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Dynamical NEC Violation in Finite Volumes

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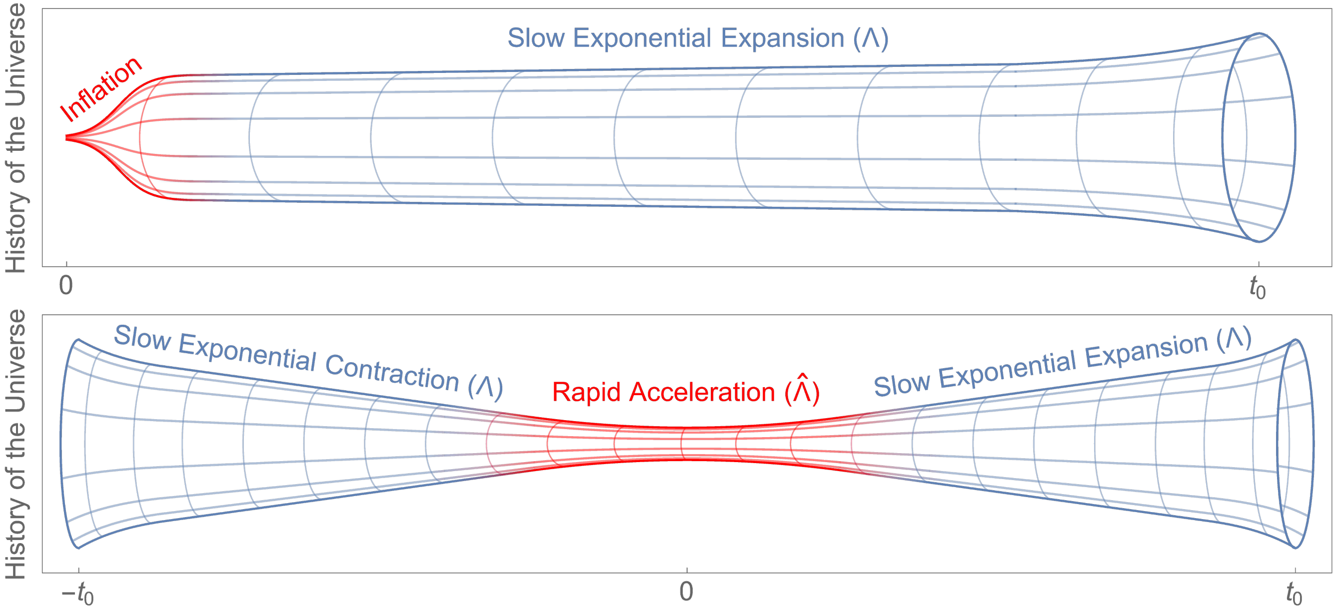
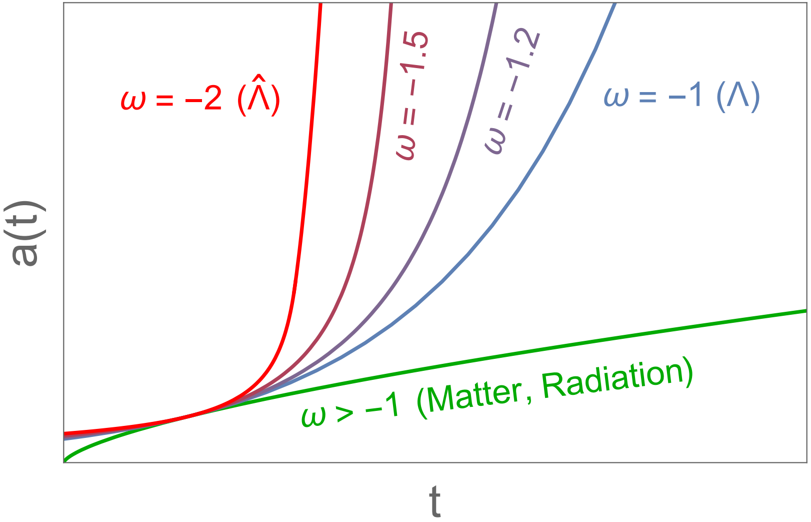
I. MOTIVATION

A. The Horizon Problem



B. Fluid Content of the Universe



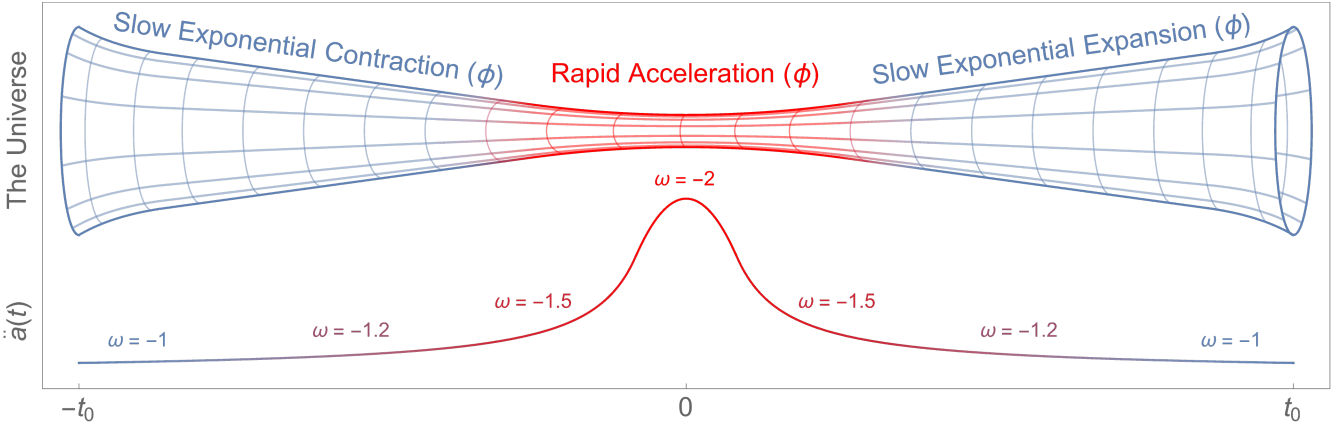
General Equation of State: $p = \omega\rho$

$$a(t, \omega \neq -1) = \left[(1+\omega)\frac{t}{t_0} - \omega\right]^{\frac{2}{3(1+\omega)}}$$

$$a(t, \omega = -1) = e^{\frac{2}{3t_0}(t-t_0)}$$

C. Time Dependent Equation of State

$$\Lambda \to \phi \qquad \text{EoS:} \quad p = \omega(t)\rho \qquad \text{Where:} \quad \omega(t) = -\left(1 + \frac{\alpha}{a^3(t)}\right)$$



$$\omega < -1 \iff \rho + p < 0$$

II. INTRODUCTION

A. Finite Volume

$$V^{(4)} = \big(La(t)\big)^3 \beta \qquad \text{where:} \quad \beta \equiv \frac{1}{T}$$

$$\beta \equiv L \implies V^{(4)} = L^4 a^3(t) \implies \mathrm{O}(4) \text{ symmetry}$$

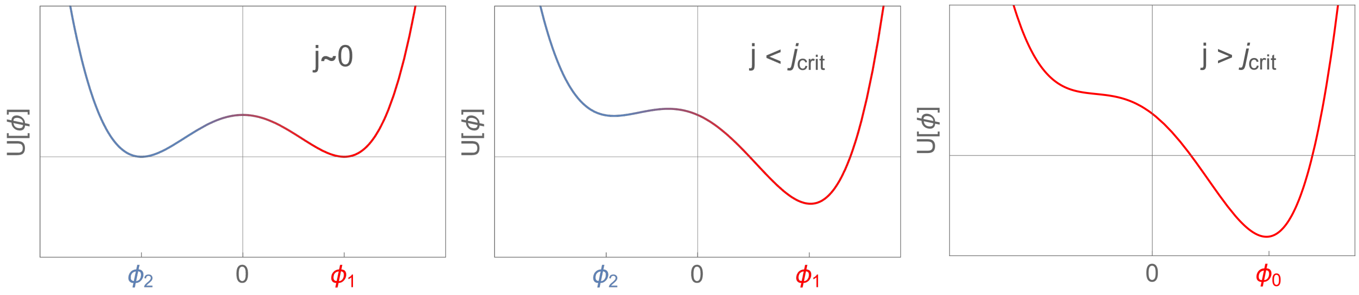
$$\text{Field Theory:} \quad L \gg \lambda_B \sim \frac{1}{\sqrt{\lambda} v} \sim 10^{-17}\text{m (Higgs)}$$

II. INTRODUCTION

B. Real Scalar Field

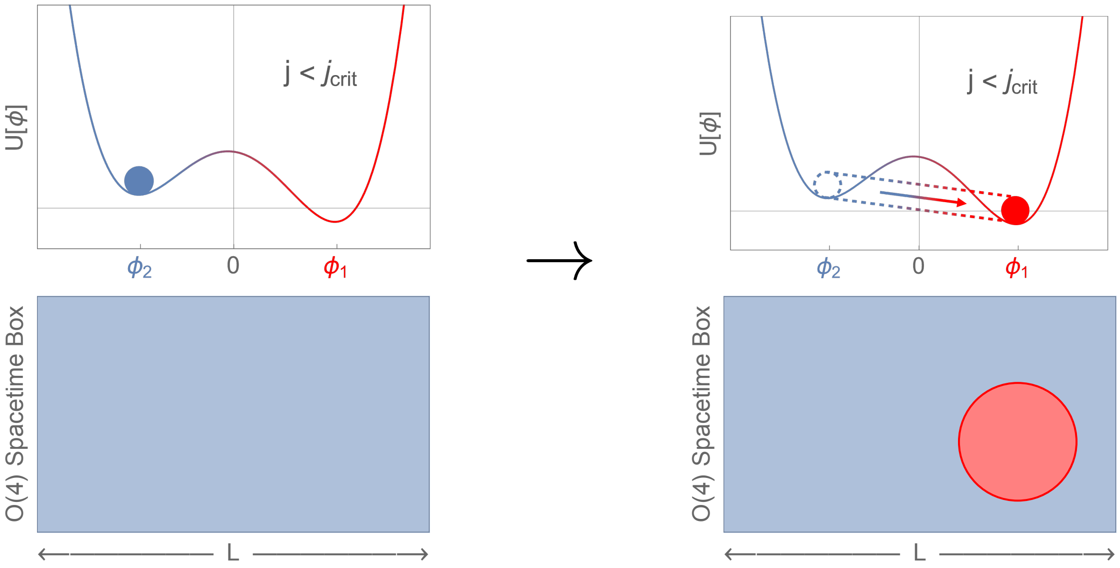
$$S[\phi] = \int d^4x \left(\frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{\lambda}{24}(\phi^2 - v^2)^2 + j\phi \right)$$

$$U[\phi] = \frac{\lambda}{24}(\phi^2 - v^2)^2 + j\phi$$



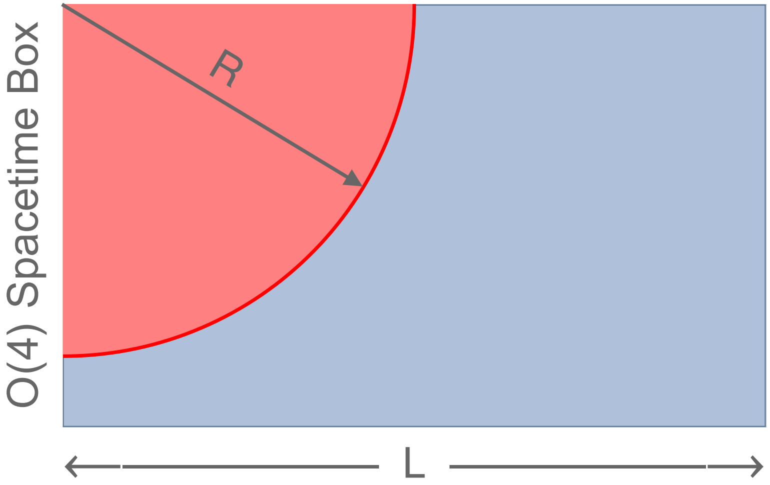
III. TUNNELING

A. Vacuum Bubble Nucleation (1/3)



III. TUNNELING

A. Vacuum Bubble Nucleation (2/3)

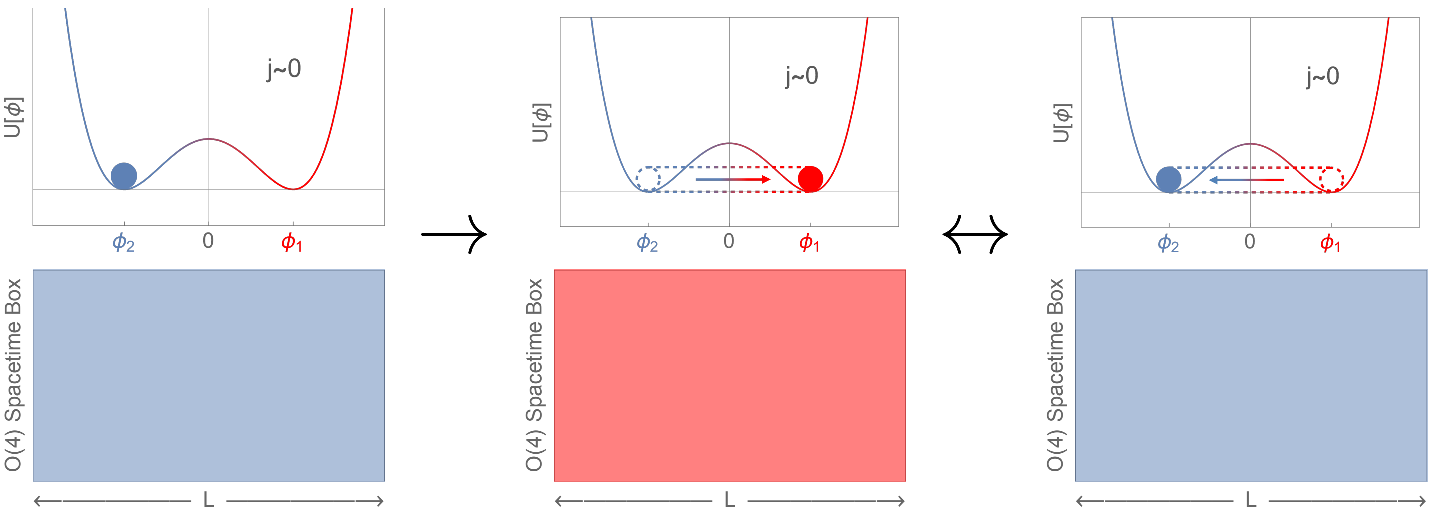


$$R \propto \frac{1}{\dot{j}}$$

[1] S. R. Coleman (1977); C. G. Callan, Jr. and S. R. Coleman (1977)

III. TUNNELING

A. Vacuum Bubble Nucleation (3/3)

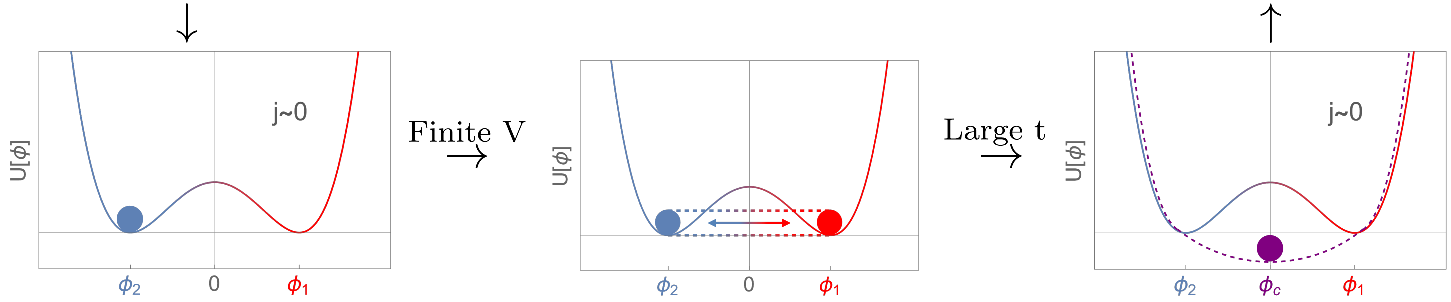


III. TUNNELING

B. Equilibrium Field Theory (1/2)

$$S[\phi] = \int d^4x \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U(\phi)\right)$$

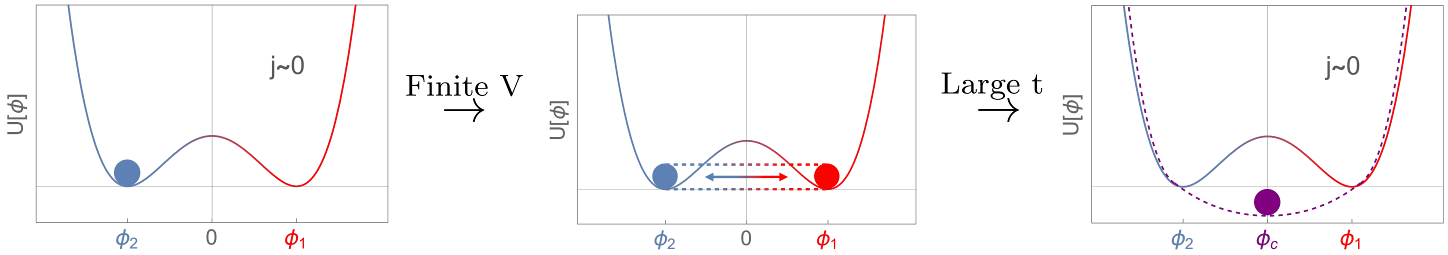
$$\Gamma[\phi_c] = V^{(4)}U_{\text{eff}}(\phi_c, V^{(4)})$$



One-to-one mapping: $\phi_c \leftrightarrow j$

III. TUNNELING

B. Equilibrium Field Theory (2/2)



Cosmological constant couples to the classical effective theory

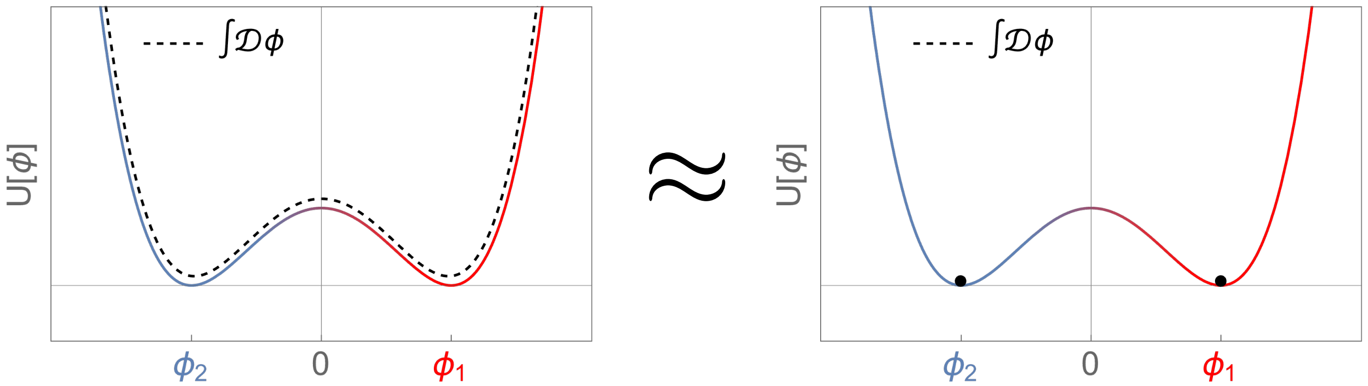
Adiabatic spacetime expansion

Symmetry restoration not via temperature fluctuations

[2] T. W. B. Kibble (1976); W. H. Zurek (1985)

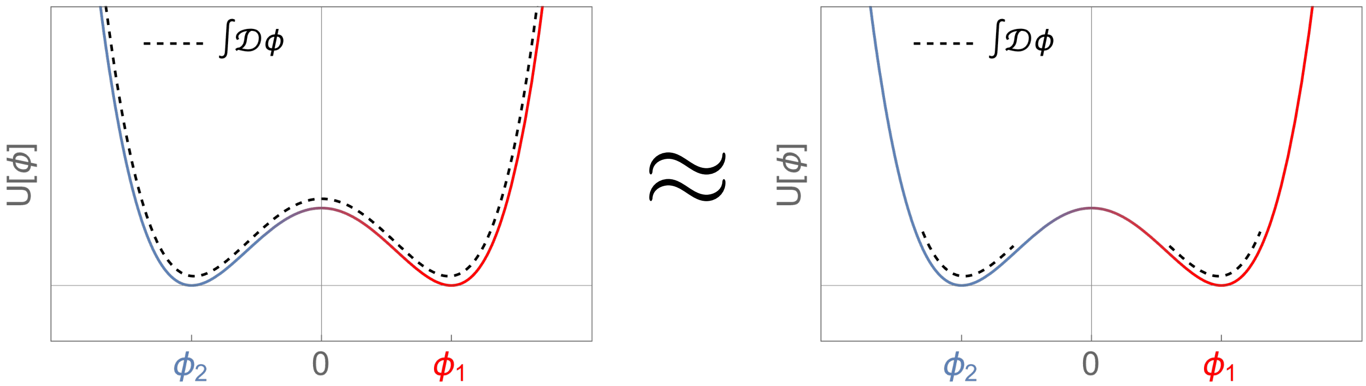
IV. CALCULATION

A. Classical Approximation



$$Z[j] = \int \mathcal{D}\phi \exp\left(-\frac{1}{\hbar}S[\phi]\right) \approx \exp\left(-\frac{1}{\hbar}S[\phi_2]\right) + \exp\left(-\frac{1}{\hbar}S[\phi_1]\right)$$

[3] J. Alexandre and A. Tsapalis (2013)

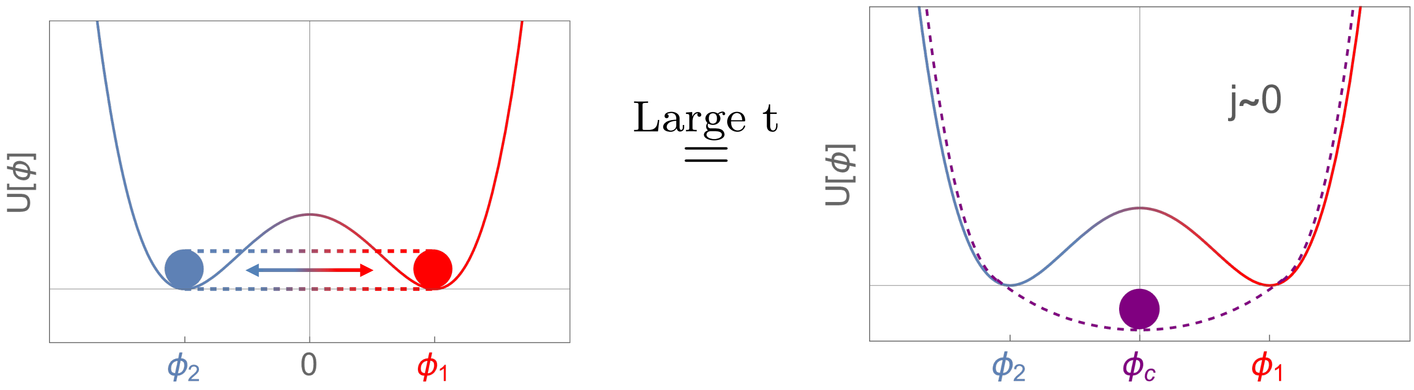


$$Z[j] \approx \int \mathcal{D}\phi \exp\left(-\frac{1}{\hbar}S[\phi_2] - \frac{1}{2}S''[\phi_2](\phi-\phi_2)^2\right) + \int \mathcal{D}\phi \exp\left(-\frac{1}{\hbar}S[\phi_1] - \frac{1}{2}S''[\phi_1](\phi-\phi_1)^2\right)$$

[4] J. Alexandre and D. Backhouse (2022)

IV. CALCULATION

C. Legendre Transformation



$$Z[j] \approx \exp\left(-\frac{1}{\hbar}\Sigma[\phi_2]\right) + \exp\left(-\frac{1}{\hbar}\Sigma[\phi_1]\right) = \exp\left(-\frac{1}{\hbar}\Sigma_c[\phi_c]\right)$$

NON-EXTENSIVE

$$\Gamma[\phi_c] = \hbar A_R \left(g_0 + \frac{g_2}{2}\left(\frac{\phi_c}{v_R}\right)^2 + \frac{g_4}{24}\left(\frac{\phi_c}{v_R}\right)^4 \right) + \mathcal{O}(\phi_c/v_R)^6$$

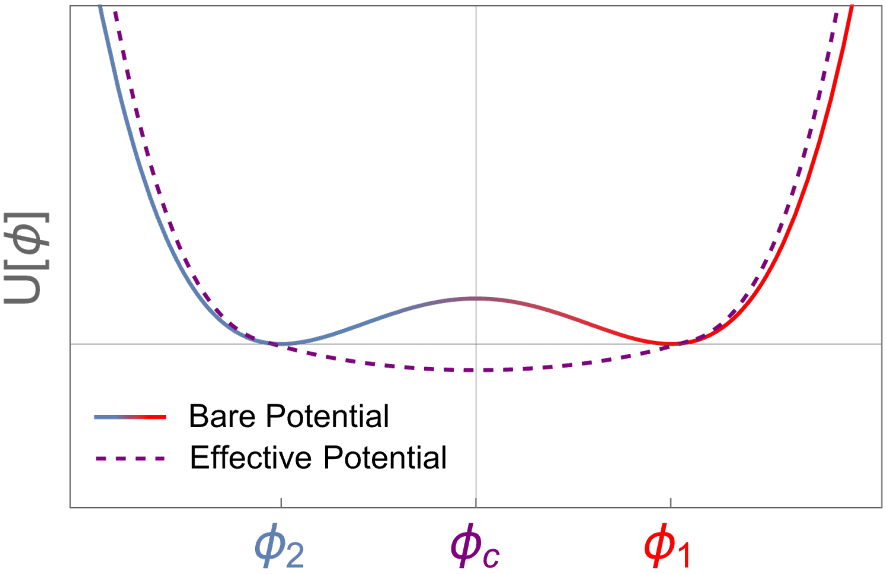
$$g_2[A_R] = \frac{8}{1+8A_R} + \frac{\hbar\lambda_R}{8\pi^2}\,\frac{16A_R+7}{(1+8A_R)^2}$$

$$A_R \equiv \frac{\lambda_R v_R^4 V^{(4)}}{24\hbar}$$

$$g_4[A_R] = \frac{64(128{A_R}^3+12A_R-3)}{(1+8A_R)^4} + \frac{\hbar\lambda_R}{4\pi^2}\,\frac{16384{A_R}^4+12288{A_R}^3+936A_R-243}{(1+8A_R)^5}$$

V. ENERGETICS

A. Effective Potential



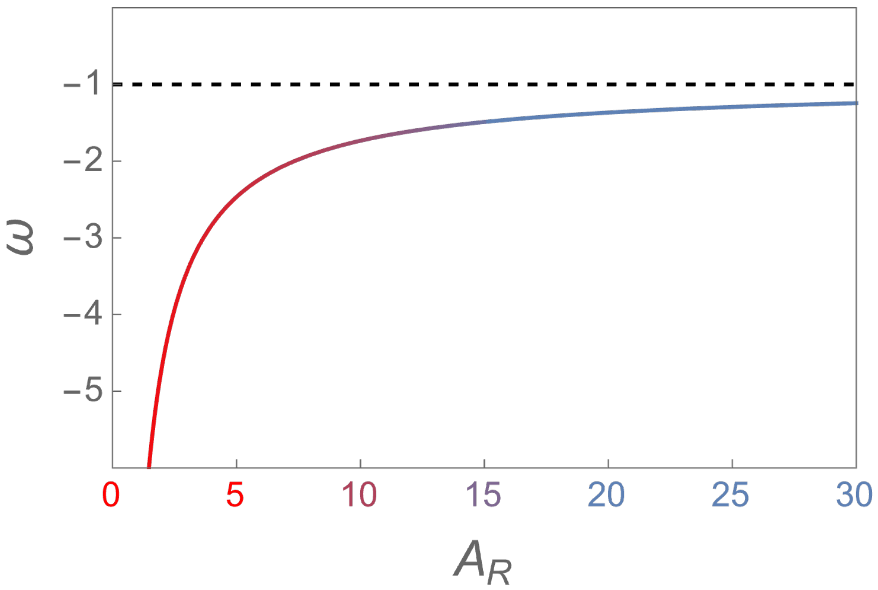
$$U(\phi) = \frac{\lambda}{24}(\phi^2 - v^2) + j\phi$$

$$U_{eff}^{(|k_R|<1)}(\phi_c) = \frac{1}{V^{(4)}}\Gamma[\phi_c]$$

$$U_{eff}^{(|k_R|>1)}(\phi_c) = U(\phi_c) + \mathcal{O}(\hbar)$$

V. ENERGETICS

B. Equation of State



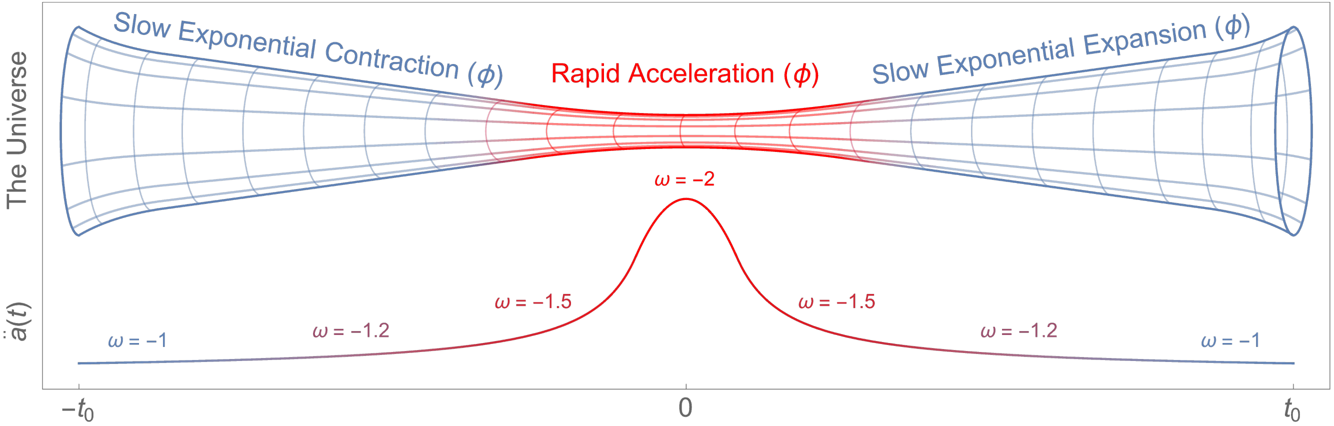
$$\rho = \frac{\Gamma[0]}{V} \qquad p = -\frac{\partial \Gamma[0]}{\partial V}$$

$$\omega[A_R] \approx -1 - \frac{22}{3A_R}$$

$$L \gg \frac{1}{\sqrt{\lambda} v} \to A_R \gg 1$$

C. Cosmological Bounce

$$A_R \equiv \frac{\lambda_R v_R^4 V^{(4)}}{24\hbar} \implies \omega[t] \approx -1 - \frac{\alpha}{a^3(t)} \qquad \text{where: } \alpha \equiv \frac{176\hbar}{\lambda_R v_R^4 L^4}$$



VI. OUTLOOK

A. Thermal Field Theory

Rather than taking $\beta \equiv L$, let T be a free parameter

Considering only a spacial average (QM not QFT):

$$\omega \sim -1 - \frac{1}{V^{(3)}\beta}$$

[5] J. Alexandre and J. Polonyi

B. Curved Spacetime

Couple to gravity $\rightarrow$ calculate stress energy tensor $\rightarrow$ properly calculate ρ and p

VII. REFERENCES

[1] S. R. Coleman, Phys. Rev. D 15 (1977), 2929-2936 [erratum: Phys. Rev. D 16 (1977), 1248];
C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D 16 (1977), 1762-1769

[2] T. W. B. Kibble, J. Phys. A 9 (1976), 1387-1398;
W. H. Zurek, Nature 317 (1985), 505-508

[3] J. Alexandre and A. Tsapalis, Phys. Rev. D 87 (2013) no.2, 025028 [arXiv:1211.0921 [hep-th]]

[4] J. Alexandre and D. Backhouse, Phys. Rev. D 105, 105018 (2022) [arXiv:2203.12543 [hep-th]]

[5] J. Alexandre and J. Polonyi, [arXiv:2205.00768 [hep-th]]

VIII. APPENDICES

A. Discrete versus continuous momentum components

$$Z[j] \approx \sum_{i=1,2} \exp\left(-\frac{1}{\hbar}S[\phi_i] - \frac{1}{2}\mathrm{Tr}\left\{\ln\left(S''[\phi_i]\right)\right\}\right) \equiv \sum_{i=1,2} \exp\left(-\frac{1}{\hbar}\Sigma[\phi_i]\right)$$

$$\mathrm{Tr}\left\{\ln\left(S''[\phi_i]\right)\right\} = \sum_{n_\mu}^{N_\mu} \ln\left(\frac{(2\pi/L)^2 n_\mu n_\mu + U''(\phi_i)}{(2\pi/L)^2 n_\mu n_\mu + U''(\phi_i(0))}\right)$$

$$\mathrm{Tr}\left\{\ln\left(S''[\phi_i]\right)\right\} \simeq V \int_0^\Lambda \frac{d^4p}{(2\pi)^4} \ln\left(\frac{p^2 + U''(\phi_i(k))}{p^2 + U''(\phi_i(0))}\right)$$

B. Renormalisation

$$Z[j] \approx \sum_{i=1,2} \exp\left(-\frac{1}{\hbar}S[\phi_i] - \frac{1}{2}V\int_0^{\Lambda} \frac{d^4p}{(2\pi)^4} \ln\left(\frac{p^2+U''(\phi_i(k))}{p^2+U''(\phi_i(0))}\right)\right) \equiv \sum_{i=1,2} \exp\left(-\frac{1}{\hbar}\Sigma[\phi_i]\right)$$

$$\boxed{Z[j] \approx \sum_{i=1,2} \exp\left(-\frac{1}{\hbar}\Sigma[\phi_i(k), V, v, \lambda, \Lambda]\right)}$$

$$\Sigma[\phi_i(k), V, v, \lambda, \Lambda] = \Sigma_R[\phi_{iR}(k_R), V, v_R, \lambda_R]$$

C. Legendre transformation

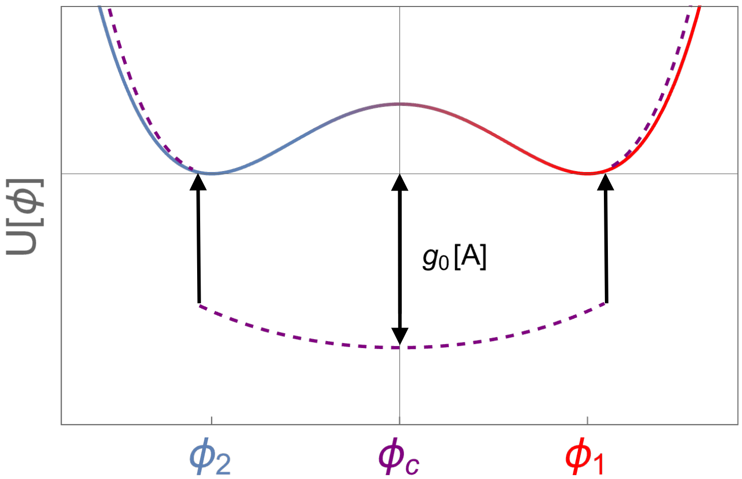
$$\langle\phi\rangle = \phi_c = \frac{1}{Z}\sum_{i=1,2}\phi_i \exp\left(-\frac{1}{\hbar}S[\phi_i]\right) = -\frac{\hbar}{Z}\frac{\delta Z[j]}{\delta j(x)} = -\frac{3\sqrt{3}}{8A_R}\frac{\partial \ln(Z(k_R))}{\partial k_R}$$

$$\frac{\phi_c}{v_R} = f_1(A_R)k_R + f_3(A_R)k_R^3 + \mathcal{O}(k_R^5)$$

$$k_R = -\frac{3\sqrt{3}}{8}g_2(A_R)\left(\frac{\phi_c}{v_R}\right) - \frac{\sqrt{3}}{16}g_4(A_R)\left(\frac{\phi_c}{v_R}\right)^3 + \mathcal{O}(\phi_c/v_R)^5$$

$$\Gamma[\phi_c] = -V\int j(\phi_c)d\phi_c = -\frac{8\hbar A_R}{3\sqrt{3}v_R}\int k_R(\phi_c)d\phi_c$$

D. Origin of Energy

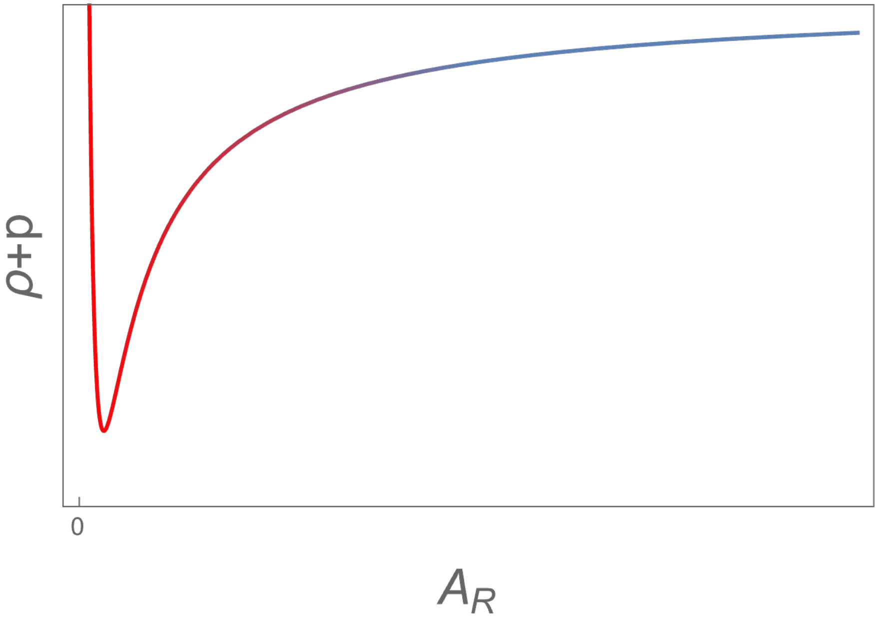


$$U_{eff}^{(|k_R|<1)}\left(\phi_c(k_R=1)\right) = U_{eff}^{(|k_R|>1)}\left(\phi_c(k_R=1)\right)$$

$$g_0[A_R] = \frac{1}{9} + \frac{3\hbar\lambda_R}{32\pi^2}\ln\left(\frac{3}{2}\right) - \frac{2}{3}g_2[A_R] - \frac{2}{27}g_4[A_R]$$

VIII. APPENDICES

E. Null Energy Condition



$$\rho = \frac{\Gamma[0]}{V} \qquad p = -\frac{\partial \Gamma[0]}{\partial V}$$

$$\rho + p = -\frac{\lambda_R v_R^4}{24} A_R \frac{\partial g_0[A_R]}{\partial A_R}$$

$$\rho + p \approx -\frac{22\hbar}{27V} + \mathcal{O}(\hbar^2)$$