PRESENTATION

U(1) gauge field & charged particles in axion inflation

Tomohiro Fujita
(Waseda Inst. Adv. Study & RESCEU Tokyo U.)

2204.01180 and 2206.12218 with
Kume (RESCEU), Mukaida (KEK), Tada (Nagoya)

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WiAS
早稲田大学高等研究所
Waseda Institute for Advanced Study

RESEARCH CENTER FOR THE EARLY UNIVERSE
THE UNIVERSITY OF TOKYO
RESCEU

Plan of Talk

1. Motivation
2. Review the case without ψ
3. Solve the system of A & ψ
4. Results
5. Summary

1 Motivation

Setup inflaton ϕ – photon A_μ – fermion ψ coupled system

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - V(\phi)}_{\text{Axionic inflaton}} \underbrace{- \frac{1}{4}FF - \frac{\alpha}{4f}\phi F\tilde{F}}_{\text{U(1) gauge field coupled to } \phi} \underbrace{+ i\bar{\psi}\not{D}\psi}_{\text{Charged fermion}}$$

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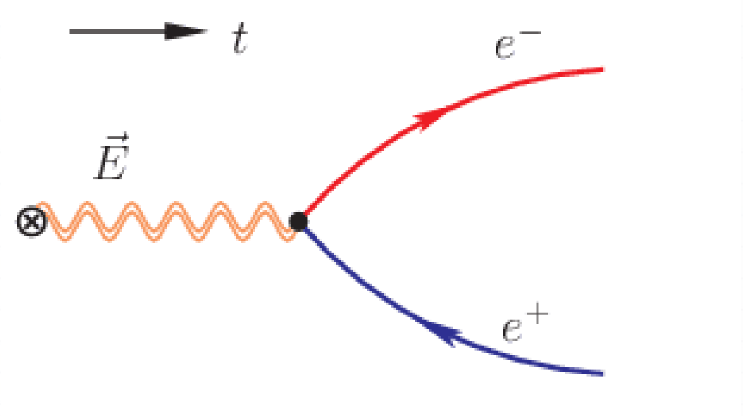
Motivations

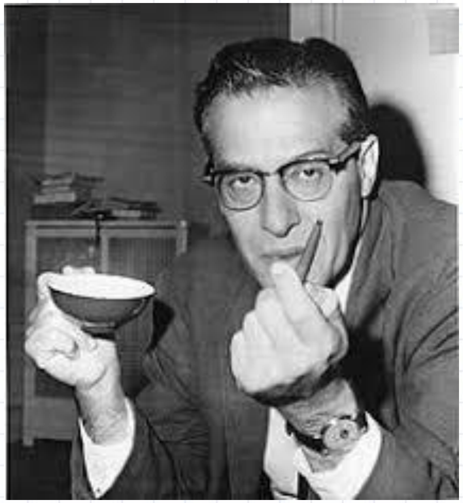
① Particle Physics: Shift symmetry of ϕ ➡ Reheating requires coupling

② Phenomenology: Helical B ➡ Baryogenesis & Magnetogenesis

③ Formal interest: Strong E ➡ Schwinger effect

Schwinger effect





Julian Schwinger(1918～1994)

- Sufficiently strong ($eE > m^2$) electric field causes a **pair production** of charged particles. It's a **non-perturbative process** in QED.
- Not yet detected. It may be observed by EBI or X-FEL etc...

G. V. Dunne, Eur. Phys. J. D55, 327-340
A. Ringwald, Phys. Lett. B510, 107-116

- In the early universe, however, It may have played an **important role**.

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Interactions

$$\begin{tikzcd}[column sep=huge]
\phi \arrow[r, bend left, "\text{Kinetic energy}"] \arrow[r, phantom, "\boxed{\text{CS coupling}}"] & A_\mu \arrow[l, bend left, "\text{Back-reaction}"] \arrow[r, bend left, "\text{Schwinger effect}"] \arrow[r, phantom, "\boxed{\text{Gauge coupling}}"] & \psi \arrow[l, bend left, "\text{EM conductivity}"]
\end{tikzcd}$$

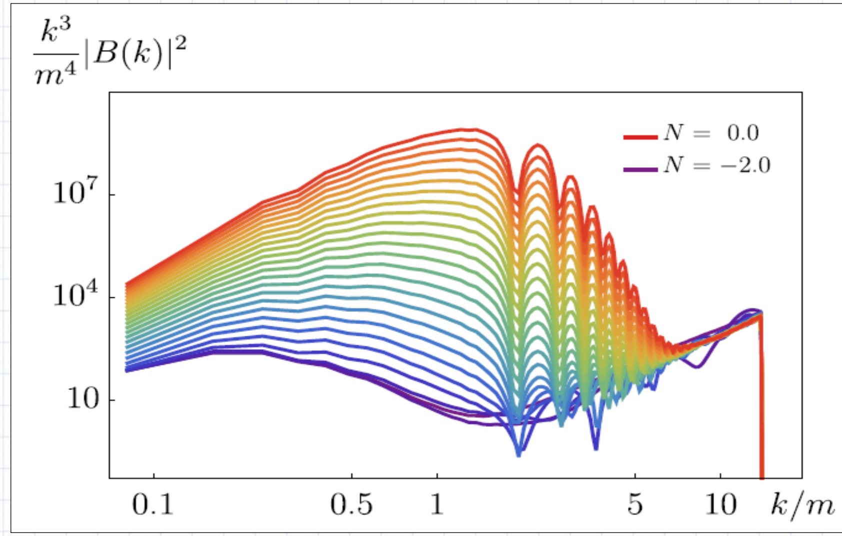
[Garretson+(1992), Field&Carroll(2000), Anber&Sorbo(2006)
Durrer+(2011), Fujita+(2015), Adshead+(2016),⋯]

Previous works

The $\phi - A_\mu$ system well studied

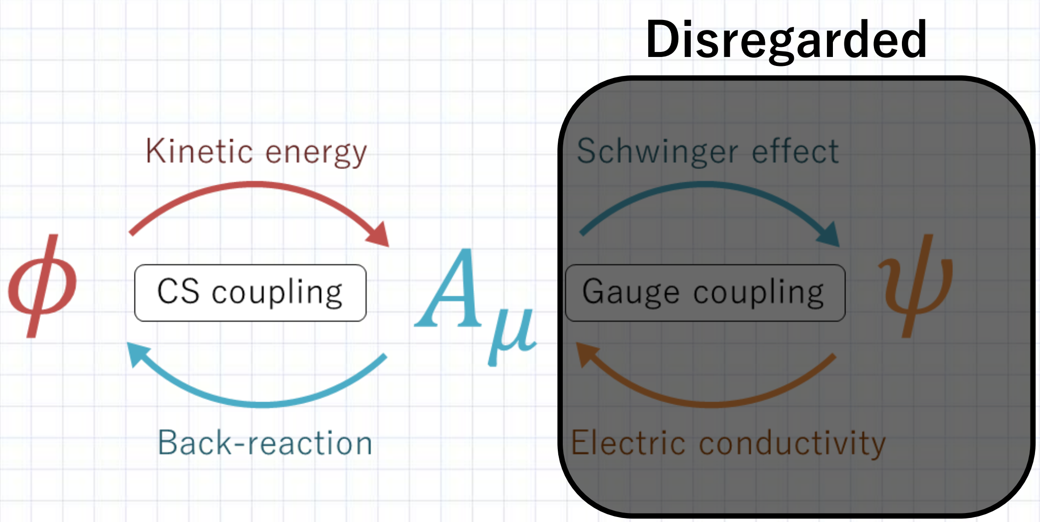
➡ A_μ production at inf. end is dominant

However, ψ is not yet included!



[Lattice simulation by Cuissa & Figueroa (2018)]

[See also Angelo's talk!!]



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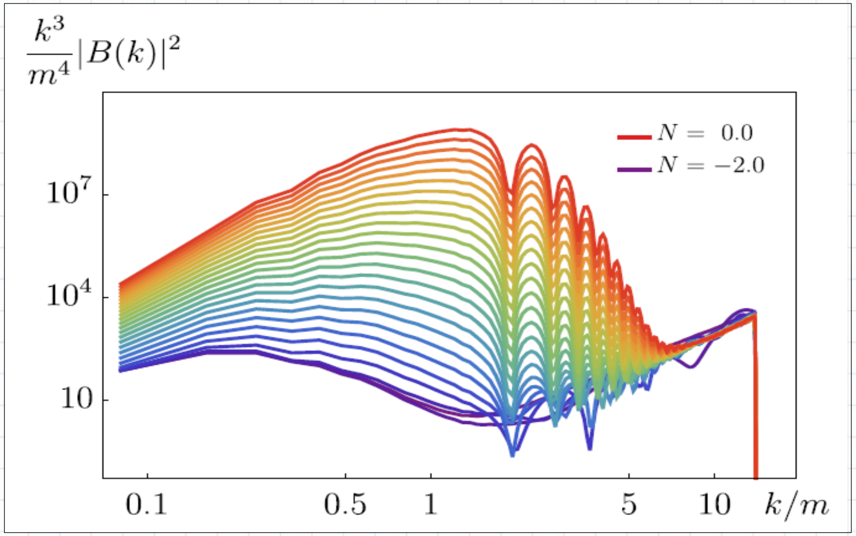

[Lattice simulation by Cuissa & Figueroa (2018)]

[See also Angelo's talk!!]

Difficulty

Non-linear & non-perturbative Dynamics

➡ $A(k), \psi(k)$: different k-modes are coupled

➡ System is close to neither free mode nor thermal equilibrium

We need a new approach to solve it

[See also Domcke, Ema, Mukaida(2019);
Gorbar, Schmitz, Sobol, Vilchinskii(2021)]

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2 Review no-charged-particle case

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - V(\phi)}_{\text{Axionic inflaton}} \underbrace{- \frac{1}{4}FF - \frac{\alpha}{4f}\phi F\tilde{F}}_{\text{U(1) gauge field coupled to } \phi} \underbrace{+ i\bar{\psi}D\psi}_{\text{Charged fermion}}$$

Assumption: the inflaton rolls at a constant velocity $\xi \equiv \dfrac{\alpha\dot{\phi}}{2fH}$

The EoM for the gauge field mode function $\mathcal{A}_\pm$ is given by

$$\left[\partial_\tau^2 + k^2 \boxed{\pm} 2k\frac{\xi}{\tau}\right]\mathcal{A}_\pm(\tau, k) = 0$$

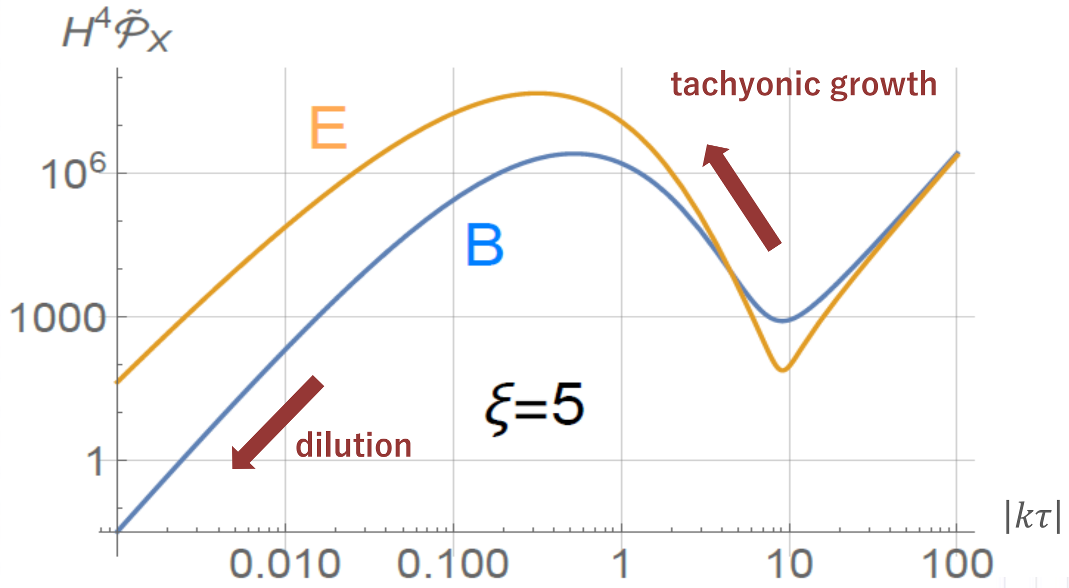
Either $\pm$ mode is amplified by the tachyonic instability.

In the slow-roll phase, an analytic solution is available.

If $\xi \equiv \frac{\alpha\dot{\phi}}{2fH} = const. > 0$ $\Rightarrow$ $\mathcal{A}_+ = \frac{1}{\sqrt{2k}} e^{\pi\xi/2} W_{-i\xi,1/2}\,(2ik\tau)$

EM field production

(without ψ)



1. Due to the exp amplification, very strong EMFs are produced, $E \gg B \gg H^2$.
2. The typical EM length scale is $L_{em} \simeq \xi/H$
3. The typical EM time scale is $t_{em} \simeq 1/H$

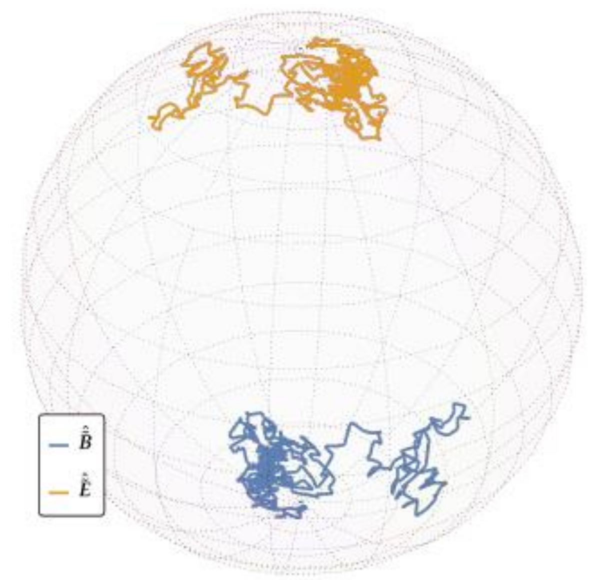
$$\tilde{\mathcal{P}}^+_{BB}(\tau,k) = a^{-4}\mathcal{P}^+_{BB}(\tau,k) = \frac{k^5}{2\pi^2 a^4}|\mathcal{A}_+(\tau,k)|^2 = \frac{|k\tau|^4 H^4}{4\pi^2}e^{\pi\xi}|W(-k\tau)|^2, \quad \tilde{\mathcal{P}}^+_{EE}(\tau,k) = a^{-4}\mathcal{P}^+_{EE}(\tau,k) = \frac{k^3}{2\pi^2 a^4}|\partial_\tau \mathcal{A}_+(\tau,k)|^2 = \frac{|k\tau|^4 H^4}{4\pi^2}e^{\pi\xi}|W'(-k\tau)|^2,$$

EM field orientation

(without ψ)

Since the parity is fully violated,
EM fields take an **anti-parallel** configuration.

$$\frac{\tilde{\mathscr{P}}^{+}_{BE}}{\sqrt{\tilde{\mathscr{P}}^{+}_{EE}\tilde{\mathscr{P}}^{+}_{BB}}} \xrightarrow{|k\tau|\ll 2\xi} -1.$$



Evolution of $\hat{\boldsymbol{E}}\cdot\hat{\boldsymbol{B}}$ for 0.5 e-folds

1. Due to the exp amplification, very strong EMFs are produced, $E \gg B \gg H^2$.
2. The typical EM length scale is $L_{em} \simeq \xi/H$
3. The typical EM time scale is $t_{em} \simeq 1/H$
4. E and B are anti-parallel, $\hat{\boldsymbol{E}}\cdot\hat{\boldsymbol{B}} = -1$

$$\tilde{\mathscr{P}}^{+}_{BB}(\tau,k) = a^{-4}\mathscr{P}^{+}_{BB}(\tau,k) = \frac{k^5}{2\pi^2 a^4}|\mathscr{A}_+(\tau,k)|^2 = \frac{|k\tau|^4 H^4}{4\pi^2}e^{\pi\xi}|W(-k\tau)|^2, \quad \tilde{\mathscr{P}}^{+}_{EE}(\tau,k) = a^{-4}\mathscr{P}^{+}_{EE}(\tau,k) = \frac{k^3}{2\pi^2 a^4}|\partial_\tau\mathscr{A}_+(\tau,k)|^2 = \frac{|k\tau|^4 H^4}{4\pi^2}e^{\pi\xi}|W'(-k\tau)|^2,$$

4 properties in the no charged particle case

1. Strong EMFs are produced: $E, B \gg H^2$

2. The EM length scale $L_{\mathrm{em}} \simeq \xi/H$

3. The EM time scale is $\tau_{\mathrm{em}} \simeq 1/H$

4. EMFs are anti-parallel: $\hat{\boldsymbol{E}} \cdot \hat{\boldsymbol{B}} = \mathbf{-1}$

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3 Solve the system of A and ψ

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - V(\phi)}_{\text{Axionic inflaton}} \underbrace{- \frac{1}{4}FF - \frac{\alpha}{4f}\phi F\tilde{F}}_{\text{U(1) gauge field}} + \underbrace{i\bar{\psi}D\psi}_{\text{Charged fermion}}$$

Assumption: the inflaton rolls at a constant velocity $\xi \equiv \frac{\alpha\dot{\phi}}{2fH}$

The EoMs for the gauge field and fermion are **coupled** and **non-linear**

$$\left[\hat{\gamma}^\mu\left(\partial_\mu + igQ\hat{A}_\mu\right) + \frac{3}{2}aH\hat{\gamma}^0\right]\hat{\psi} = 0$$

$$\partial_\tau^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau}\epsilon_{ijl}\partial_j A_l = a^2 e J_i \qquad J^\mu = \bar{\psi}\gamma^\mu\psi$$

We cannot exactly solve them… Then, we introduce two prescriptions

1. **Integrating out ψ**: Reduce the coupled EoMs into a single non-linear eq.
2. **Mean-field approx**: linear eq. for perturbation and consistency eq.

3 Integrating out ψ

[Domcke&Mukaida(2018)]

Remember the properties of the produced EMFs

1 $E, B \gg H^2$ 2 $L_{\rm em} \simeq \xi/H$ 3 $\tau_{\rm em} \simeq 1/H$

Typical momentum of the Schwinger produced fermion is $\boldsymbol{p_\psi \simeq \sqrt{eE}}$

Thus, a **hierarchy of scales** exists

$$\boxed{L_\psi \sim t_\psi \sim (eE)^{-1/2} \ll L_{\rm em} \sim t_{\rm em} \sim H^{-1}}$$

For fermions, EMFs look static and homogeneous, $\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{B}} \simeq const.$

Schwinger current induced by static, homogeneous & anti-parallel EMFs is known:

$$\partial_\tau (a^2 e J_i) = \frac{e^3 B E_i}{2\pi^2} \coth\left(\frac{\pi B}{E}\right).$$

NB; this current satisfies the chiral anomaly equation. Assumption: the fermion's mass is negligible, $m_\psi \ll H$

3 Integrating out ψ

We need not $\partial_\tau J_i$ but J_i itself.

Assumption: the physical EMFs are static, $E, B \propto a^2$, for $t \gtrsim H^{-1}$

$$\partial_\tau (a^2 e J_i) = \frac{e^3 B E_i}{2\pi^2} \coth\left(\frac{\pi B}{E}\right). \quad \Rightarrow \quad \boxed{e J_i \simeq \frac{e^3 B E_i}{6\pi^2 a^3 H} \coth\left(\frac{\pi B}{E}\right)}$$

Since $t_{\rm em} \simeq H^{-1}$, this expression may not be very accurate.
But on average, E and B amplitudes should be constant,
because the energy injection from the insflaton is constant, $\xi = const$.

This assumption may lead to O(1) Error?

We obtain a single non-linear EoM for A!!

$$\partial_\tau^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau} \epsilon_{ijl} \partial_j A_l = a^2 e J_i$$

3 Mean-field approximation

How to solve a full non-linear equation??

$$\partial_\tau^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau}\epsilon_{ijl}\partial_j A_l = a^2 e J_i \qquad\qquad eJ_i \simeq \frac{e^3 B E_i}{6\pi^2 a^3 H}\coth\left(\frac{\pi B}{E}\right)$$

We introduce **mean-field approx**. and split EMFs into a mean and a perturbation

$$\boldsymbol{E}(\tau,\boldsymbol{x}) \simeq \boldsymbol{E}_0 + \delta\boldsymbol{E}(\tau,\boldsymbol{x}), \qquad \boldsymbol{B}(\tau,\boldsymbol{x}) \simeq \boldsymbol{B}_0 + \delta\boldsymbol{B}(\tau,\boldsymbol{x}).$$

The Schwinger current is accordingly decomposed. $(\hat{\boldsymbol{E}}_0 \cdot \hat{\boldsymbol{B}}_0 = -1, \text{but } \delta\boldsymbol{E}\cdot\delta\boldsymbol{B} \neq -1)$

$$\begin{aligned}
a^2 e\boldsymbol{J} &= a^2 e(\boldsymbol{J}_0 + \delta\boldsymbol{J}),\\
a^2 e\boldsymbol{J}_0 &= \frac{e^3 B_0 E_0}{6\pi^2 aH}\coth\left(\frac{\pi B_0}{E_0}\right)\boldsymbol{e}_z,\\
a^2 e\delta\boldsymbol{J} &= \frac{e^3}{6\pi^2 aH}\Bigg[\left(\frac{B_0^3\delta E_z - E_0^3\delta B_z}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right) + (B_0\delta E_z + E_0\delta B_z)\frac{\pi B_0}{E_0}\operatorname{csch}^2\left(\frac{\pi B_0}{E_0}\right)\right)\boldsymbol{e}_z\\
&\quad + \frac{E_0^2 B_0\delta\boldsymbol{E} - B_0^2 E_0\delta\boldsymbol{B}}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right)\Bigg].
\end{aligned}$$

3 Linearized eq. for perturbation

The EoM for the perturbation is

$$\boxed{\left[\partial_z^2 - \frac{\Sigma}{z}\partial_z + 1 - \frac{2\xi_{\rm eff}}{z}\right]\mathscr{A}_+^{(\sigma)} = 0}$$

with **the electric and magnetic conductivity**:

$$\Sigma \equiv \Sigma_E + \Sigma_{E'}\sin^2\theta_{\boldsymbol{k}}, \qquad \xi_{\rm eff} \equiv \xi - \frac{1}{2}\left(\Sigma_B + \Sigma_{B'}\sin^2\theta_{\boldsymbol{k}}\right)$$

$$\hat{\boldsymbol{E}}_0 \cdot \boldsymbol{e}^{\pm}(\hat{\boldsymbol{k}}) = -\sin\theta_{\boldsymbol{k}}/\sqrt{2}.$$

$$\Sigma_E \equiv \frac{e^3 B_0}{6\pi^2 a^2 H^2}\left(\frac{E_0^2}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right)\right), \qquad \Sigma_{E'} \equiv \frac{e^3 B_0}{12\pi^2 a^2 H^2}\left(\frac{B_0^2}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right) + \frac{\pi B_0}{E_0}\operatorname{csch}^2\left(\frac{\pi B_0}{E_0}\right)\right)$$

$$\Sigma_B \equiv \frac{e^3 E_0}{6\pi^2 a^2 H^2}\left(\frac{B_0^2}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right)\right), \qquad \Sigma_{B'} \equiv \frac{e^3 E_0}{12\pi^2 a^2 H^2}\left(\frac{E_0^2}{E_0^2+B_0^2}\coth\left(\frac{\pi B_0}{E_0}\right) - \frac{\pi B_0}{E_0}\operatorname{csch}^2\left(\frac{\pi B_0}{E_0}\right)\right)$$

Fortunately, an analytic solution is available!

$$\mathscr{A}_+^{(\sigma)}(\tau, \boldsymbol{k}) = \frac{1}{\sqrt{2k}} e^{\pi\xi_{\rm eff}/2} z^{\Sigma/2}\left[c_1 W_{-i\xi_{\rm eff},(\Sigma+1)/2}(-2iz) + c_2 M_{-i\xi_{\rm eff},(\Sigma+1)/2}(-2iz)\right],$$

3 Consistent equation

We impose the **consistent equation** to determine the mean-field value,

Require the integration over the perturbation reproduces the mean field amplitude

Mean-field

Perturbation

$$\tilde{E}_0 = \sqrt{2\rho_E(\tilde{E}_0, \tilde{B}_0)}, \quad \tilde{B}_0 = \sqrt{2\rho_B(\tilde{E}_0, \tilde{B}_0)},$$

$$\rho_B = \frac{1}{4}\int_{-1}^{1} \mathrm{d}\cos\theta \int_0^{2\xi} \frac{\mathrm{d}z}{z}\, \tilde{\mathscr{P}}_{BB}^{+(\sigma)}(z,\theta), \qquad \rho_E = \frac{1}{4}\int_{-1}^{1} \mathrm{d}\cos\theta \int_0^{2\xi} \frac{\mathrm{d}z}{z}\, \tilde{\mathscr{P}}_{EE}^{+(\sigma)}(z,\theta),$$

$$\tilde{\mathscr{P}}_{BB}^{+(\sigma)}(z,\theta_{\boldsymbol{k}}) = \frac{H^4}{4\pi^2} e^{\pi\xi_{\mathrm{eff}}} z^{4+\Sigma} \left| c_1 W_\Sigma + c_2 M_\Sigma \right|^2, \quad \tilde{\mathscr{P}}_{EE}^{+(\sigma)}(z,\theta_{\boldsymbol{k}}) = \frac{H^4}{4\pi^2} e^{\pi\xi_{\mathrm{eff}}} z^{4+\Sigma} \left| c_1 W'_\Sigma + c_2 M'_\Sigma + \frac{\Sigma}{2z}(c_1 W_\Sigma + c_2 M_\Sigma) \right|^2,$$

We **numerically found** the consistent amplitudes of EMFs for given ξ

NB: This matching doesn't take into account the direction of EMFs.

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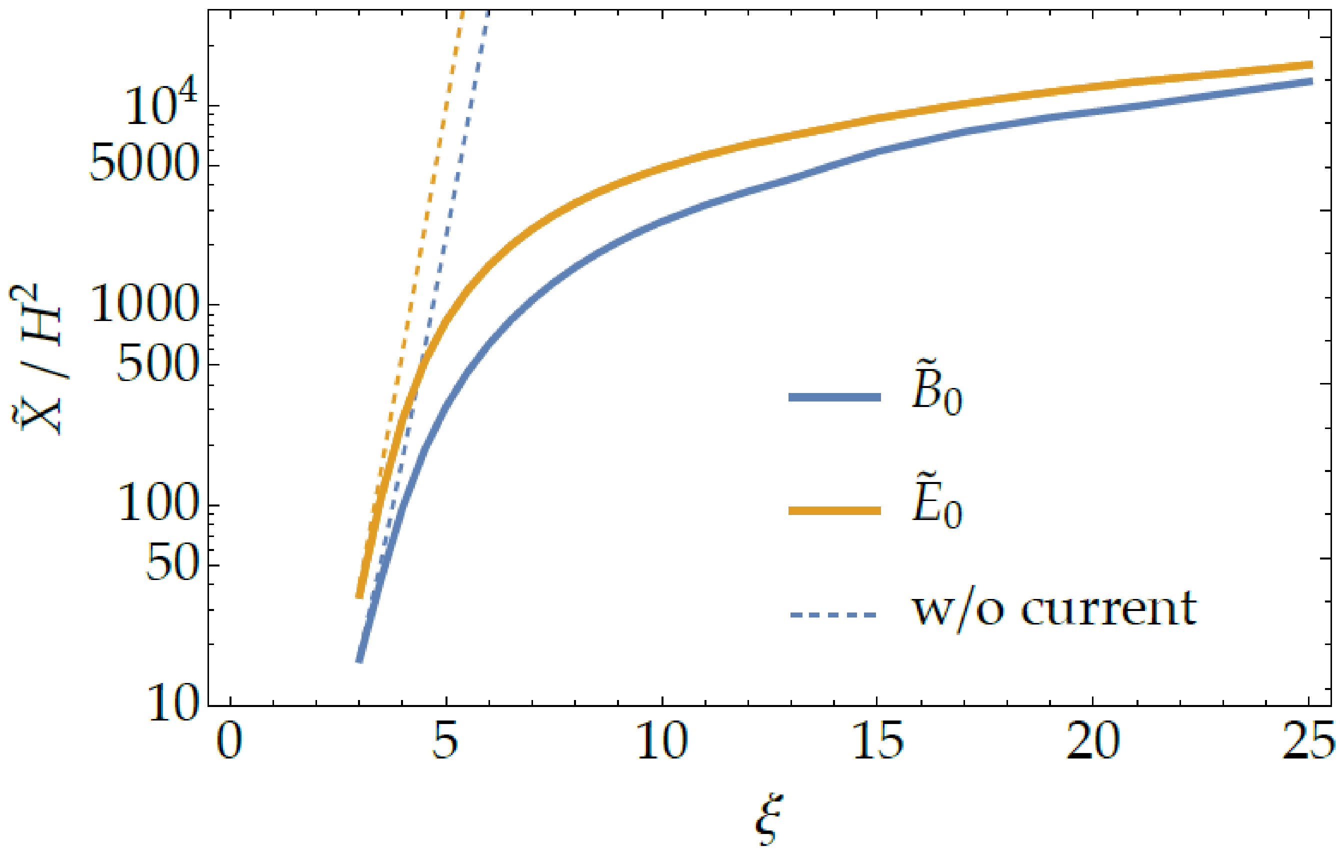
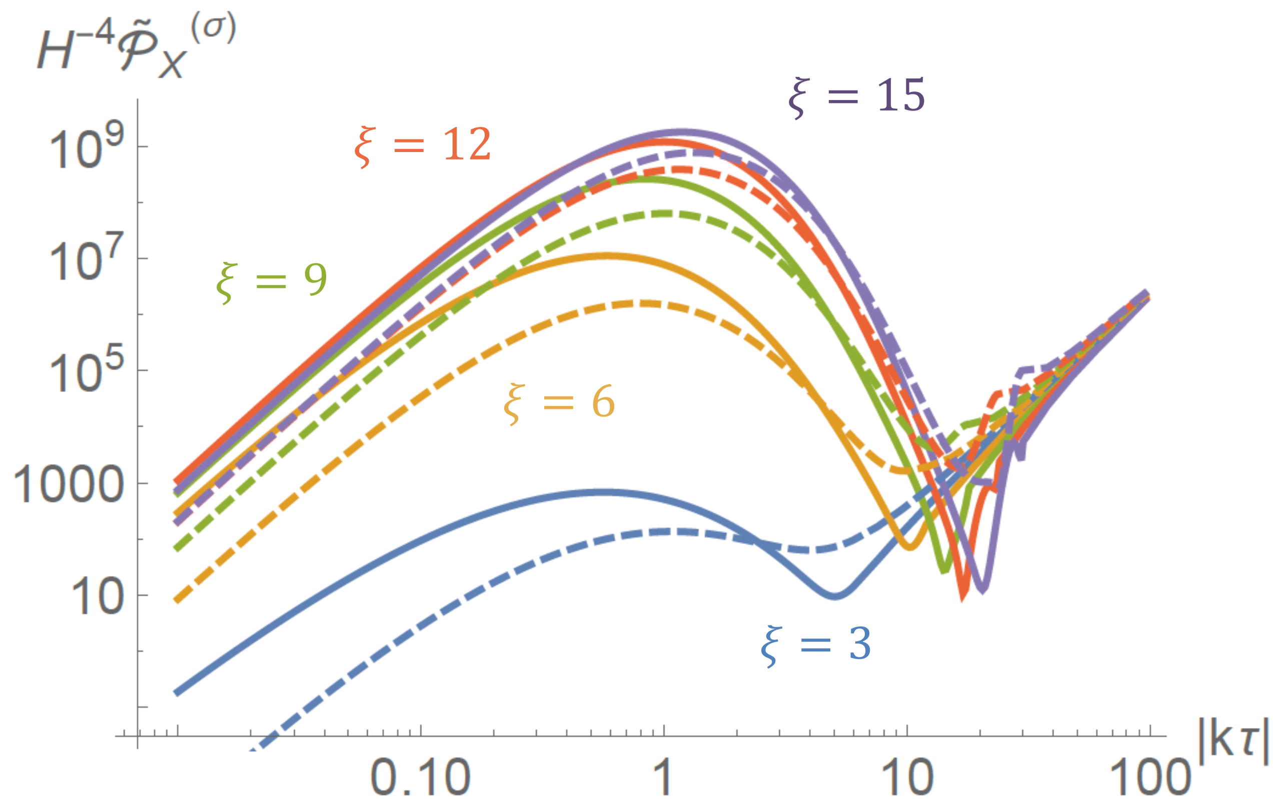
Self-consistent mean-field amplitudes for EMFs



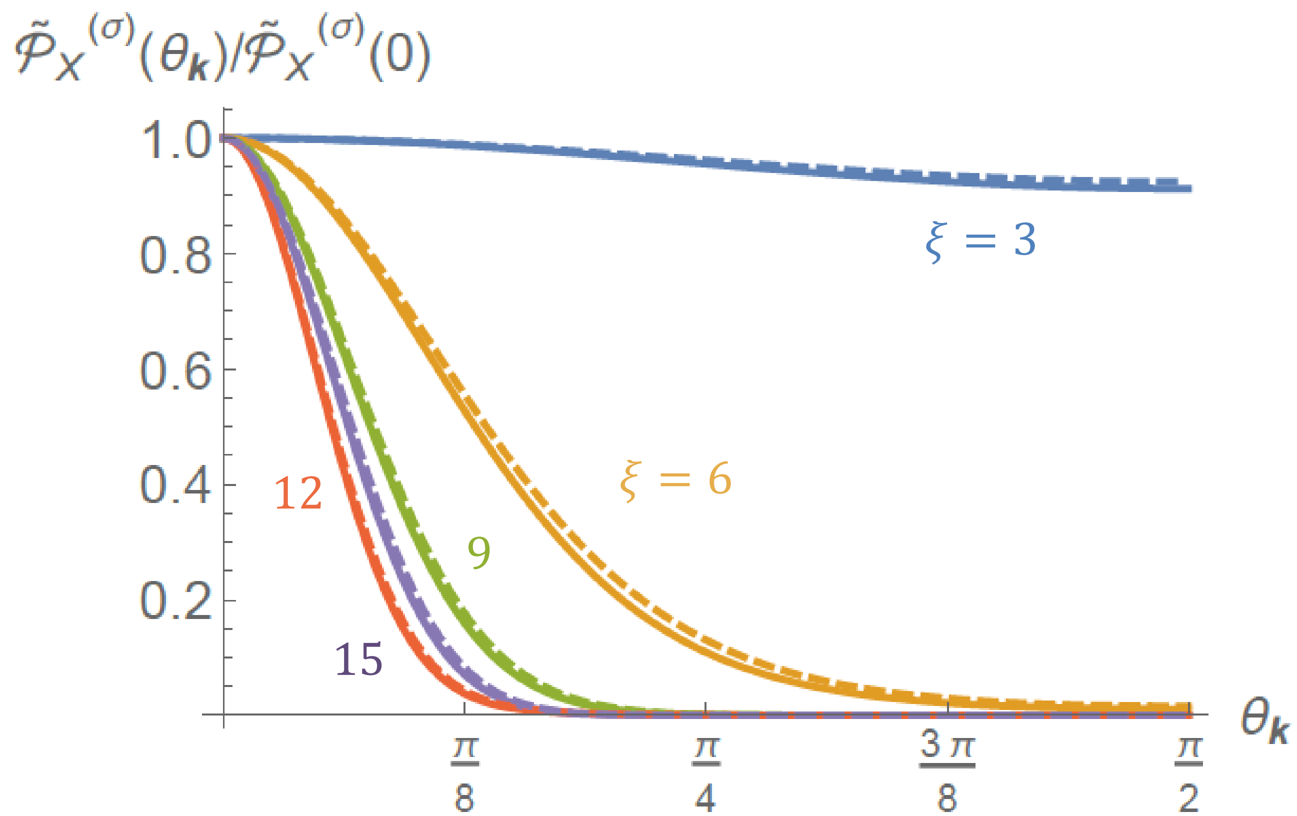
Charged fermions **drastically suppress** the EMF amplitudes.

E,B power spectra



- The spectra reach their peaks **earlier** due to the **effective friction**.
- EMFs keep the **4 properties**, which verifies our argument.

Direction dependence of the power spectra



Schwinger current prevents the EMF production in similar directions

perpendicular production is favored ⇒ **Rotation** of the EMFs??

4 Energy conservation

The energy density of EMFs evolves as

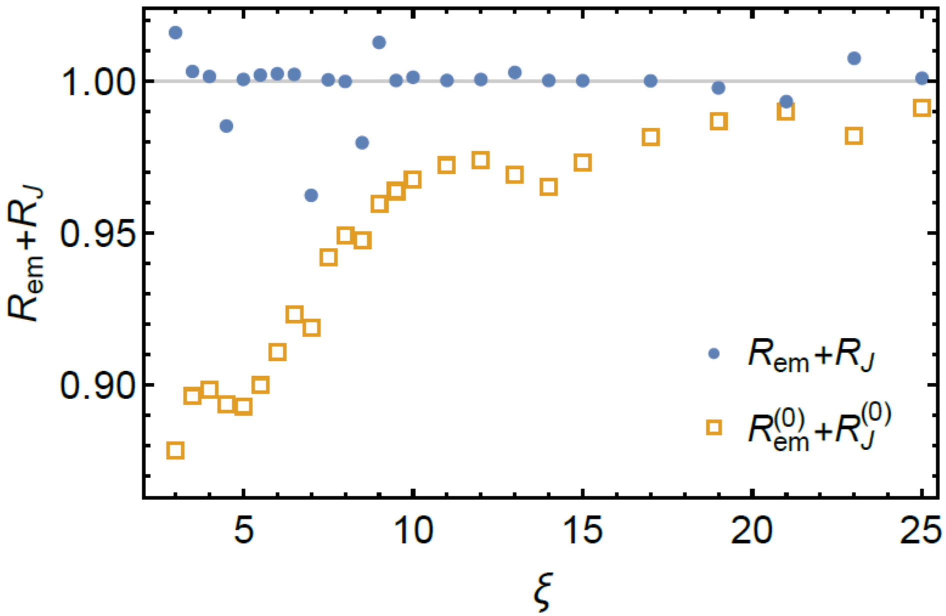
$$\langle \dot{\rho}_A \rangle = \underbrace{-2H\langle \tilde{\boldsymbol{E}}^2 + \tilde{\boldsymbol{B}}^2 \rangle}_{\text{Hubble dilution}} \underbrace{- 2\xi H \langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{B}} \rangle}_{\text{Energy injection from } \phi} \underbrace{- e \langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{J}} \rangle}_{\text{produce\&accelerate charged fermions}},$$

Since we consider a **static** system, $\langle \dot{\rho}_A \rangle$ should vanish and the 3 terms should be **balanced.**

$$\mathcal{R}_{\rm em} + \mathcal{R}_J = 1,$$

$$\mathcal{R}_{\rm em} \equiv \frac{\langle \tilde{\boldsymbol{E}}^2 + \tilde{\boldsymbol{B}}^2 \rangle}{\xi |\langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{B}} \rangle|},$$

$$\mathcal{R}_J \equiv \frac{e \langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{J}} \rangle}{2\xi H |\langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{B}} \rangle|}$$

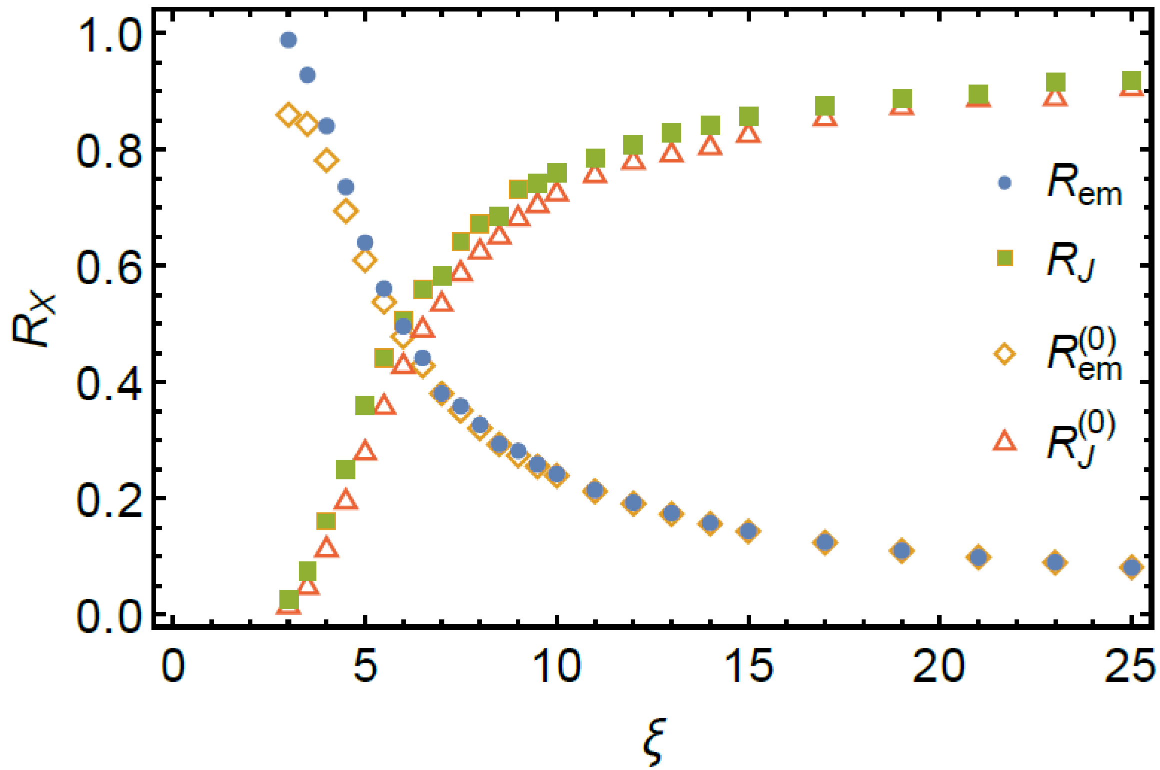


4 Energy distribution

$$R_{\rm em} + R_J = 1,$$

$$R_{\rm em} \equiv \frac{\langle \tilde{\boldsymbol{E}}^2 + \tilde{\boldsymbol{B}}^2 \rangle}{\xi |\langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{B}} \rangle|},$$

$$R_J \equiv \frac{e\langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{J}} \rangle}{2\xi H |\langle \tilde{\boldsymbol{E}} \cdot \tilde{\boldsymbol{B}} \rangle|}$$



- For $\xi \gtrsim 10$, the energy transfer to the **fermions is dominant**.
- We don't know **why**… But it may have an interesting consequences.

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SUMMARY

- Inflaton ϕ – photon A_μ – fermion ψ coupled system is well motivated but difficult. We need a **new approach** to solve this.

- We **integrated out** ψ by using the scale separation $L_\psi \ll L_{\rm em}$, and introduced **mean-field approx**. to solve non-linear eq. for A_μ. EM conductivities provide effective friction and reduction of ξ.

- We numerically solve the **consistent equation** to find the mean fields. The EM amplitudes are **drastically suppressed** compared to no- ψ case.

- Interestingly, the **dominant part** of the injected energy from ϕ goes to the charged fermions for $\xi \gtrsim 10$, which changes the conventional picture and may leads **new consequences**.

Future Work

- Relax the $\xi = const.$ assumption.

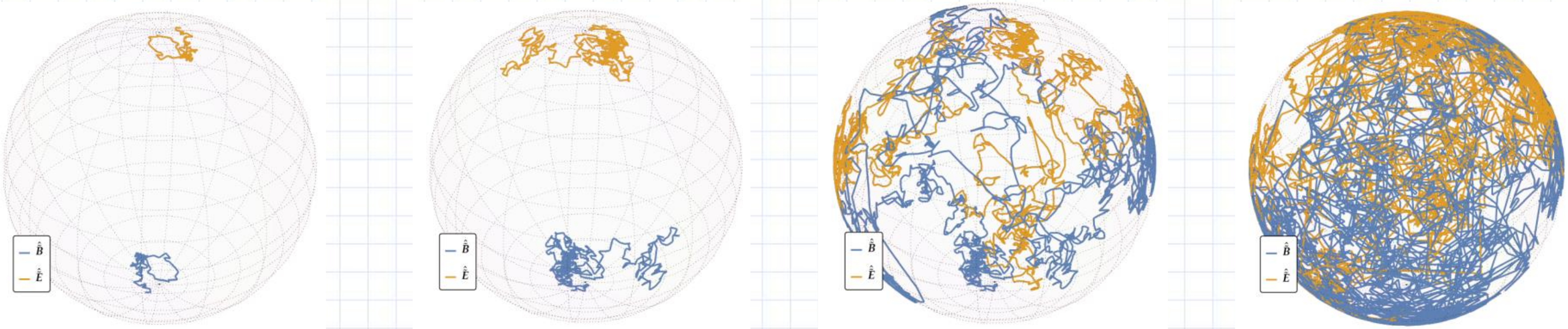
  Then we can explore the **inflation end** where ξ becomes maximum.

- Two unsatisfactory points of this work:

  1. Static EM assumption for $t \gtrsim H^{-1}$
  2. Consistent eq. is imposed only on the EMF amplitude not direction.

  ⇨ We cannot incorporate the **rotation** of EMFs

[TF, Mukaida, Tada, 2206.12218]



$\Delta N = 0.1$ $\Delta N = 0.5$ $\Delta N = 2$ $\Delta N = 10$

THE THEME
OF CHAPTER IS...

Thank you ！