SU(N)-natural inflation

Kai Murai
University of Tokyo

Based on: T. Fujita, K. Mukaida, KM, H. Nakatsuka, *PRD* 105 103519, arXiv: 2110.03228
T. Fujita, KM, R. Namba, *PRD* 105 103518, arXiv: 2203.03977

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Introduction

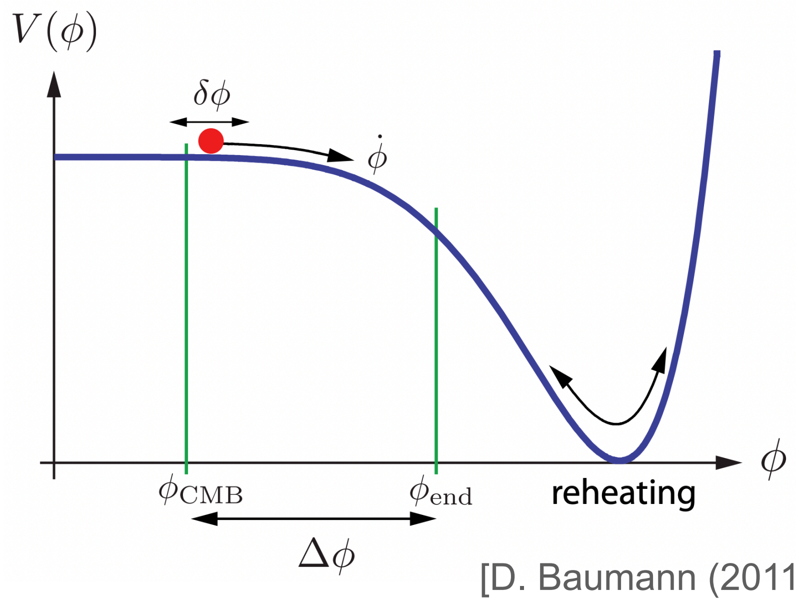
■ Single-field slow-roll inflation

Inflation explains the origin of the cosmological fluctuations:

$$\mathcal{P}_\zeta \simeq \frac{V}{24\pi^2 \epsilon_V M_{\rm Pl}^4} \simeq 2.2 \times 10^{-9},$$

$$n_s \simeq 1 - 6\epsilon_V + 2\eta_V \simeq 0.96, \quad r \simeq 16\epsilon_V \lesssim 0.03$$

$$\epsilon_V \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta_V \equiv M_{\rm Pl}^2 \frac{V''}{V}$$



[D. Baumann (2011)]

Slow-roll inflation requires a flat potential.

Considering radiative corrections, a fine-tuning problem arises.

→ “Natural inflation”

[K. Freese, J. A. Frieman, and A. V. Olinto (1990)]

Natural inflation [K. Freese, J. A. Frieman, and A. V. Olinto (1990)]

Pseudo Nambu-Goldstone boson such as an axion

→ The shift symmetry protects the flatness of the potential.

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right) \right]$$

The success of natural inflation requires

$$f \gtrsim M_{\mathrm{Pl}}, \quad \Lambda \sim m_{\mathrm{GUT}} \sim 10^{15}\,\mathrm{GeV}$$

[K. Freese and W. H. Kinney (2004)]

Axion inflation with a sub-Planckian decay constant?

One of the possibilities is

"Chromo-natural inflation"

[P. Adshead and M. Wyman (2012)]

Contents

Chromo-natural inflation

- Chromo-natural inflation (CNI)

[P. Adshead and M. Wyman (2012)]

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu,$$

$$\tilde{F}^{a\mu\nu} = \frac{1}{2\sqrt{-\tilde{g}}}\epsilon^{\mu\nu\rho\sigma}F^a_{\rho\sigma},$$

$$\mathscr{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}_{\text{axion/inflaton}} \underbrace{- \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}}_{\text{SU(2) gauge fields}} + \underbrace{\frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}}_{\text{Axion-gauge fields coupling}}$$

Source gauge fields

Slow-rolling axion

Homogeneous isotropic SU(2) gauge fields

$A^a_i \propto \delta^a_i$

Effective potential

SU(2) gauge fields induce an effective "friction" of the axion.

→ More slow-roll inflation

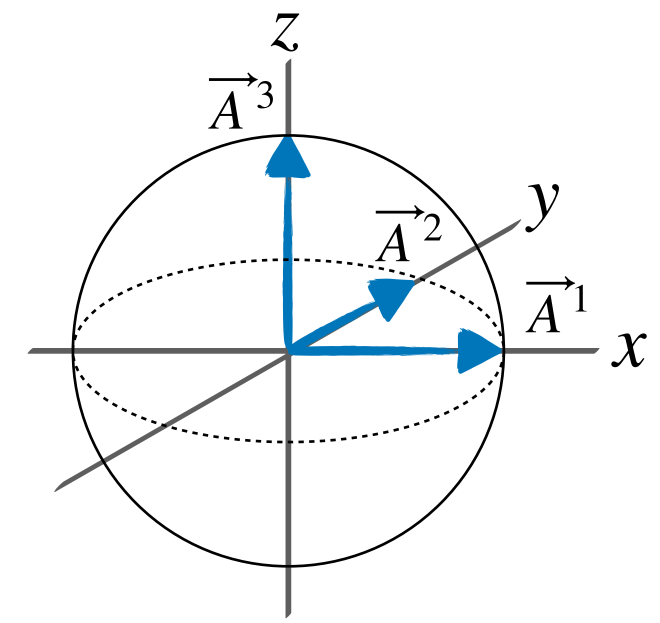
We can obtain compatible $\mathscr{P}_\zeta$ and n_s with $f < M_{\rm Pl}$.

Chromo-natural inflation

Points of the BG solution

$$A_i^a(t) = \delta_i^a a(t) Q(t), \quad m_Q \equiv \frac{gQ}{H} \simeq \left(\frac{-g^2 f \partial_\phi V}{3H^4} \right)^{1/3},$$

$$\xi \equiv \frac{\dot{\phi}}{2fH} \simeq m_Q + m_Q^{-1}.$$



- The gauge fields have a rotationally invariant configuration:

$$A_i^a \propto \delta_i^a \;\; \Rightarrow \;\; {}^\forall R, {}^\exists G : R_{ij} A_j^a = G^{ab} A_i^b$$

- SSB: spatial SO(3) × gauge SU(2) → diagonal SO(3)
- This solution is an attractor solution. [I. Wolfson, *et al.* (2021)]

Chromo-natural inflation

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu,$$

$$\tilde{F}^{a\mu\nu} = \frac{1}{2\sqrt{-\tilde{g}}}\epsilon^{\mu\nu\rho\sigma} F^a_{\rho\sigma},$$

- Chromo-natural inflation (CNI)

[P. Adshead and M. Wyman (2012)]

$$\mathscr{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}_{\text{axion/inflaton}} \underbrace{- \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}}_{\text{SU(2) gauge fields}} + \underbrace{\frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}}_{\text{Axion-gauge fields coupling}}$$

Due to the SU(2) gauge field background,
gauge field perturbations experience a tachyonic instability.

→ Chiral and non-Gaussian GWs are overproduced.
Chromo-natural inflation fails… [P. Adshead, E. Martinec, and M. Wyman (2013)]

If ϕ is a spectator, this mechanism can produce observable GWs.

[E. Dimastrogiovanni, M. Fasiello, T. Fujita (2016)
K. Ishiwata, E. Komatsu, I. Obata (2021)
M. Kakizaki, M. Ogata, O. Seto (2021)]

Chromo-natural inflation

- Extend Chromo-natural inflation

SU(2) group is the minimal choice for nonzero gauge field VEV.

How about other groups?

- Background solution
- GW production (inflationary model / spectator model)

“SU(N)-natural inflation”

Contents

Decomposition of gauge fields

$$\mathscr{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}_{\text{Axion}} \underbrace{- \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}}_{\text{SU(N) gauge field}} + \underbrace{\frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}}_{\text{Interaction}}$$

According to the SU(2) subalgebra, we decompose A_i :

$$A_i = A_i^a \mathscr{T}_a = A_i^j \mathscr{T}_j + A_i^A \mathscr{T}_A$$

SU(N) generators
$a = 1, \ldots, N^2 - 1$
$\mathrm{Tr}\left[\mathscr{T}_a \mathscr{T}_b\right] = \frac{\delta_{ab}}{2}$

SU(2) generators
$i = 1,2,3$
$[\mathscr{T}_i, \mathscr{T}_j] = i\lambda\epsilon_{ijk}\mathscr{T}_k$

the other SU(N) generators
$A = 4, \ldots, N^2 - 1$

λ is determined by a choice of SU(2)

SU(N)-natural inflation

- Examples: $N = 3$

SU(3) ⊃ SU(2) × U(1) $\quad \mathbf{3} = \mathbf{2} + \mathbf{1}, \quad \mathbf{8} = \mathbf{3} + \mathbf{2} + \mathbf{2} + \mathbf{1}$

$$\mathcal{T}_i = \frac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix} \qquad \lambda = 1$$

SU(3) ⊃ SU(2) $\quad \mathbf{3} = \mathbf{3}, \quad \mathbf{8} = \mathbf{3} + \mathbf{5}$

$$\mathcal{T}_i = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\lambda = \frac{1}{2}$$

SU(N)-natural inflation

Background solutions

$$A_i = A_i^j \underline{\mathcal{T}_j} + A_i^A \mathcal{T}_A$$

SU(2) generators
$i = 1,2,3$

$[\mathcal{T}_i, \mathcal{T}_j] = i\lambda\epsilon_{ijk}\mathcal{T}_k$

$$A_i^j(t) = \delta_i^j \frac{aHm_Q}{g\lambda}, \qquad A_i^A(t) = 0,$$

$$m_Q \simeq \left(\frac{-g^2\lambda^2 f\partial_\phi V}{3H^4}\right)^{1/3}, \quad \xi \equiv \frac{\dot{\phi}}{2fH} \simeq m_Q + m_Q^{-1}.$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\underbrace{f^{abc}}_{\to\lambda} A_\mu^b A_\nu^c$$

- The same as CNI except for $g \to g\lambda$
- Different amplitudes depending on λ or the choice of SU(2)

Contents

Perturbations in SU(N)

■ "Universality" of the linear perturbations

At the linear level,

- δA_i^a decouples according to the SU(2) reps.
- δg_{ij} couples to only the $\mathbf{3}$ rep. of δA_i^j (= SU(2) generators)
- λ is degenerate with g in the $\mathbf{3}$ rep. part.

Example

SU(3) ⊃ SU(2) × U(1) $\mathbf{3} = \mathbf{2} + \mathbf{1}, \ \ \mathbf{8} = \mathbf{3} + \mathbf{2} + \mathbf{2} + \mathbf{1}$

$\boxed{\boxed{\delta g} \longleftrightarrow \boxed{\delta A^{(\mathbf{3})}}}$ $\boxed{\delta A^{(\mathbf{2+2})}}$ $\boxed{\delta A^{(\mathbf{1})}}$

Almost the same as CNI   decouple   decouple

Perturbations in SU(N)

- "Universality" of the linear perturbations

At the linear level,

- δA_i^a decouples according to the SU(2) reps.
- δg_{ij} couples to only the $\mathbf{3}$ rep. of δA_i^j (= SU(2) generators)
- λ is degenerate with g in the $\mathbf{3}$ rep. part.

The metric perturbations in the SU(N)-natural inflation are the same as in CNI except for $g \to g\lambda$

Non-linear effects of δA_i^A
Transitions of BG solutions
can break the universality.

Contents

Summary

- We extend the CNI model into SU(N)-natural inflation and find multiple background solutions.
- BG solutions are labeled by λ, which is degenerate with g.
- λ is determined by a choice of SU(2) subalgebra in SU(N).
- The prediction for δg has a universality at the linear level.
- Higher-order effects or transitions of BG solutions can break this universality.
- This model cannot avoid the GW overproduction as an inflationary model.
- On the other hand, it might show a distinct signal in GWs as a spectator model generating GWs.

- "Tensor" perturbations

The linear EoMs for the tensor perturbations are

small

$$\psi_{ij}'' - \partial_k\partial_k\psi_{ij} - \frac{2}{\tau^2}\psi_{ij} = \frac{2\sqrt{\epsilon_B}}{m_Q\tau}t_{ij}' + \frac{2\sqrt{\epsilon_B}}{\tau}\epsilon^{ikl}\partial_l t_{jk} + \frac{2\sqrt{\epsilon_B}m_Q}{\tau^2}t_{ij},$$

$$t_{ij}'' - \partial_k\partial_k t_{ij} + \frac{2(2m_Q + m_Q^{-1})}{\tau}\epsilon^{ikl}\partial_l t_{jk} + \frac{2(m_Q^2+1)}{\tau^2}t_{ij} = \mathcal{O}(\psi_{ij})\,.$$

$$\psi_{ij} \equiv \frac{aM_{\rm Pl}}{2}h_{ij}, \quad \delta A_i^a = t_{ai} + \cdots, \quad a\mathrm{d}\tau = \mathrm{d}t,$$

$$\tau \simeq -\frac{1}{aH}, \quad \epsilon_B \equiv \frac{g^2Q^4}{M_{\rm Pl}^2H^2},$$

Back up

- Tensor perturbations

$$\partial_\tau^2 \Psi_1^{R/L} + \left[k^2 - \frac{2}{\tau^2}\right]\Psi_1^{R/L} = \frac{2\sqrt{\epsilon_B}}{m_Q \tau}\partial_\tau T_1^{R/L} \pm \frac{2k\sqrt{\epsilon_B}}{\tau} T_1^{R/L} + \frac{2\sqrt{\epsilon_B}m_Q}{\tau^2} T_1^{R/L},$$

$$\partial_\tau^2 T_1^{R/L} + \left[k^2 \pm \frac{2k(2m_Q + m_Q^{-1})}{\tau} + \frac{2(m_Q^2+1)}{\tau^2}\right] T_1^{R/L} = \mathcal{O}\left(\Psi_1^{R/L}\right).$$

$$T_1^R(\tau, k) = \frac{1}{\sqrt{2k}} e^{\pi(2m_Q + m_Q^{-1})/2} \underbrace{W_{\beta,\alpha}(2ik\tau)}_{\text{Whittaker function}},$$

$$\Psi_1^R(\tau, k) = \int_{-\infty}^{\infty} \mathrm{d}\eta \, G_\psi(\tau, \eta, k)\mathcal{D}(\eta, k) T_1^R(\eta, k),$$

$$\alpha \equiv -i\sqrt{2m_Q^2 + 7/4}, \quad \beta \equiv -i(2m_Q + m_Q^{-1}), \quad \mathcal{D}(\eta, k) \equiv \frac{2\sqrt{\epsilon_B}}{m_Q \eta}\partial_\eta + \frac{2\sqrt{\epsilon_B}}{\eta^2}(m_Q + k\eta),$$

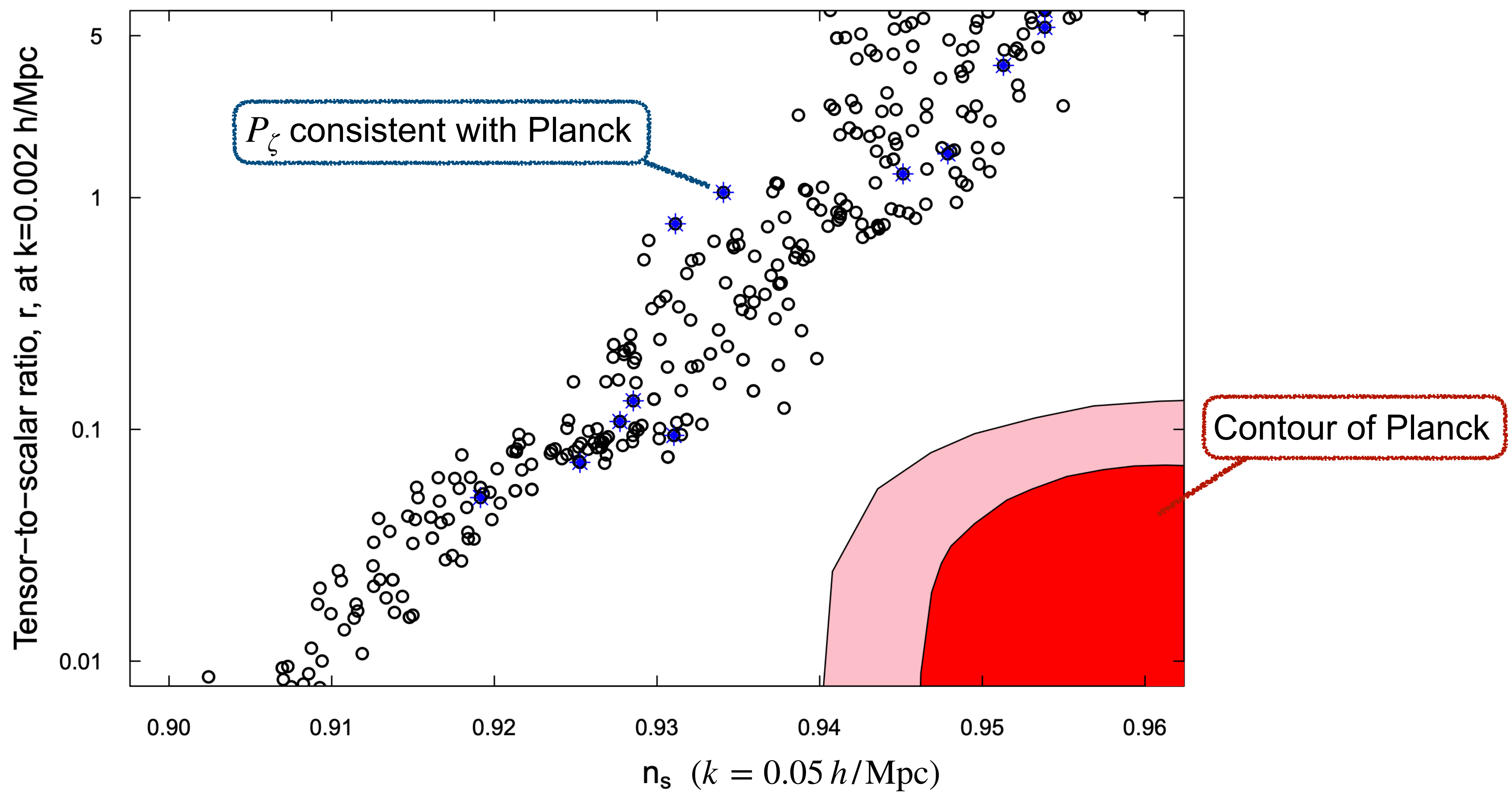
$$G_\psi(\tau, \eta, k) \equiv \frac{\Theta(\tau - \eta)}{k^3 \tau \eta}\left[k(\eta - \tau)\cos\left(k(\tau - \eta)\right) + (1 + k^2\tau\eta)\sin\left(k(\tau - \eta)\right)\right],$$

Back up

- Constraints on CNI [P. Adshead, E. Martinec, and M. Wyman (2013)]

CNI is excluded by (n_s, r).

For acceptable scalar spectra, the gravitational waves are overproduced.



Back up

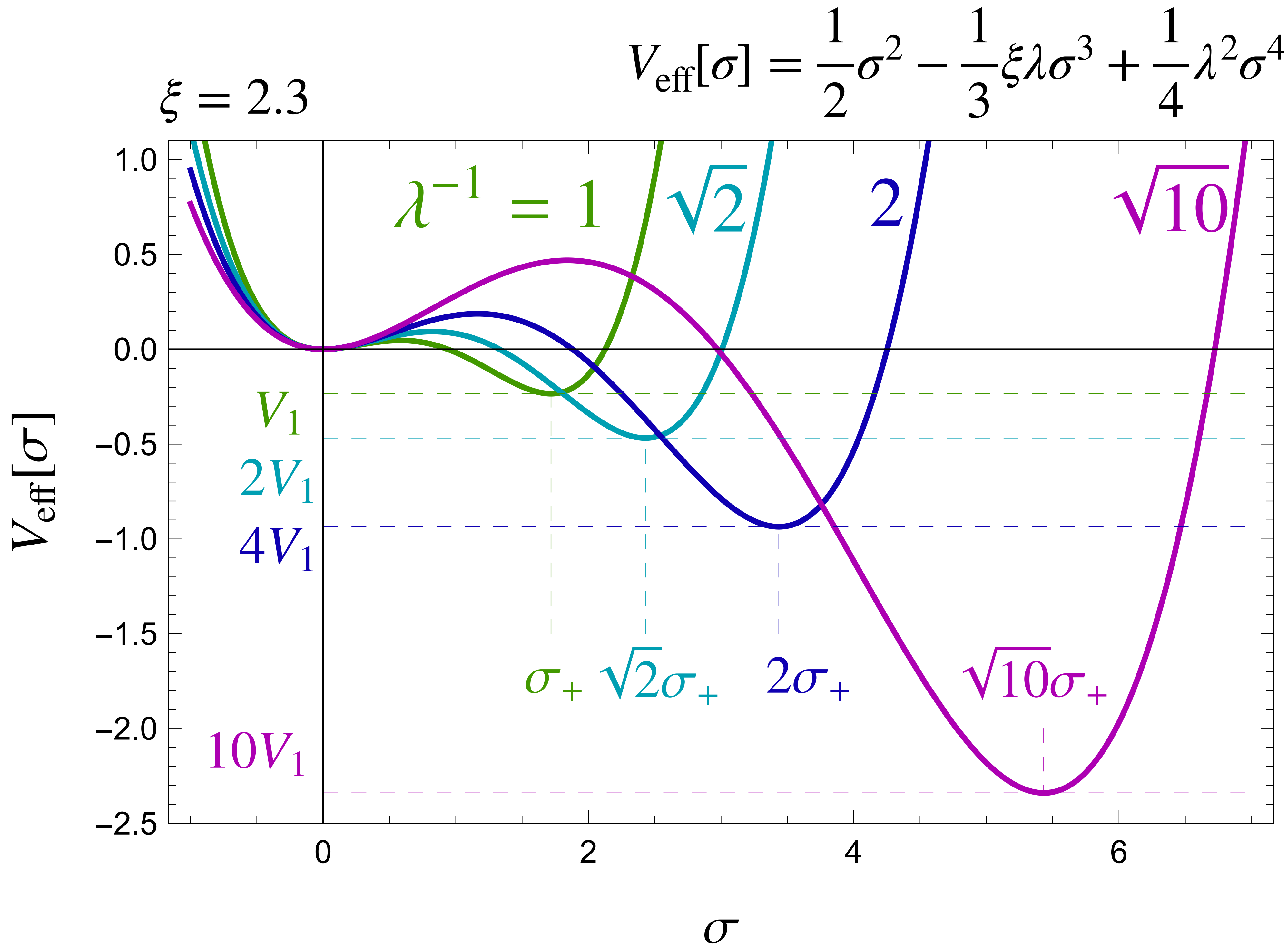
- SU(N) "electromagnetic" field

$$E_i^a = -F_{0i}^a = -\frac{aH^2}{g}M_i^a \equiv \frac{aH^2}{g}\mathcal{E}_i^a$$
$$B_i^a = \frac{1}{2}\epsilon_{ijk}F_{jk}^a = -\frac{a^2H^2}{2g}\epsilon_{ijk}f^{abc}M_j^b M_k^c \equiv \frac{a^2H^2}{g}\mathcal{B}_i^a$$

In terms of the EM fields, the EoM becomes

$$-2\mathcal{E}_i^a + \frac{1}{2}\epsilon_{ijk}f^{abc}\mathcal{E}_j^b\mathcal{B}_k^c + \xi\mathcal{B}_i^a = 0.$$

- Stability


$V_{\rm eff}[\sigma] = \frac{1}{2}\sigma^2 - \frac{1}{3}\xi\lambda\sigma^3 + \frac{1}{4}\lambda^2\sigma^4$
$\xi = 2.3$
$\lambda^{-1} = 1$
$\sqrt{2}$
2
$\sqrt{10}$
V_1
$2V_1$
$4V_1$
$10V_1$
σ_+
$\sqrt{2}\sigma_+$
$2\sigma_+$
$\sqrt{10}\sigma_+$
$V_{\rm eff}[\sigma]$
σ
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0
2
4
6

- Other perturbations

SU(3) ⊃ SU(2) × U(1), $\mathbf{8} = \mathbf{3}_0 + \mathbf{2}_3 + \mathbf{2}_{-3} + \mathbf{1}_0$

$$A_i^a T^a = A_i^j T^j + A_i^{(q,s)} E^{(q,s)} + A_i^8 T^8,$$

$$E^{(+,\uparrow)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad E^{(+,\downarrow)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$E^{(-,\uparrow)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad E^{(-,\downarrow)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

- Other perturbations

SU(3) ⊃ SU(2) × U(1), $\mathbf{8} = \mathbf{3}_0 + \mathbf{2}_3 + \mathbf{2}_{-3} + \mathbf{1}_0$

$$
\begin{aligned}
L_{2,\mathbf{2}+\mathbf{2}} = & \left(\delta A_i^{+\prime}\right)^\dagger \delta A_i^{+\prime} - \left(\partial_i \delta A_j^+\right)^\dagger \partial_i \delta A_j^+ + \left(\partial_i \delta A_j^+\right)^\dagger \partial_j \delta A_i^+ \\
& + gaQ \left[\left(\delta A_j^+\right)^\dagger \frac{i\sigma_i}{2} \partial_i \delta A_j^+ - \left(\partial_i \delta A_j^+\right)^\dagger \frac{i\sigma_i}{2} \delta A_j^+ - \left(\delta A_j^+\right)^\dagger \frac{i\sigma_i}{2} \partial_j \delta A_i^+ + \left(\partial_j \delta A_i^+\right)^\dagger \frac{i\sigma_i}{2} \delta A_j^+\right] \\
& - \frac{(gaQ)^2}{2} \left[\left(\delta A_i^+\right)^\dagger \delta A_i^+ - 3\epsilon^{ijk} \left(\delta A_i^+\right)^\dagger \frac{i\sigma_j}{2} \delta A_k^+\right] \\
& - a\xi H \epsilon^{ijk} \left[\left(\delta A_i^+\right)^\dagger \partial_j \delta A_k^+ - \partial_j \left(\delta A_i^+\right)^\dagger \delta A_k^+ + 2gaQ \left(\delta A_i^+\right)^\dagger \frac{i\sigma_j}{2} \delta A_k^+\right]
\end{aligned}
$$

$$
L_{2,\mathbf{1}} = \frac{1}{2}\left[(\delta A_i^{8\prime})^2 - (\partial_i \delta A_j^8)^2 + \partial_i \delta A_j^8 \partial_j \delta A_i^8\right] - a\xi H \epsilon^{ijk} \delta A_i^8 \partial_j \delta A_k^8
$$

Chromo-natural inflation

- Equations of motion

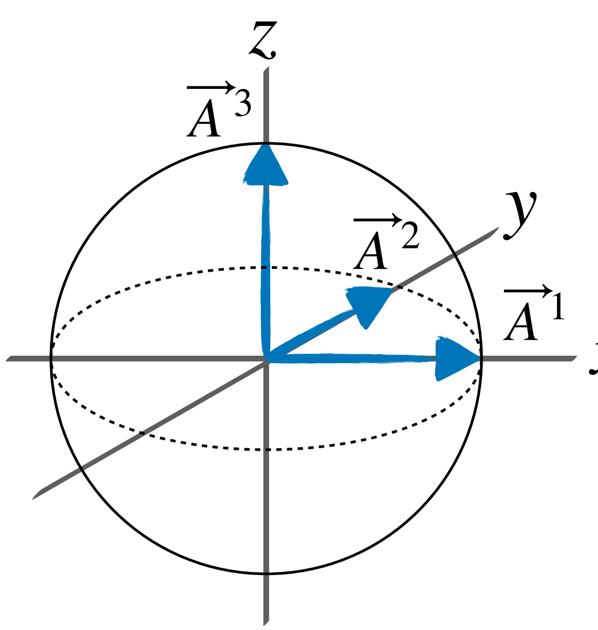
$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$

Using the temporal gauge $A^a_0 = 0$ and an ansatz $A^a_i(t) = \delta^a_i a(t)Q(t)$,
we obtain the background EoMs:

gauge field amplitude

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = -\frac{3g}{f}Q^2\left(\dot{Q} + HQ\right),$$

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H} + 2H^2\right)Q + 2g^2Q^3 = \frac{g}{f}Q^2\dot{\phi}$$



The motion of ϕ sources the gauge field Q.

- Equations of motion

In the slow-roll limit, we parametrize

$$\underbrace{\xi}_{\text{Axion velocity}} \equiv \frac{\dot{\phi}}{2fH}, \qquad \underbrace{m_Q}_{\text{Amplitude of } A_i^a} \equiv \frac{gQ}{H}.$$

$$\boxed{\begin{aligned} \cancel{\ddot{\phi} + 3H\dot{\phi}} + \partial_\phi V(\phi) &= -\frac{3g}{f}Q^2\left(\cancel{\dot{Q}} + HQ\right), \\ \cancel{\ddot{Q} + 3H\dot{Q}} + \left(\cancel{\dot{H}} + 2H^2\right)Q + 2g^2Q^3 &= \frac{g}{f}Q^2\dot{\phi} \end{aligned}}$$

$$m_Q - \xi m_Q^2 + m_Q^3 = 0$$

Then, the background solution is

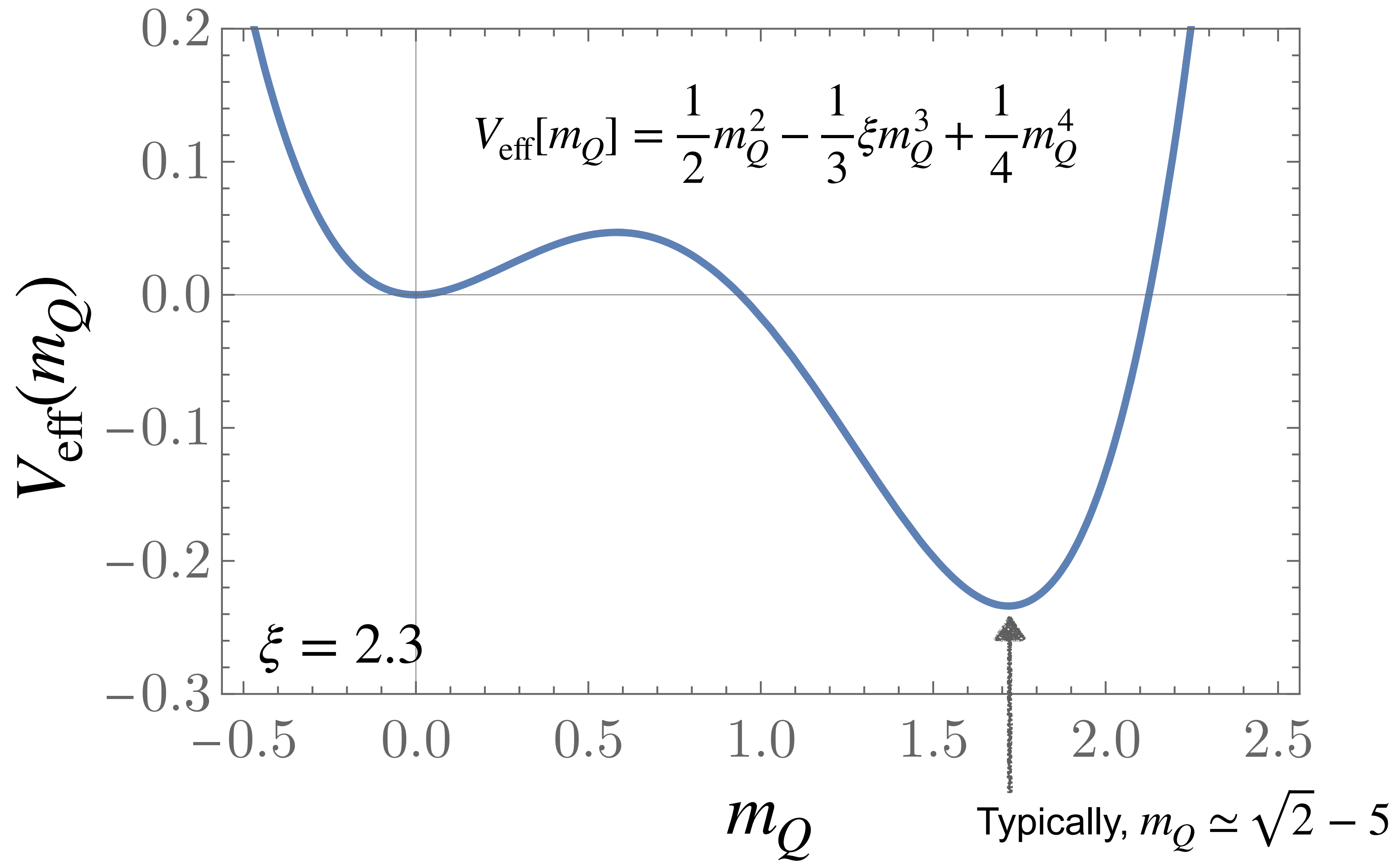
$$\boxed{m_Q \simeq \left(\frac{-g^2 f\partial_\phi V}{3H^4}\right)^{1/3}, \qquad \xi \equiv \frac{\dot{\phi}}{2fH} \simeq m_Q + m_Q^{-1}}$$

Chromo-natural inflation

- Stability

Consider the effective potential for m_Q.

→ $m_Q \neq 0$ is the true vacuum for $\xi > \dfrac{3}{\sqrt{2}}$.



- Examples: $N = 4$

SU(4) ⊃ SU(3) × U(1) $\quad \mathbf{4} = \mathbf{3} + \mathbf{1}$

$$\mathscr{T}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \lambda = \frac{1}{2}$$

SU(4) ⊃ SU(2) $\quad \mathbf{4} = \mathbf{4}$

$$\mathscr{T}_z = \frac{1}{\sqrt{10}}\begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \qquad \lambda = \frac{1}{\sqrt{10}}$$

SU(N)-natural inflation

- Examples: $N = 4$

SU(4) ⊃ SU(2) × SU(2) $\quad \mathbf{4} = (\mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$

$$\mathcal{T}_z = \frac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \sigma_i \end{pmatrix}$$

$$\bar{A}_z \propto \frac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix} \text{ or } \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \sigma_i \end{pmatrix} \qquad \lambda = 1$$

$$\bar{A}_z \propto \frac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \qquad \lambda = \frac{1}{\sqrt{2}}$$

SU(N)-natural inflation

- Examples: $N = 4$

SU(4) ⊃ SU(2) × SU(2) $\mathbf{4} = (\mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$

$$\mathcal{T}_z = \frac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \sigma_i \end{pmatrix} \qquad \lambda = 1$$

$$\mathcal{T}_z = \frac{1}{2\sqrt{2}}\begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \qquad \lambda = \frac{1}{\sqrt{2}}$$

For $N \geq 4$, multiple SU(2)s can embedded in SU(N).

SU(N)-natural inflation

- Examples: $N = 4$

SU(4) ⊃ SU(2) × SU(2) $\quad \mathbf{4} = (\mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$

$$\mathscr{T}_z = \frac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \sigma_i \end{pmatrix} \qquad \lambda = 1, \frac{1}{\sqrt{2}}$$

SU(4) ⊃ SU(2) × SU(2) $\quad \mathbf{4} = (\mathbf{2}, \mathbf{2})$

$$\mathscr{T}_z = \underbrace{\frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}}_{1_{2\times2} \otimes \sigma_z}, \quad \underbrace{\frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}}_{\sigma_z \otimes 1_{2\times2}} \qquad \lambda = \frac{1}{\sqrt{2}}, \frac{1}{2}$$

SU(N)-natural inflation

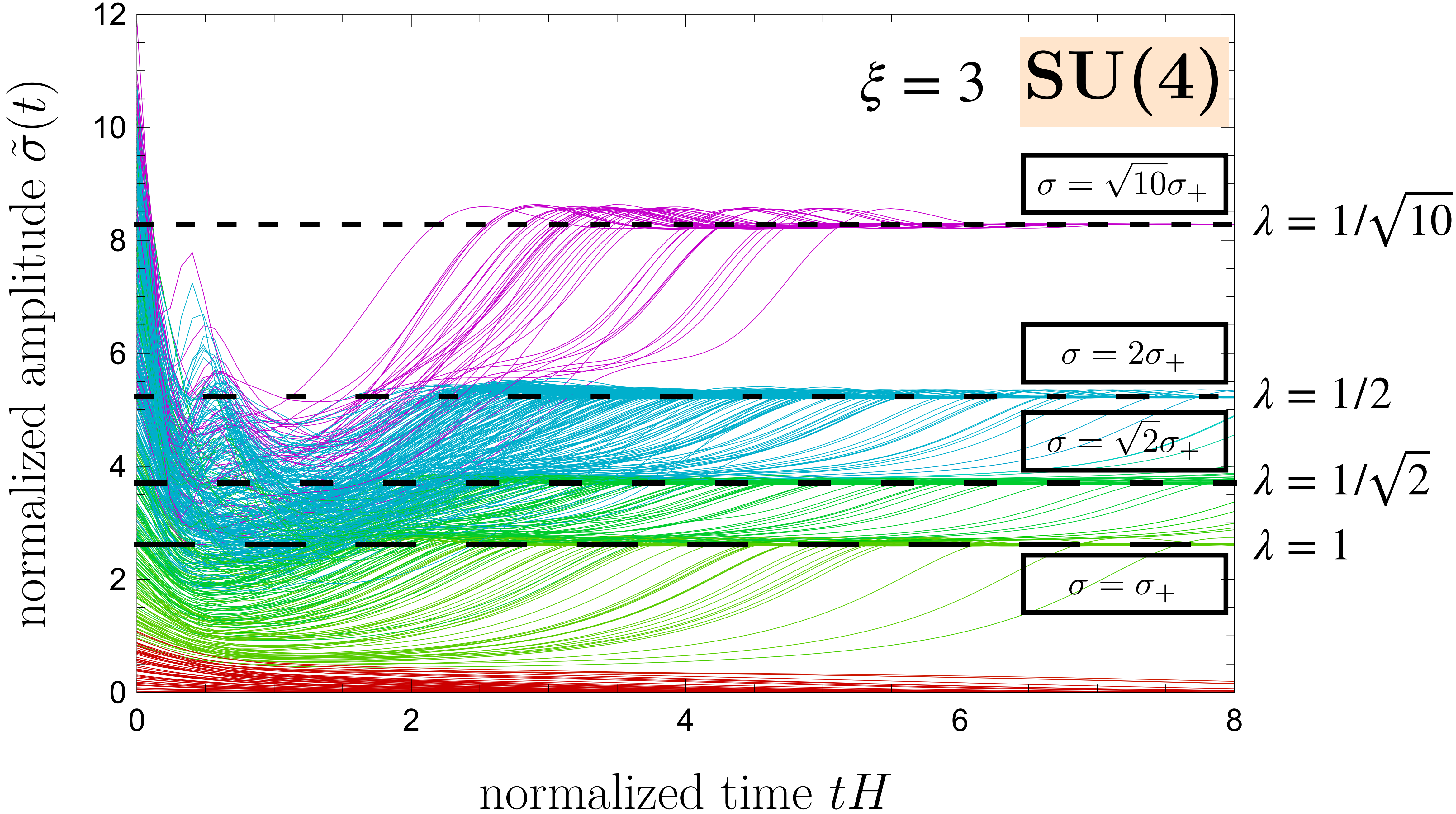
- General N

$$\mathrm{SU(N)} \supset \mathrm{SU(2)} \times \cdots \qquad \mathbf{N} = \mathbf{m} + \cdots \qquad (m = 2, \cdots, N)$$

$$\mathcal{T}_z = \lambda(m)\,\mathrm{diag}\left(\frac{m-1}{2}, \frac{m-3}{2}, \cdots, -\frac{m-1}{2}, 0, \cdots, 0\right)$$

$$\lambda(m) = \left(\frac{m(m^2-1)}{6}\right)^{-1/2}$$

- Numerical simulation



Chromo-natural inflation

- "Tensor" perturbations

Since $\bar{A}^a_i$ is invariant under the diagonal rotation of i and a:

$$\bar{A}^a_i \to U^{ij} U^{ab} \underbrace{\bar{A}^b_j}_{\propto \delta^b_j} = \bar{A}^a_i, \qquad (\ U\text{: rotation matrix}\ )$$

we can consider a as a spatial index.

Then, SVT decomposition can be applied to A^a_j.

To see the GW production, we focus on the tensor component:

$$g_{ij} = -a^2(\delta_{ij} + h_{ij}), \quad A^a_i = \bar{A}^a_i + \delta A^a_i = \bar{A}^a_i + t_{ia} + \cdots,$$

Traceless and transverse

Chromo-natural inflation

- "Tensor" perturbations

$$\psi_{ij}(\tau, \mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left[e^R_{ij}(\hat{\mathbf{k}})\Psi^R_{\mathbf{k}}(\tau) + e^L_{ij}(\hat{\mathbf{k}})\Psi^L_{\mathbf{k}}(\tau) \right],$$

$$t_{ij}(\tau, \mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left[e^R_{ij}(\hat{\mathbf{k}})T^R_{\mathbf{k}}(\tau) + e^L_{ij}(\hat{\mathbf{k}})T^L_{\mathbf{k}}(\tau) \right],$$

$$i\epsilon^{ikl}k^l e^{R/L}_{jk}(\mathbf{k}) = \pm\, k e^{R/L}_{ij}(\mathbf{k})$$

$$\psi_{ij} \equiv \frac{aM_{\mathrm{Pl}}}{2} h_{ij},$$

$$a\mathrm{d}\tau = \mathrm{d}t, \quad \epsilon_B \equiv \frac{g^2Q^4}{M_{\mathrm{Pl}}^2 H^2},$$

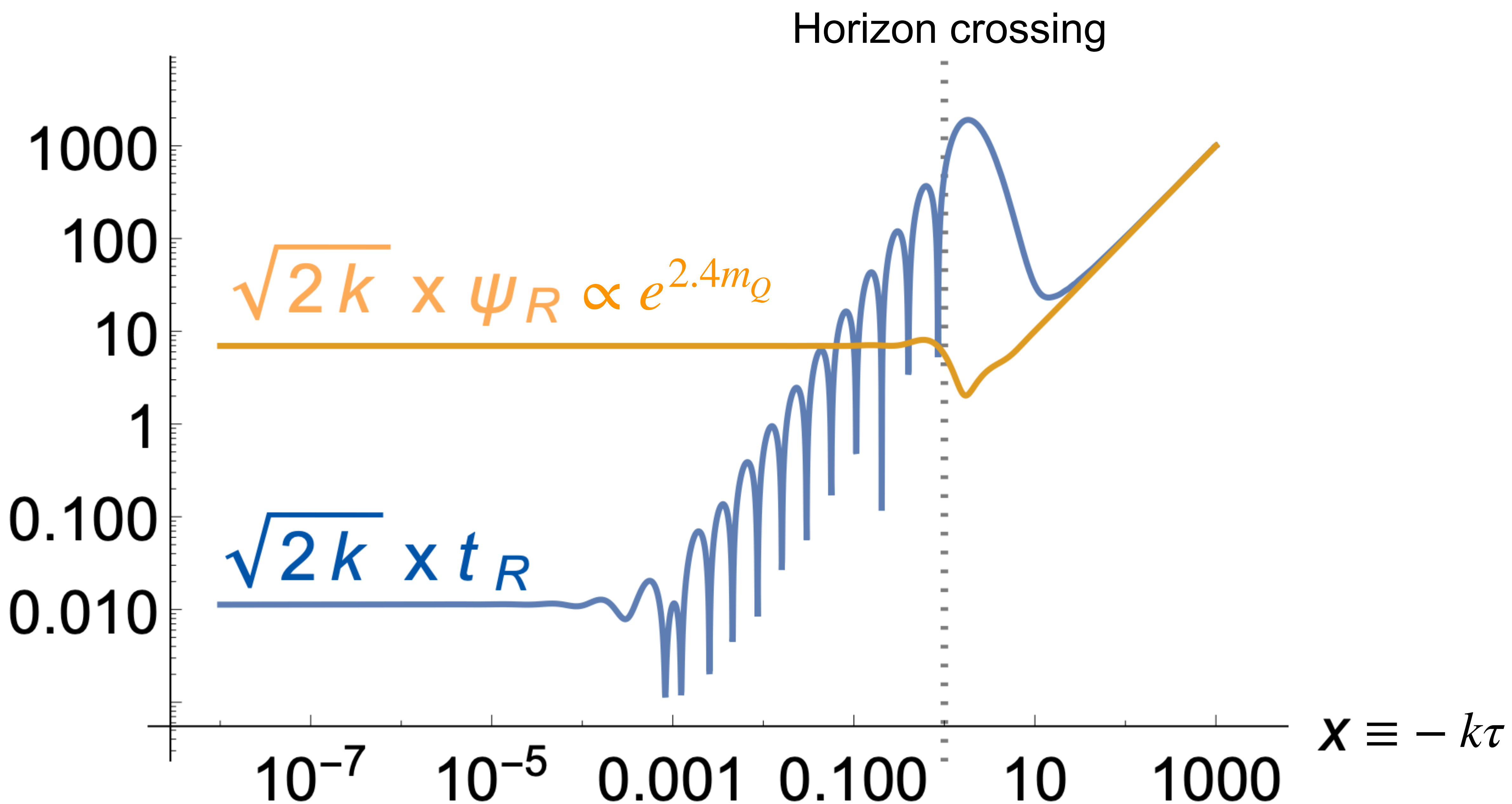
$$\partial_\tau^2 \Psi^{R/L} + \left[k^2 - \frac{2}{\tau^2} \right] \Psi^{R/L} = \frac{2\sqrt{\epsilon_B}}{m_Q \tau} \partial_\tau T^{R/L} \pm \frac{2k\sqrt{\epsilon_B}}{\tau} T^{R/L} + \frac{2\sqrt{\epsilon_B} m_Q}{\tau^2} T^{R/L},$$

$$\partial_\tau^2 T^{R/L} + \underbrace{\left[k^2 \pm \frac{2k(2m_Q + m_Q^{-1})}{\tau} + \frac{2(m_Q^2 + 1)}{\tau^2} \right]} T^{R/L} = \mathcal{O}\left(\Psi^{R/L}\right).$$

T^R experiences a tachyonic instability.

Ψ^R is sourced by T^R.

- "Tensor" perturbations



[E. Dimastrogiovanni, M. Fasiello, and T. Fujita (2016)]

Background gauge fields

Consider SU(N) gauge group:

$a = 1, \ldots, N^2 - 1$

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$

Assume $a \propto e^{Ht}$ and $\xi \equiv \dfrac{\dot{\phi}}{2fH} = \mathrm{const}\,.$

Parametrize A^a_i by $M^a_i(t) \equiv \dfrac{g}{a(t)H}A^a_i(t)$.

cf.) In CNI, $M^a_i = m_Q\delta^a_i$

EoM for background gauge fields

$$\frac{\ddot{M}^a_i}{H^2} + \frac{3}{H}\dot{M}^a_i + 2M^a_i + f^{bac}f^{bde}M^c_jM^d_iM^e_j - \xi\epsilon_{ijk}f^{abc}M^b_jM^c_k = 0,$$

Structure constant of SU(N)

SU(N)-natural inflation

■ Background gauge fields

EoM of static background gauge fields

$$\cancel{\frac{\ddot{M}_i^a}{H^2}} + \cancel{\frac{3}{H}\dot{M}_i^a} + 2M_i^a + f^{bac} f^{bde} M_j^c M_i^d M_j^e - \xi \epsilon_{ijk} f^{abc} M_j^b M_k^c = 0,$$

$$\rho_A = \frac{H^4}{4g^2} \left[f^{abc} f^{ade} M_i^b M_j^c M_i^d M_j^e + 2 \left(\cancel{\frac{\dot{M}_i^a}{H}} + M_i^a \right)^2 \right] \to \mathrm{const}\,.$$

We have considered homogeneous and static solutions.

But this is still difficult to solve…

→ We assume that “electromagnetic” fields are “parallel”.

SU(N)-natural inflation

- Parallelism of electromagnetic fields

E_i and B_i are coupled through the Chern-Simons coupling:

$$\phi F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \to \propto \xi E^a_i B^a_i$$

→ E_i and B_i source each other.

→ We assume $\underline{E_i \propto B_i \quad (i = x, y, z)}$

$E_i = E^a_i T^a, B_i = B^a_i T^a$

T^a : SU(N) generators

$$\mathrm{Tr}\left[T^a T^b\right] = \frac{\delta^{ab}}{2}$$

On the other hand, for constant M^a_i,

$$E^a_i = -\frac{aH^2}{g}M^a_i, \quad B^a_i = -\frac{a^2H^2}{2g}\epsilon_{ijk}f^{abc}M^b_j M^c_k$$

Then,

$$\boxed{B_i = \frac{ig\epsilon^{ijk}}{H^2}\left[E_j, E_k\right]}$$

SU(N)-natural inflation

- Parallelism of electromagnetic fields

E_i and B_i are coupled through the Chern-Simons coupling:

$$\phi F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \rightarrow \propto \xi E^a_i B^a_i$$

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$E_i = E^a_i T^a, B_i = B^a_i T^a$

T^a : SU(N) generators

$$\mathrm{Tr}\left[T^a T^b\right] = \frac{\delta^{ab}}{2}$$

Using $E_i \propto B_i \propto M_i \propto \mathscr{T}_i$,

$$B_i = \frac{ig\epsilon^{ijk}}{H^2}\left[E_j, E_k\right] \rightarrow \boxed{[\mathscr{T}_i, \mathscr{T}_j] = i\lambda\epsilon_{ijk}\mathscr{T}_k}$$

The basis of M_i, $\{\mathscr{T}_i\}$ generates an SU(2) subalgebra.

- Background solution

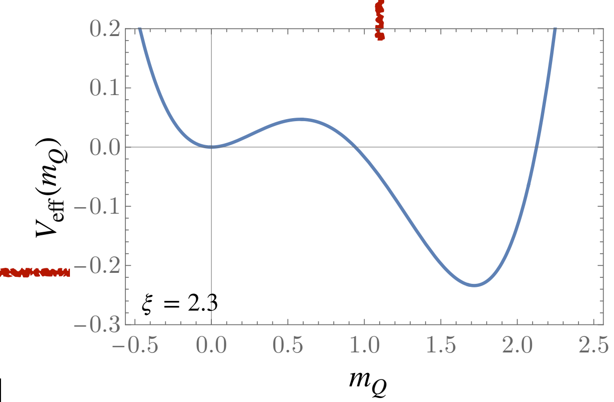
EoM

$$2M_i^a + f^{bac} f^{bde} M_j^c M_i^d M_j^e - \xi \epsilon_{ijk} f^{abc} M_j^b M_k^c = 0$$

$$M_i^j = \sigma_i \delta_i^j, \quad M_i^A = 0, \quad f^{ijk} = \lambda \epsilon_{ijk}$$

$$2\sigma_i + \lambda^2 \sigma_i \sum_{l \neq i} \sigma_l^2 - \xi\lambda \sum_{j,k} |\epsilon_{ijk}| \sigma_j \sigma_k = 0 \quad (i = x, y, z)$$

$$\sigma = \sigma_x = \sigma_y = \sigma_z, \quad \sigma = 0, \frac{\xi \pm \sqrt{\xi^2 - 4}}{2\lambda}$$



cf.) $\sigma = \dfrac{\xi + \sqrt{\xi^2 - 4}}{2}$ for CNI

SU(N)-natural inflation

- Isotropy of the solution

Since the gauge fields have nonzero VEV only in SU(2) part:

$$M_i = \sigma \mathscr{T}_i, \quad [\mathscr{T}_i, \mathscr{T}_j] = i\lambda\epsilon_{ijk}\mathscr{T}_k,$$

This solution is also isotropic as in CNI:

$$^\forall R, ^\exists G : R_{ij} M_j^a = G^{ab} M_i^b$$

SSB pattern is: $\mathrm{SO}(3) \times \underbrace{\mathrm{SU}(2)}_{\subset \mathrm{SU(N)}} \to \mathrm{SO}(3)$

The only difference from CNI is λ.

What values can λ take?

SU(N)-natural inflation

Numerical simulation

- Solve the EoM with time derivatives : $\left(a \propto e^{Ht}, \quad \xi \equiv \frac{\dot{\phi}}{2fH} = \mathrm{const}\,.\right)$

$$\frac{\ddot{M}_i^a}{H^2} + \frac{3}{H}\dot{M}_i^a + 2M_i^a + f^{bac}f^{bde}M_j^c M_i^d M_j^e - \xi\epsilon_{ijk}f^{abc}M_j^b M_k^c = 0,$$

- Initial condition is random variables :

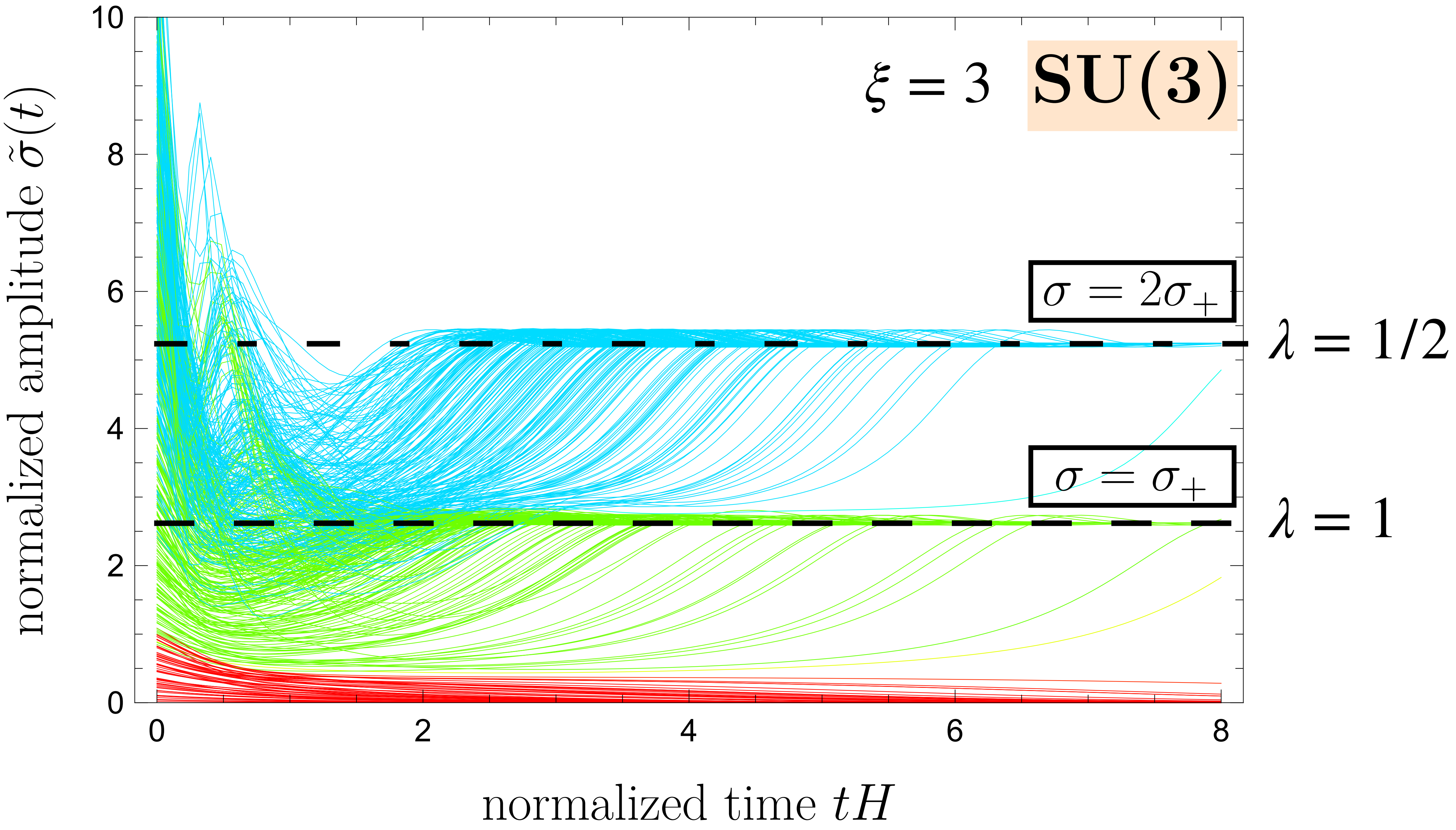
$$\dot{A}_i^a = 0, \quad M_i^a(t_0) = (\text{Gaussian distribution})$$

- We characterize the results by the averaged amplitude:

$$\tilde{\sigma}^2 = \frac{1}{3}\mathrm{Tr}[M_i M_i]$$

SU(N)-natural inflation

- Numerical simulation



Perturbations in SU(N)

- "Universality" of the linear perturbations

$$A_i = (\bar{A}_i^j + \delta A_i^j)\mathcal{T}_j + \delta A_i^A \mathcal{T}_A$$

With this decomposition, we have

$$F^a_{\mu\rho}F^a_{\nu\sigma} = \bar{F}^i_{\mu\rho}\bar{F}^i_{\nu\sigma} + \mathcal{O}(\delta A^j_\mu) + \mathcal{O}(\delta A^j_\mu \delta A^k_\rho) + \mathcal{O}(\delta A^B_\mu \delta A^C_\rho)$$

Then, the quadratic action of the perturbations has

$$\mathcal{O}(\delta g_{ij}\delta A^k_\mu), \quad \mathcal{O}(\delta A^j_\mu \delta A^k_\rho), \quad \mathcal{O}(\delta A^B_\mu \delta A^C_\rho)$$

Especially, $\mathcal{O}(\delta A^B_\mu \delta A^C_\rho)$ terms have forms of

$$\delta A^B_\mu \delta A^B_\rho \quad \text{or} \quad \underbrace{f^{iBC}}_{\text{action of } \mathcal{T}_i \text{ on } \mathcal{T}_B}\delta A^B_\mu \delta A^C_\rho \longleftarrow \text{close under SU(2)}$$