Inflation with 2-form field:

the production of primordial black holes and gravitational waves

Ippei Obata (Max-Planck-Institute for Astrophysics, JSPS fellow)

arXiv: 2202.02401

In collaboration with
Tomohiro Fujita, Hiromasa Nakatsuka, Sam Young

科研費
KAKENHI

ダークマターの正体は何か？
広大なディスカバリースペースの網羅的研究
文部科学省
科学研究費助成事業
学術変革領域研究
（2020–2024）
What is dark matter? - Comprehensive study of the huge discovery space in dark matter

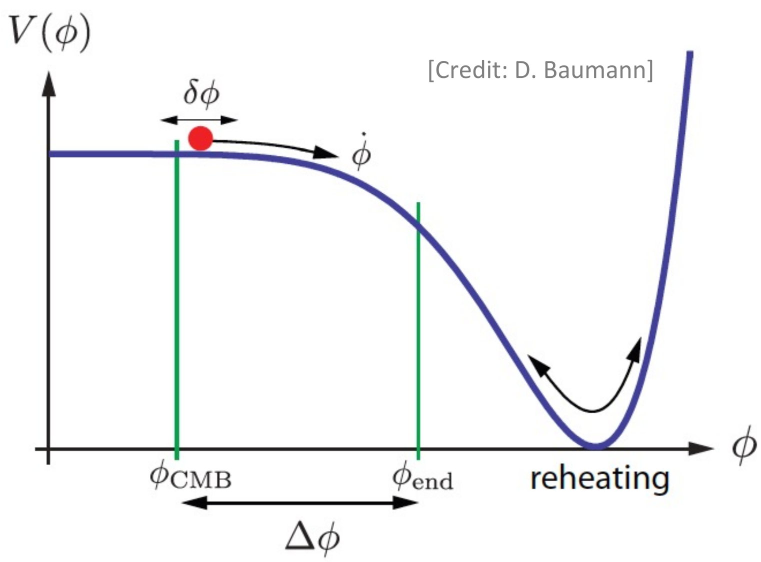
MAX-PLANCK-INSTITUT
FÜR ASTROPHYSIK

25.7.2022 PASCOS (Heidelberg)

Inflationary universe

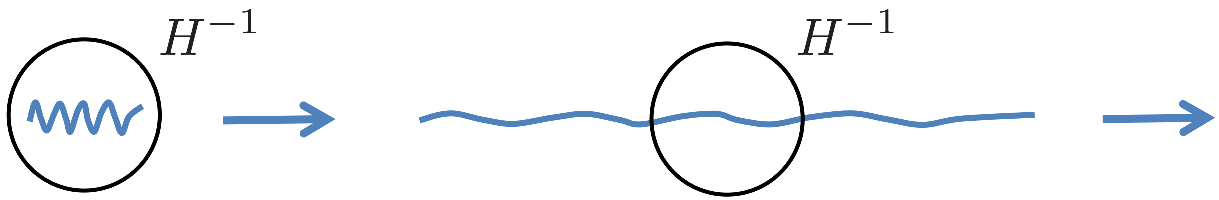
Sato; Guth; (1981), ...

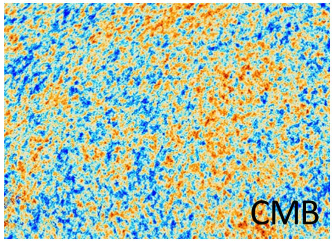
- Era of (nearly) exponential expansion in early universe: $a(t) \propto e^{Ht}$



- Scalar field (inflaton) plays a role of inflation

- Mechanism for providing **the seed of cosmological fluctuations**





Standard prediction

- Primordial density perturbation (curvature perturbation)

$$dl^2 = a(t)^2\,[1+2\zeta(t,\boldsymbol{x})]\,[\delta_{ij}+h_{ij}(t,\boldsymbol{x})]\,dx^i dx^j$$

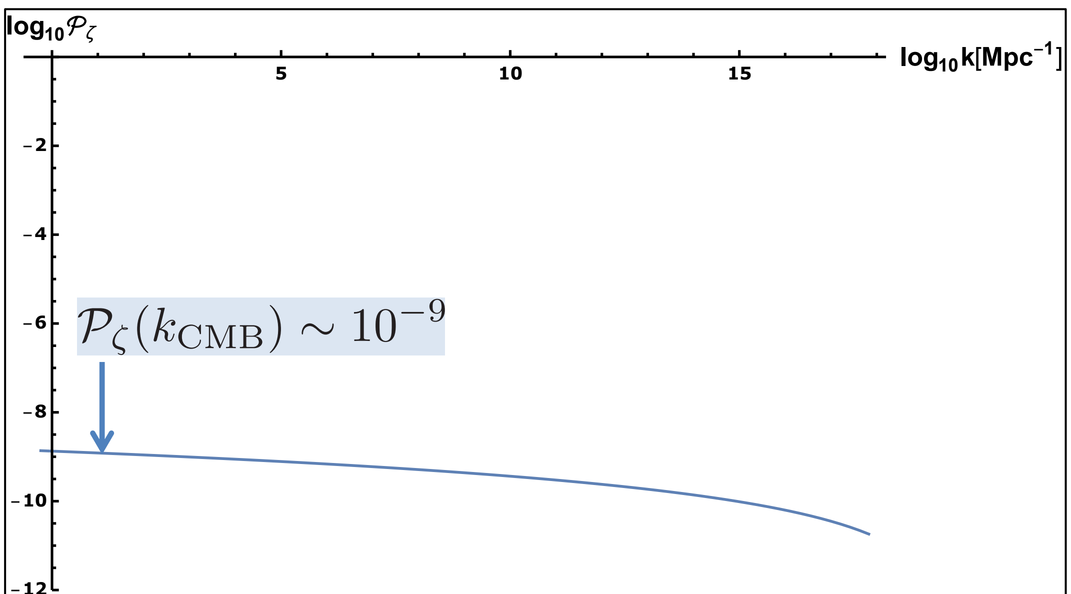
provides a **nearly scale-invariant** power spectrum:

$$\langle \zeta_{\boldsymbol{k}}\zeta_{\boldsymbol{k}'}\rangle \equiv (2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k}')\frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k)$$

$$\mathcal{P}_\zeta = \left.\frac{H^2}{8\pi^2 M_p^2 \epsilon_H}\right|_{k=aH} \equiv A_s\left(\frac{k}{k_*}\right)^{n_s-1}$$

(CMB scales)

$$A_s \simeq 2\times 10^{-9}\;,\;\; n_s \simeq 0.96$$



- Obtained by the **vacuum fluctuation**

(no matter sectors are included)

Inflation with form fields

- In higher dimensional theories, scalar sectors are naturally coupled to matter sectors (e.g. form fields):

$$\mathcal{L} \supset I(\varphi)^2 F_{\mu\nu}F^{\mu\nu}, \ I(\varphi)^2 H_{\mu\nu\rho}H^{\mu\nu\rho}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$: vector field (1-form field)

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$$: antisymmetric tensor field (2-form field)

- **Time variation of the kinetic function** could trigger the **particle production of form fields** during inflation
  → enhance the coupled cosmological perturbations

Rich cosmological phenomena

$I^2 FF$ model

- Generation of primordial magnetic fields ***Ratra (1992); Martin, Yokoyama (2008); Fujita, Mukohyama (2012);… (a lot)***
- Anisotropic inflation models ***Watanabe, Kanno, Soda (2009)…; Ito, Soda (2015);***
- Statistically-anisotropic primordial GWs ***Fujita, IO, Tanaka, Yokoyama (2018); Hiramatsu, Murai, IO, Yokoyama (2020);***
- Generation of primordial BHs & GWs ***Kawasaki, Nakatsuka, IO (2019)***

$I^2 HH$ model

- Anisotropic inflation models ***Ohashi, Tsujikawa, Soda (2013)…; Ito, Soda (2015);***
- Statistically-anisotropic primordial GWs ***IO, Fujita (2018);***
- **Generation of Primordial BHs & GWs** ***Fujita, Nakatsuka, IO, Young (2022);***

Inflation with two-form field

- Consider the two-form field kinetically coupled to inflaton

$$\mathcal{L} = \frac{1}{2}M_p^2 R - \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi) - \frac{1}{12} I(\varphi)^2 H_{\mu\nu\rho} H^{\mu\nu\rho}$$

FLRW Metric: $ds^2 = -dt^2 + a(t)^2 d\boldsymbol{x}^2 = a(\tau)^2(-d\tau^2 + d\boldsymbol{x}^2)$

Gauge conditions: $\partial_i B_{ij} = 0 \ , \quad \partial_i B_{0i} = 0$

- Consider the Fourier decomposition of dynamical component:

$$B_{ij}(t, \boldsymbol{x}) = \int \frac{d\boldsymbol{k}}{(2\pi)^3} \hat{B}_{\boldsymbol{k}} \epsilon_{ij}(\hat{\boldsymbol{k}}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$k_i \epsilon_{ij}(\hat{\boldsymbol{k}}) = 0 \ , \quad \epsilon_{ij}(-\hat{\boldsymbol{k}}) = \epsilon^*_{ij}(\hat{\boldsymbol{k}}) \ , \quad \epsilon_{ij}(\hat{\boldsymbol{k}})\epsilon^*_{ij}(\hat{\boldsymbol{k}}) = 2$

$$\hat{B}_{\boldsymbol{k}} = B_k a_{\boldsymbol{k}} + B_k^* a^\dagger_{-\boldsymbol{k}}$$

$[a_{\boldsymbol{k}}, \ a^\dagger_{-\boldsymbol{k}'}] = (2\pi)^3 \delta(\boldsymbol{k} + \boldsymbol{k}')$

↑ find a solution of mode function

Solution of mode function (1)

- EOM for the mode function:

$$\left[\partial_\tau^2 + k^2 - \frac{\partial_\tau^2 I}{I} - \frac{2\partial_\tau I}{\tau I}\right]\left(\frac{IB_k}{a}\right) = 0$$

- Define the following index: $n \equiv -\frac{\dot{I}}{HI} \rightarrow I(\varphi(\tau)) \propto a(\tau)^{-n}$

when $n = n_0 \ (\mathrm{const.})$, EOM leads to

$$\left[\partial_\tau^2 + k^2 - \frac{n_0(n_0+1)}{\tau^2}\right]\left(\frac{IB_k}{a}\right) = 0$$ (Bessel-type equation)

Then, we obtain $$\frac{IB_k}{a} = \frac{e^{i(n_0+1)\pi/2}}{\sqrt{2k}}\sqrt{\frac{-\pi k\tau}{2}} H^{(1)}_{n_0+1/2}(-k\tau)$$

(Banch-Davies initial condition is chosen)

Solution of mode function (2)

- Then, using the following asymptotic form in the super-horizon limit:

$$H^{(1)}_{\nu>0}(x) \to -\frac{i}{\pi}\Gamma(\nu)\left(\frac{2}{x}\right)^{\nu} \quad (x \equiv -k\tau \to 0)$$

The "electromagnetic-like" component of two-form field

$$E_k = \frac{I\dot{B}_k}{a^2}\ , \quad M_k = \frac{kIB_k}{a^3}$$

are computed as

$$E_k = \frac{H^2 e^{i\frac{n_0+1}{2}\pi}}{\sqrt{2k^3}}\sqrt{\frac{\pi x^5}{2}}H^{(1)}_{n_0+3/2}(x) \propto \left(\frac{a_k}{a}\right)^{1-n_0}$$

$$M_k = \frac{H^2 e^{i\frac{n_0+1}{2}\pi}}{\sqrt{2k^3}}\sqrt{\frac{\pi x^5}{2}}H^{(1)}_{n_0+1/2}(x) \propto \left(\frac{a_k}{a}\right)^{2-n_0}$$

$$(a_k \equiv k/H \ll a)$$

- Electric (magnetic) field is amplified when $n_0 > 1\ (n_0 > 2)$

However…

- **In most cases, the index "n" is not a constant but a dynamical value**

Ex) consider the simplest configuration

$$I(\varphi) = I_0 \exp\left(\frac{\varphi}{\Lambda}\right) \quad \rightarrow \quad n = \frac{\dot{\varphi}}{H\Lambda} \neq \text{const.}$$

Since the speed of scalar field naturally increases in time,
we need to solve the EOM with dynamical "n":

$$\partial_\tau^2 V_k + \left(k^2 - \frac{n(\tau)(n(\tau)+1)}{\tau^2}\right) V_k = 0$$

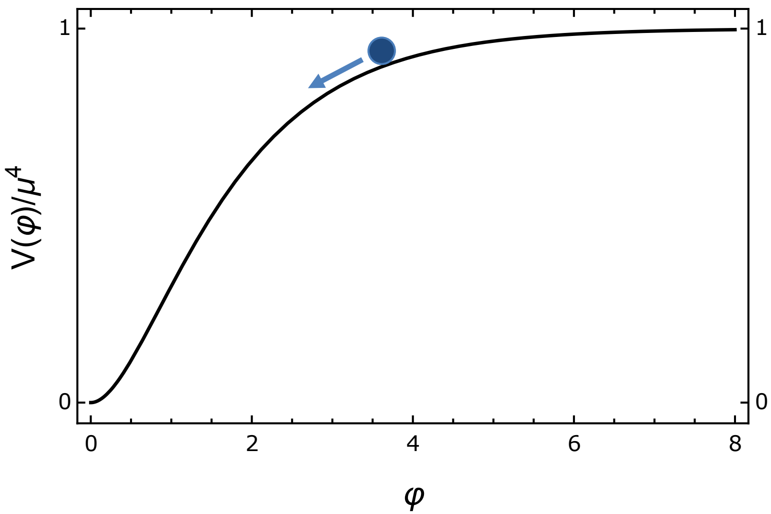
- The occurrence of particle production could be scale-dependent depending on the time evolution of $n(t)$

Model setup

Fujita, Nakatsuka, IO, Young (2022);

Inflaton potential

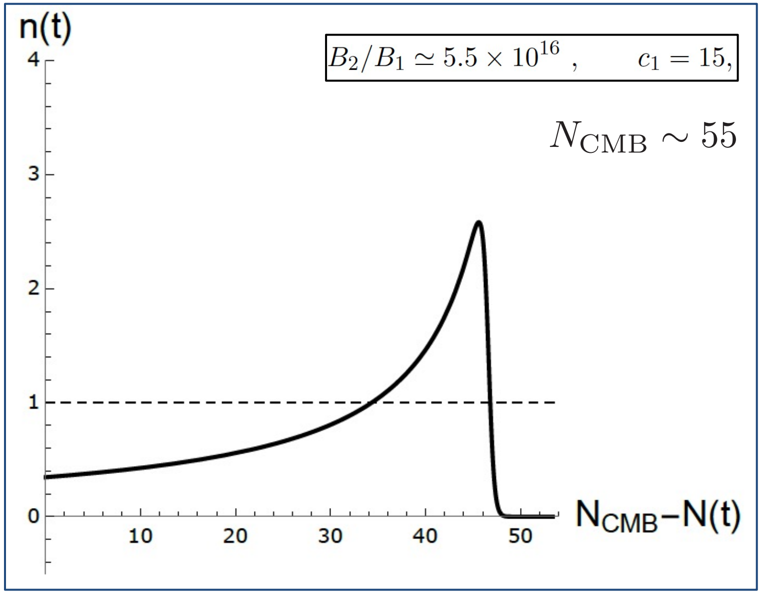
$$V(\varphi) = \mu^4 (1 - e^{-\gamma\varphi})^2 \,, \qquad \gamma = \sqrt{\frac{2}{3}} M_{\rm Pl}^{-1}$$



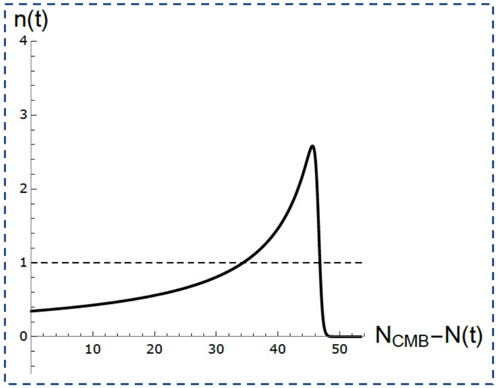
Kinetic function

$$I(\varphi) = B_1 \exp\left(c_1 \frac{\varphi}{M_{\rm Pl}} \right) + B_2$$

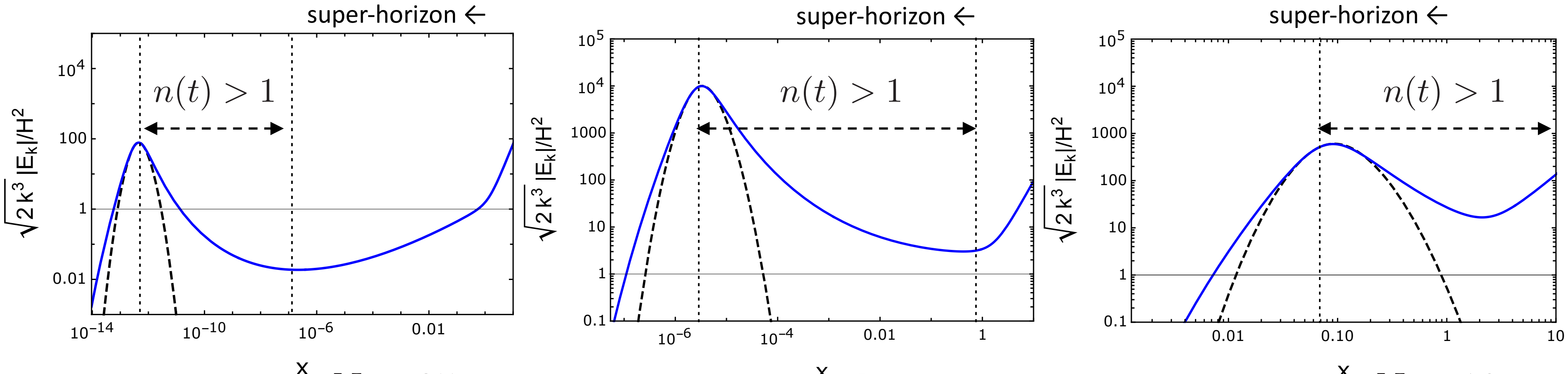
$$n(t) \equiv \frac{d \ln \bar{I}}{dN} = -\frac{\bar{I}_\varphi}{\bar{I}} \frac{\dot{\varphi}}{H}$$



Perturbation dynamics



Evolutions of the electric field component on super-horizon regime

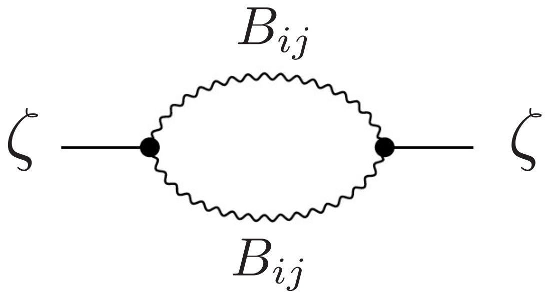


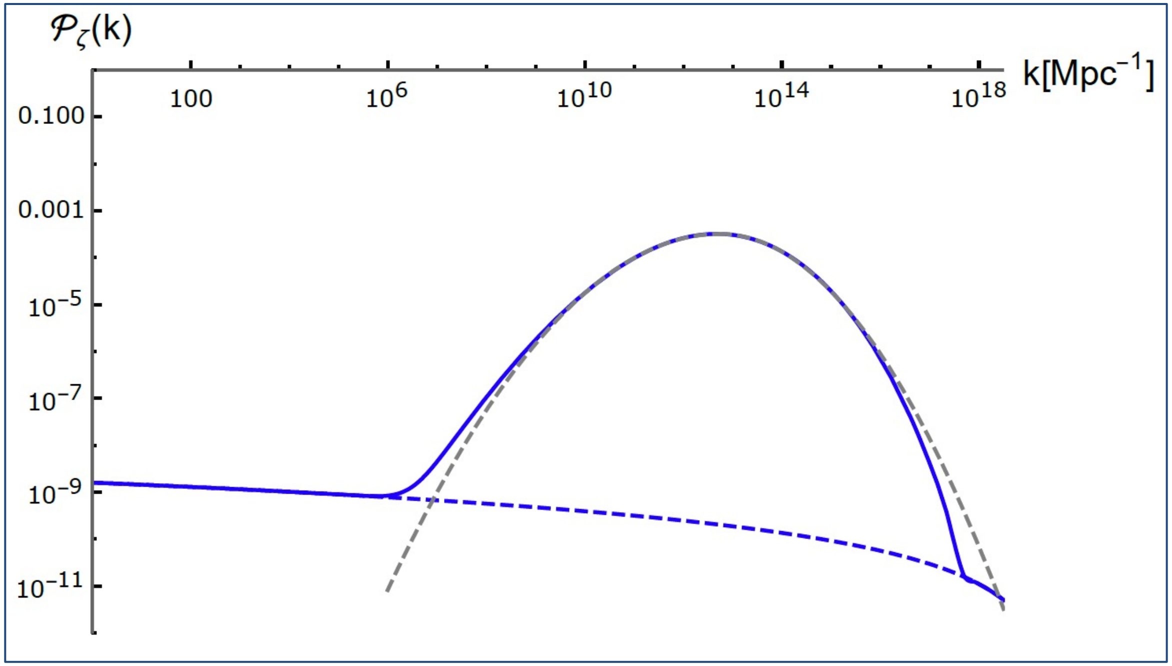
exiting horizon at $N \sim 35$ $N \sim 20$ $N \sim 10$

around peak: $E_k \simeq \frac{H^2}{\sqrt{2k^3}} E_{\rm peak}(k) \exp\left[-\frac{(\ln(\tau/\tau_{\rm peak}))^2}{\sigma^2}\right]$

Generation of scalar mode

- Curvature perturbation is sourced by the two-form field at second-order level





The fitting function around the peak:

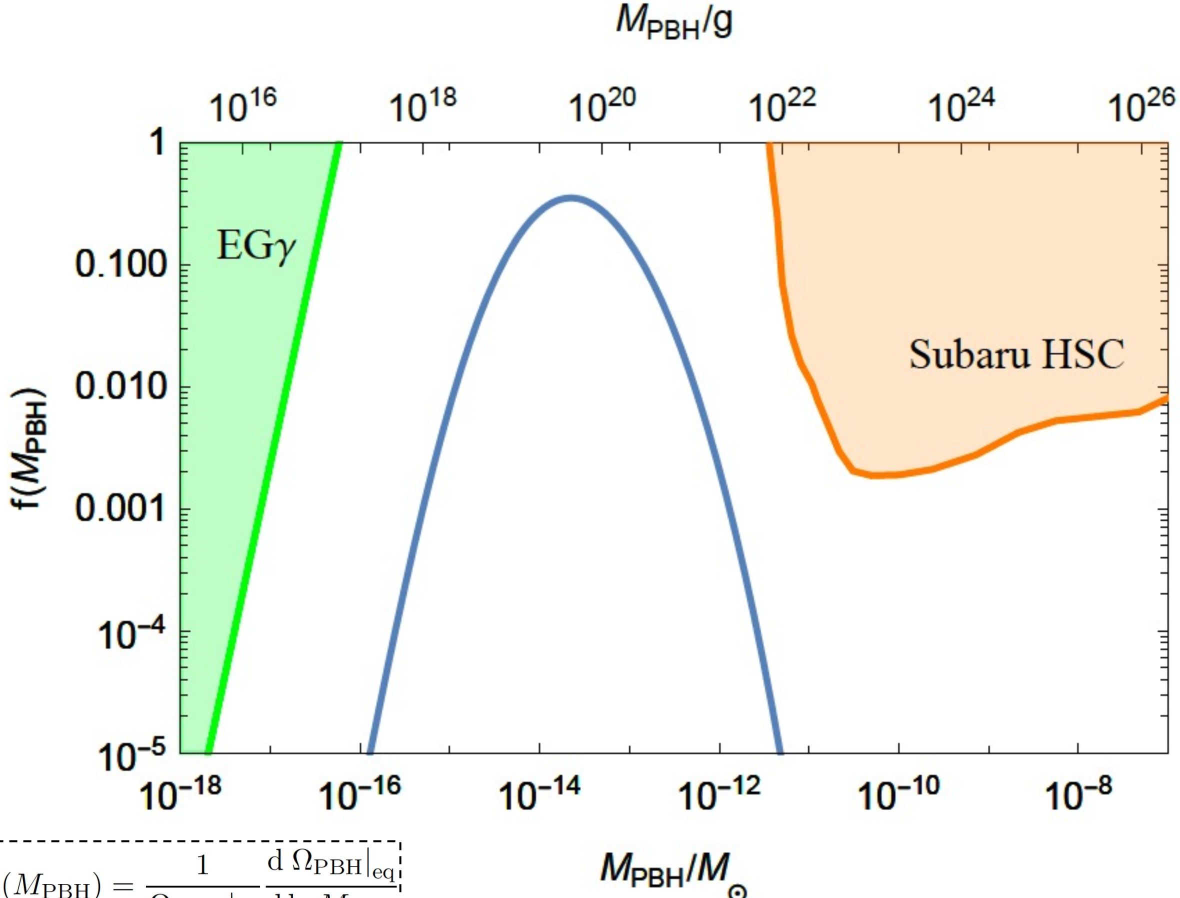
$$\mathcal{P}_\zeta(k) \simeq A \exp\left[-\frac{(\ln(k/k_p))^2}{\sigma_\zeta^2}\right]$$

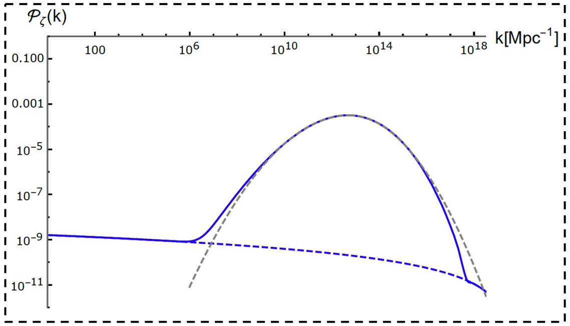
$$A \simeq 3.2 \times 10^{-4}\ , \quad k_p \simeq 5.6 \times 10^{12}\mathrm{Mpc}^{-1}\ , \quad \sigma_\zeta^2 \simeq 3.7^2\Theta(k_p - k) + 3.1^2\Theta(k - k_p)$$

Generation of PBHs as DM

Hawking (1971); Carr, Hawking (1974); …

- Assuming ζ obeys χ-squared distribution: $\zeta = \chi^2$





$$f(M_{\rm PBH}) = \frac{1}{\Omega_{\rm CDM}|_{\rm eq}} \frac{{\rm d}\,\Omega_{\rm PBH}|_{\rm eq}}{{\rm d}\ln M_{\rm PBH}}$$

Generation of tensor modes

$$g_{ij}(t,\boldsymbol{x}) = a(t)^2\left(\delta_{ij} + \frac{1}{2}h_{ij}(t,\boldsymbol{x})\right) \qquad h_{ij} = \sum_{\lambda=+,\times}\int \frac{d\boldsymbol{k}}{(2\pi)^3}\hat{h}^{\lambda}_{\boldsymbol{k}} e^{\lambda}_{ij}(\hat{\boldsymbol{k}}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

(1) Primordial GWs sourced by two-form field

EOM: $$(\partial_t^2 + 3H\partial_t - \nabla^2)h_{ij} \simeq -\frac{4I^2}{M_p^2 a^4}\Pi^{lm}_{ij}\dot{B}_{ln}\dot{B}_{mn}$$

(in Fourie space) $$\left[\partial_x^2 + 1 - \frac{2}{x^2}\right](a\hat{h}^s_{\boldsymbol{k}}) = -e^s_{ij}(\hat{\boldsymbol{k}})\frac{4a^3}{k^2 M_{\rm Pl}^2}\int\frac{d\boldsymbol{p}}{(2\pi)^3}E_{\boldsymbol{p}}E_{\boldsymbol{k}-\boldsymbol{p}}\epsilon_{in}(\hat{\boldsymbol{p}})\epsilon_{jn}(\widehat{\boldsymbol{k}-\boldsymbol{p}})$$

The following identities

$$e^{+}_{ij}(\hat{\boldsymbol{k}})\epsilon_{in}(\hat{\boldsymbol{p}})\epsilon_{jn}(\widehat{\boldsymbol{k}-\boldsymbol{p}}) = -\frac{1}{\sqrt{2}}\sin\theta_{\hat{\boldsymbol{p}}}\sin\theta_{\widehat{\boldsymbol{k}-\boldsymbol{p}}}\cos 2\phi_{\hat{\boldsymbol{p}}}$$

$$e^{\times}_{ij}(\hat{\boldsymbol{k}})\epsilon_{in}(\hat{\boldsymbol{p}})\epsilon_{jn}(\widehat{\boldsymbol{k}-\boldsymbol{p}}) = -\frac{1}{\sqrt{2}}\sin\theta_{\hat{\boldsymbol{p}}}\sin\theta_{\widehat{\boldsymbol{k}-\boldsymbol{p}}}\sin 2\phi_{\hat{\boldsymbol{p}}}$$

vanishes by integration in $\phi_{\hat{\boldsymbol{p}}}$!

(Two-form field does not source primordial GWs at leading order!)

Generation of tensor modes

(2) Induced GWs by PBHs ***Saito, Yokoyama (2008);…***

- Secondary GWs are sourced by scalar perturbations after re-entering the horizon

$$\left[\partial_\tau^2 - \nabla^2\right](ah_{ij}) = -4a\Pi_{ij}^{lm}\mathcal{S}_{lm} \ ,$$

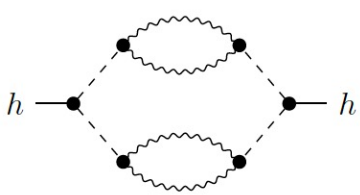
$$\mathcal{S}_{ij} \equiv 4\Psi\partial_i\partial_j\Psi + 2\partial_i\Psi\partial_j\Psi - \frac{1}{\mathcal{H}^2}\partial_i(\Psi' + \mathcal{H}\Psi)\partial_j(\Psi' + \mathcal{H}\Psi)$$

$$\Psi_{\boldsymbol{k}}(\tau) = -\frac{2}{3}\zeta_{\boldsymbol{k}}\Psi(k\tau)$$

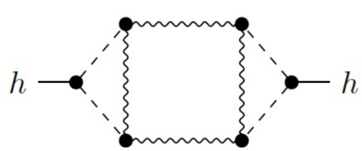
- 2-point function of induced GWs is given by the 8-point function of two-form field

$$\langle hh\rangle \propto \langle\zeta\zeta\zeta\zeta\rangle \propto \langle BBBBBBBB\rangle$$

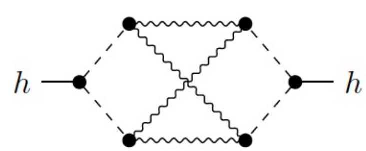
→ Three diagram contributes the spectrum



"Reducible"



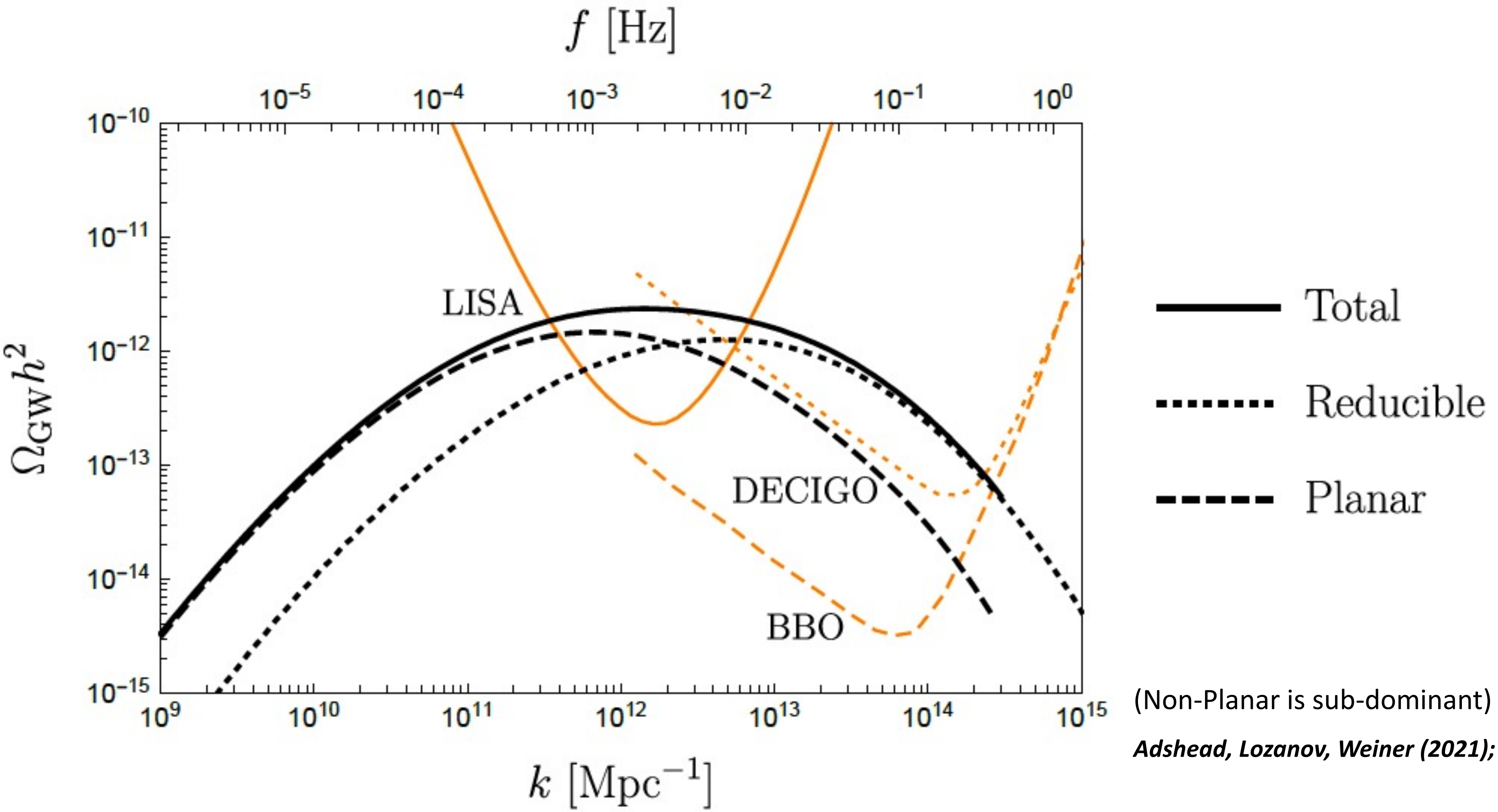
"Planar"



"Non-Planar"

Garcia-Bellido, Peloso, Unal (2017);
Cai, Pi, Sasaki (2019);
Adshead, Lozanov, Weiner (2021);

Power spectrum of indued GWs



(Non-Planar is sub-dominant)

Adshead, Lozanov, Weiner (2021);

Summary

- We proposed an inflationary model where a two-form field is kinetically coupled with an inflaton, and explored the particle production of two-form field occurring at an intermediate scale during inflation.
- The amplified two-form field enhances the curvature perturbation at second order and produces the sizable amount of PBHs as dark matter after inflation.
- The enhanced curvature perturbation also provides induced GWs after inflation and the spectral amplitudes are potentially testable with future laser interferometers.

Appendix

The power spectrum of 2-form field