Improved indirect limits on muon EDM

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Based on 2108.05398 and 2207.01679 with T. Gao and M. Pospelov

Electric dipole moment

- Electric dipole moment of a particle is proportional to spin:

$$\mathscr{H} = -\overrightarrow{B}\cdot\overrightarrow{\mu} - \overrightarrow{E}\cdot\overrightarrow{d} = -2\vec{s}\cdot\left(\mu\overrightarrow{B} + d\overrightarrow{E}\right).$$

* μ : magnetic dipole moment, d :electric dipole moment.

➡ EDM violates P and T (or CP).

	$\overrightarrow{B}$	$\overrightarrow{E}$	$\vec{s}$
P	+	−	+
T	−	+	−

- Flavor diagonal: standard model contribution extremely suppressed.

e.g. $d_e^{(\mathrm{equiv})}(\delta_{\mathrm{CKM}}) \simeq 10^{-35}\, e\,\mathrm{cm}\,.$ [YE, Gao, Pospelov 22]

➡ Background free probe of CP-odd new physics.

- CP violation motivated by baryogenesis, BSM such as 2HDM, SUSY, ...

Muon EDM

- Recently FNAL confirms BNL muon g-2 result:

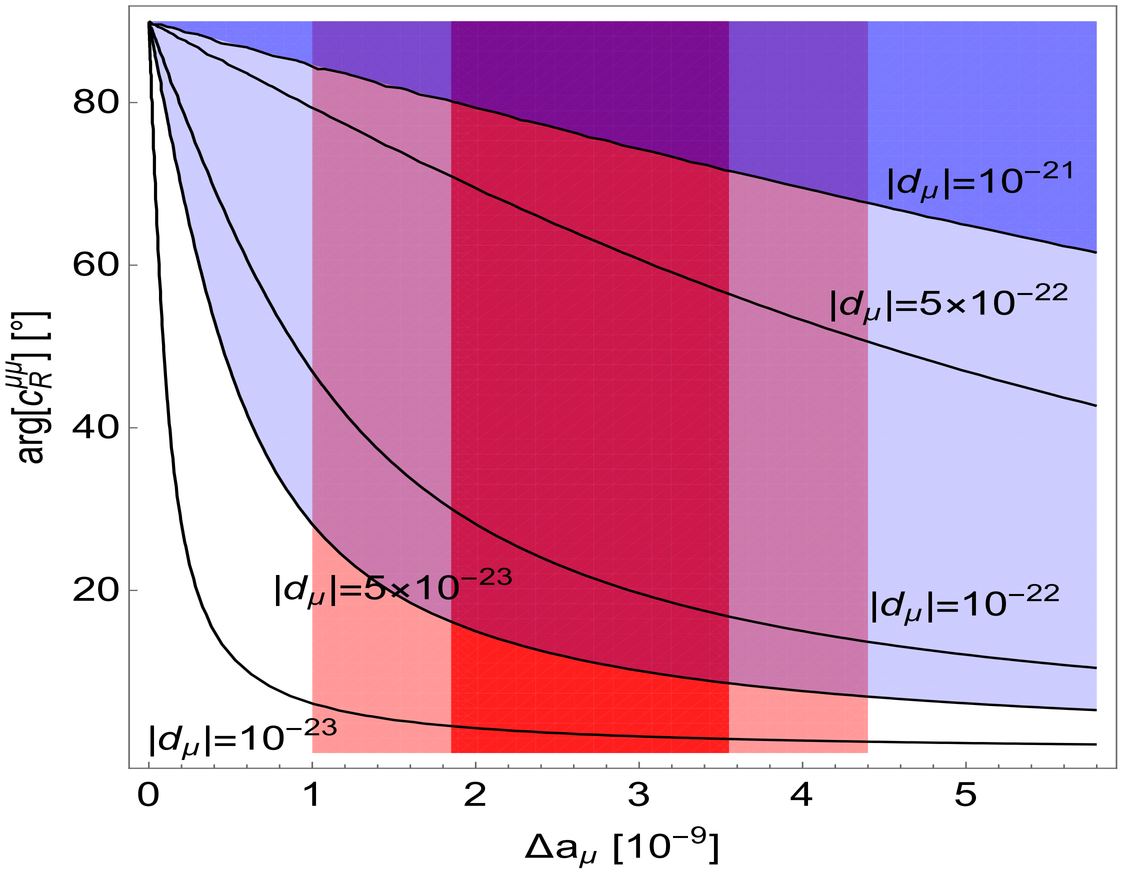
$$a_\mu(\text{exp}) - a_\mu(\text{SM}) = \left(251 \pm 59\right) \times 10^{-11} \quad (4.2\sigma)$$ [FNAL muon g-2 21]

- Muon g-2 and EDM can be closely related:

$$\mathscr{L} = -\frac{c}{2}\bar{\psi}_R \sigma \cdot F \psi_L + \text{h.c.} \quad \Rightarrow \quad \text{Re}[c] = \frac{e a_\mu}{2m}, \quad \text{Im}[c] = d_\mu .$$

- $\mathcal{O}(1)$ phase directly probed in near future.

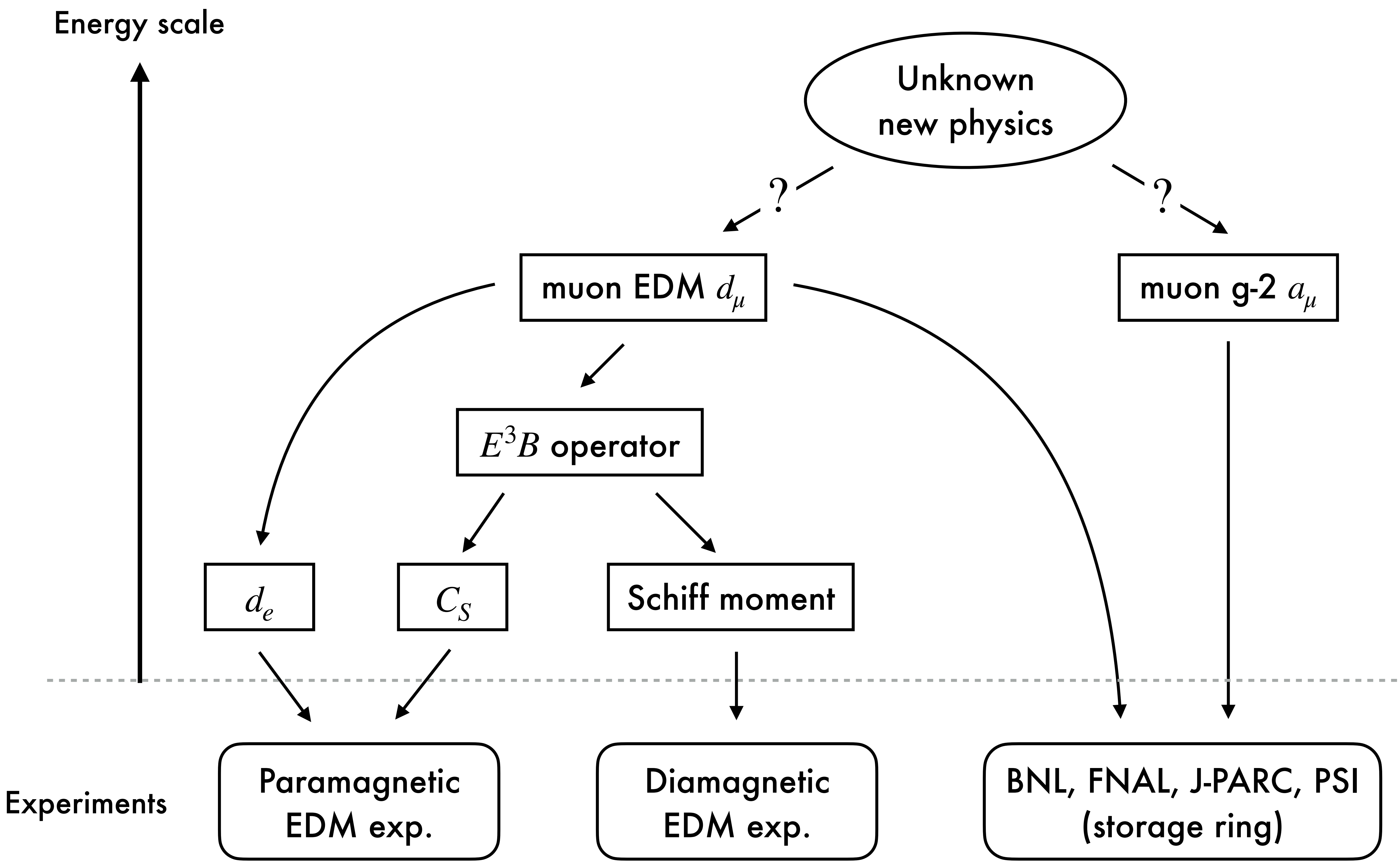
understand indirect limits from atomic/molecular EDM experiments.



[Figure taken from Crivellin et.al.18]

Toward observables

Many observables and many ways to achieve them



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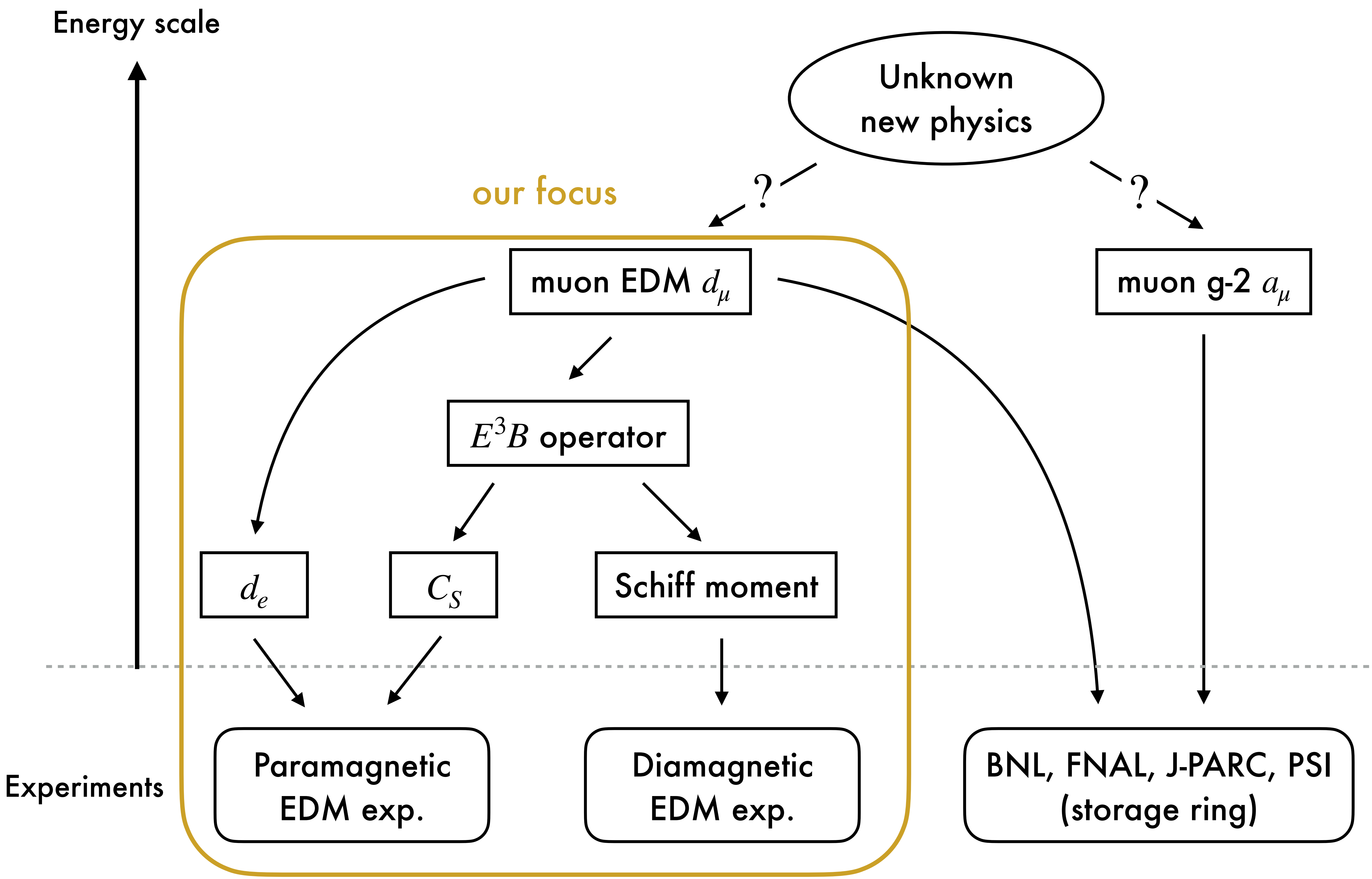

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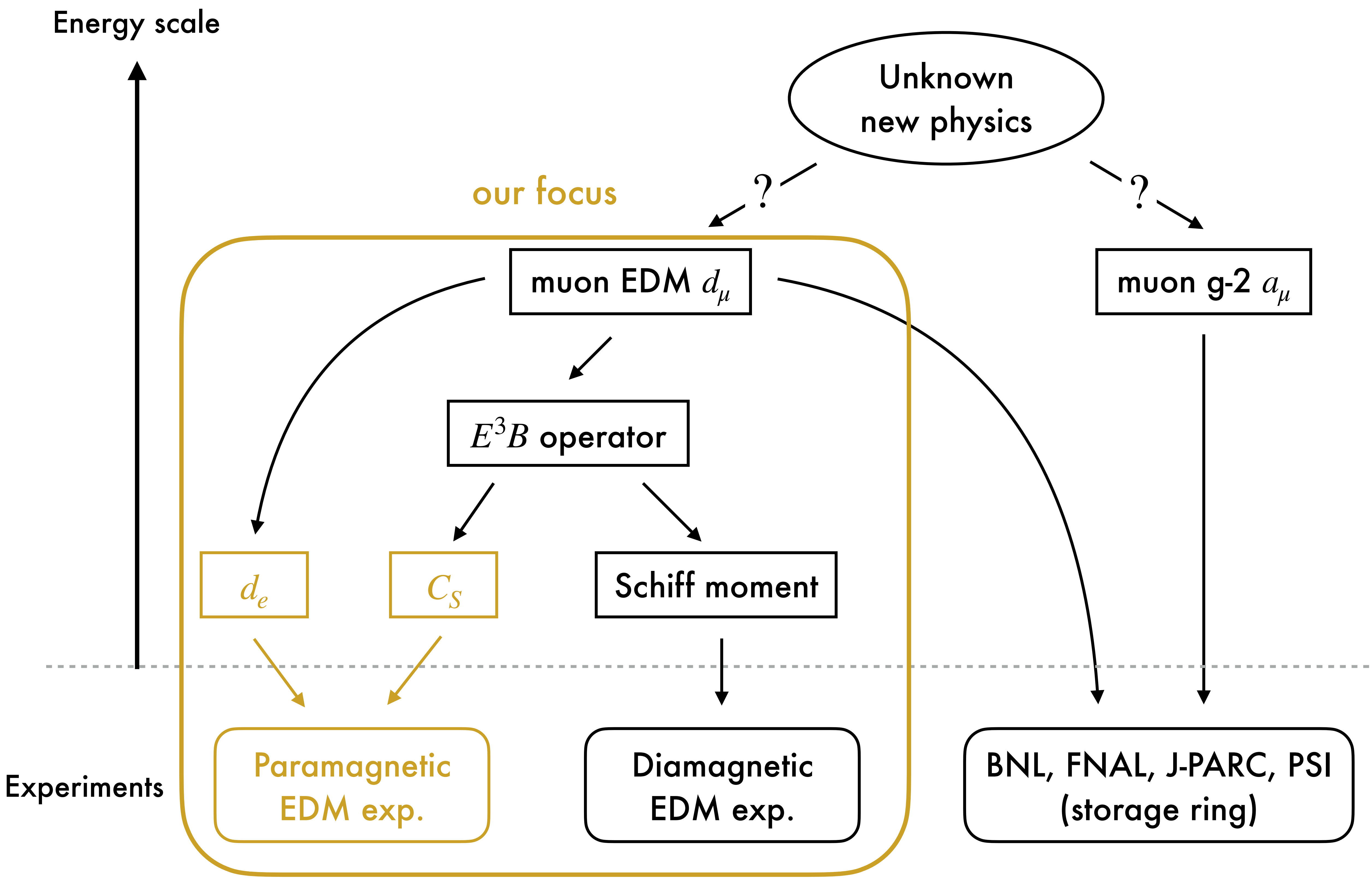
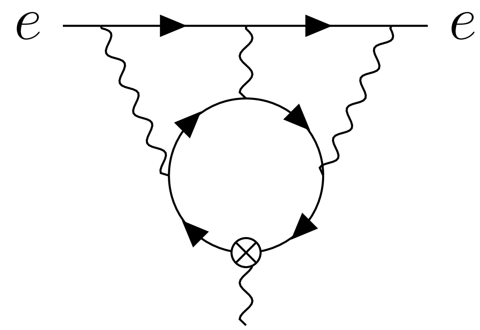


Electron EDM

- Muon EDM induces electron EDM at three-loop:



+ permutations

* cross-dot: EDM operator insertion

- There are two types of contributions:

* [Grozin, Khriplovich, Rudenko 08] computed only $S^{(1)}$.

$$i\mathscr{M} = i\tilde{F}^{\mu\nu}\,\bar{e}(p)\left[S^{(1)}m_e\sigma_{\mu\nu} + S^{(2)}\left\{\sigma_{\mu\nu}, \not{p}\right\}\right]e(p)\,.$$

- Combining two, the result is $\sim 40\,\%$ larger:

[YE, Gao, Pospelov 22]

$$d_e = 2.75 \times d_\mu \left(\frac{\alpha}{\pi}\right)^3 \frac{m_e}{m_\mu} \sim 2 \times 10^{-10} \times d_\mu\,.$$

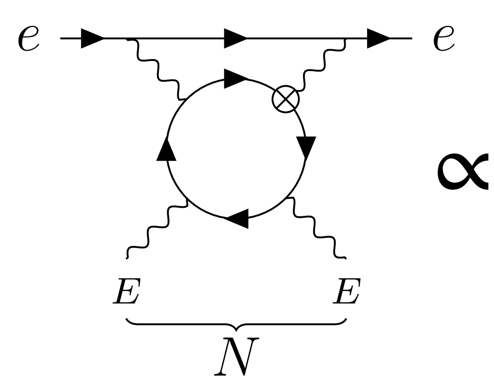
- But paramagnetic atom sensitive only to linear combination of d_e and C_S:

$$d_e^{\rm equiv} = d_e + C_S \times 1.5 \times 10^{-20}\, e\,{\rm cm} \;\text{ for }\; {\rm ThO}.$$

➡ Need to evaluate semi-leptonic CP-odd operator C_S .

Semi-leptonic CP-odd operator

- Paramagnetic atom EDM depends on C_S : $\mathscr{L} = C_S \frac{G_F}{\sqrt{2}} \bar{e} i\gamma_5 e \, \bar{N}N$.
- Muon EDM induces



$\propto \; d_\mu \times \bar{e} i\gamma_5 e \times E_N^2$.

* We evaluated this with leading-log accuracy.

- Nuclear electric field E_N^2 localized around nucleus.

$\bar{e} i\gamma_5 e \times E_N^2 \;\sim\; \bar{e} i\gamma_5 e \times \bar{N}N$: equivalent to C_S.

* Fudge factor included in our actual computation.

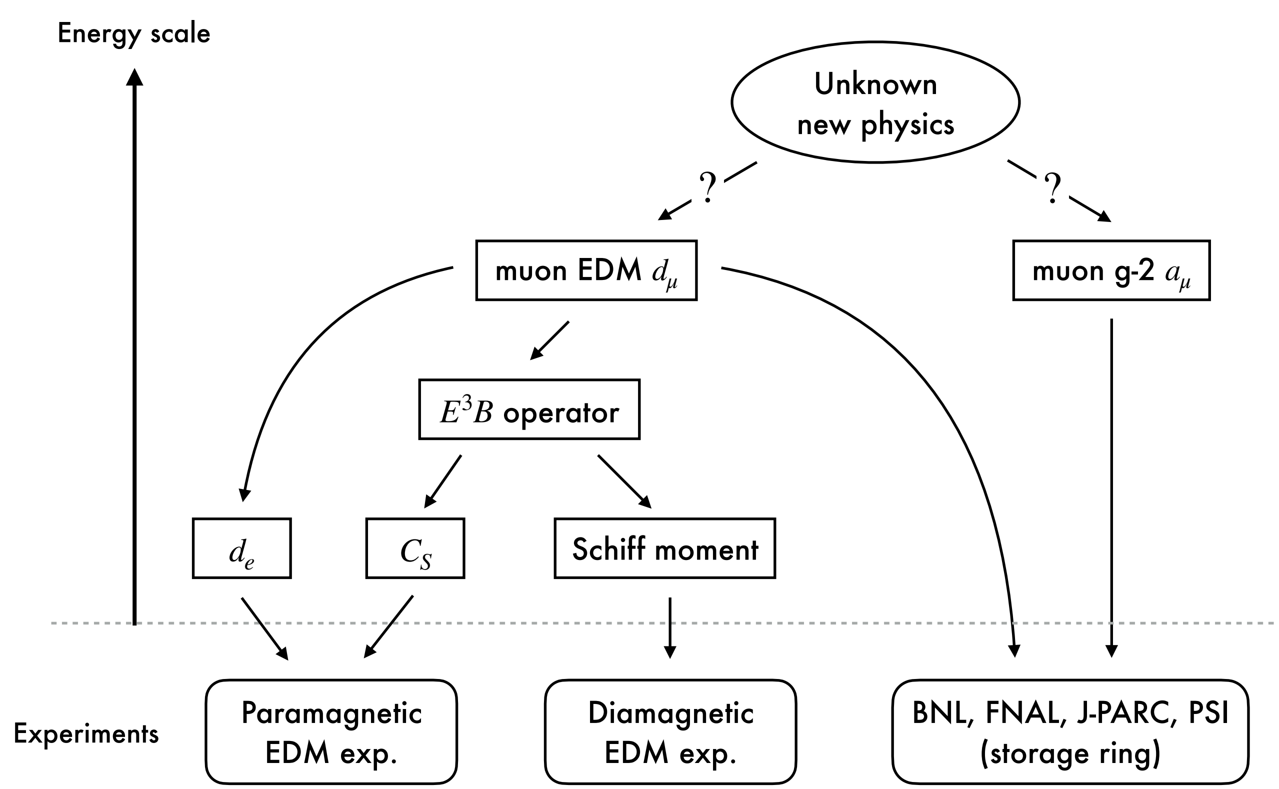
- ACME experiment: $|\, d_e^{\rm equiv} \,| < 1.1 \times 10^{-29}\, e\,{\rm cm}$ [ACME 18]

$|\, d_\mu({\rm ThO}) \,| < 1.7 \times 10^{-20}\, e\,{\rm cm}$ [YE, Gao, Pospelov 21, 22]

* C_S dominates over d_e by a factor 4, better than direct bound by BNL $|\, d_\mu \,| < 1.8 \times 10^{-19}\, e\,{\rm cm}$.

Summary

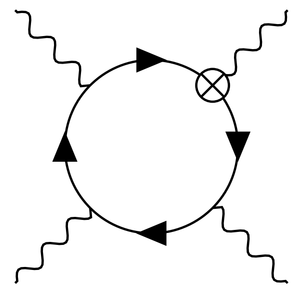
- We derived indirect limits on muon EDM, motivated by muon g-2.
- $|d_\mu(\mathrm{ThO})| < 1.7 \times 10^{-20}\, e\,\mathrm{cm}$ from ThO.
- $|d_\mu(\mathrm{Hg})| < 6.4 \times 10^{-20}\, e\,\mathrm{cm}$ from $^{199}\mathrm{Hg}$.
- Two different observables, so cancellation by chance less likely.
- Can be applied to tau EDM: $|d_\tau| < 1.1 \times 10^{-18}\, e\,\mathrm{cm}$ (dominantly from d_e).



Back up

CP-odd photon operator

- Muon EDM induces CP-odd photon operator at one-loop:



$$= -\frac{e^3 d_\mu}{96\pi^2 m_\mu^3}\tilde{F}_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$$

where cross-dot: muon EDM $d_\mu \bar{\mu}\sigma \cdot \tilde{F}\mu$ insertion.

- Atomic EDM exp. has large $Z \rightarrow$ strong nuclear electric field.

$\tilde{F}_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} \ni E^3 B$ can induce sizable CP-odd effects.

- In particular this operator induces

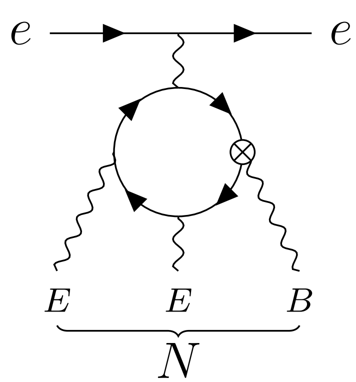
  - semi-leptonic CP-odd operator $\rightarrow$ paramagnetic EDM (ThO)
  - Schiff moment $\rightarrow$ diamagnetic EDM (Hg)

Schiff moment

- Schiff moment: $\mathscr{H}_{\text{int}} = -4\pi\alpha(\vec{S}/e)\cdot\vec{\nabla}_e\delta^{(3)}(\vec{r}_e)\,.$

$$\Rightarrow \quad \vec{d}_A = \sum_{i=1}^{Z}\langle\Psi|e\vec{r}_i|\Psi\rangle = -\sum_{n\neq 0}\frac{2}{E_0-E_n}\sum_{i=1}^{Z}\langle 0_e|e\vec{r}_i|n_e\rangle\langle n_e|4\pi\alpha(\vec{S}/e)\cdot\vec{\nabla}_i\delta^{(3)}(\vec{r}_i)|0_e\rangle\,.$$

- E^3B with two E_N and one B_N induces effective EDM distribution:



$$= \int d^3r\left(\vec{\nabla}_e\frac{\alpha}{|\vec{r}-\vec{r}_e|}\right)\cdot\frac{\vec{d}_N(\vec{r})}{e}\,,\qquad \vec{d}_N \propto d_\mu\left(2\vec{E}_N(\vec{E}_N\cdot\vec{B}_N)+\vec{B}_N E_N^2\right).$$

- Difference between EDM and charge distribution gives Schiff moment:

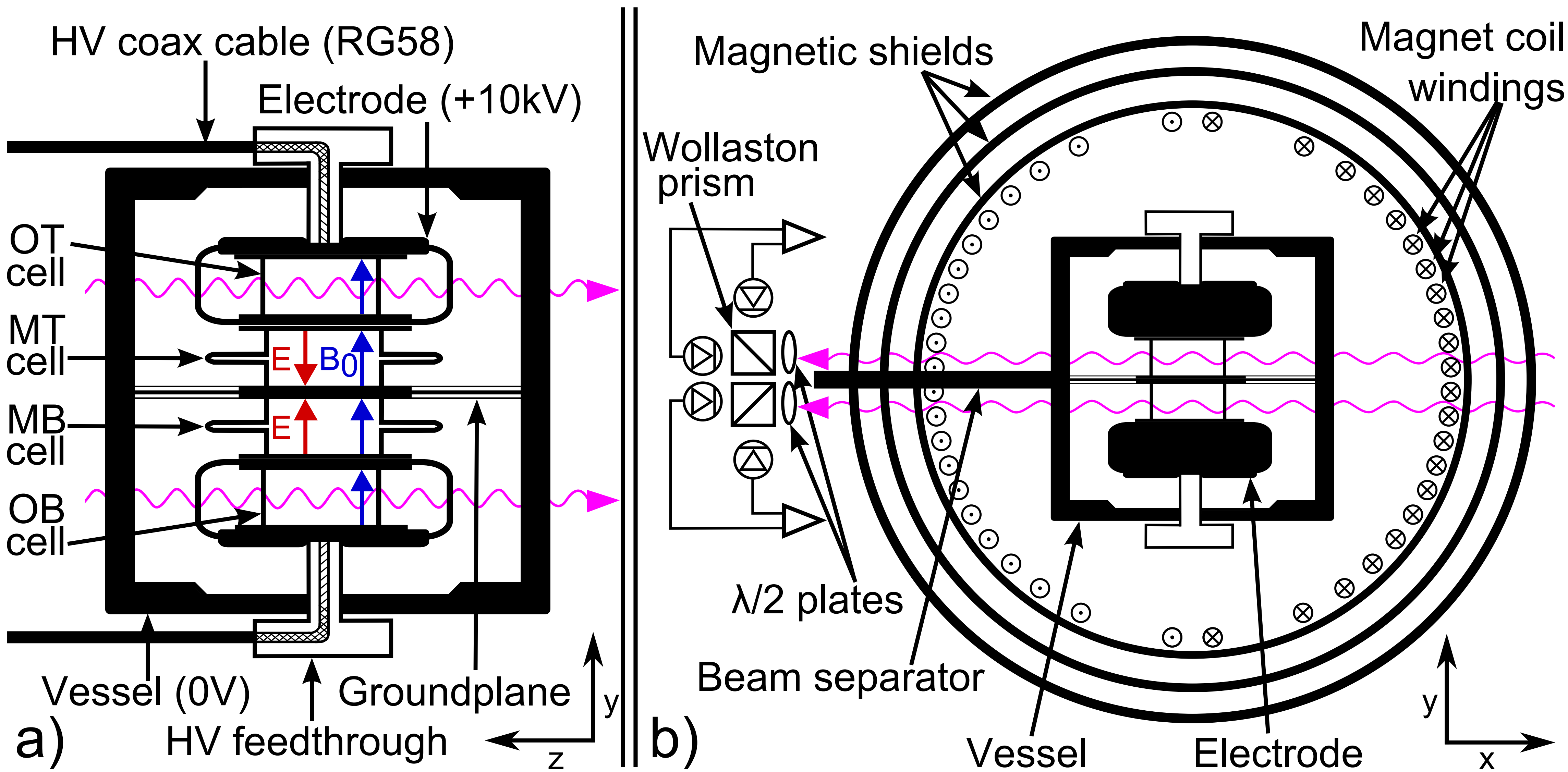
$$\mathscr{H}_{\text{eff}} = \int d^3r\left(\frac{\vec{d}_N}{e}-\rho_q\frac{\langle\vec{d}_N\rangle}{e}\right)\cdot\vec{\nabla}_e\frac{\alpha}{|\vec{r}-\vec{r}_e|} = -4\pi\alpha\frac{\vec{S}}{e}\cdot\vec{\nabla}_e\delta^{(3)}(\vec{r}_e)+\cdots\,.$$

- ^{199}Hg constraint: $|S_{^{199}\text{Hg}}| < 3.1\times 10^{-13}\,e\text{fm}^3\,.$ [Graner et.a. 16]

$$\Rightarrow \quad \boxed{|d_\mu(\text{Hg})| < 6.4\times 10^{-20}\,e\text{cm}}$$

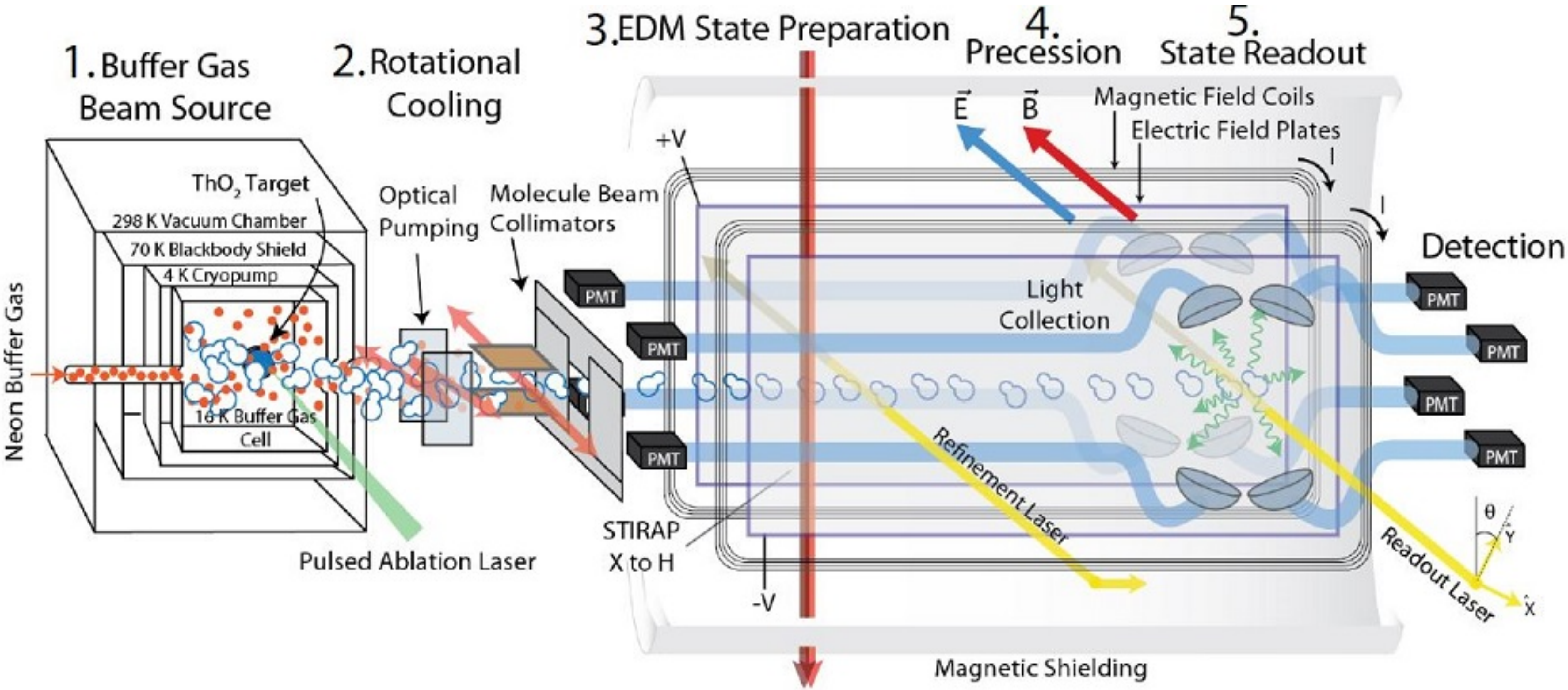
[YE, Gao, Pospelov 21]

^{199}Hg experiment



[Graner et.a. 16]

ACME ThO experiment



[ACME 18]

Nuclear electric field

- Nuclear electric field given by the charge distribution inside nuclei:

$$e\overrightarrow{E}_N(\vec{r}) = \frac{Ze^2}{4\pi}\int d^3r_N\rho_q(\vec{r}_N)\,\overrightarrow{\nabla}\frac{1}{|\vec{r}-\overrightarrow{r_N}|}\,.$$

- We simply take the charge distribution as

$$\rho_q(r_N) = \frac{3}{4\pi R_N^3}\Theta\left(R_N - r_N\right), \quad R_N = \sqrt{\frac{5}{3}}r_c, \quad r_c = 5.45\,\mathrm{fm}\ \text{ for } {}^{199}\text{Hg}.$$

- The Woods-Saxon shape different only within $10\,\%$ in the final result.

Nuclear magnetic field

- ^{199}Hg has an unpaired outermost neutron with $2p_{1/2}$ $(n=2, l=1, j=1/2)$.

  ➡ $\overrightarrow{B}_N$ dominantly provided by this neutron.

- As a result $\overrightarrow{B}_N$ is given by

$$e\overrightarrow{B}_N(\vec{r}) = \frac{2e\mu_n}{3}\psi_n^\dagger(\vec{r})\overrightarrow{\sigma}\psi_n(\vec{r}) + \frac{e\mu_n}{4\pi}\left[\vec{\nabla}\left(\vec{\nabla}\cdot\right) - \frac{\vec{\nabla}^2}{3}\right]\int d^3r_n \frac{\psi_n^\dagger(\vec{r}_n)\overrightarrow{\sigma}\psi_n(\vec{r}_n)}{|\vec{r}_n - \vec{r}|}$$

$$= e\mu_n \frac{\left|R(r)\right|^2}{4\pi}\chi^\dagger\left[\left(\overrightarrow{n}\cdot\overrightarrow{\sigma}\right)\overrightarrow{n} - \overrightarrow{\sigma}\right]\chi + \frac{e\mu_n}{4\pi}\int_0^\infty dr_n\, r_n^2 \left|R(r_n)\right|^2 \chi^\dagger\overrightarrow{g}(\vec{r}, r_n)\chi,$$

where $\mu_n \simeq -1.91\dfrac{e}{2m_p}$: neutron magnetic moment, ψ_n : neutron wave function,

R_n: neutron radial wave function, χ neutron spinor, $\overrightarrow{n} = \vec{r}/r$.

- We use the nuclear shell model to obtain ψ_n.

Fudge factor from E_N^2 to $\bar{N}N$

- E_N^2 and $\bar{N}N$ are both localized around nuclei, but not exactly the same.

- Relevant electron transition btw $s_{1/2}$ and $p_{1/2}$ states.

  ➡ matrix element:

$$\begin{cases} \int d^3r \rho_N \psi_p^\dagger \gamma^0 \gamma_5 \psi_s \propto \int dr\, r^2 \bar{\rho}_N \left(f_p g_s + f_s g_p \right) & \text{for} \quad \bar{N}N\bar{e}i\gamma_5 e, \\ \int d^3r \, |\overrightarrow{E}_N|^2 \psi_p^\dagger \gamma^0 \gamma_5 \psi_s \propto \int dr\, r^2 \bar{\rho}_{E^2} \left(f_p g_s + f_s g_p \right) & \text{for} \quad E_N^2 \bar{e} i \gamma_5 e \,. \end{cases}$$

- We compute the fudge factor κ by solving the Dirac equation and get

$$\kappa = \frac{\int dr\, r^2 \bar{\rho}_{E^2} \left(f_p g_s + f_s g_p \right)}{\int dr\, r^2 \bar{\rho}_N \left(f_p g_s + f_s g_p \right)} \simeq 0.66.$$

Magnetic quadrupole moment

- Magnetic quadrupole moment (MQM) also violates P and CP:

$$\mathcal{H}_{\rm eff} = -\frac{M}{6}\nabla_j B_i I_{ij}, \quad I_{ij} \equiv \frac{3}{2I(I-1)}\left(I_i I_j + I_j I_i - \frac{2}{3}\delta_{ij} I(I+1)\right).$$

- The E^3B operator converts EQM Q to MQM as

$$\frac{M}{e} \simeq -\frac{Z^2\alpha^3 d_\mu/e}{5\pi m_\mu^2 R_N^3}\frac{Q}{e} \simeq 1.1\times 10^{-4}\,{\rm fm}\left(\frac{Q/e}{300\,{\rm fm}^2}\right)\times d_\mu/e\,.$$

- Q can be large in nuclei with $I \geq 1$ and large deformation.

➡ can be an interesting observable in future.