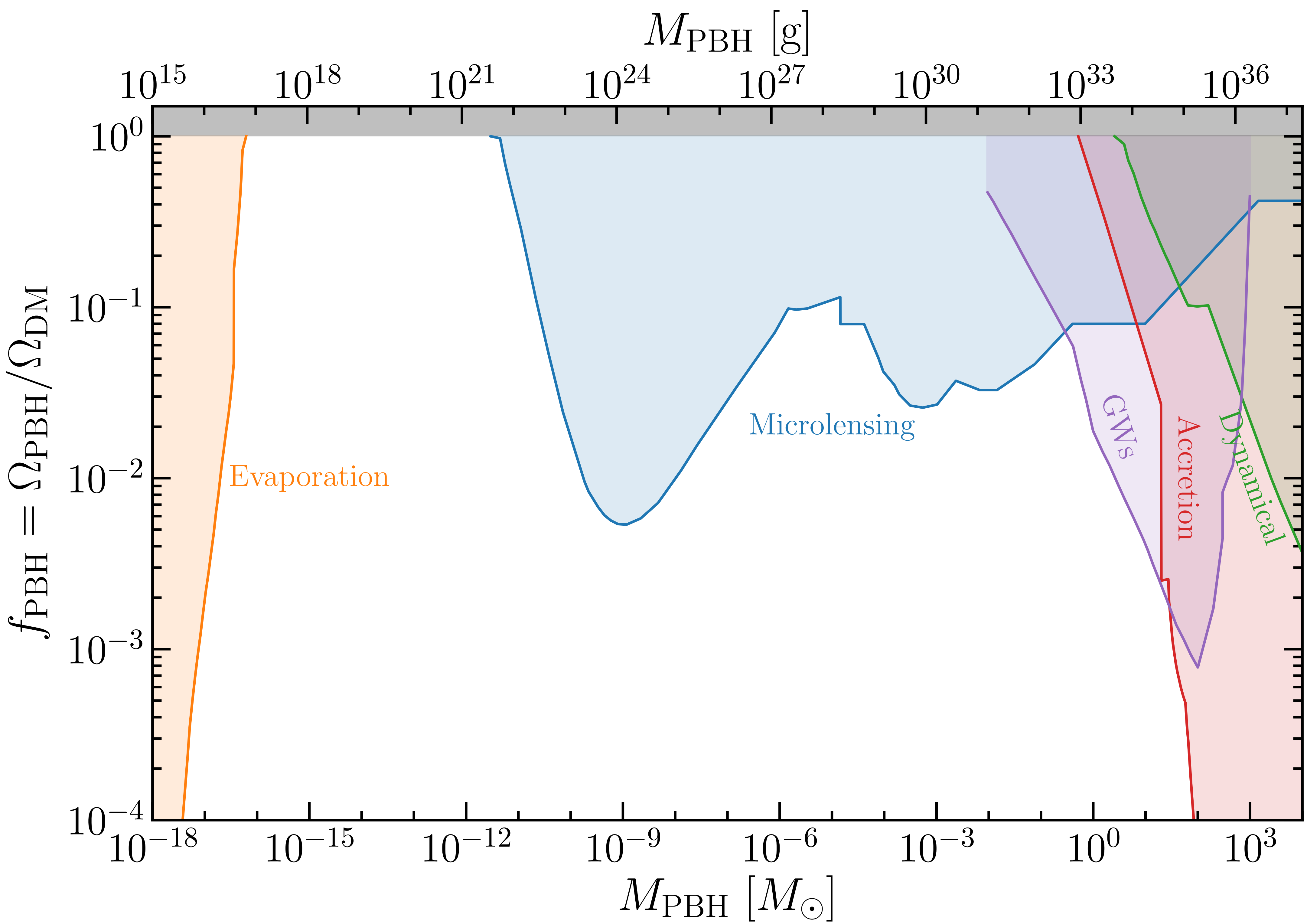


Summary

- Non perturbative effects can be obtained using stochastic techniques
- On multi field models we have computed that interactions between fields leads to non perturbative effects
- Many things to understand, generalisation to other models, factorial enhancements, observation consequences

Fokker-Planck equation including all terms

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \psi}\left(\frac{V'(\psi)}{3H}P + \frac{D_\psi}{2}\frac{\partial P}{\partial \psi}\right) + H\Delta_\zeta^2(t)\frac{\partial^2}{\partial\psi\partial\zeta}\left(\left(\frac{2\Omega^2}{H^2}\psi + \frac{2f_\zeta\Omega}{3H}\right)P\right)$$
$$+ \frac{H\Delta_\zeta(t)^2}{2}\frac{\partial^2 P}{\partial \zeta^2} + \frac{6\Omega^2}{f_\zeta^2 H}\frac{\partial}{\partial\zeta}\left(\psi^2 P\right) - \frac{\Omega\Delta_\zeta^2(t)}{f_\zeta}\frac{\partial^2}{\partial\zeta^2}(\psi P) - \frac{1}{4}\Omega f_\zeta \Delta_\zeta^4(t)\frac{\partial^3 P}{\partial^2\zeta\partial\psi}$$



Green and Kavanagh ‘20