More on Fake GUT

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Introduction of the SU(5) GUT

Goal : force unification and charge quantization
by a single group

Why was the SU(5) selected ?

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}$$

$$\bar{u} \quad \bar{d} \quad \bar{e}$$

→

just enough!

$$\bar{5} = (\bar{d}_{\mathrm{R}} \quad \bar{d}_{\mathrm{G}} \quad \bar{d}_{\mathrm{B}} \quad e \quad -\nu),$$

$$10 = \begin{pmatrix} 0 & \bar{u}_{\mathrm{B}} & -\bar{u}_{\mathrm{G}} & u_{\mathrm{R}} & d_{\mathrm{R}} \\ -\bar{u}_{\mathrm{B}} & 0 & \bar{u}_{\mathrm{R}} & u_{\mathrm{G}} & d_{\mathrm{G}} \\ \bar{u}_{\mathrm{G}} & -\bar{u}_{\mathrm{R}} & 0 & u_{\mathrm{B}} & d_{\mathrm{B}} \\ -u_{\mathrm{R}} & -u_{\mathrm{G}} & -u_{\mathrm{B}} & 0 & \bar{e} \\ -d_{\mathrm{R}} & -d_{\mathrm{G}} & -d_{\mathrm{B}} & -\bar{e} & 0 \end{pmatrix}$$

A mysterious fact in the SM

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}$$

$$\bar{u} \quad \bar{d} \quad \bar{e}$$

$$\longrightarrow$$

remarkable!

$$\bar{5} = (\bar{d}_{\mathrm{R}} \quad \bar{d}_{\mathrm{G}} \quad \bar{d}_{\mathrm{B}} \quad e \quad -\nu),$$

$$10 = \begin{pmatrix} 0 & \bar{u}_{\mathrm{B}} & -\bar{u}_{\mathrm{G}} & u_{\mathrm{R}} & d_{\mathrm{R}} \\ -\bar{u}_{\mathrm{B}} & 0 & \bar{u}_{\mathrm{R}} & u_{\mathrm{G}} & d_{\mathrm{G}} \\ \bar{u}_{\mathrm{G}} & -\bar{u}_{\mathrm{R}} & 0 & u_{\mathrm{B}} & d_{\mathrm{B}} \\ -u_{\mathrm{R}} & -u_{\mathrm{G}} & -u_{\mathrm{B}} & 0 & \bar{e} \\ -d_{\mathrm{R}} & -d_{\mathrm{G}} & -d_{\mathrm{B}} & -\bar{e} & 0 \end{pmatrix}$$

There are many models composed of chiral fermions with SM gauge charges, which do not satisfy this property.

R. Foot et al., PRD 39 (1989)

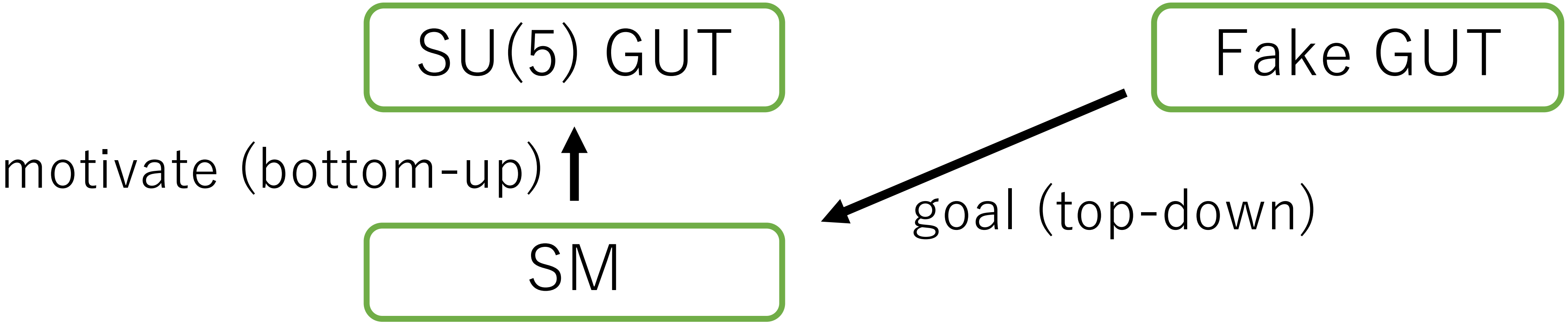
Why is the above structure unified into $\bar{5}$, 10 realized ?

$$\longrightarrow$$

Fake GUT !

Contents

Position of the fake GUT



SU(5) GUT

Fake GUT

motivate (bottom-up)

goal (top-down)

SM

Different framework from GUT where f_{SM}'s are unified into common multiplets at the HE.

“Fake” means the fake GUT explains the SM matter structure as well as GUT, but this is not GUT.

What is Fake GUT ?

fermions		SM fermions at the HE
chiral	$\bar{5}$, 10	**Not necessarily** embedded into $\bar{5}$, 10
vector-like	$\boldsymbol{H, \bar{H}}$	**can be** embedded into $H, \bar{H}$

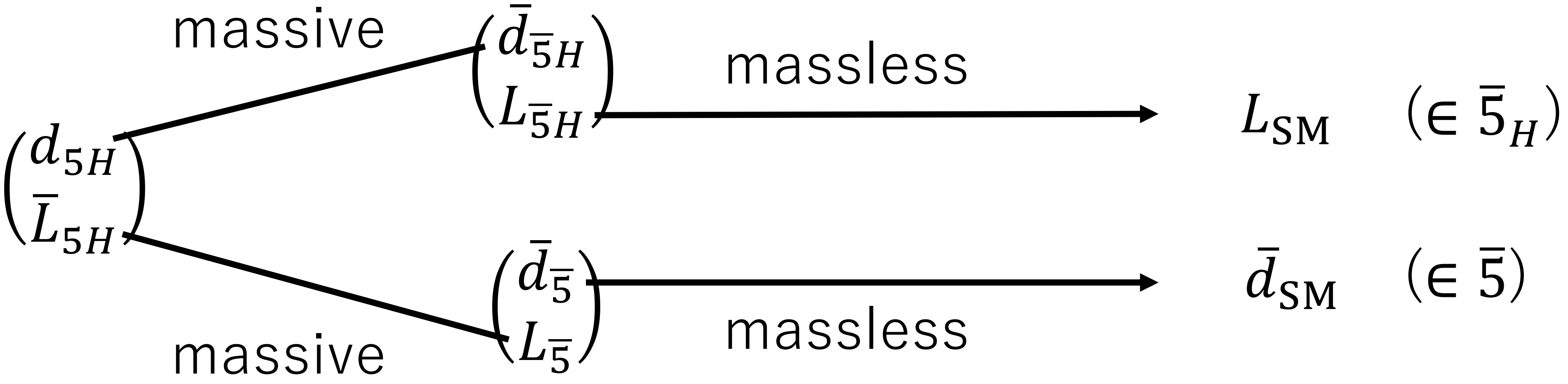
Even if f_{SM}'s are not embedded into $\bar{5}$ and 10 at the HE,
only f_{SM}'s remain automatically at the LE.

How SM fermions, f_{SM}, remain at the low energy (Fake GUT, G=SU(5))

Fermions chiral $\bar{5}, 10$ **vector-like $5_H, \bar{5}_H$**

$\bar{d}_{\mathrm{SM}}$ or L_{SM} can be contained in $\bar{5}_H$

– simple case



$\begin{pmatrix} d_{5H} \\ \bar{L}_{5H} \end{pmatrix}$

massive

$\begin{pmatrix} \bar{d}_{\bar{5}H} \\ L_{\bar{5}H} \end{pmatrix}$

massless

L_{SM} $(\in \bar{5}_H)$

massive

$\begin{pmatrix} \bar{d}_{\bar{5}} \\ L_{\bar{5}} \end{pmatrix}$

massless

$\bar{d}_{\mathrm{SM}}$ $(\in \bar{5})$

How SM fermions, f_{SM}, remain at the low energy

- Mass term

$$-\mathcal{L}_{mass} = \mathbf{5}_{\boldsymbol{H}}\,(M_{5V} + \langle 24_H \rangle)\,\bar{\mathbf{5}}_{\boldsymbol{H}} \;+\; \mathbf{5}_{\boldsymbol{H}}\,(M_{5C} + \langle 24_H \rangle)\,\bar{5}$$

previous simple case,

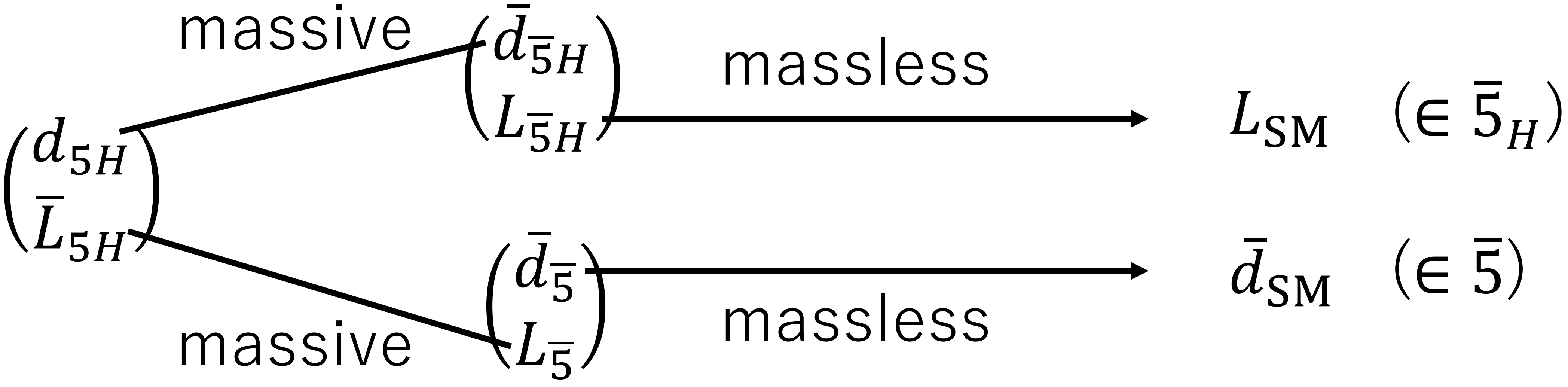
$$M_{5V} + \langle 24_H \rangle = diag(v, v, v, 0, 0),$$
$$M_{5C} + \langle 24_H \rangle = diag(0, 0, 0, v, v)$$

In this time,

$$\begin{aligned} -\mathcal{L}_{mass} &= 5_H\,(M_{5V} + \langle 24_H \rangle)\,\bar{5}_H \quad + \quad 5_H\,(M_{5C} + \langle 24_H \rangle)\,\bar{5} \\ &= v\, d_{5H}\, \bar{d}_{\bar{5}H} + \mathbf{0} \times \bar{L}_{5H}\, L_{\bar{5}H} \quad + \quad \mathbf{0} \times d_{5H}\, \bar{d}_{\bar{5}} + v\, \bar{L}_{5H}\, L_{\bar{5}} \end{aligned}$$

How SM fermions, f_{SM}, remain at the low energy

$$-\mathcal{L}_{mass} = v\, d_{5H}\, \bar{d}_{\bar{5}H} + \underline{\mathbf{0} \times \bar{L}_{5H}\, L_{\bar{5}H}} \quad + \quad \underline{\mathbf{0} \times d_{5H}\, \bar{d}_{\bar{5}}} + v\, \bar{L}_{5H}\, L_{\bar{5}}$$



$\begin{pmatrix} d_{5H} \\ \bar{L}_{5H} \end{pmatrix}$

massive $\begin{pmatrix} \bar{d}_{\bar{5}H} \\ L_{\bar{5}H} \end{pmatrix}$ massless $\longrightarrow$ $L_{\mathrm{SM}} \quad (\in \bar{5}_H)$

massive $\begin{pmatrix} \bar{d}_{\bar{5}} \\ L_{\bar{5}} \end{pmatrix}$ massless $\longrightarrow$ $\bar{d}_{\mathrm{SM}} \quad (\in \bar{5})$

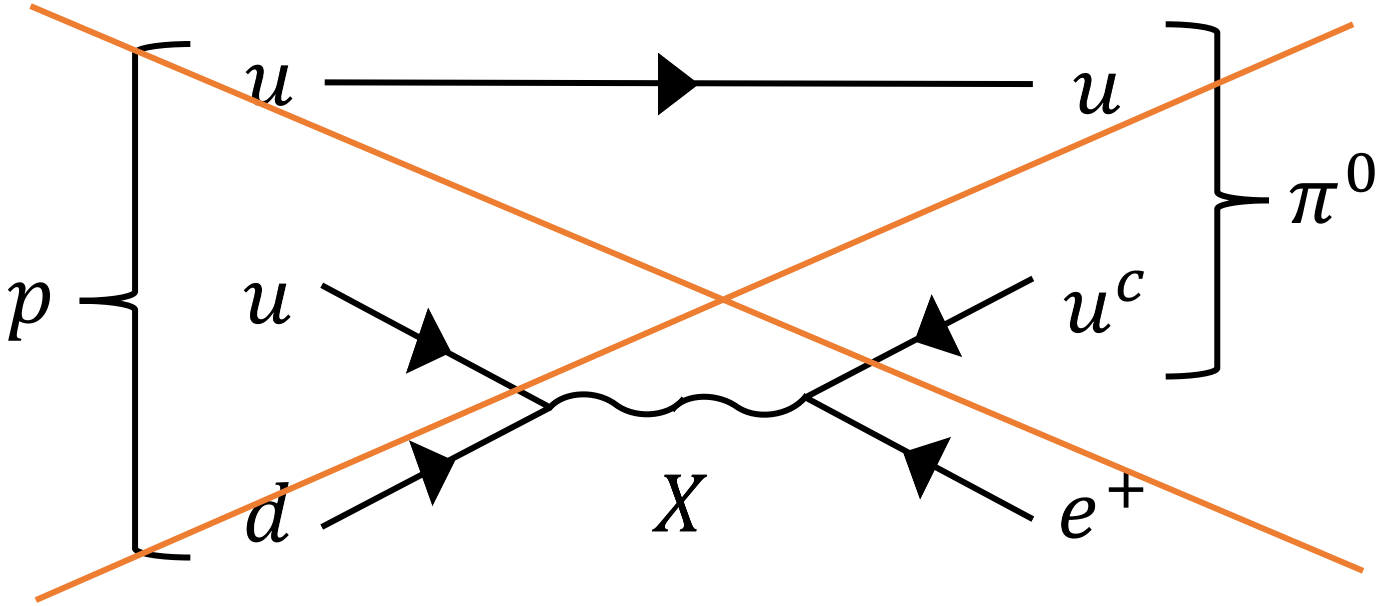
Even if f_{SM}'s are not embedded into $\bar{5}$ and 10 at the high energy, only f_{SM}'s remain automatically.

Proton decay in the Fake GUT

If the quarks and leptons have the completely different origin,

$\longrightarrow$ Proton does not decay.

$$p \to \pi^0 + e^+$$



The case of mixing

Consider the more general case,

$$M_{5V} + \langle 24_H \rangle = diag(v, v, v, \delta, \delta),$$

$$M_{5C} + \langle 24_H \rangle = diag(0, 0, 0, v, v)$$

In this time,

$$-\mathcal{L}_{mass} = 5_H\,(M_{5V} + \langle 24_H \rangle)\bar{5}_H \quad + \quad 5_H\,(M_{5C} + \langle 24_H \rangle)\,\bar{5}$$

$$= \underline{v\, d_{5H}\, \bar{d}_{\bar{5}H}} + \delta \times \bar{L}_{5H}\, L_{\bar{5}H} \quad + \quad \underline{0 \times d_{5H}\, \bar{d}_{\bar{5}}} + v\, \bar{L}_{5H}\, L_{\bar{5}}$$

$$\longrightarrow \qquad \bar{d}_{\bar{5}} = \bar{d}_{\mathrm{SM}}$$

The case of mixing

- Mass terms of L component

$$-\mathcal{L}_{L\,mass} = v\,\bar{L}_{5H}\,L_{\bar{5}} + \delta \times \bar{L}_{5H}\,L_{\bar{5}H}$$

$$= \bar{L}_{5H}\,(v \quad \delta)\begin{pmatrix} L_{\bar{5}} \\ L_{\bar{5}H} \end{pmatrix} = \bar{L}_{5H}\,(M \quad \mathbf{0})\begin{pmatrix} L_M \\ \boldsymbol{L_{\mathrm{SM}}} \end{pmatrix}$$

Even in the mixing case ($L_{\bar{5}}$ - $L_{\bar{5}H}$),
only f_{SM}'s remain automatically.

- Fate of the proton ($\bar{d}_{\bar{5}}$, Q_{10}, $\bar{u}_{10}$ = SM quarks)

Proton can decay ($\because L_{\bar{5}} \ni L_{\mathrm{SM}}$).

$$O_{p-\mathrm{decay}} \propto (\bar{d}^{\dagger}_{\bar{5}}\,\bar{u}^{\dagger}_{10})(L_{\bar{5}}\,Q_{10}) \ni \sin\theta \times \underline{(\bar{d}^{\dagger}_{\mathrm{SM}}\,\bar{u}^{\dagger}_{\mathrm{SM}})(L_{\mathrm{SM}}\,Q_{\mathrm{SM}})}$$

The case of mixing

mass terms of L component

$$-\mathcal{L}_{L\,mass} = v\,\bar{L}_{5H}\,L_{\bar{5}} + \delta \times \bar{L}_{5H}\,L_{\bar{5}H}$$

$$= \bar{L}_{5H}\begin{pmatrix} v & \delta \end{pmatrix}\begin{pmatrix} L_{\bar{5}} \\ L_{\bar{5}H} \end{pmatrix} = \bar{L}_{5H}\begin{pmatrix} M & 0 \end{pmatrix}\begin{pmatrix} L_M \\ L_{\mathrm{SM}} \end{pmatrix}$$

$$\xrightarrow{\text{generally}} \quad \bar{L}_{5H,i}\begin{pmatrix} M_{ij} & M'_{ik} \end{pmatrix}\begin{pmatrix} L_{\bar{5},j} \\ L_{\bar{5}H,k} \end{pmatrix} = \bar{L}_{5H,i}\begin{pmatrix} M_i\delta_{ij} & 0 \end{pmatrix}\begin{pmatrix} L_{M,j} \\ L_{\mathrm{SM},k} \end{pmatrix}$$

Generally,

different predictions of proton decay from SU(5) GUT

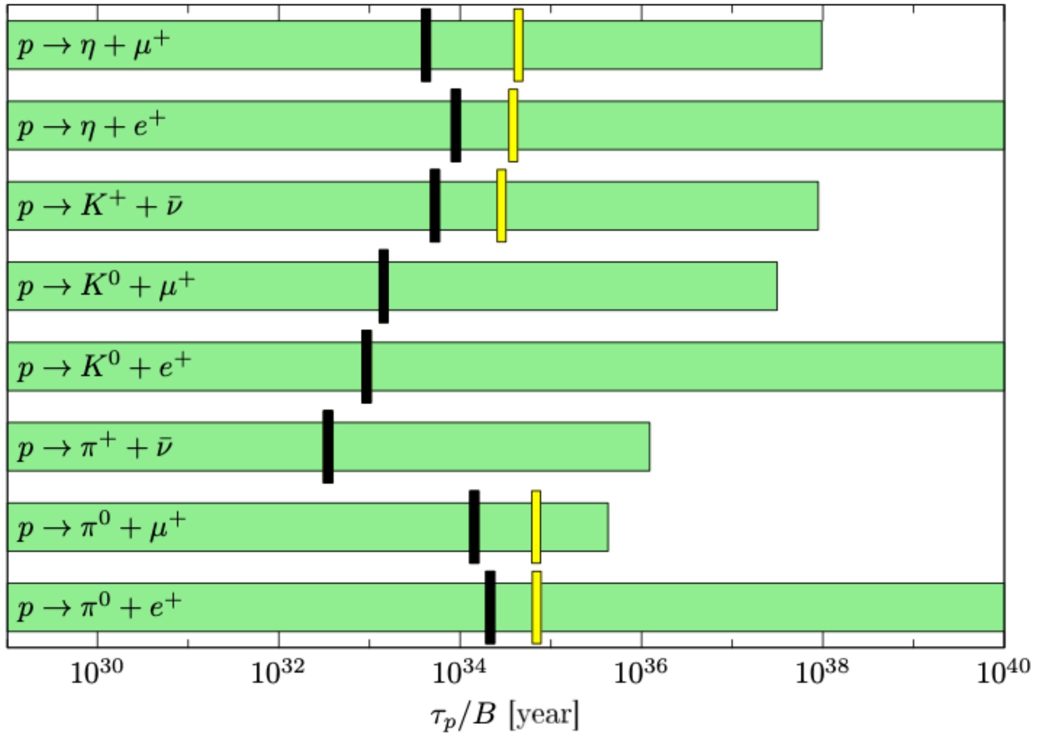
(∵ generation mixing, e.g. $\boldsymbol{L_{\bar{5},1} \ni L_{\mathrm{SM},\mu}}$).

Proton lifetime

($\bar{5}$, 10 ∋ SM quarks, $L_{\bar{5},1}$ $(\bar{e}_{10,1})$ ∋ $L_{\mathrm{SM},\mu}$ $(\bar{e}_{\mathrm{SM},\mu})$)

black : SK constraint
yellow : HK prospect

$$p \rightarrow \pi^0 + \mu^+$$



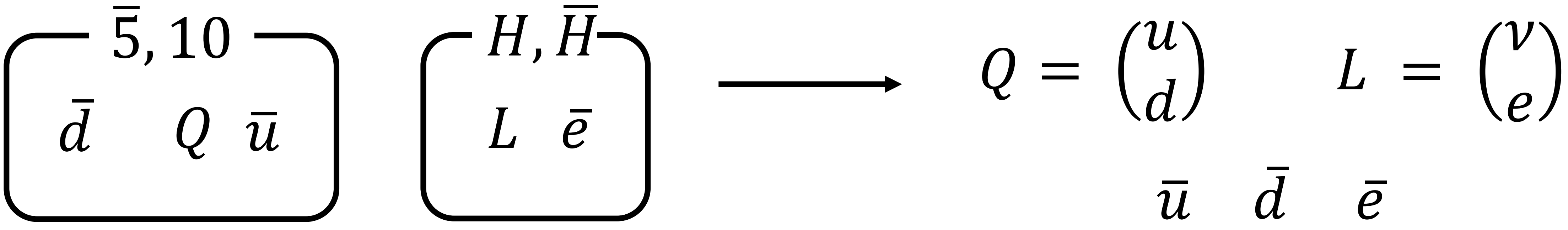
M. Ibe et al., JHEP 07 (2022) 087

Note that proton lifetime depends on size of mixings and proton decay is not necessarily invisible in the HK.

Contents

3. Conclusion

- At HE At LE



$\bar{5}, 10$: $\bar{d}$ Q $\bar{u}$

$H, \bar{H}$: L $\bar{e}$

$\longrightarrow$

$Q = \begin{pmatrix} u \\ d \end{pmatrix}$ $L = \begin{pmatrix} \nu \\ e \end{pmatrix}$

$\bar{u}$ $\bar{d}$ $\bar{e}$

Let's consider without being deceived by the appearance of the matter unification.

- $$\Gamma_{\text{fake GUT}}\,(p \to A + \cdots) \neq \Gamma_{\text{conventional GUT}}\,(p \to A + \cdots),$$
$$\frac{\Gamma_{\text{fake GUT}}\,(p \to A + \cdots)}{\Gamma_{\text{fake GUT}}\,(p \to B + \cdots)} \neq \frac{\Gamma_{\text{conventional GUT}}\,(p \to A + \cdots)}{\Gamma_{\text{conventional GUT}}\,(p \to B + \cdots)}$$

THANK YOU FOR YOUR ATTENTION!

BACK UP

Definition of the fake GUT

- $$\boxed{\mathrm{SU}(5)\times \mathrm{H}} \xrightarrow{\text{Symmetry breaking}} \boxed{\mathrm{SM}}$$
- Cartan of SU(5) ⊃ Cartan of ($\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$, $\mathrm{U}(1)_Y$)
- Fermions

  Chiral fermion $\bar{\mathbf{5}}, \mathbf{10}$

  Vector-like fermion $H, \bar{H}$ (under $\mathrm{SU}(5)\times \mathrm{H}$)
- (Some of $\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ may be diagonal subgroup of $\mathrm{SU}(5)\times \mathrm{H}$)

Definition of the fake GUT

- $$\mathrm{SU(5)} \times \mathrm{H} \xrightarrow{\text{Symmetry breaking}} \mathrm{SM}$$

- Fermions

  Chiral fermion $\bar{\mathbf{5}}, \mathbf{10}$

  In the SU(5) GUT, all f_{SM}'s are contained in $\bar{\mathbf{5}}$ and $\mathbf{10}$.

  Vector-like fermion $H, \bar{H}$

Definition of the fake GUT

- $$\boxed{\mathrm{SU(5)} \times \mathrm{H}} \xrightarrow{\text{Symmetry breaking}} \boxed{\mathrm{SM}}$$

- Fermions

  Chiral fermion $\bar{\mathbf{5}}, \mathbf{10}$

  Vector-like fermion $H, \bar{H}$

In the fake GUT, f_{SM}’s can be contained in vector-like fermions.

An example of anomaly-free chiral fermion set

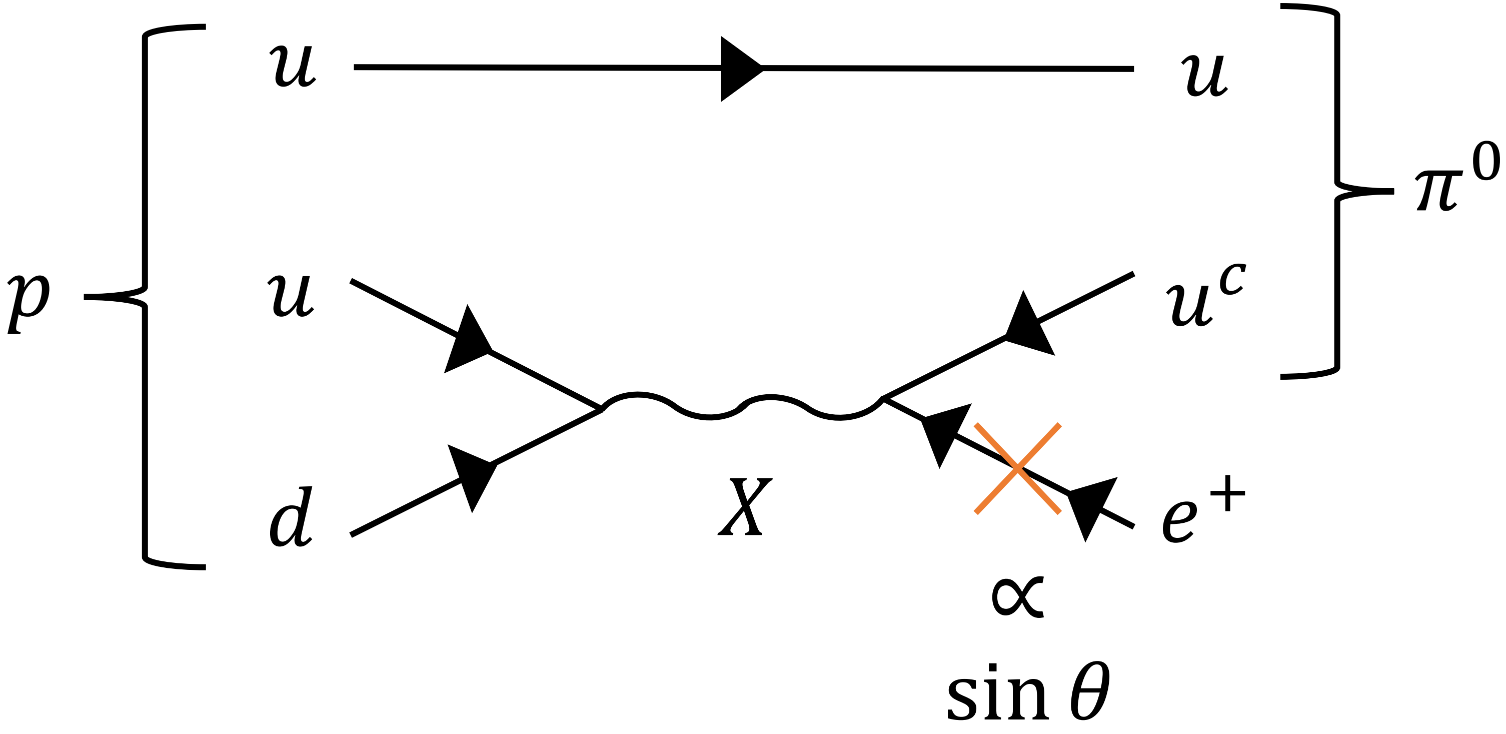
R. Foot et al, PRD 39 (1989) 3411-3424

	Q_1	Q_2	Q_3	Q_4	L_1	L_2	L_3	L_4
SU(3)$_{\text{c}}$	$\bar{3}$	3	3	$\bar{3}$	1	1	1	1
SU(2)$_{\text{L}}$	3	2	2	1	3	2	2	1
U(1)$_{\text{Y}}$	$-1/3$	$5/6$	$-1/6$	$-1/3$	1	$-3/2$	$-1/2$	1

Above chiral fermion set cannot be embedded into $\bar{5}$ and 10.

Diagram of proton decay ($p \to \pi^0 + e^+$)

If $L_{\bar{5}}$ and $\bar{e}_{10}$ contain $\sin\theta\, L_{\mathrm{SM}}$ and $\sin\theta\ \bar{e}_{\mathrm{SM}}$ respectively,



Proton lifetime

($L_{\bar{5},1}$ and $\bar{e}_{10,1}$ contain $\sin\theta\, L_{\mathrm{SM},e}$ and $\sin\theta\ \bar{e}_{\mathrm{SM},e}$)

$$\tau(p \to \pi^0 + e^+) \cong 5 \times 10^{26}\ \frac{1}{\sin^2\theta}\left(\frac{M_X/g_5}{10^{14}\ \mathrm{GeV}}\right)^4\ \mathrm{yrs}$$

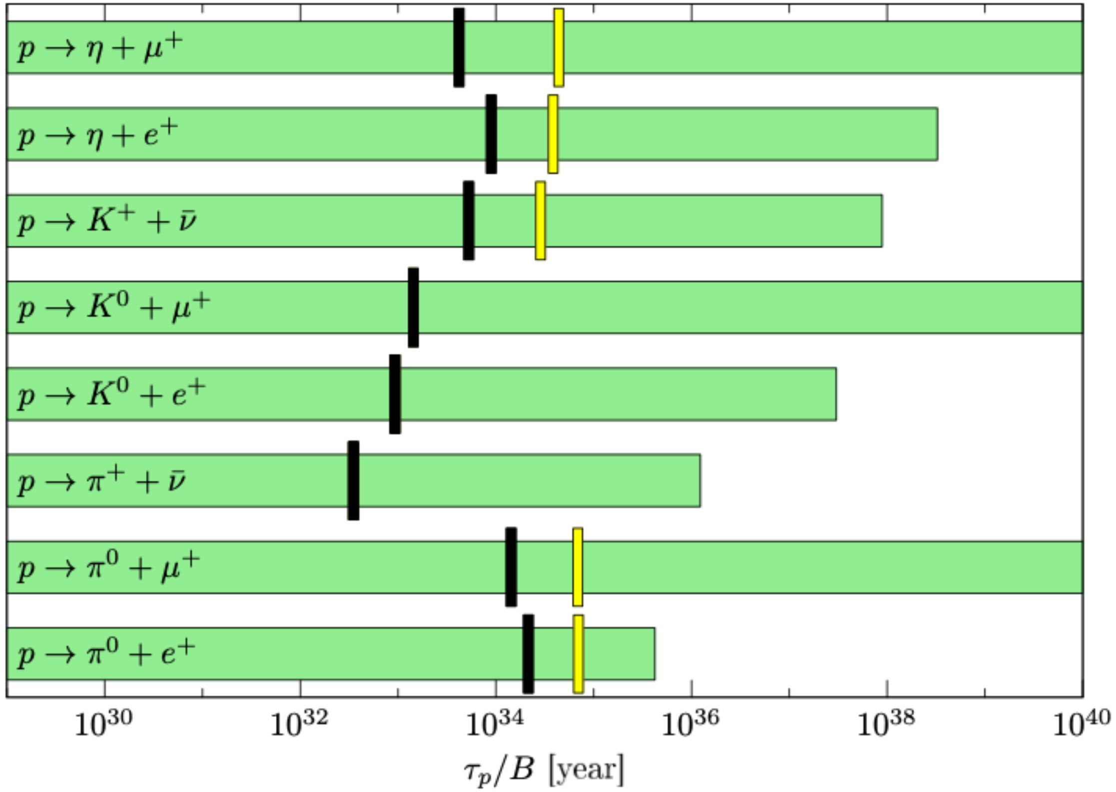
$\longrightarrow$ $\sin\theta \lesssim 10^{-4}$ due to $\tau(p \to \pi^0 + e^+) > 2.4 \times 10^{34}$ yrs

A. Takenaka et al. (SK collaboration) PRD102, 112011 (2020)

Proton lifetime

$$-\mathcal{L}_{mass} = v\left(\bar{L}_{H,1}\ \ \bar{L}_{H,2}\right)\begin{pmatrix}1 & 0 & 10^{-4} & 0\\ 0 & 1 & 0 & 0\end{pmatrix}\left(L_{\bar{5},1}\ \ L_{\bar{5},2}\ \ L_{H,1}\ \ L_{H,2}\right)^T$$

$$= v\,\bar{L}_{H,1}\left(1\ \ 10^{-4}\right)\left(L_{\bar{5},1}\ \ L_{H,1}\right)^T + v\,\bar{L}_{H,2}\,L_{\bar{5},2}$$



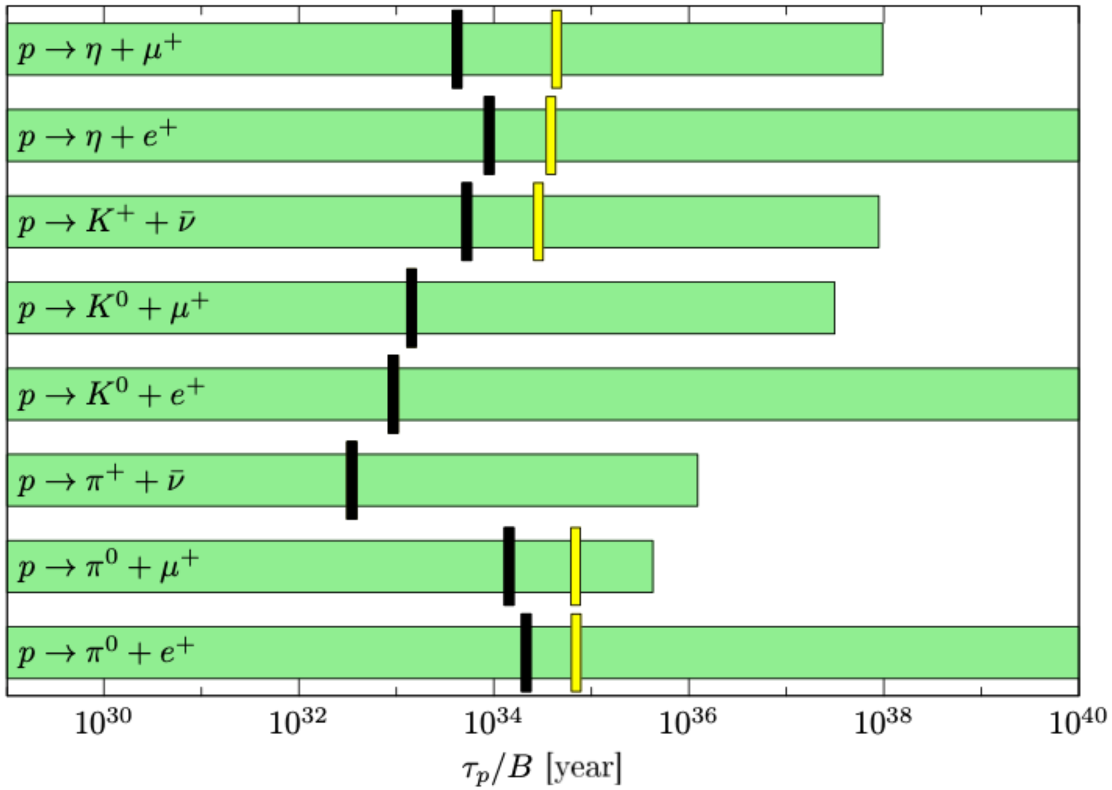
black : SK

yellow : HK

Proton lifetime

$$-\mathcal{L}_{mass} = v\left(\bar{L}_{H,1}\ \ \bar{L}_{H,2}\right)\begin{pmatrix} 1 & 0 & 0 & 10^{-4} \\ 0 & 1 & 0 & 0 \end{pmatrix}\left(L_{\bar{5},1}\ \ L_{\bar{5},2}\ \ L_{H1}\ \ L_{H,2}\right)^{T}$$

$$= v\,\bar{L}_{H,1}\left(1\ \ 10^{-4}\right)\left(L_{\bar{5},1}\ \ L_{H,2}\right)^{T} + v\,\bar{L}_{H,2}\,L_{\bar{5},2}$$



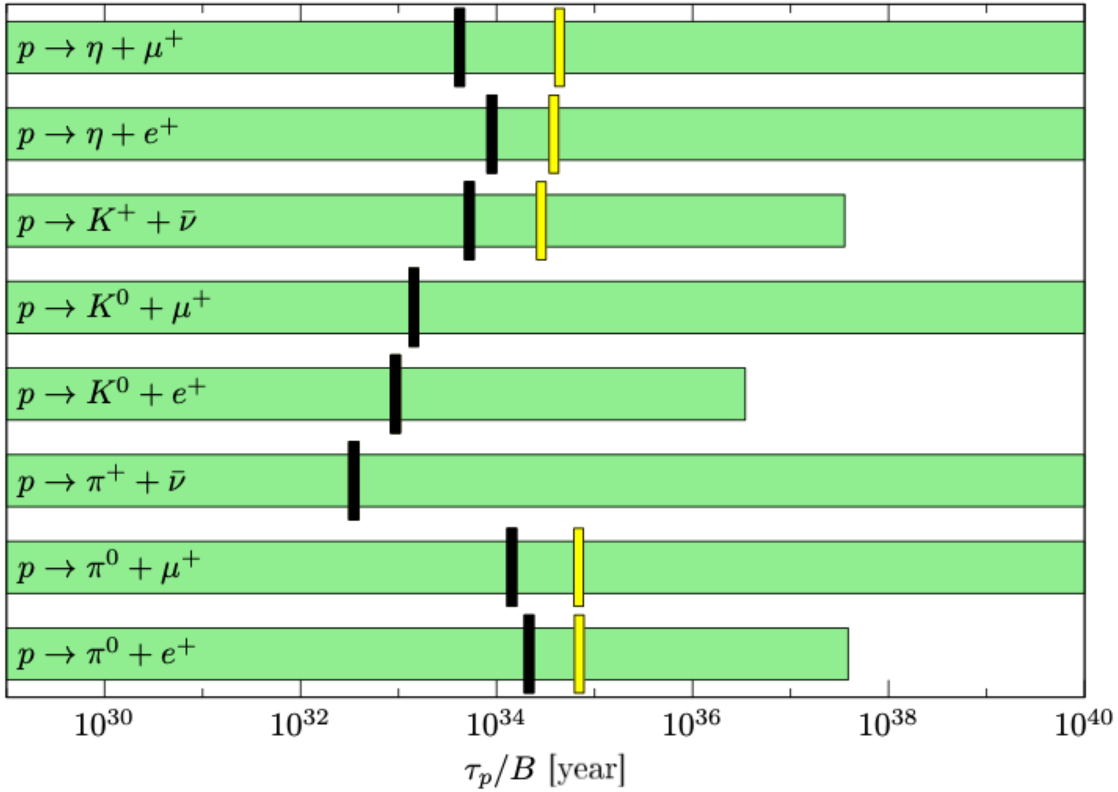
black : SK

yellow : HK

Proton lifetime

$$-\mathcal{L}_{mass} = v\left(\bar{L}_{H,1}\ \ \bar{L}_{H,2}\right)\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 10^{-4} & 0\end{pmatrix}\left(L_{\bar{5},1}\ \ L_{\bar{5},2}\ \ L_{H1}\ \ L_{H,2}\right)^T$$

$$= v\,\bar{L}_{H,2}\left(1\ \ 10^{-4}\right)\left(L_{\bar{5},2}\ \ L_{H,1}\right)^T + v\,\bar{L}_{H1}\,L_{\bar{5},1}$$



black : SK

yellow : HK

$SU(5) \times U(2)_H$ model

Fermions ($SU(5), SU(2)_H, U(1)_H$)

$\bar{5} : (\bar{5}, 1, 0)$ $\qquad$ $10 : (10, 1, 0)$

$L_H : (1, 2, -1/2)$ $\qquad$ $\bar{L}_H : (1, 2, 1/2)$

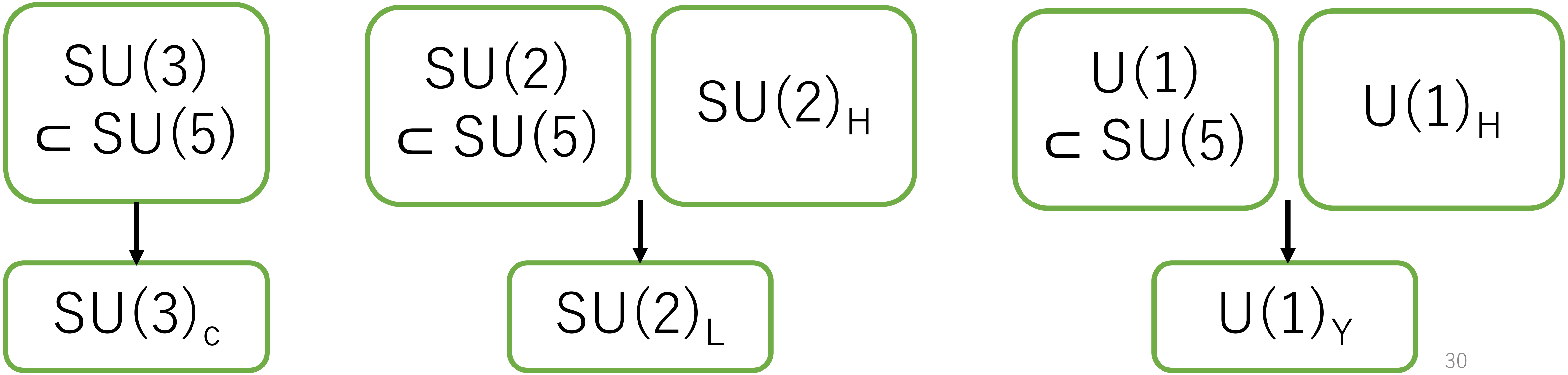
$E_H : (1, 1, -1)$ $\qquad$ $\bar{E}_H : (1, 1, 1)$

- As we will show later, SM leptons are mostly contained in L_H and $\bar{E}_H$.
- SM quarks are all contained in $\bar{5}$ and 10.

$\mathrm{SU(5)} \times \mathrm{U(2)_H}$ model

Scalar ϕ_2 : $(5, 2, -1/2)$ of ($\mathrm{SU(5)}$, $\mathrm{SU(2)_H}$,$\mathrm{U(1)_H}$)

$$\langle \phi_2 \rangle = \begin{pmatrix} \underbrace{\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}}_{\mathrm{SU(3)}} & \underbrace{\begin{matrix} v_2 & 0 \\ 0 & v_2 \end{matrix}}_{\mathrm{SU(2)}} \end{pmatrix} \left.\vphantom{\begin{matrix} 0 \\ 0 \end{matrix}}\right\} \mathrm{SU(2)_H}$$



$\mathrm{SU(3)} \subset \mathrm{SU(5)}$ → $\mathrm{SU(3)_c}$

$\mathrm{SU(2)} \subset \mathrm{SU(5)}$, $\mathrm{SU(2)_H}$ → $\mathrm{SU(2)_L}$

$\mathrm{U(1)} \subset \mathrm{SU(5)}$, $\mathrm{U(1)_H}$ → $\mathrm{U(1)_Y}$

$SU(5) \times U(2)_H$ model

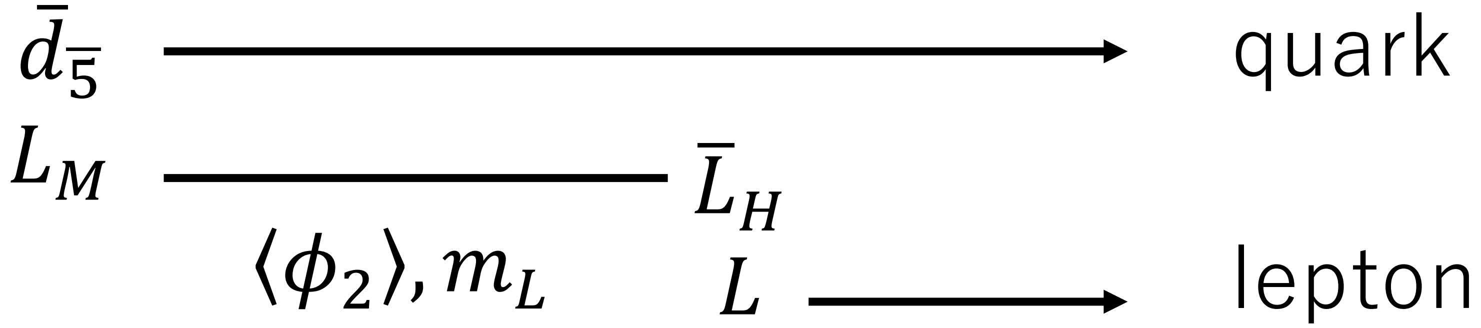
Lagrangian

$$\mathcal{L} = m_L \, L_H \, \bar{L}_H + \lambda_L \, \bar{5} \, \phi_2 \bar{L}_H + m_E \, E_H \, \bar{E}_H + \frac{\lambda_E}{\Lambda} E_H \, \phi_2^\dagger \phi_2^\dagger \, 10$$

(E_H and $\bar{E}_H$ are omitted) $\downarrow$

$$\bar{L}_H \begin{pmatrix} \lambda_L \frac{v_2}{\sqrt{2}} & m_L \end{pmatrix} \begin{pmatrix} L_{\bar{5}} \\ L_H \end{pmatrix} \longrightarrow \bar{L}_H \, (M_L \ \ 0) \begin{pmatrix} L_M \\ L \end{pmatrix}$$

Only SM fermions remain massless at the low energy.



$\bar{d}_{\bar{5}}$ → quark

L_M — $\bar{L}_H$

$\langle \phi_2 \rangle, m_L$

L → lepton

$\mathrm{SU}(5) \times \mathrm{U}(2)_\mathrm{H}$ model

Mixing of lepton components

L : SM lepton

L_M : heavy lepton

$$\begin{pmatrix} L_{\bar{5}} \\ L_H \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} L_M \\ L \end{pmatrix} \qquad \tan\theta = \frac{m_L}{\lambda_L\, v_2}$$

$\mathrm{SU}(5) \times \mathrm{U}(2)_{\mathrm{H}}$ model

Yukawa interactions

We consider a case one SM Higgs remains in the low energy.

Scalar containing the SM Higgs

$H_5 : (5, 1, 0\ )$

$H_2 : (1, 2, 1/2\ )$

$$H_5 = \begin{pmatrix} h_5^{color} \\ h_5^{SM} \end{pmatrix}$$

$$H_2 = h_2^{SM}$$

Higgs mixing term

$$\mathcal{L}_{52\, mix} = \ \mu_{mix} H_2 \phi_2 H_5^* + h.c.$$

$$h^{SM} \ = \cos\theta_h \, h_2^{SM} - \sin\theta_h \, h_5^{SM}$$

$\mathrm{SU}(5) \times \mathrm{U}(2)_{\mathrm{H}}$ model

Yukawa interactions

$$\mathcal{L}_{YQ} = -(y_5)_{ij}\,\bar{5}_i 10_j H_5^* - (y_{10})_{ij} 10_i 10_j H_5 + h.c.$$

$$\mathcal{L}_{YL} = -(y_{LE})_{ij} L_{Hi} \bar{E}_{Hj} H_2^* + h.c.$$

$$(y_u^{SM})_{ij} = -\sin\theta_h\,(y_{10})_{ij}$$

$$\left(y_d^{SM}\right)_{ij} = -\sin\theta_h\,(y_5)_{ij}$$

$$(y_e^{SM})_{ij} = \cos\theta_h\,(y_{LE})_{ij} + \mathcal{O}(\theta_L\theta_E)\sin\theta_h\,(y_5)_{ij}$$

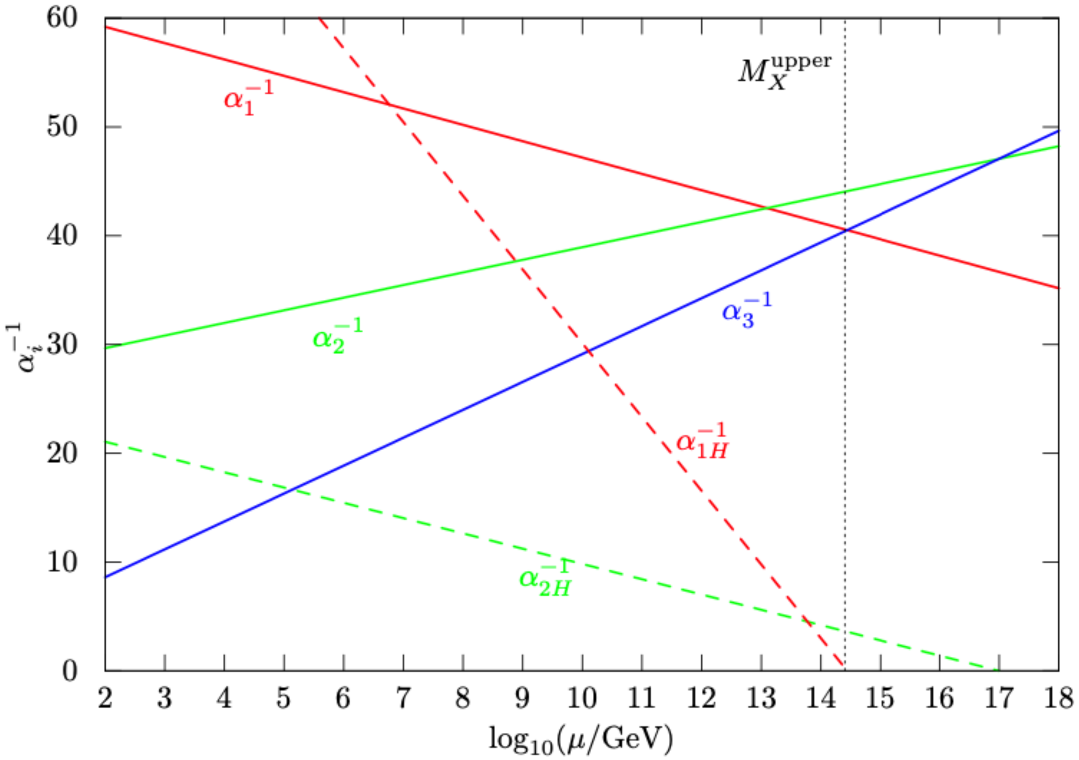
$\mathrm{SU}(5) \times \mathrm{U}(2)_{\mathrm{H}}$ model

Gauge couplings

$$\alpha_1^{-1}(M_X) = \alpha_5^{-1}(M_X) + \frac{3}{5}\alpha_{1H}^{-1}(M_X)$$

$$\alpha_2^{-1}(M_X) = \alpha_5^{-1}(M_X) + \alpha_{2H}^{-1}(M_X)$$

$$\alpha_3^{-1}(M_X) = \alpha_5^{-1}(M_X)$$



$$M_X^{\mathrm{upper}} \cong 10^{14.4}\ \mathrm{GeV}$$