Co-financed by the Connecting Europe Facility of the European Union
INTERACTIONS

Einstein-Cartan Portal to Dark Matter

Inar Timiryasov
Niels Bohr Institute, University of Copenhagen

PASCOS 2022
July 26th 2022, Heidelberg

References

Mikhail Shaposhnikov, Andrey Shkerin, IT, and Sebastian Zell:

- Einstein-Cartan gravity, matter, and scale-invariant generalization, 2007.16158, JHEP 10 (2020) 177
- Higgs inflation in Einstein-Cartan gravity 2007.14978, JCAP 02 (2021) 008
- Einstein-Cartan Portal to Dark Matter, 2008.11686, Phys.Rev.Lett. 126 (2021) 16

Dark matter

- What is known about DM:
  - It *gravitates*

Metric and Palatini gravity

Lowest order action*

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{|g|} R$$

Riemann curvature tensor is expressed via connection $\Gamma^{\rho}_{\nu\sigma}$ as

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Metric gravity

- $\Gamma^{\rho}_{\nu\sigma}$ is **symmetric** with respect to lower indices
- $\Gamma^{\rho}_{\nu\sigma}$ is expressed in terms of metric via $g_{\mu\nu\,;\,\alpha} = 0$
- The dynamical variable is $g_{\mu\nu}$, variation with respect to $g_{\mu\nu}$ gives the Einstein equations

* we only colder 3+1 dimensions in this talk

Metric and Palatini gravity

Lowest order action $$S = \frac{M_P^2}{2}\int d^4x\sqrt{|g|}R$$

Riemann curvature tensor is expressed via connection $\Gamma^\rho_{\nu\sigma}$ as

$$R^\rho_{\sigma\mu\nu} = \partial_\mu\Gamma^\rho_{\nu\sigma} - \partial_\nu\Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}$$

Palatini gravity

- $\Gamma^\rho_{\nu\sigma}$ is **symmetric** with respect to lower indices
- The dynamical variables are $g_{\mu\nu}$ and $\Gamma^\rho_{\nu\sigma}$,
- Variation with respect to $\Gamma^\rho_{\nu\sigma}$ gives the relation between $\Gamma^\rho_{\nu\sigma}$ and $g_{\mu\nu}$ variation with respect to $g_{\mu\nu}$ gives the Einstein equations

Metric and Palatini gravity

Lowest order action $$S = \frac{M_P^2}{2} \int d^4x \sqrt{|g|} R$$

Riemann curvature tensor is expressed via connection $\Gamma^{\rho}_{\nu\sigma}$ as

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Metric gravity

$\Gamma^{\rho}_{\nu\sigma}$ is compatible with metric

$$\nabla_\alpha g^{\mu\nu} = 0$$

Palatini gravity

$\Gamma^{\rho}_{\nu\sigma}$ is independent

$$\frac{\delta S}{\delta \Gamma^{\alpha}_{\mu\nu}} \propto \nabla_\alpha g^{\mu\nu}$$

$$\frac{\delta S}{\delta g^{\mu\nu}} \propto R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

without matter Palatini gravity is equivalent to metric gravity

Einstein-Cartan gravity

Einstein-Cartan(-Sciama–Kibble) theory
gauging of the Poincaré group, Utiyama '56, Kibble '61

Riemann curvature tensor is expressed via connection $\Gamma^{\rho}_{\nu\sigma}$ as

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Symmetry of $\Gamma^{\rho}_{\nu\sigma}$ with respect to lower indices is not assumed.

Torsion tensor: $T^{\rho}_{\nu\sigma} = \Gamma^{\rho}_{\nu\sigma} - \Gamma^{\rho}_{\sigma\nu}$

Variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$

Variation with respect to $g_{\mu\nu}$ gives the Einstein equations

On the solution $T^{\rho}_{\nu\sigma} = 0$

Einstein-Cartan pure gravity is equivalent to metric gravity

Fermions in Einstein-Cartan gravity

- Fermions source torsion:

$$\epsilon_{\mu\lambda\rho\sigma} T^{\lambda\rho\sigma} + 3\, A_\mu = 0, \quad A^\mu = \bar{\psi}\, \gamma^\mu\, \gamma^5\, \psi$$

- Plugging back into action:

$$\frac{3}{16\, M_P^2} A^\mu A_\mu$$

- Torsion does not propagate

- Non-vanishing torsion allows introducing new couplings:

$$S = \frac{i}{2} \int \mathrm{d}^4 x \sqrt{-g}\, \left(\bar{\Psi} \left(1 - \boxed{i\alpha - i\beta\gamma^5} \right) \gamma^\mu D_\mu \Psi - \mathrm{h.c.} \right)$$

α, β — real parameters

Kibble '61 [https://aip.scitation.org/doi/10.1063/1.1703702]

For details of calculation see, e.g.
S. Mercuri gr-qc/0601013,
L. Freidel, D. Minic, T. Takeuchi, hep-th/0507253.

Einstein-Cartan theory

- Without matter — equivalent to GR
  (more general perspective — metric-affine theory,
  see, e.g. Hehl, Kerlick and Von Der Heyde Phys. Lett. B 63 (1976)
  also a book "Gravity and Strings" by Tomás Ortín;
  Rigouzzo and Zell Phys.Rev.D 106 (2022))

- More allowed terms in the action

- Fermions source torsion

- Torsion does not propagate

Can we distinguish EC theory from GR?

Fermonic action in Einstein-Cartan gravity

Integrating out torsion one arrives at
new universal four-fermion interaction

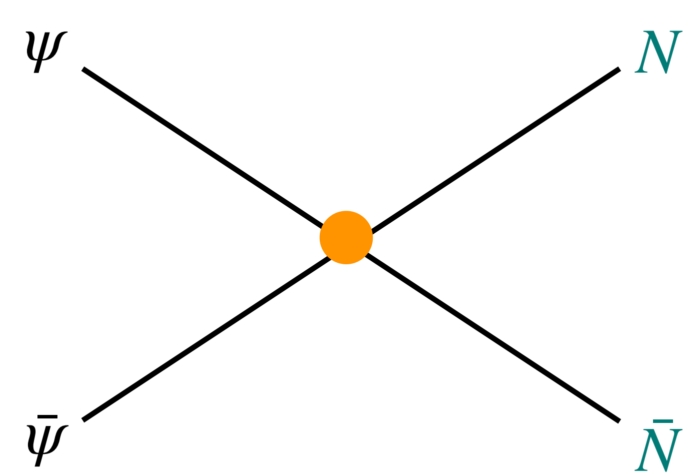
$$\mathcal{L}_{4f} = \frac{-3\alpha^2}{16M_P^2} V^\mu V_\mu - \frac{3\alpha\beta}{8M_P^2} V^\mu A_\mu + \frac{3-3\beta^2}{16M_P^2} A^\mu A_\mu$$

$$V^\mu = \bar{N}\gamma^\mu N + \sum_{SM} \bar{\psi}\gamma^\mu\psi$$

$$A^\mu = \bar{N}\gamma^5\gamma^\mu N + \sum_{SM} \bar{\psi}\gamma^5\gamma^\mu\psi$$

This interaction is universal and also affects a new hypothetical singlet particle— DM candidate

X are the SM fermions



Freeze-in DM production via Four-Fermion Interactions

$$\mathscr{L}_{4f} = \frac{-3\alpha^2}{16M_P^2} V^\mu V_\mu - \frac{3\alpha\beta}{8M_P^2} V^\mu A_\mu + \frac{3-3\beta^2}{16M_P^2} A^\mu A_\mu$$

Allows for annihilation of the SM particles
$\bar{X} + X \to \bar{N} + N$

Kinetic description of N production:

$$\left(\frac{\partial}{\partial t} - H q_i \frac{\partial}{\partial q_i}\right) f_N(t, \vec{q}) = R(\vec{q}, T)$$

number density

$$R = \frac{1}{2|q|} \sum_X \int \frac{\mathrm{d}^3\vec{p}_1}{(2\pi)^3\, 2E_1} \frac{\mathrm{d}^3\vec{p}_2}{(2\pi)^3\, 2E_2} \frac{\mathrm{d}^3\vec{p}_3}{(2\pi)^3\, 2E_3} \times (2\pi)^4 \delta^{(4)}\left(p_1 + p_2 - q - p_3\right) |\overline{\mathcal{M}}_X|^2 f_X(p_1) f_{\bar{X}}(p_2)$$

we assume thermal distributions of the SM particles

DM abundance:

$$\frac{\Omega_N}{\Omega_{DM}} \simeq 3.6 \cdot 10^{-2} C_f \left(\frac{M_N}{10\mathrm{keV}}\right)\left(\frac{T_{\mathrm{prod}}}{M_P}\right)^3$$

$$C_M = \frac{9}{4}\left\{24\left(1+\alpha^2-\beta^2\right)^2 + 21\left(1-(\alpha+\beta)^2\right)^2\right\}$$

Einstein-Cartan portal to dark matter

- **Higgs inflation** in EC theory
  [Långvik, Ojanperä, Raatikainen, Räsänen, arXiv:2007.12595
  Shaposhnikov, Shkerin, IT, and Zell, arXiv:2007.14978]

- Almost **instantaneous** preheating in Higgs inflation
  [DeCross, Kaiser, Prabhu, Prescod-Weinstein, Sfakianakis; Ema, Jinno, Mukaida, Nakayama ; Rubio, Tomberg; Bezrukov, Shepherd, Dux, Florio, Klarić, Shkerin, IT]

- We take $T_{prod} = T_{reh}$

$$T_{\rm reh} \simeq \left(\frac{15\lambda}{2\pi^2 g_{\rm eff}}\right)^{\frac{1}{4}} \frac{M_P}{\sqrt{\xi}}$$

$$\frac{\Omega_N}{\Omega_{DM}} \simeq 1.4 \frac{\sqrt{\xi}\lambda^{3/4}}{g_{\rm eff}^{3/4}} \frac{(\alpha+\beta)^4}{\xi^2} \left(\frac{M_N}{10\mathrm{keV}}\right) \left(\frac{T_{\rm prod}}{T_{\rm reh}}\right)^3$$

Einstein-Cartan portal to dark matter

- Two "natural" choices of α and β:

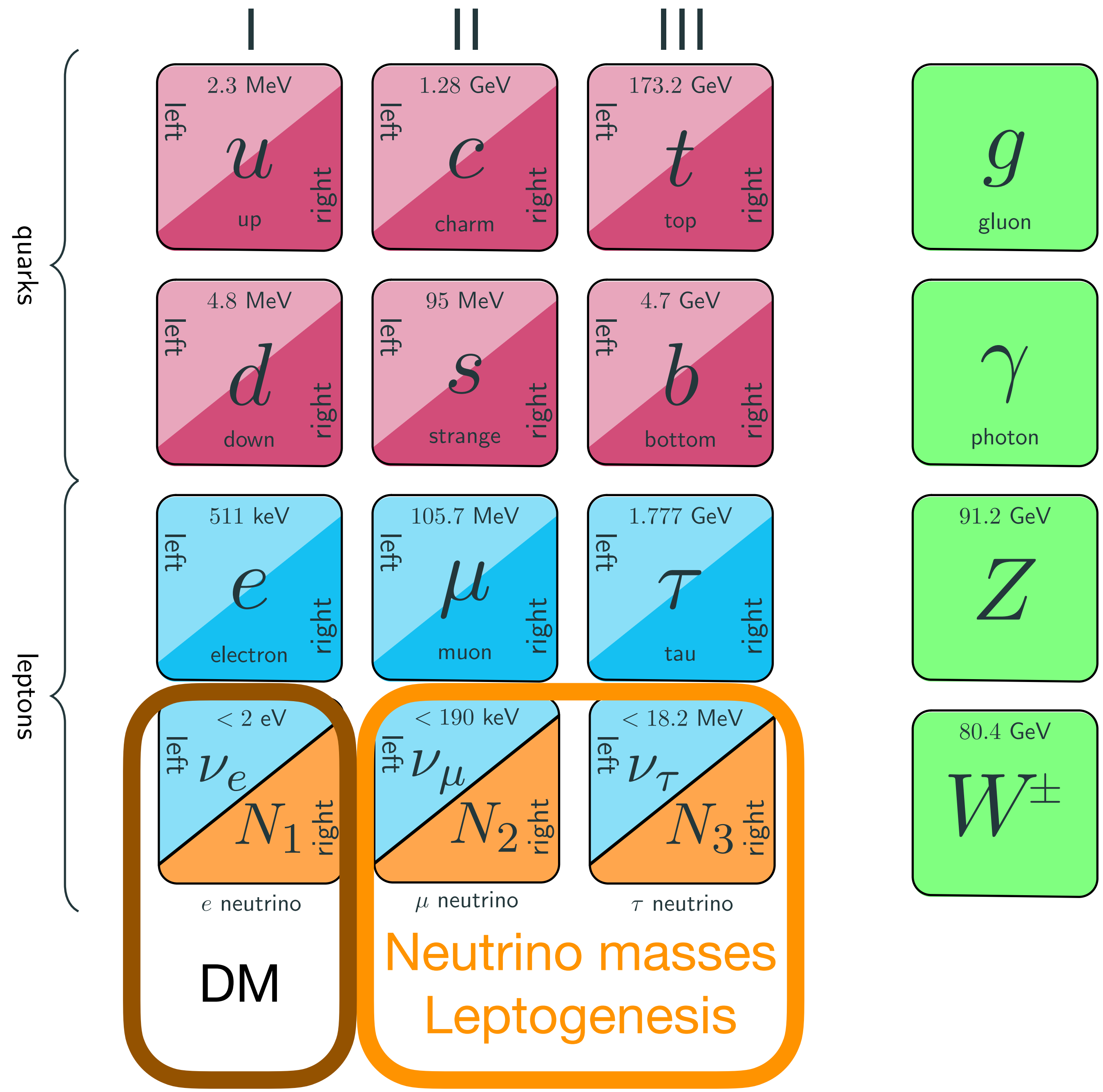
- $\alpha = \beta = 0$
  the correct DM abundance is obtained for $(3-6)\times 10^8$ GeV fermion in Palatini Higgs inflation

- $\alpha \sim \beta \sim \sqrt{\xi}$ (universal UV cutoff $\Lambda \sim M_P/\sqrt{\xi}$)
  the correct DM abundance is obtained for a keV fermion in Palatini Higgs inflation

$$\frac{\Omega_N}{\Omega_{DM}} \simeq 1.4 \frac{\sqrt{\xi}\lambda^{3/4}}{g_{\rm eff}^{3/4}} \frac{(\alpha+\beta)^4}{\xi^2} \left(\frac{M_N}{10\rm keV}\right)\left(\frac{T_{\rm prod}}{T_{\rm reh}}\right)^3$$

Einstein-Cartan portal to dark matter

and the νMSM



125.1 GeV
H
Higgs

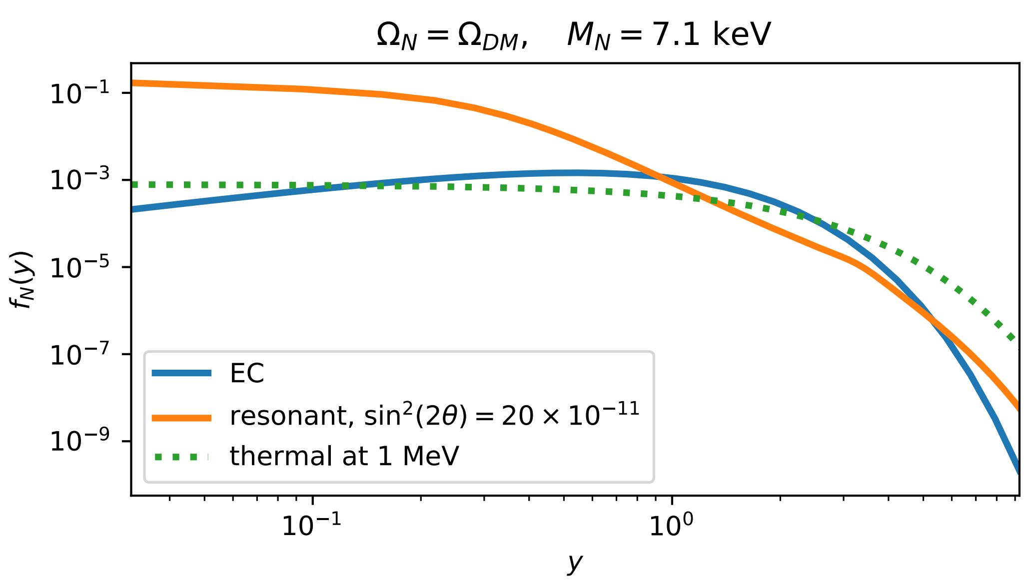
EW symmetry breaking
inflation

direct searches!

Asaka, Blanchet, Shaposhnikov 2005
Asaka, Shaposhnikov 2005

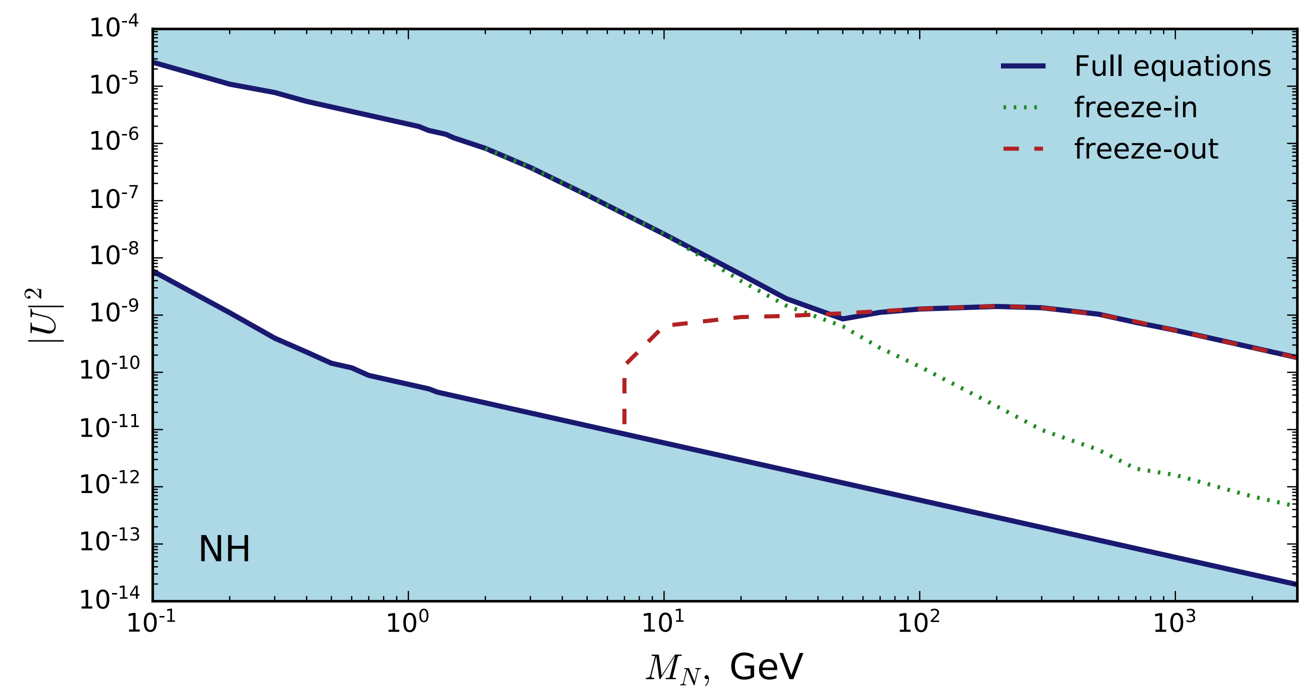
Einstein-Cartan portal to dark matter
and the νMSM

N_1



N_1 momentum distribution

$N_{2,3}$



Juraj Klarić, Mikhail Shaposhnikov, IT
2008.13771, Phys.Rev.Lett. 127 (2021)
2103.16545, Phys.Rev.D 104 (2021)

$N_{2,3}$ are also produced by EC,
but $n_{2,3} \simeq 10^{-2} n_{eq} \left(10\text{keV}/M_1\right)$
So leptogenesis is not affected

See the talk on leptogenesis by Juraj Klarić

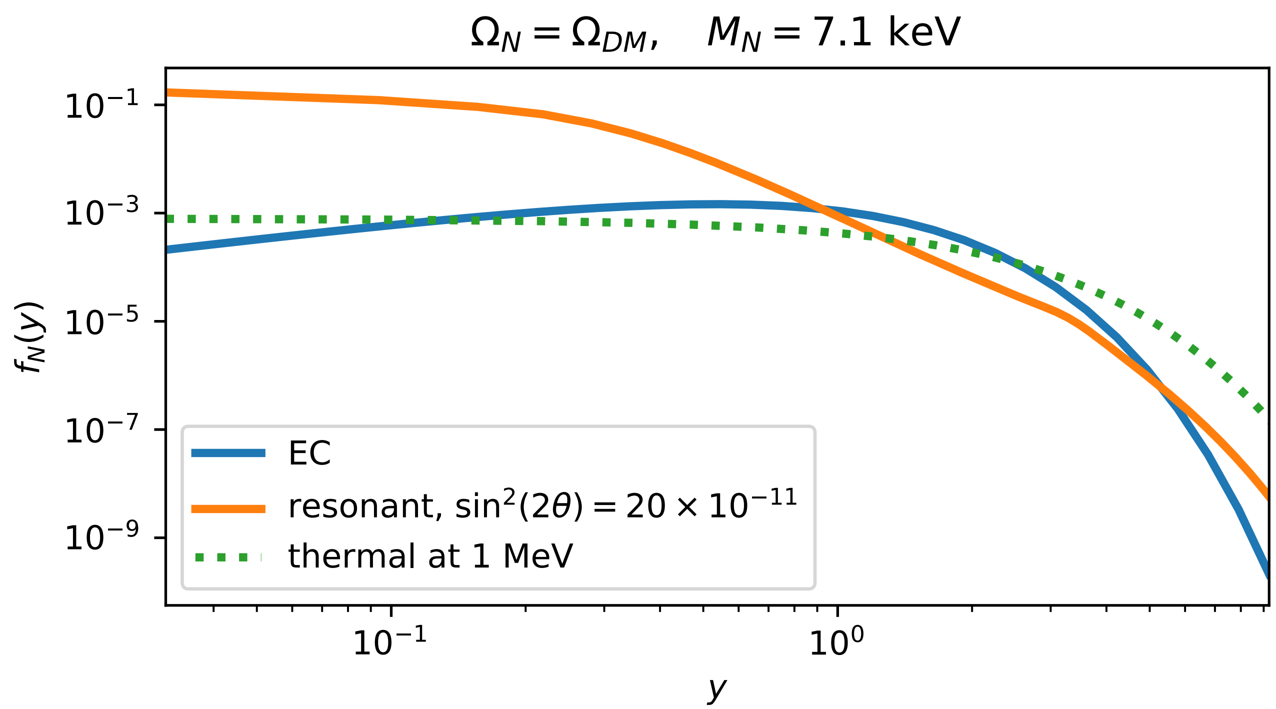
Summary

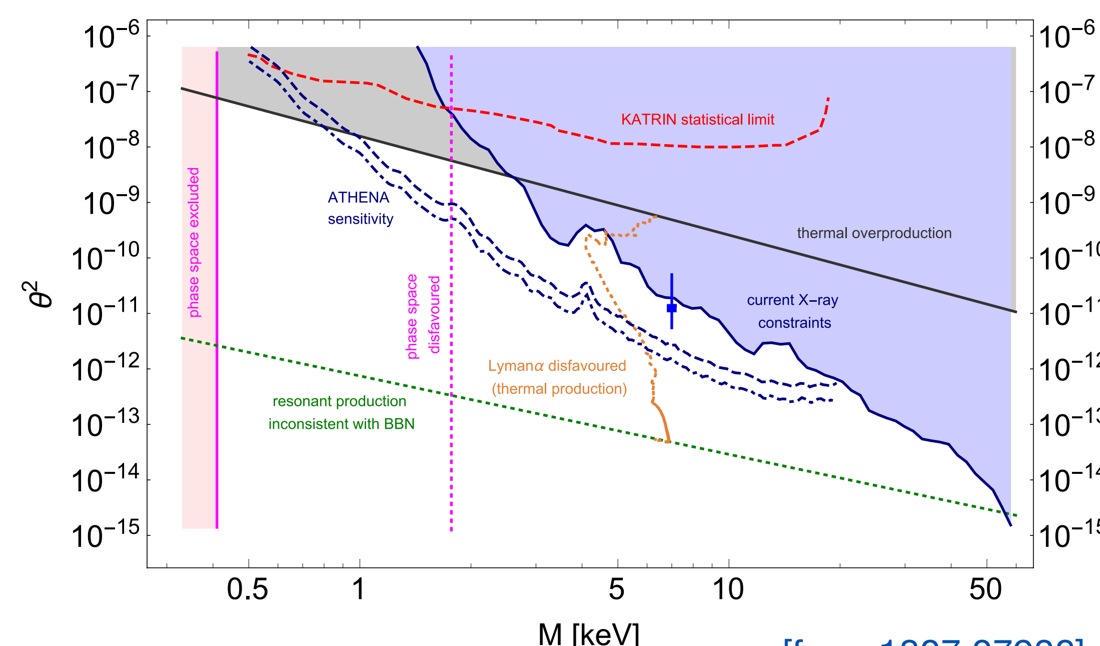
- Different formulations of gravity are equivalent without matter.
- This changes once gravity is coupled to matter, such as fermions or a non-minimally coupled scalar field.
    - A new universal mechanism for fermion dark matter production.
    - Properties of fermion dark matter may be able to discern the Einstein-Cartan theory of gravity from the most commonly used metric formulation.

backup slides

Einstein-Cartan portal to dark matter
and the νMSM

N_1 — momentum distribution





[from 1807.07938]

Fermions in Einstein-Cartan gravity

- Appropriate variables:
  - e_μ^a - tetrad field (translations)
  - ω_μ^{ab} - spin connection (local Lorentz transformations)
- Fermionic action

$$S = \frac{i}{2}\int \mathrm{d}^4x\sqrt{-g}\left(\bar{\Psi}\gamma^\mu D_\mu\psi - \mathrm{h.c.}\right)$$

$$D_\mu\Psi = \left(\partial_\mu + \frac{1}{8}\omega_{\mu ab}\left[\gamma^a, \gamma^b\right]\right)\Psi$$

Higgs inflation in EC gravity

Non-minimal coupling to Higgs can be added

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{|g|}R + \frac{M_P^2}{2\bar{\gamma}}\int \mathrm{d}^4x\sqrt{|g|}\,\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} + M^2\int \mathrm{d}^4x\,\partial_\mu\left(\sqrt{|g|}\,\epsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)$$

M. Långvik, J. Ojanperä, S. Raatikainen, S. Räsänen,
Higgs inflation with the Holst and the Nieh-Yan term, arXiv:2007.12595

M. Shaposhnikov, A. Shkerin, IT, and Sebastian Zell:,
Higgs inflation in Einstein-Cartan gravity, arXiv:2007.14978

Higgs inflation in EC gravity: Nieh-Yan invariant

Metric/Palatini:

$$S = \int \mathrm{d}^4x\sqrt{-\hat{g}}\left\{-\frac{1}{2}\left(\frac{1}{\Omega^2}+\frac{6\xi^2h^2}{M_p^2\Omega^4}\right)\left(\partial_\mu h\right)^2 - \frac{\lambda}{4}\frac{h^4}{\Omega^4} + \frac{M_P^2}{2}\hat{R}\right\}$$

only metric

flat potential

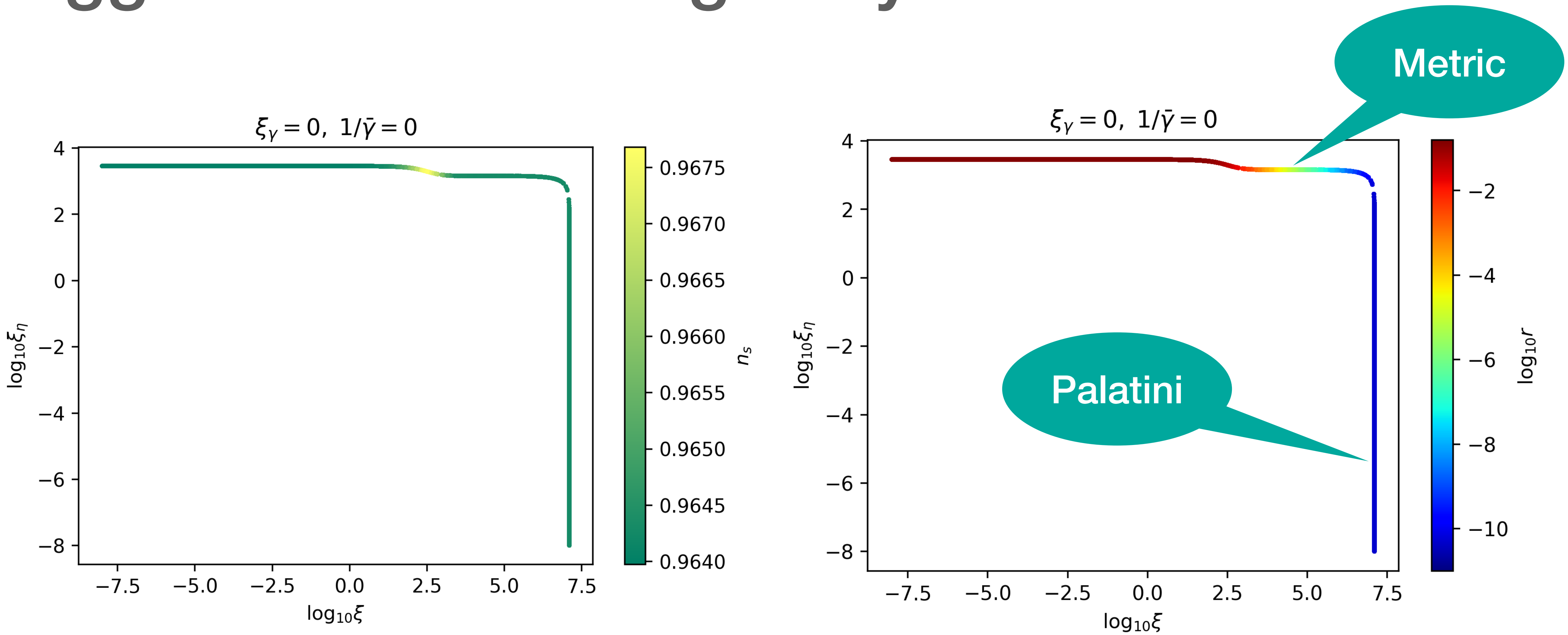
EC with Nieh-Yan invariant

$$S = \int \mathrm{d}^4x\sqrt{-\hat{g}}\left\{-\frac{1}{2}\left(\frac{1}{\Omega^2}+\frac{3\xi_\eta^2h^2}{2M_P^2\Omega^4}\right)\left(\partial_\mu h\right)^2 - \frac{\lambda}{4}\frac{h^4}{\Omega^4} + \frac{M_P^2}{2}\hat{R}\right\}$$

One can interpolate between metric and Palatini Higgs inflation by varying ξ_η

$$S_{\mathrm{grav}} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(M_P^2+\xi h^2\right)R$$
$$+\frac{1}{2\bar{\gamma}}\int \mathrm{d}^4x\sqrt{-g}\left(M_P^2+\xi_\gamma h^2\right)\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$$
$$+\frac{1}{2}\int \mathrm{d}^4x\,\xi_\eta h^2\partial_\mu\left(\sqrt{-g}\,\epsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)$$

Higgs inflation in EC gravity: Nieh-Yan invariant



(a) (b)

Figure 1. Spectral tilt (a) and tensor-to-scalar ratio (b) in Nieh-Yan inflation. We take $N_\star = 55$ and $\lambda = 10^{-3}$. The regions of Palatini (the right vertical segment) and metric (the "ankle" at which n_s and r vary considerably) Higgs inflation are clearly distinguishable. The transition between the two regions is smooth and stays within the observational bounds. The left horizontal segment has $r > 0.1$ and is not compatible with observations.

$$S_{\rm grav} = \frac{1}{2}\int d^4x\sqrt{-g}\,\left(M_P^2+\xi\,h^2\right)R$$
$$+\frac{1}{2\bar{\gamma}}\int d^4x\sqrt{-g}\left(M_P^2+\xi_\gamma\,h^2\right)\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$$
$$+\frac{1}{2}\int d^4x\,\xi_\eta\,h^2\partial_\mu\left(\sqrt{-g}\,\epsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)$$

Generic Einstein-Cartan Higgs inflation

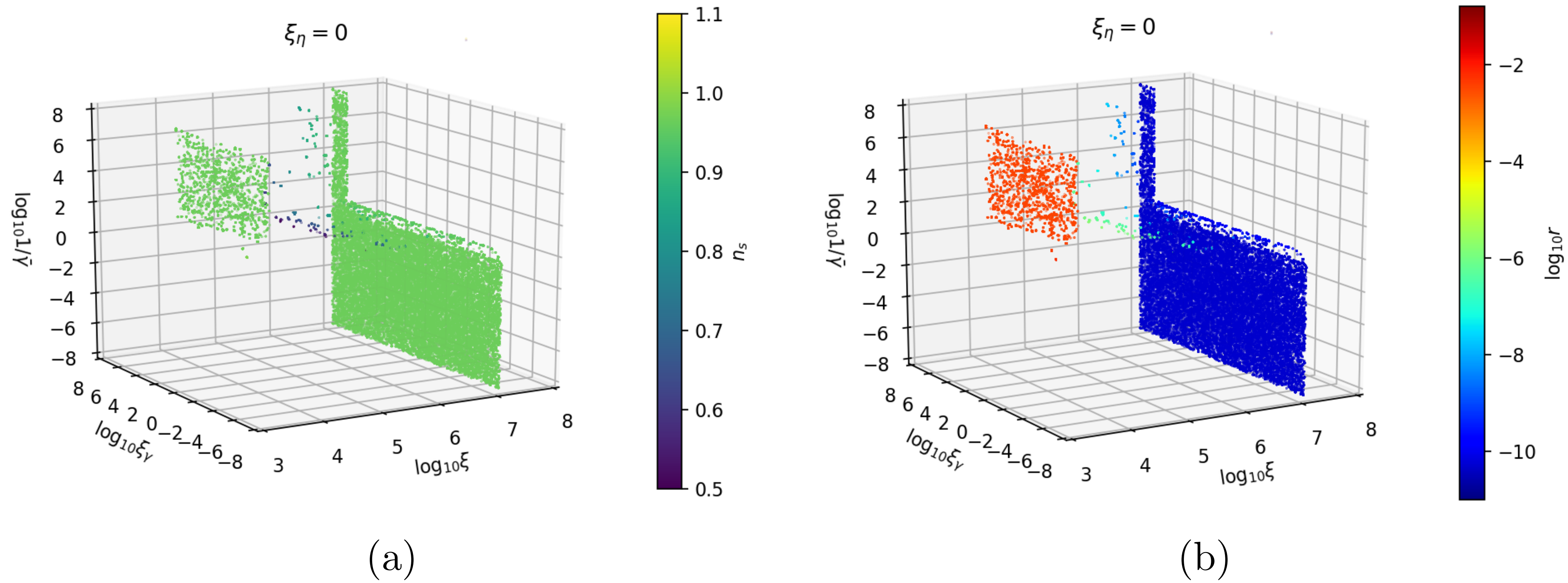


Figure 6. Spectral tilt (a) and tensor-to-scalar ratio (b) in the case $\xi_\eta = 0$. The right and left parts of the plots correspond to generalisations of the Palatini and metric Higgs inflation, respectively.

$$S_{\mathrm{grav}} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\left(M_P^2 + \xi\, h^2\right) R$$
$$+\frac{1}{2\bar{\gamma}}\int \mathrm{d}^4x\sqrt{-g}\,\left(M_P^2 + \xi_\gamma\, h^2\right)\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$$
$$+\frac{1}{2}\int \mathrm{d}^4x\,\xi_\eta\, h^2\partial_\mu\left(\sqrt{-g}\,\epsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)$$