HIDDeν

Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

Axion quality from the symmetric of SU(N)

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arXiv [2007.12663]

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Introduction

The Peccei-Quinn solution to the Strong CP problem is *problematic*

↓

Peccei-Quinn must be an extremely good symmetry of high-energy physics

↓

Peccei-Quinn (axion) quality problem

→ New **gauge symmetries** can solve the issue

The Strong CP problem

The QCD Lagrangian violates CP symmetry

$$\mathcal{L}_{\rm QCD} \supset \bar{\theta} \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$

→ Gauge-invariant + renormalizable

Non-perturbative QCD (istantons) →

Natural expectation

$$\bar{\theta} \sim \mathcal{O}(1)$$

Prediction: electric dipole moment for the neutron

$$d_n \approx 10^{-16} |\bar{\theta}| e \text{ cm}$$

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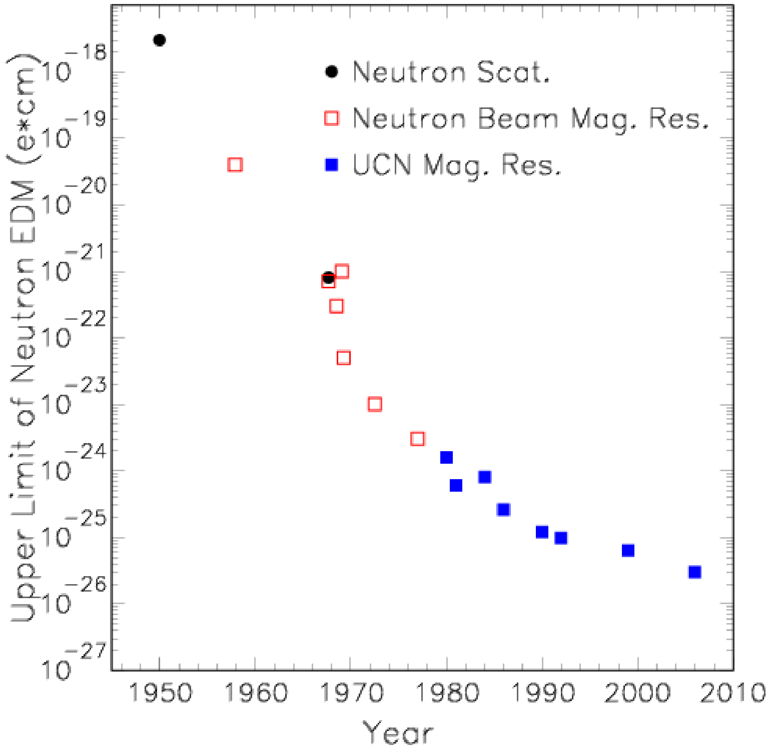


Upper bound $\bar{\theta} \leq 10^{-10}$

Why so small??

Peccei-Quinn mechanism

A new *global chiral* U(1) symmetry + charged scalar field Φ

1) Spontaneously broken → a new Goldstone boson: the *axion*

$$\Phi(x) \simeq f_a e^{ia(x)/f_a}$$

$$a(x) \stackrel{PQ}{\to} a(x) + \gamma f_a$$

2) QCD anomaly → generates a potential for the axion

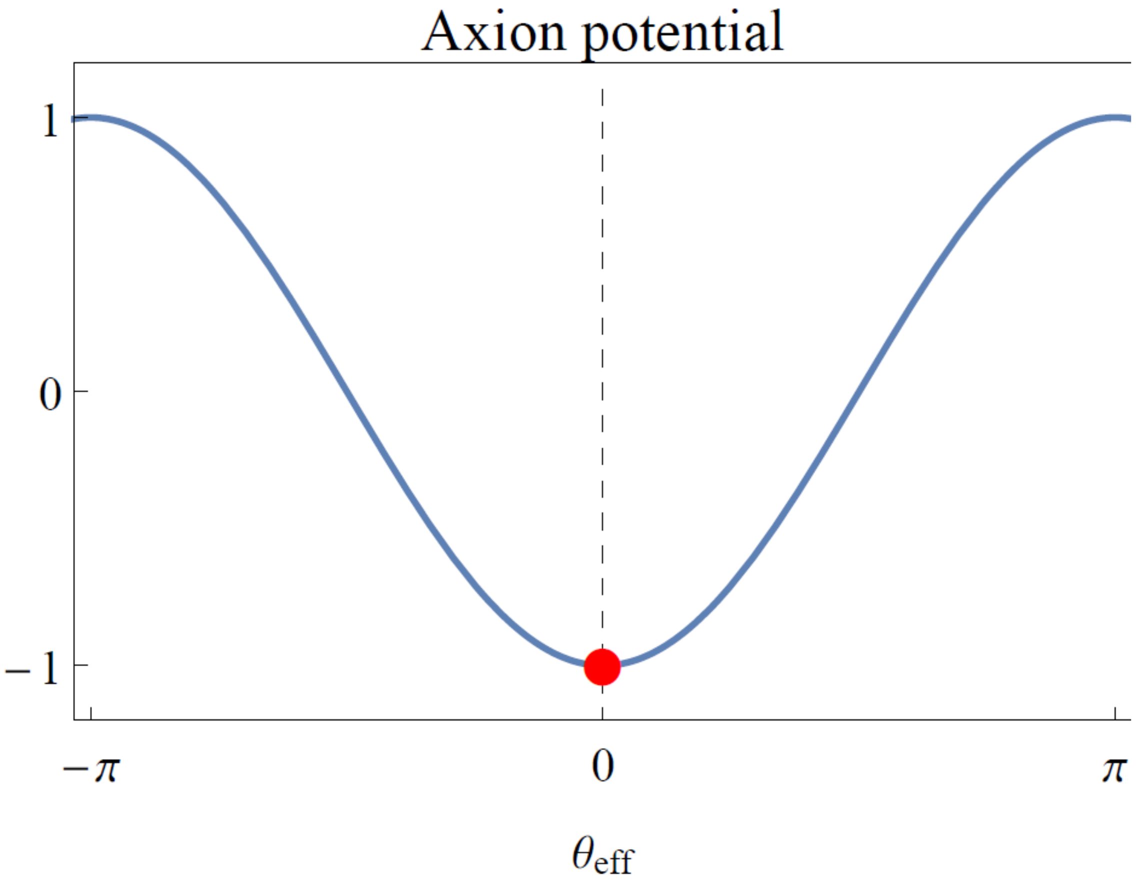
Colored fermions charged under U(1)

$$\mathcal{L}^{\rm PQ}_{\rm QCD} \supset \theta_{\rm eff}(x) \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$

$$\theta_{\rm eff}(x) \equiv \bar{\theta} - \frac{a(x)}{f_a}$$

Peccei-Quinn mechanism

The QCD potential relaxes to the CP-conserving minimum



$$V \approx -m_a^2 f_a^2 \cos\theta_{\rm eff}$$

$$m_a \sim \Lambda_{\rm QCD}^2 / f_a$$

$$\langle \bar{\theta}_{\rm eff} \rangle = 0$$ (Vafa-Witten theorem)

Global symmetries

We assume that *global symmetries* are not fundamental but *accidental*

They arise at low-energy (e.g. from gauge invariance)

$$\mathcal{L} = \mathcal{L}^{(4)}$$

Ex: baryon/lepton number in the SM

They are broken by higher-dimensional operators in EFT (UV physics)

$$\Delta\mathcal{L}_{\rm UV} \sim \frac{1}{\Lambda_{\rm UV}^{d-4}}\mathcal{O}^{[d]}$$

Ex: Towers of new states in string theory

+ Quantum gravity conjectures: gravity violates all global symmetries

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UV cut-off scale

+ Quantum gravity conjectures: gravity violates all global symmetries

The quality problem

Peccei-Quinn breaking operators are generated at UV scale

$$\Delta\mathcal{L}_{\rm UV} \sim \frac{1}{\Lambda_{\rm UV}^{d-4}}\mathcal{O}^{[d]}$$

They generate an extra potential for the axion

$$\Delta V_{\rm UV}(\theta_{\rm eff}) \sim \Lambda_{\rm UV}^4 \left(\frac{f_a}{\Lambda_{\rm UV}}\right)^d$$

The quality problem

Peccei-Quinn breaking operators are generated at UV scale

$$\Delta\mathcal{L}_{\rm UV} = \frac{\lambda_n |\Phi|^{2m} e^{-i\delta_n}\Phi^n}{\Lambda_{\rm UV}^{d-4}} + h.c.$$

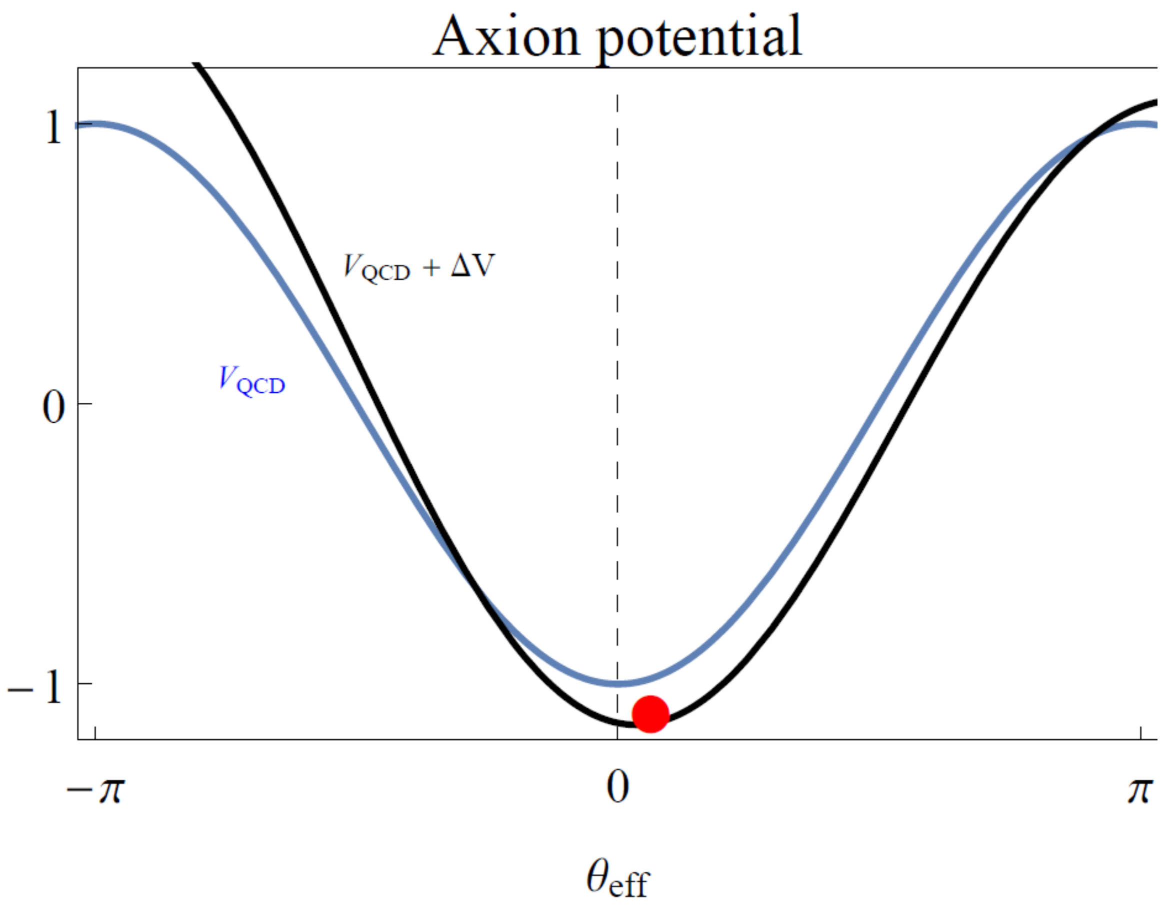
They generate an extra potential for the axion

$$\Phi(x) \simeq f_a e^{ia(x)/f_a}$$

$$\Delta V_{\rm UV} \approx \lambda_n \Lambda_{\rm UV}^4 \left(\frac{f_a}{\Lambda_{\rm UV}}\right)^d \cos\left(\frac{na}{f_a} - \delta_n\right)$$

The quality problem

The minimum of the potential is shifted



$\langle \theta_{\rm eff} \rangle \neq 0$ → CP – violating minimum!

The quality problem

We solve the Strong CP problem only if

$$\langle \theta_{\rm eff} \rangle < 10^{-10}$$ (neutron EDM)

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UV physics

QCD

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For physically well motivated scales

$$f_a \sim 10^{11}\ \text{GeV}$$
$$\Lambda_{\text{UV}} \sim M_{\text{Pl}}$$

PQ must be **preserved** up to operators of dimension **d ≥ 12** (high-quality symmetry)

The symmetric model

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(\mathcal{N})$	$U(1)_{PQ}$	$U(1)_{\mathcal{Q}}$	$U(1)_{\mathcal{L}}$
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New dark SU(N) gauge group

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A new scalar field in the symmetric representation of SU(N)

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Accidental U(1) global symmetry at the renormalizable level

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A new colored fermion provides the QCD anomaly

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We introduce color-singlet fermions to cancel gauge anomalies

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<u>SSB</u>: $\mathcal{S}(x) = \left(\sqrt{\mathcal{N}/2} f_a + \cdots\right) e^{ia(x)/f_a}$ ⟶ $SU(N) \otimes U(1)_{\mathrm{PQ}} \to SO(N)$

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SO(N) *confinement*: **2 energy scales** in the model → $\Lambda_{\mathrm{SO}} \ll f_a$

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SO(N) *confinement*: **2 energy scales** in the model ⟶ $\Lambda_{\mathrm{SO}} \ll f_a$

Extra *accidental* Z_2 symmetry preserved by SO(N) dynamics

Solving the quality problem

What are the dangerous operators?

Gauge invariant

PQ breaking

The lowest dimensional gives the strongest condition

$$\left(\frac{f_a}{\Lambda_{\mathrm{UV}}}\right)^{d-4} f_a^4 \lesssim 10^{-10}\Lambda_{\mathrm{QCD}}^4$$

Solving the quality problem

What are the dangerous operators?

Gauge invariant

PQ breaking

The lowest dimensional arises at dimension N

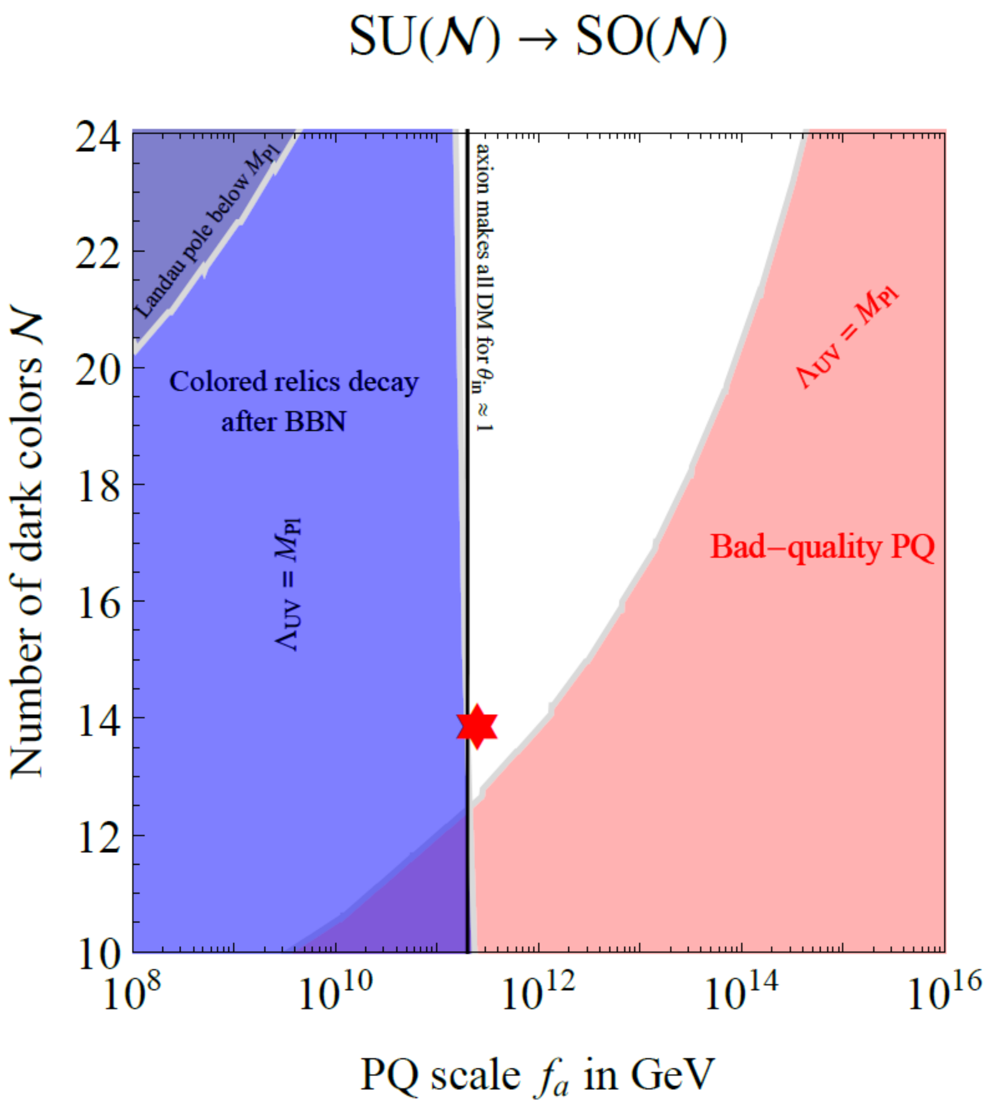
$$\det \mathcal{S} = \epsilon_{\mathcal{N}} \epsilon_{\mathcal{N}} \mathcal{S}^{\mathcal{N}}$$

If N is large enough the PQ quality problem is solved!

Gauge invariance forbids PQ breaking below dimension N and allows for *arbitrary high protection*

Parameter space

- Solution to the quality problem
- Colored relics decay before BBN
- No Landau Poles below the cut-off scale

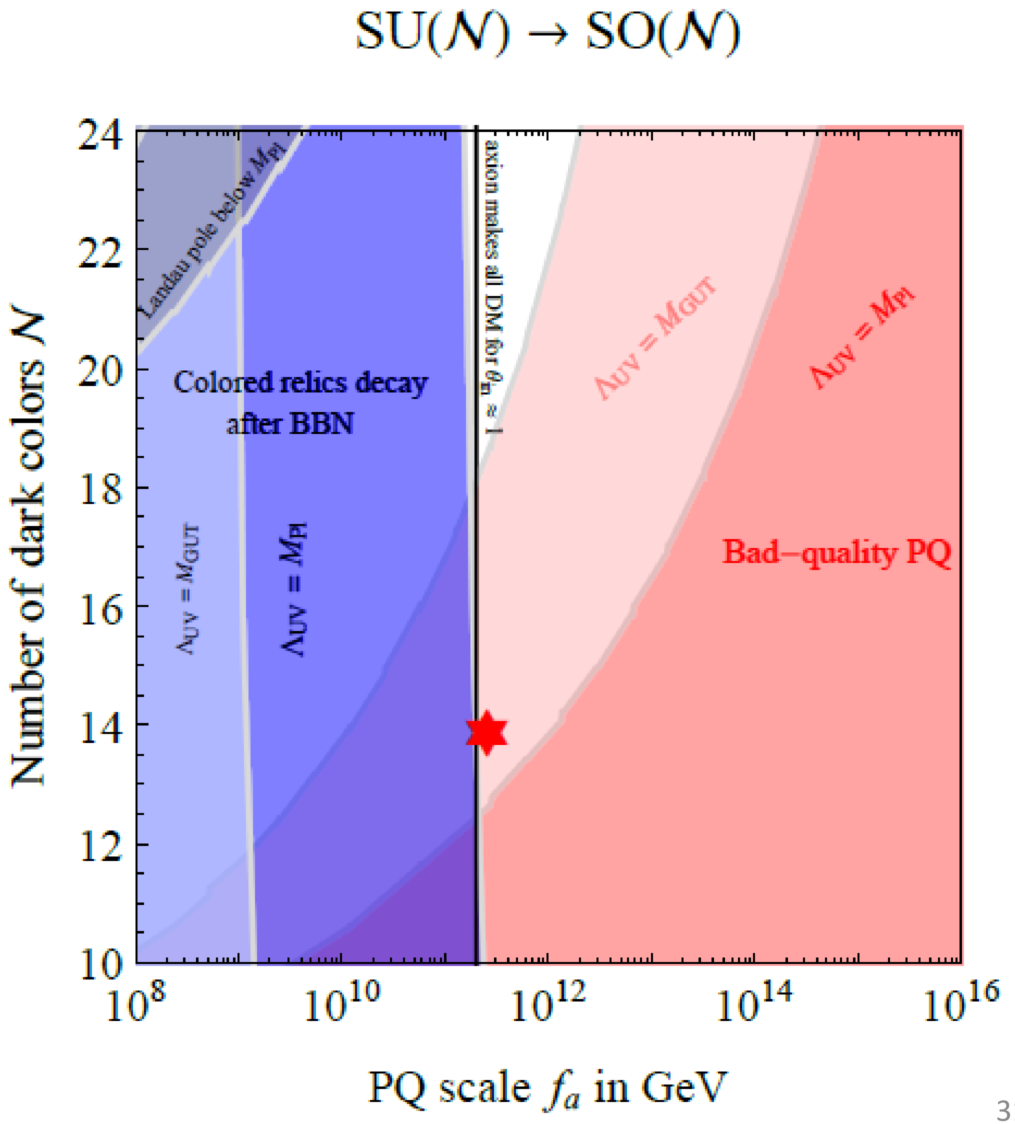


- We need $\mathcal{N} > 12$ if $\Lambda_{UV} = M_{Pl}$

Parameter space

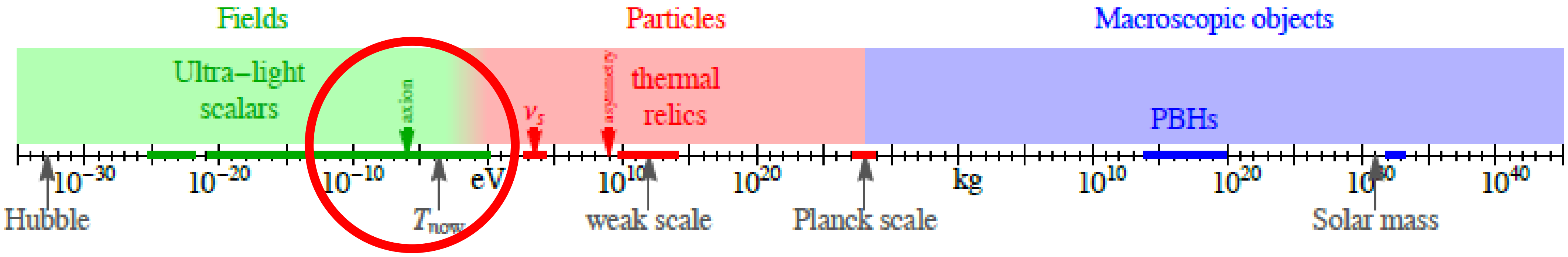
- Solution to the quality problem
- Colored relics decay before BBN
- No Landau Poles below the cut-off scale

- We need $\mathcal{N} > 12$ if $\Lambda_{\rm UV} = M_{\rm Pl}$
- Larger values for smaller cut-off scales



Axion Dark Matter

The mass range of DM candidates spans over 80 order of magnitudes



Axions are excellent ultra-light DM candidates

They are produced through vacuum misalignment mechanism

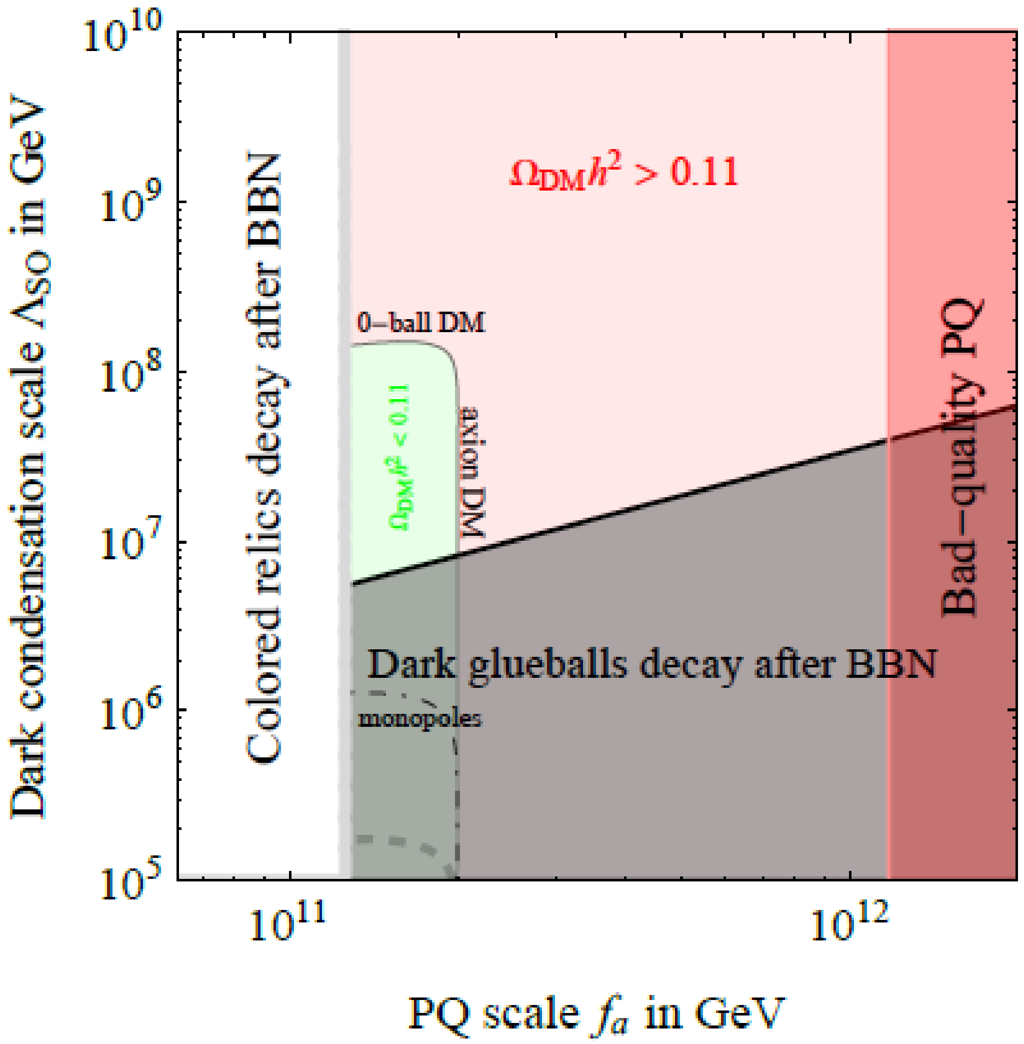
Dark Matter

Axion DM
+
Composite SO(N) bound states

- They emerge after SO(N) confinement
- Most of them just decay...

- ...but the Z_2 odd are stable DM candidates!

PQ broken after inflation



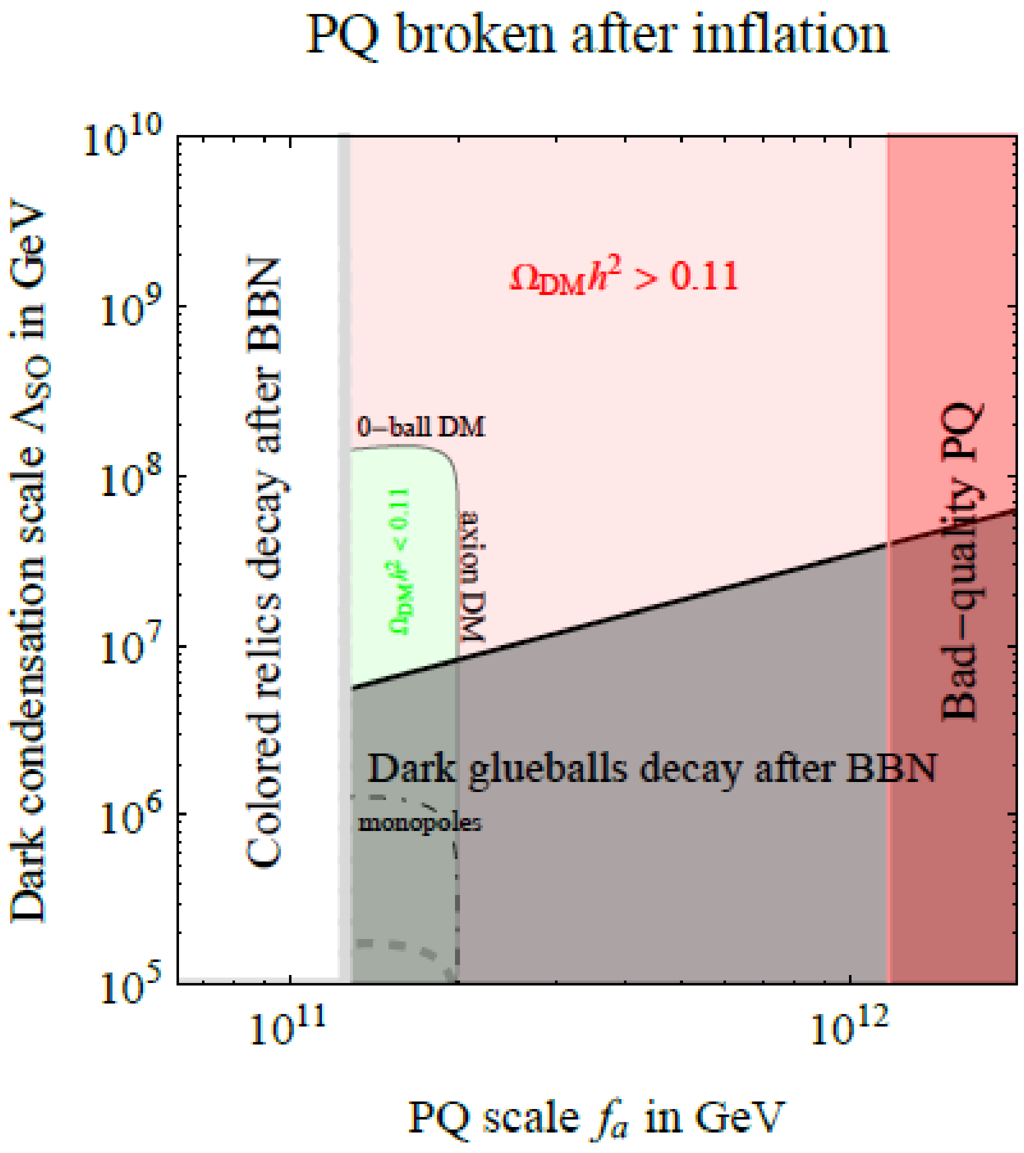
Dark Matter

Z_2 : reflection in group space along an arbitrary color direction

$$\mathcal{A}_{IJ} \to (-1)^{\delta_{1I}+\delta_{1J}} \mathcal{A}_{IJ}$$

$$\mathcal{L}^I \to (-1)^{\delta_{1I}} \mathcal{L}$$

if $\mathcal{N}$ is odd	$\epsilon_{\mathcal{N}} \mathcal{A}^{(\mathcal{N}-1)/2} \mathcal{L}$
if $\mathcal{N}$ is even	$\epsilon_{\mathcal{N}} \mathcal{A}^{\mathcal{N}/2}$



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- Their mass is $\sim \Lambda_{\rm SO}$
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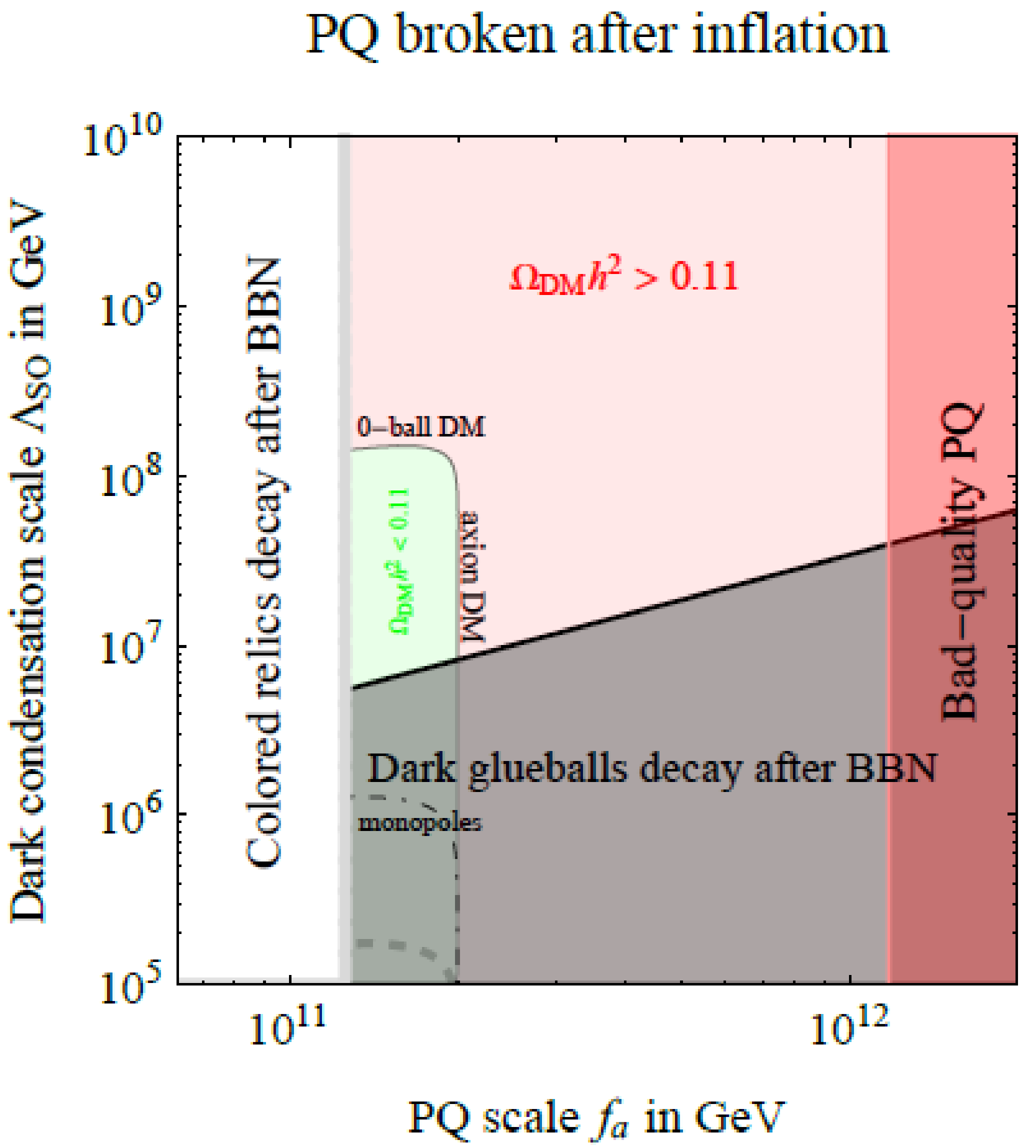

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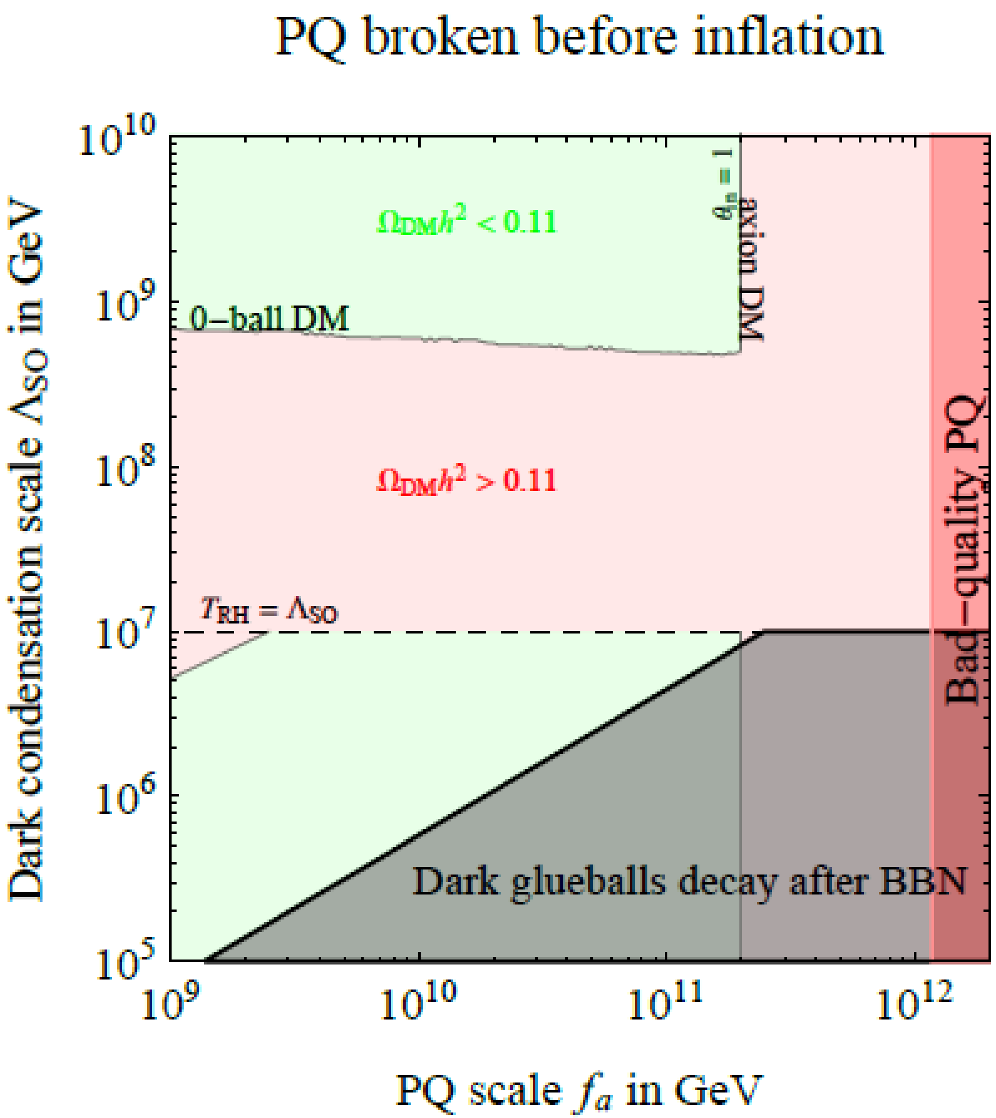

Conclusions

The Peccei-Quinn solution has a problem of *UV sensitivity*

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Introducing a new gauge symmetry we can build a consistent model

↓

We can solve the **quality problem** and get **multicomponent DM**

Backup slides

N-ality rule (1)

What are the dangerous operators?

Gauge invariant

$$\delta_i^j \quad \epsilon_{i_1 \cdots i_N}$$

PQ breaking

PQ charge is proportional to ***N-ality***

$$\mathrm{PQ}[\Phi^{\{j\}}_{\{i\}}] \propto \#_{\{i\}} - \#^{\{j\}}$$

N-ality rule (2)

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$$\mathrm{PQ}[\mathcal{S}_{ij}] = \frac{1}{2} \times 2 = 1 \qquad \mathrm{PQ}[\mathcal{Q}_i] = \frac{1}{2} \times 1 = \frac{1}{2} \qquad \mathrm{PQ}[\mathcal{L}^i] = \frac{1}{2} \times (-1) = -\frac{1}{2}$$

N-ality rule (3)

What are the dangerous operators?

Gauge invariant

$\delta_i^j \quad \epsilon_{i_1 \cdots i_N}$

PQ breaking

PQ charge is proportional to ***N-ality***

$$\mathrm{PQ}[\Phi^{\{j\}}_{\{i\}}] \propto \#_{\{i\}} - \#^{\{j\}}$$

PQ breaking = N-ality breaking

$$\#_{\{i\}} - \#^{\{j\}} \neq 0$$

PQ is only broken by operators containing one $\epsilon_{i_1 \cdots i_N}$ tensor

Lagrangian and symmetries

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm yuk} - V(\mathcal{S})$$

$$\mathcal{L}_{\rm yuk} = y_{\mathcal{Q}} \mathcal{Q}_L \mathcal{S}^\dagger \mathcal{Q}_R + y_{\mathcal{L}}^{ij} \mathcal{L}_L^i \mathcal{S} \mathcal{L}_R^j + h.c.$$

$U(1)_{\mathcal{Q}}$

$$\mathcal{Q}_{L(R)} \to e^{(-)i\alpha} \mathcal{Q}_{L(R)}$$

if $Y_{\mathcal{L}} \neq 0$ $\quad U(1)_{\mathcal{L}}$

$$\mathcal{L}_{L(R)} \to e^{(-)i\beta} \mathcal{L}_{L(R)}$$

$$V(S) = M_{\mathcal{S}}^2 Tr\left[\mathcal{S}^\dagger \mathcal{S}\right] + \lambda_{\mathcal{S}} Tr\left[(\mathcal{S}^\dagger \mathcal{S})^2\right] + \lambda'_{\mathcal{S}} Tr\left[\mathcal{S}^\dagger \mathcal{S} \mathcal{S}^\dagger \mathcal{S}\right] - \lambda_{H\mathcal{S}} (H^\dagger H) Tr\left[\mathcal{S}^\dagger \mathcal{S}\right]$$

Lagrangian and symmetries

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{yuk}} - V(\mathcal{S})$$

$$\mathcal{L}_{\text{yuk}} = y_{\mathcal{Q}} \mathcal{Q}_L \mathcal{S}^\dagger \mathcal{Q}_R + y^{ij}_{\mathcal{L}} \mathcal{L}^i_L \mathcal{S} \mathcal{L}^j_R + h.c.$$

$$U(1)_{\mathcal{Q}}$$

$$\mathcal{Q}_{L(R)} \to e^{(-)i\alpha} \mathcal{Q}_{L(R)}$$

$$\text{if } Y_{\mathcal{L}} = 0 \qquad Z_2$$

No distinction between L and R

$$\mathcal{L} \to -\mathcal{L}$$

$$V(S) = M^2_{\mathcal{S}} Tr\left[\mathcal{S}^\dagger \mathcal{S}\right] + \lambda_{\mathcal{S}} Tr\left[(\mathcal{S}^\dagger \mathcal{S})^2\right] + \lambda'_{\mathcal{S}} Tr\left[\mathcal{S}^\dagger \mathcal{S} \mathcal{S}^\dagger \mathcal{S}\right] - \lambda_{H\mathcal{S}} (H^\dagger H) Tr\left[\mathcal{S}^\dagger \mathcal{S}\right]$$

U-parity

SU(N) and SO(N) are invariant under a discrete symmetry: *U-parity*

$$\mathcal{S}_{ij} \to (-1)^{\delta_{1i}+\delta_{1j}} \mathcal{S}_{ij} \qquad \mathcal{A}_{ij} \to (-1)^{\delta_{1i}+\delta_{1j}} \mathcal{A}_{ij} \qquad \mathcal{Q}_i \to (-1)^{\delta_{1i}} \mathcal{Q}_i$$

Reflection in group space along an arbitrary direction

↓

We can build gauge-invariant bound states odd under U-parity

$$\epsilon_{i_1 \cdots i_N} \mathcal{A}_{i_1 i_2} \cdots \mathcal{A}_{i_{N-1} i_N}$$

↓

Stable DM candidates

Stable SO(N) bound states

Stable states because of O-parity or dark lepton number

	if $Y_{\mathcal{L}} \neq 0$	if $Y_{\mathcal{L}} = 0$
if N is odd	$\epsilon_N \mathcal{A}^{(N-1)/2}\mathcal{L}$ and $\mathcal{L}\mathcal{L}$	$\epsilon_N \mathcal{A}^{(N-1)/2}\mathcal{L}$.
if N is even	$\epsilon_N \mathcal{A}^{N/2}$ and $\mathcal{L}\mathcal{L}$	$\epsilon_N \mathcal{A}^{N/2}$

To avoid charged relics we must fix

$$Y_{\mathcal{L}} = 0$$

Decay of colored relics

Gauge invariance allows dimension-6 operators

$$(q_R \mathcal{Q})(e_R \mathcal{L}) \qquad (q_R \mathcal{Q})(q'_R \mathcal{L})$$

They are allowed for $Y_{\mathcal{Q}} \pm Y_{\mathcal{L}} = \{-1/3, 2/3, -4/3\}$

We assume that $m_{\mathcal{Q}} > m_{\mathcal{L}}$

We require that decays of colored fermions occur before BBN

$$\Gamma_{\mathcal{Q}} \simeq \frac{1}{13\ \mathrm{sec}} \left(\frac{m_{\mathcal{Q}}}{2 \times 10^{11}\ \mathrm{GeV}} \right)^5 \left(\frac{M_{\mathrm{Pl}}}{\Lambda_{\mathrm{UV}}} \right)^4 \qquad f_a > \frac{1}{y_{\mathcal{Q}}} \sqrt{\frac{10}{N}} \left(\frac{\Lambda_{\mathrm{UV}}}{M_{\mathrm{Pl}}} \right)^{4/5} \times 10^{11}\ \mathrm{GeV}$$

Axion photon coupling

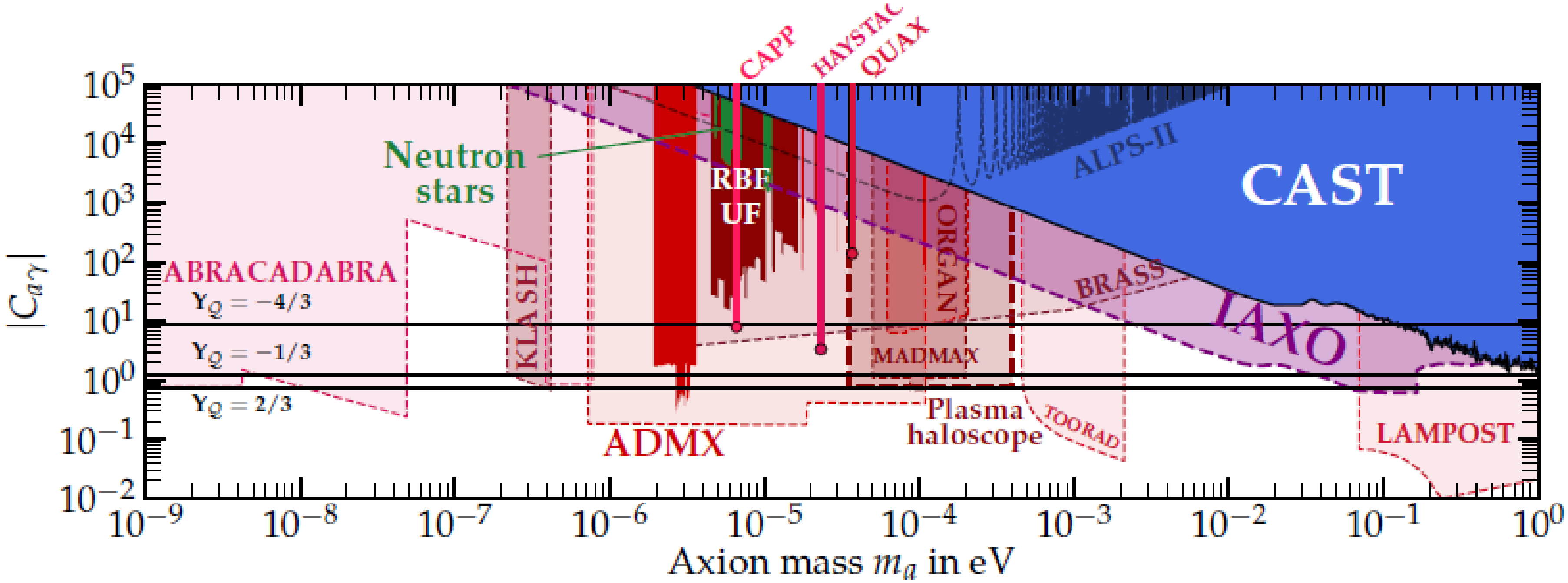
$$\mathcal{L}_{\text{eff}} = \frac{\alpha_{\text{em}} C_{a\gamma}}{8\pi f_a} a F^{\mu\nu} \tilde{F}_{\mu\nu}, \qquad C_{a\gamma} = 6(Y_Q^2 - Y_{\mathcal{L}}^2) - 1.92(4)$$

$6(Y_Q^2 - Y_{\mathcal{L}}^2)$ ↓ model dependent $U(1)_{PQ}\, U(1)_Y^2$

$1.92(4)$ ↓ model independent $\pi_0 - a$ mixing

$Y_{\mathcal{L}} = 0$ is needed to avoid charged relics

$Y_Q = \frac{\{2,-1,-4\}}{3}$ is required to allow Q decays



Thermal production of composite DM

$$M_{\rm BS} \approx \Lambda_{\rm SO} \approx 10^8 \text{ GeV} \qquad \frac{\Omega_{\rm DM} h^2}{0.12} \approx \left(\frac{M_{\rm BS}}{100 \text{ TeV}}\right)^2$$

Efficient dilution mechanisms

Post-inflationary

Freeze-out production
+
Early matter domination and late decays of SO(N) glue-balls

Pre-inflationary

Freeze-in production

Production suppressed by $e^{-M_{\rm BS}/T_{\rm RH}}$

→ Huge entropy injection/DM dilution

Scalar in antisymmetric of SU(N)

Same idea but SU(N) is broken to Sp(N)

PQ is broken by dimension N/2 operator $\epsilon_{\mathcal{N}/2} S^{\mathcal{N}/2}$

DM: axions (+ mesons $\mathcal{L}\gamma_{\mathcal{N}}\mathcal{L}$)