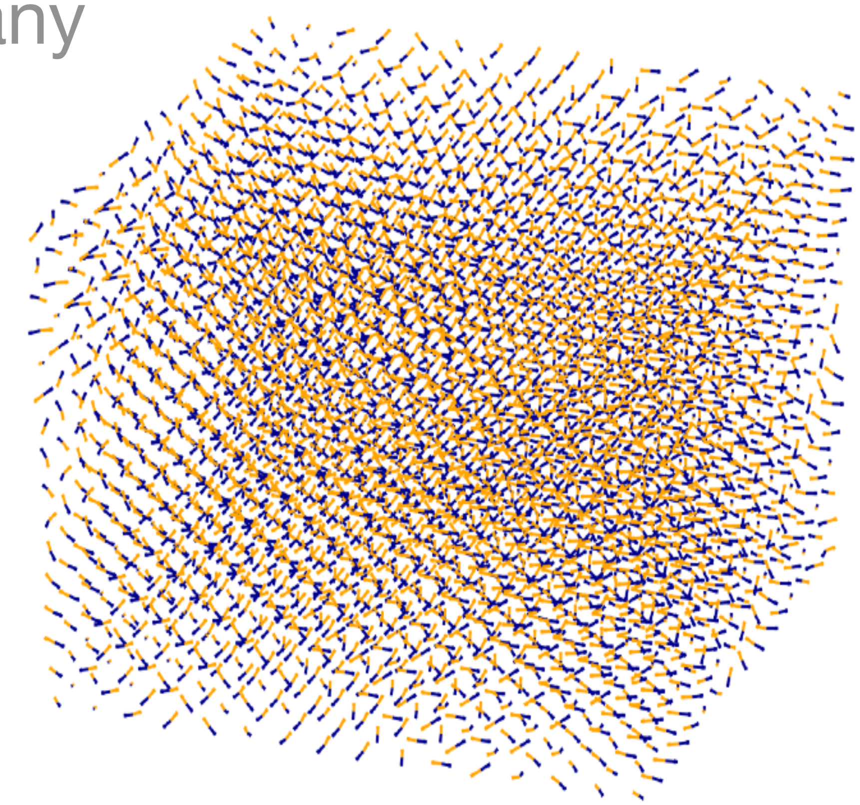
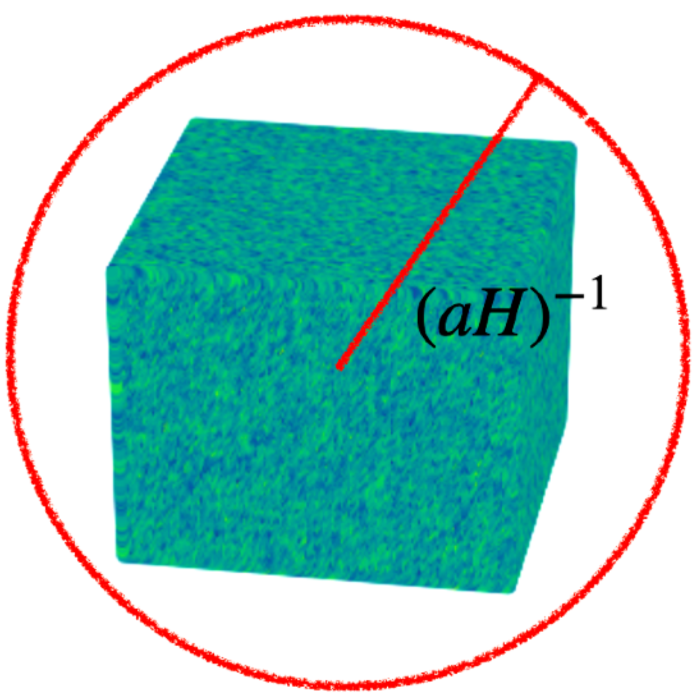
Lattice Simulations of Axion-U(1) Inflation

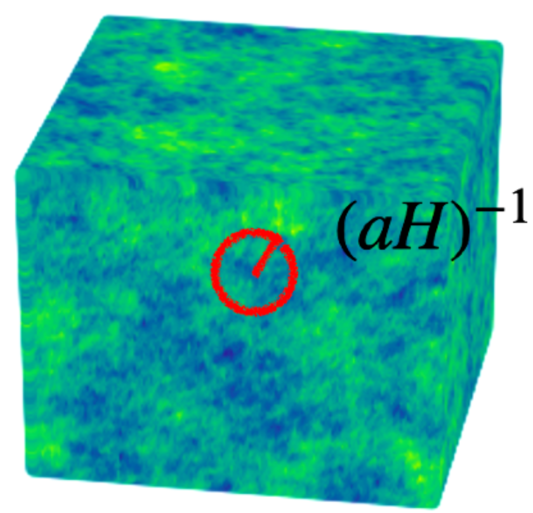
Angelo Caravano LMU & MPI, Munich, Germany



A. Caravano, E. Komatsu, K. D. Lozanov, J. Weller

arXiv:2204.12874





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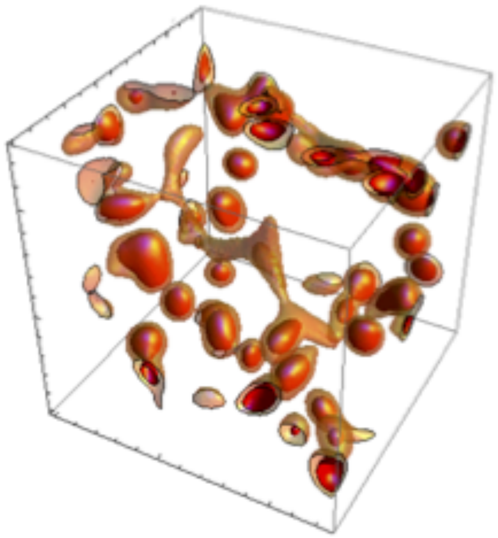
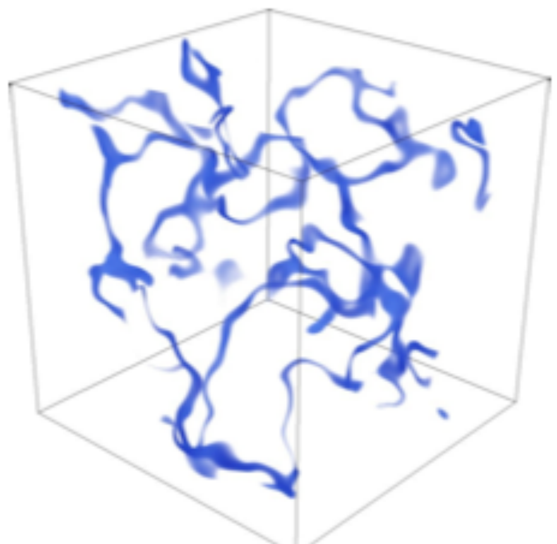
Lattice Simulations

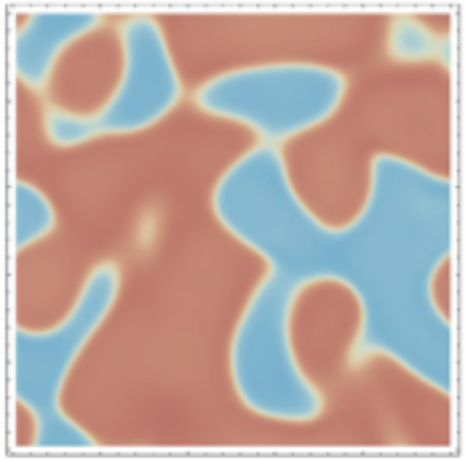
- Numerical tool to study non-linear phenomena involved in inflation.
- Typically associated with the reheating phase after inflation.



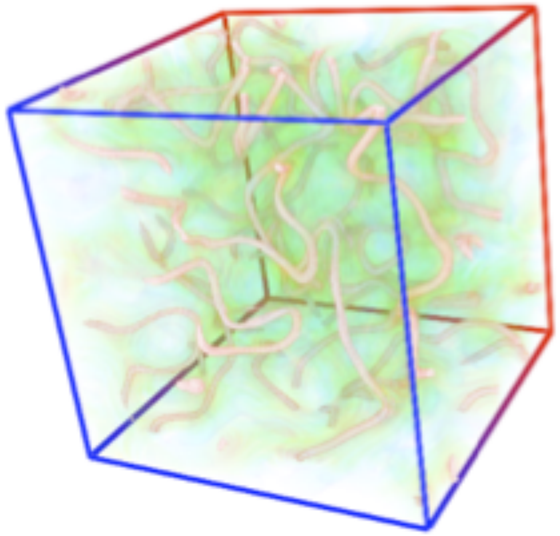
[M. A. Amin, R. Easther, H. Finkel, arXiv:1009.2505]



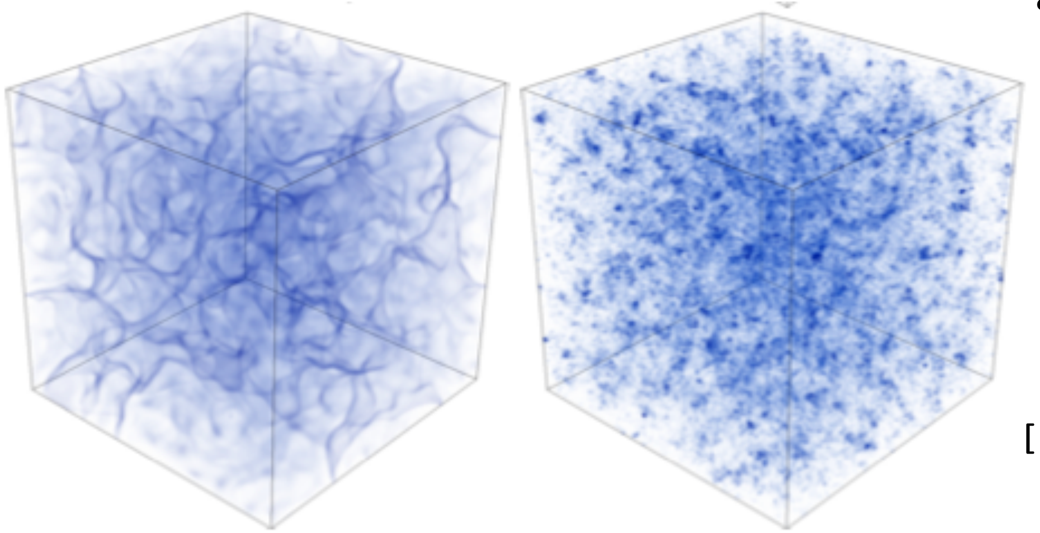
[A. V. Frolov, arXiv:1004.3559]



[M. A. Amin, J. Fan, K. D. Lozanov, M. Reece, arXiv:1802.00444]



[J. Dufaux, D.G. Figueroa, J. Garcia-Bellido, arXiv:1006.0217]



[A. V. Frolov, arXiv:0809.4904]

Lattice Simulations

- Numerical tool to study non-linear phenomena involved in inflation.
- Typically associated with the reheating phase after inflation.

Our goal:

Generalise this machinery to inflationary dynamics

A. Caravano, E. Komatsu, K. D. Lozanov, J. Weller arXiv:2102.06378
arXiv:2110.10695
arXiv:2204.12874

In this talk: **focus on axion-U(1) model.**

Axion-U(1) inflation

Adding an electromagnetic U(1) field that interacts with the inflation:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_{\rm Pl}^2 \mathscr{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{4f} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Ingredients:

- Pseudoscalar (axion) inflaton ϕ
- U(1) gauge field A_μ
- Interaction $\phi F \tilde{F}$

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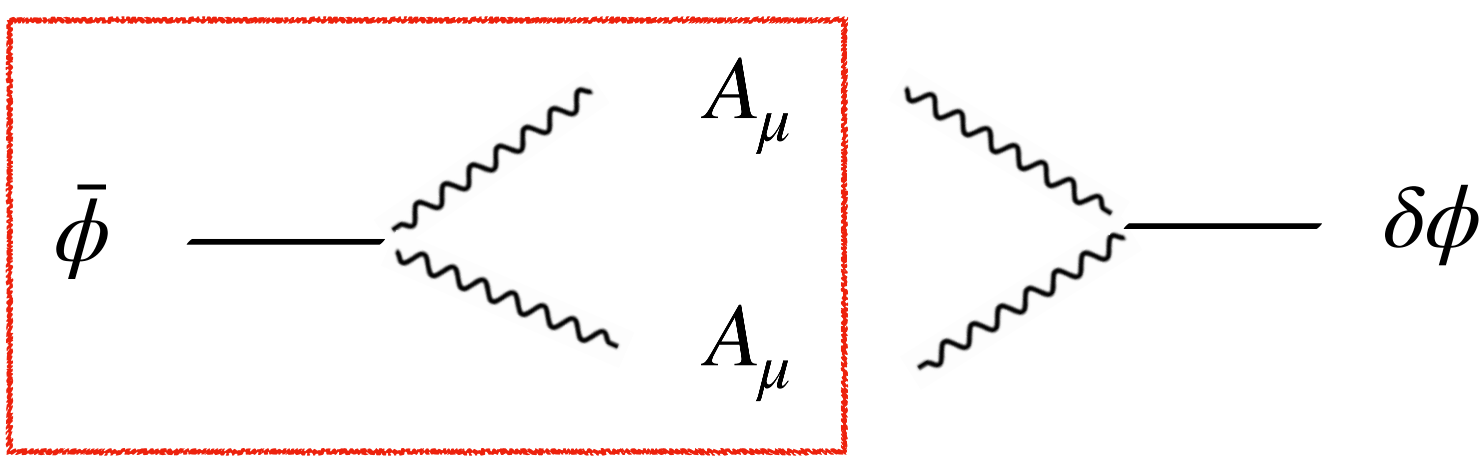
Consequences of the interaction:

1. Production of gauge field particles.

2. ⇒ these act as a source for inflation perturbation (and GWs)



Axion-U(1) inflation

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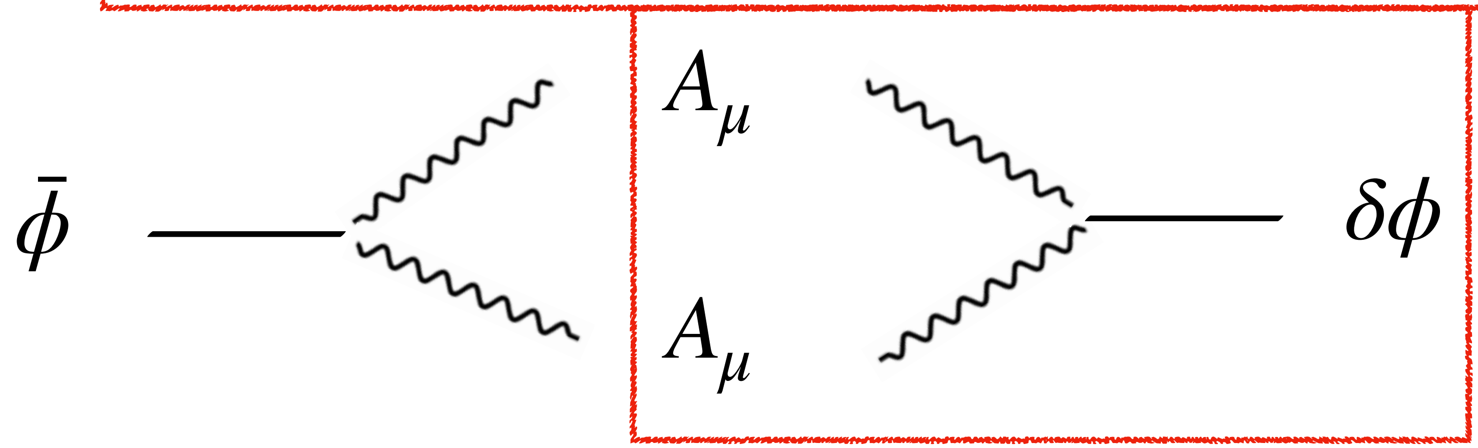


This particle production is observable

For $k \ll aH$

M. Anber, L. Sorbo 0908.4089
N. Barnaby, M. Peloso 1011.1500

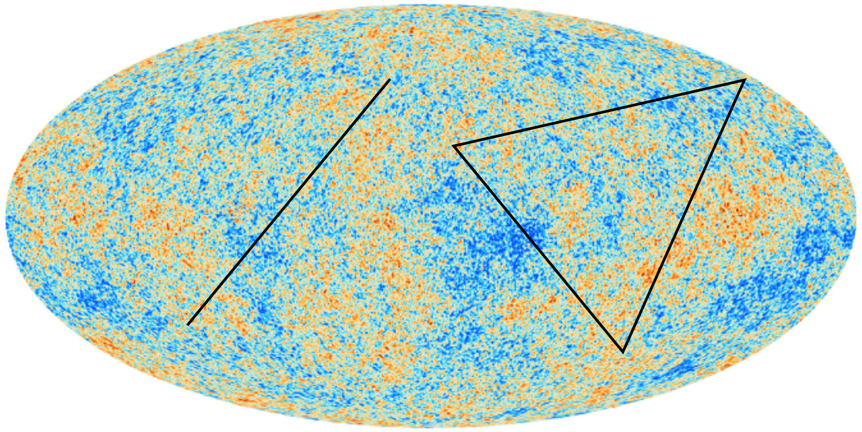
- $\langle \delta\phi_{\mathbf{k}} \delta\phi_{\mathbf{k}'} \rangle \simeq \frac{H^2}{2k^3} \left(1 + f_2(\xi) e^{4\pi\xi}\right) \delta(\mathbf{k} + \mathbf{k}')$

$$\xi = \frac{\alpha \dot{\phi}}{2fH}$$

vacuum
(single-field)

sourced

- $\langle \delta\phi_{\mathbf{k}_1} \delta\phi_{\mathbf{k}_2} \delta\phi_{\mathbf{k}_3} \rangle \neq 0 \quad \simeq \frac{9}{80}(2\pi)^{5/2} \frac{H^6}{k^6} f_3\left(\xi, \frac{k_2}{k_1}, \frac{k_3}{k_1}\right) e^{6\pi\xi} \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$



CMB fluctuations from Planck

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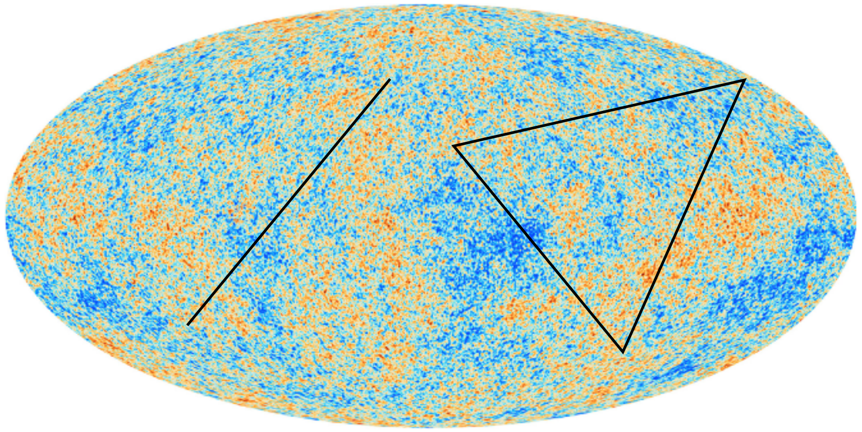
(+ Gravitational waves)



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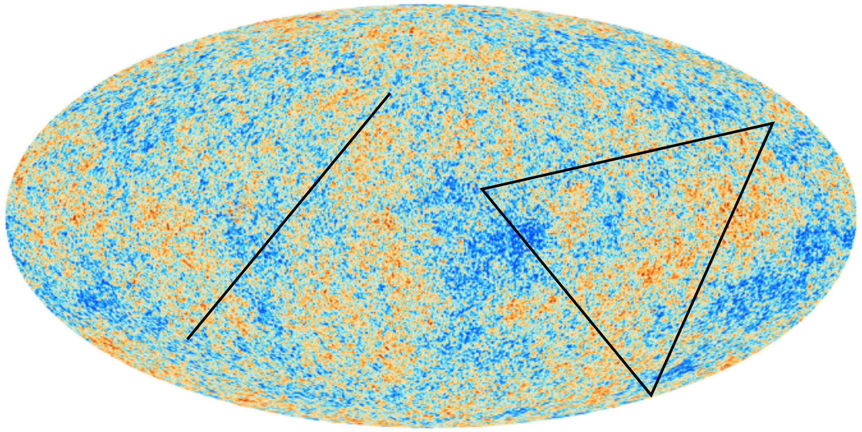


CMB fluctuations from Planck

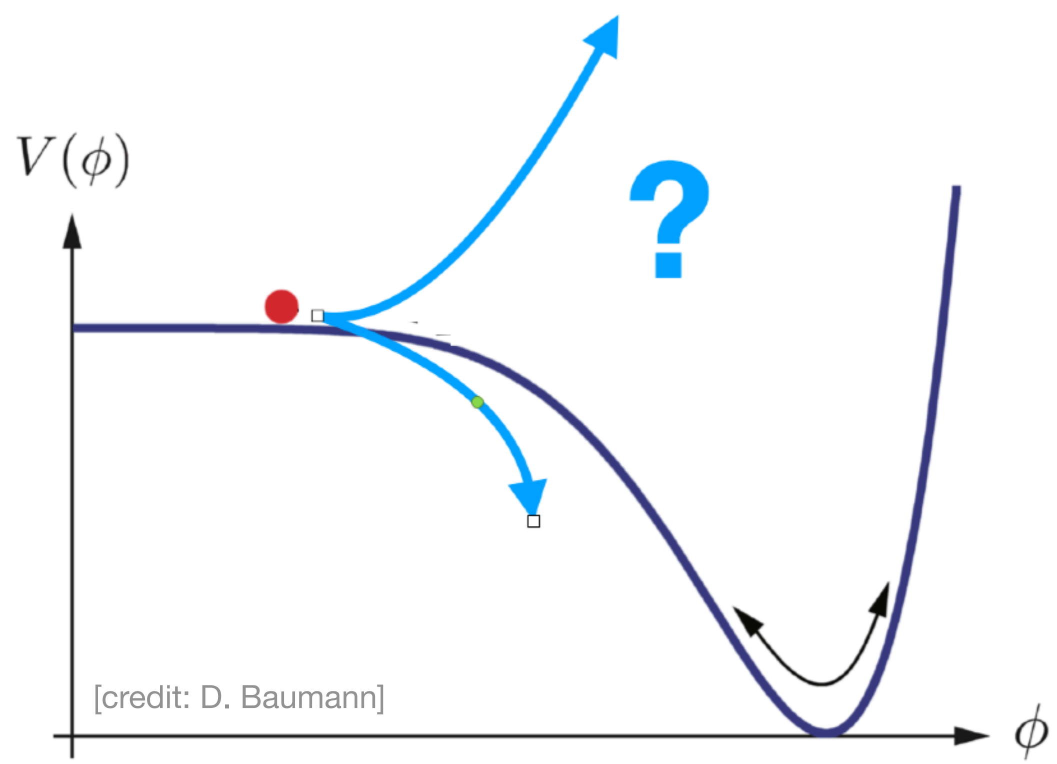
Backreaction

$$\xi = \frac{\alpha\dot{\phi}}{2fH}$$

If ξ is large

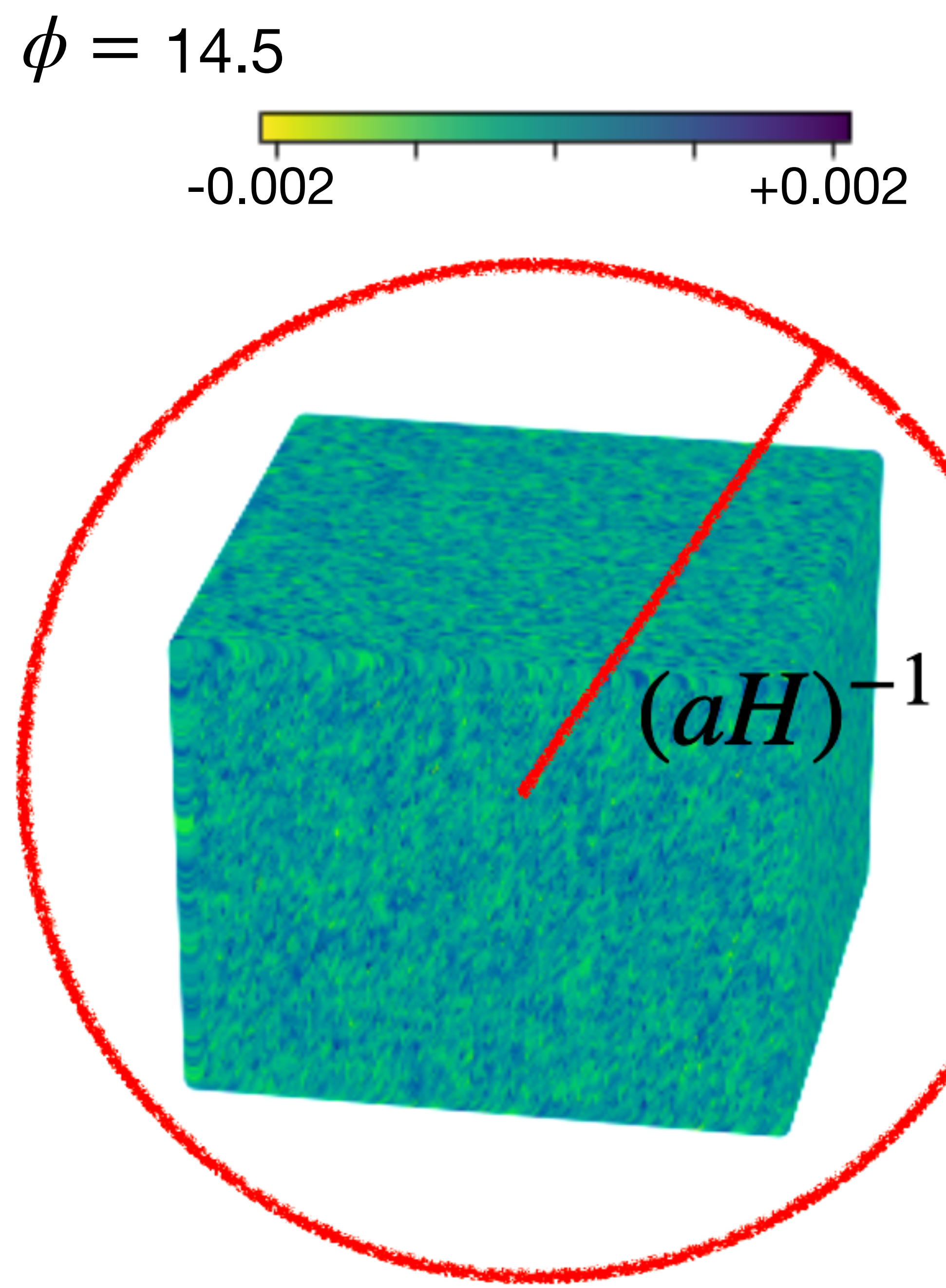
→ Need nonlinear tools

$$\frac{H^2}{26\pi|\dot{\phi}|}\xi^{-3/2}e^{\pi\xi} \ll 1.$$

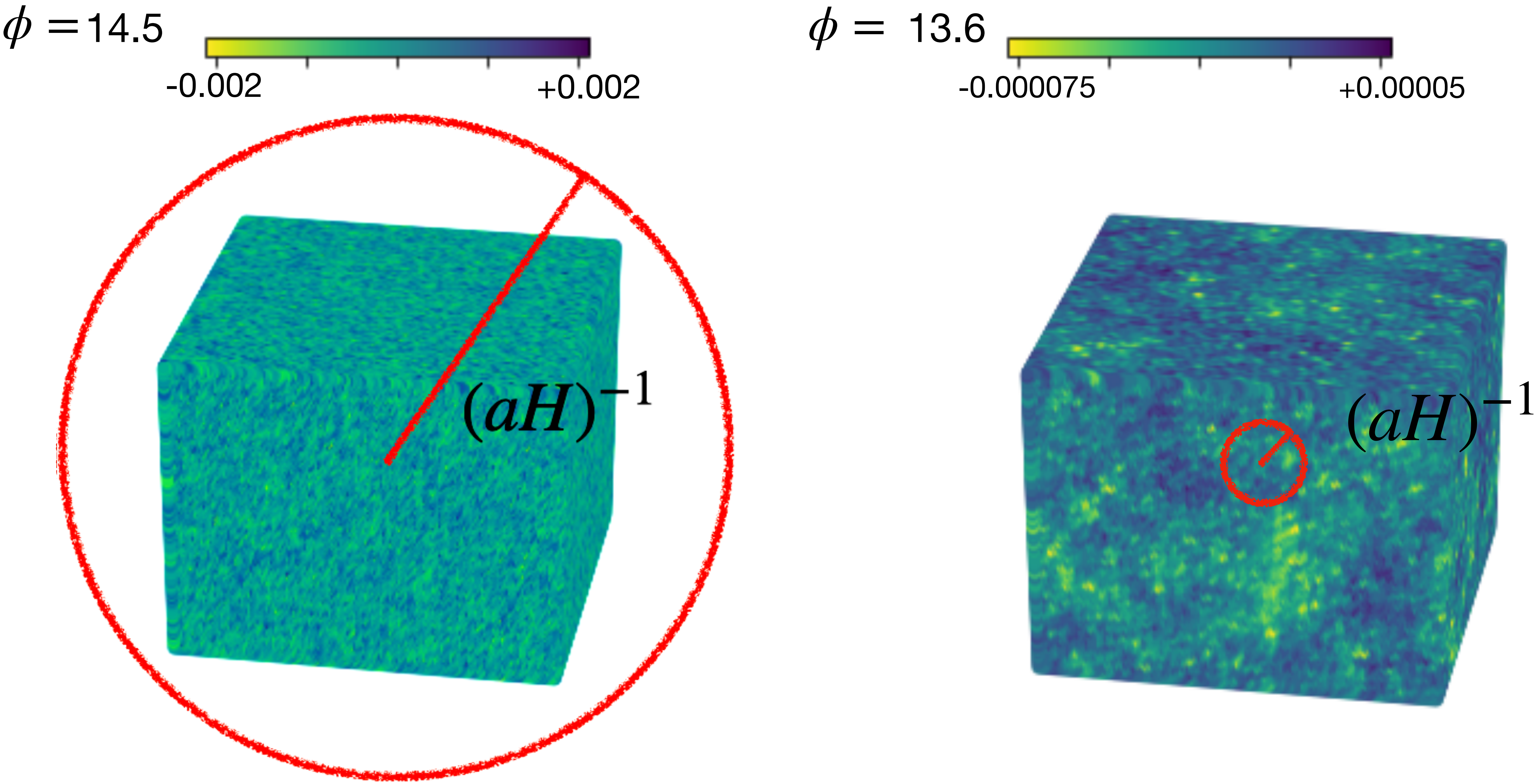


Lattice approach

Start with a **sub-horizon** box



Lattice approach

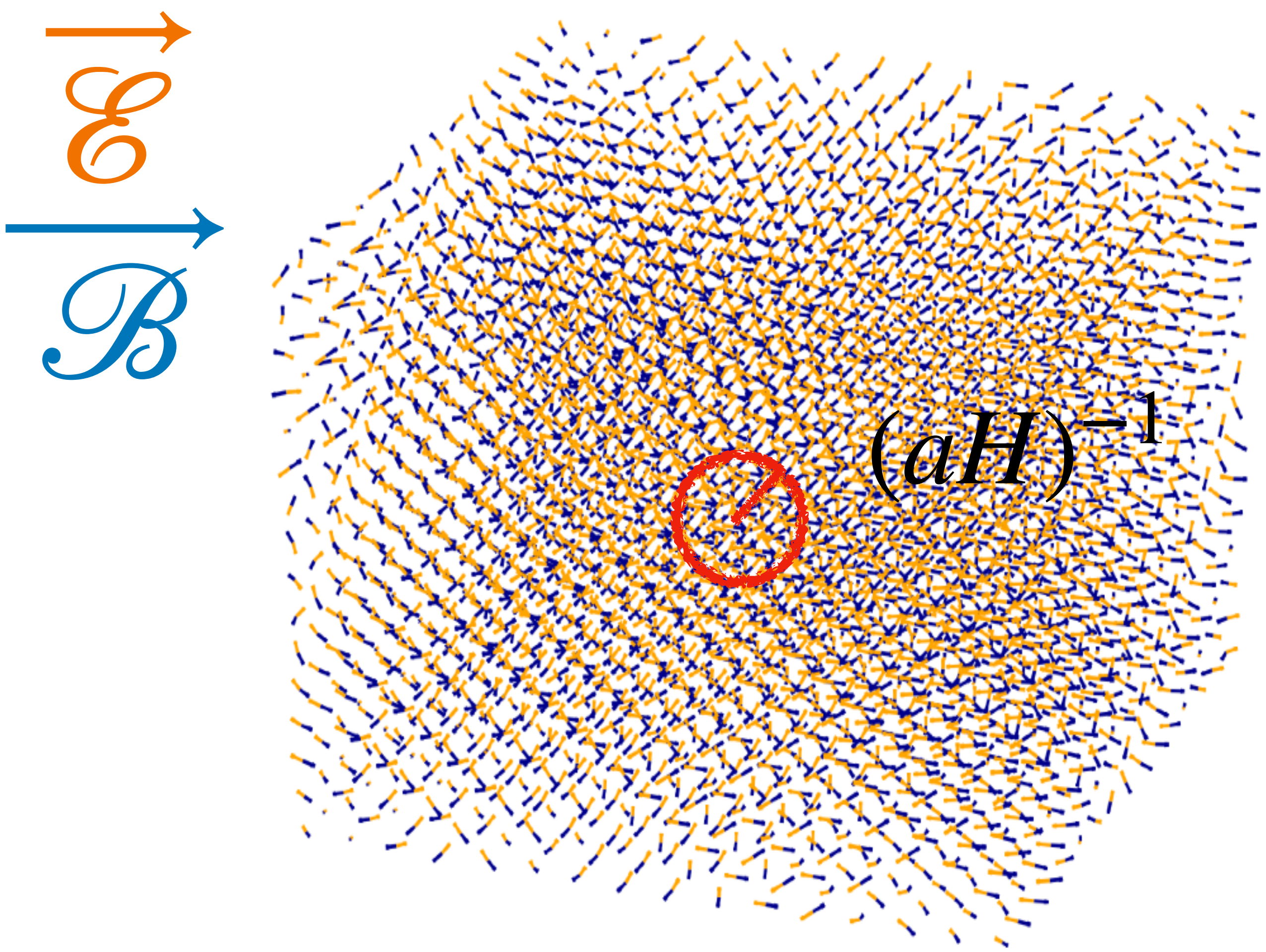


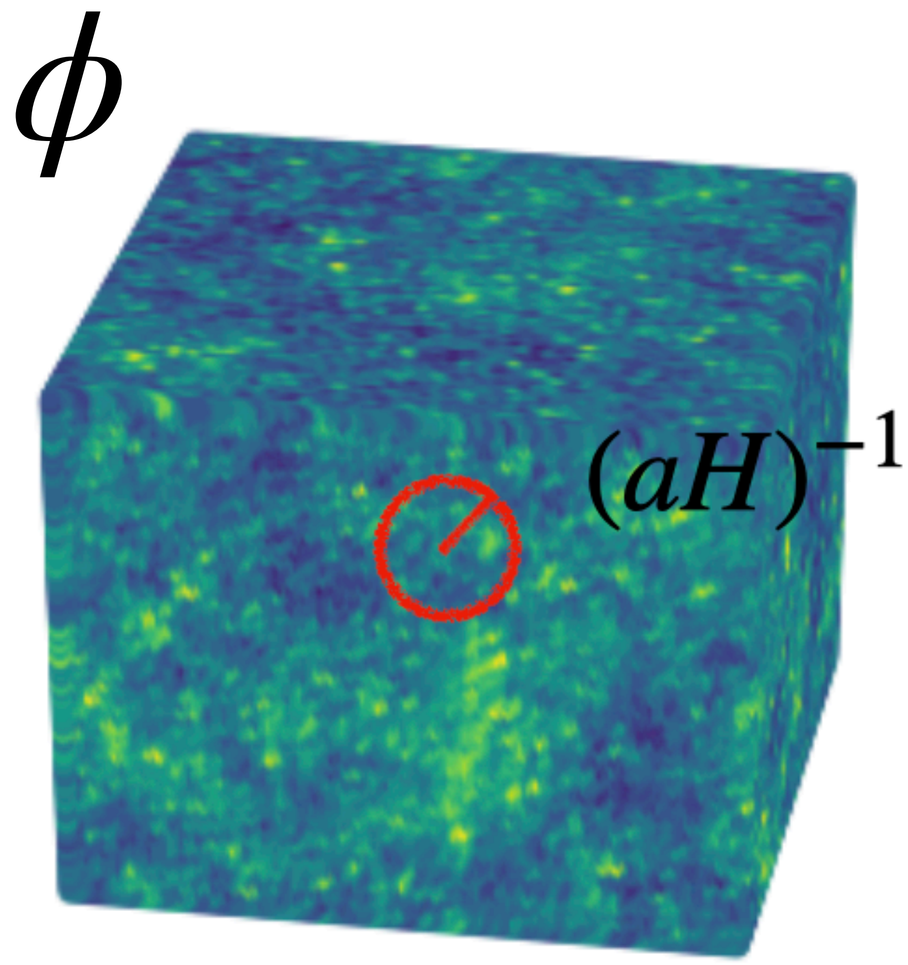
Sub-horizon box —Evolution→ **Super-horizon** box

$a_f/a_i = 10^3$

Lattice approach

We are interested in the statistical properties of the super-horizon box:





Results of the simulation:

1. Linear regime

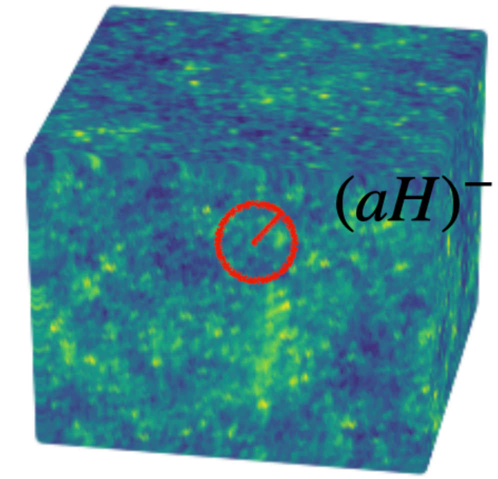
small $\xi = \frac{\alpha \dot{\phi}}{2fH}$

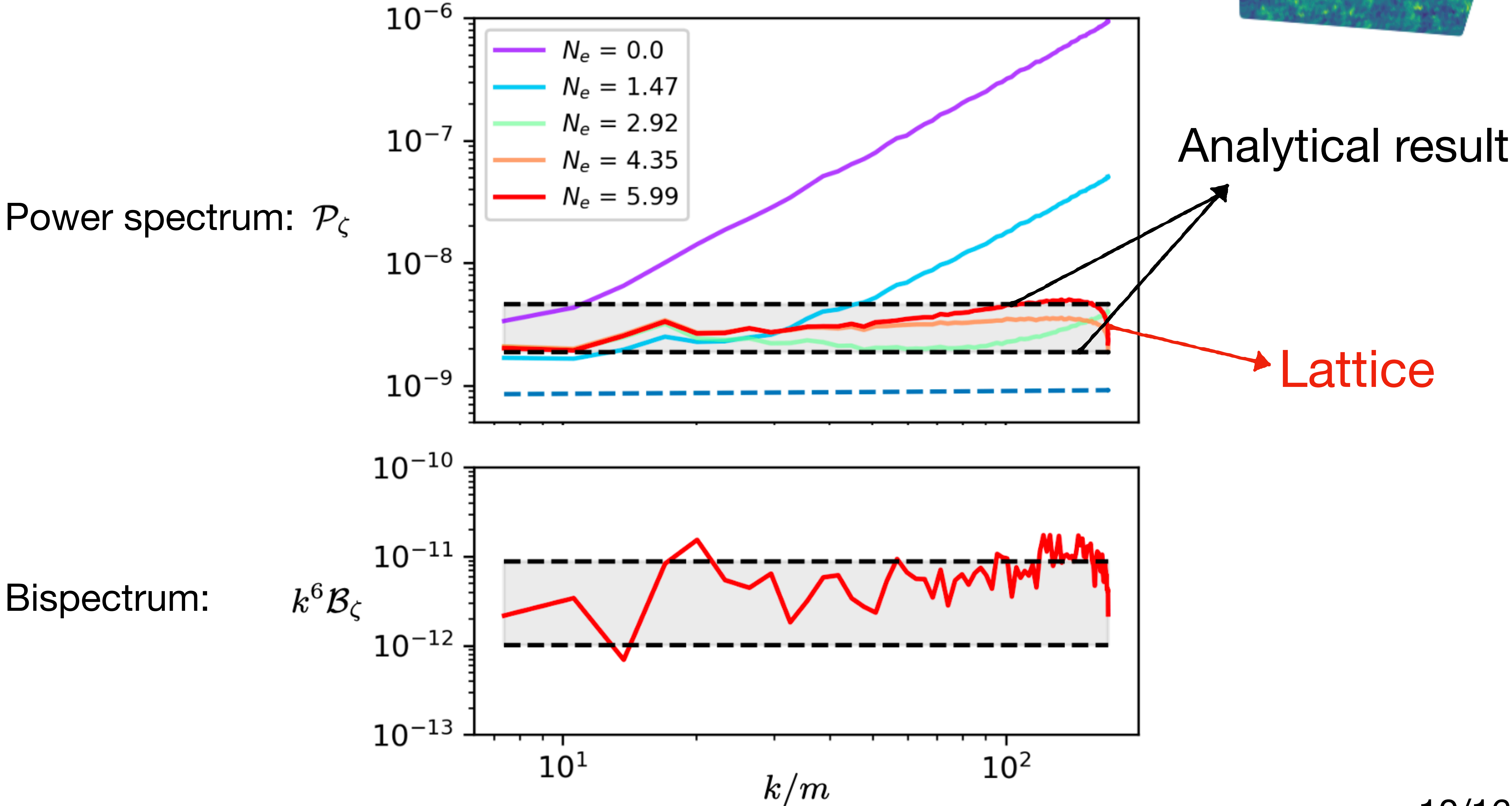
2. Nonlinear regime

large $\xi = \frac{\alpha \dot{\phi}}{2fH}$

The linear case:

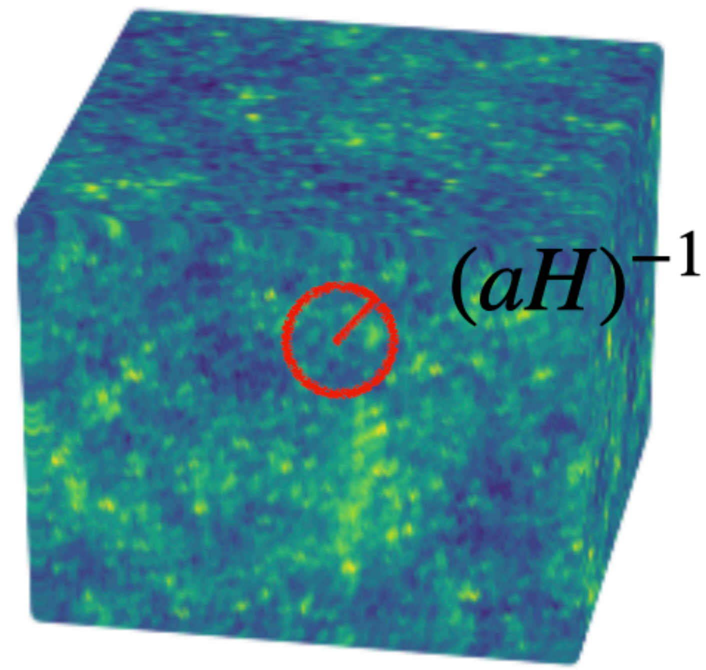
Power spectrum and bispectrum agree
with previous analytical estimates

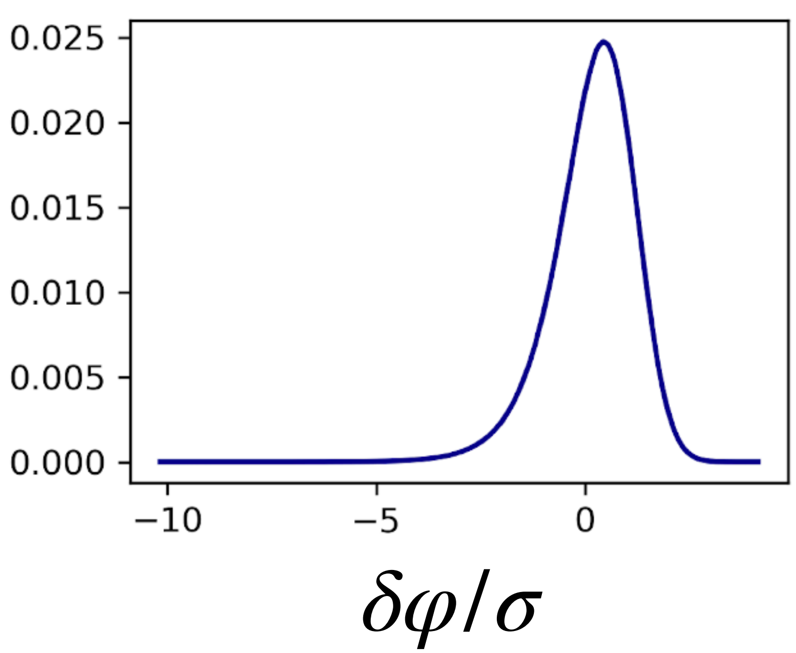


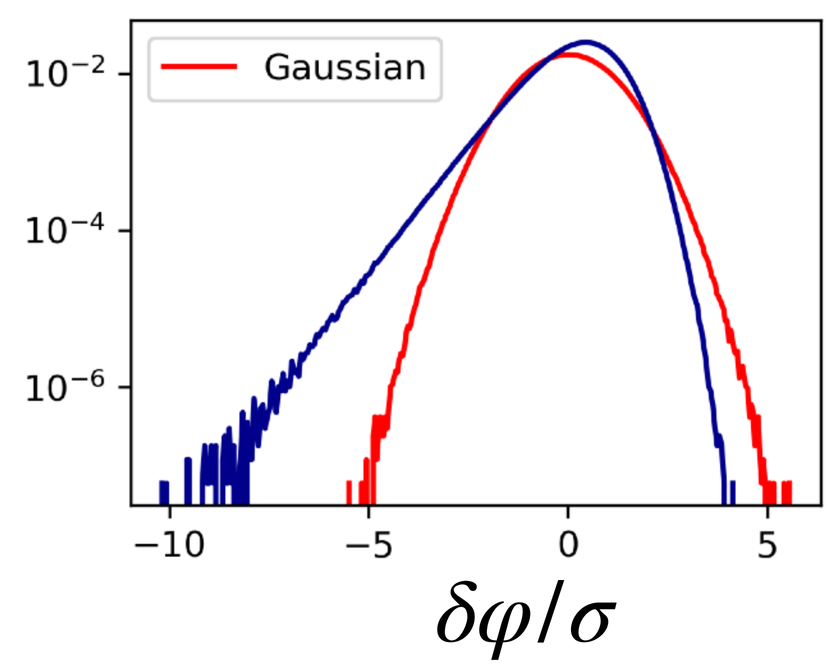


The linear case: what's new?

Thanks to the lattice,
we know the full distribution of $\delta\varphi(\mathbf{x})$ in real space!





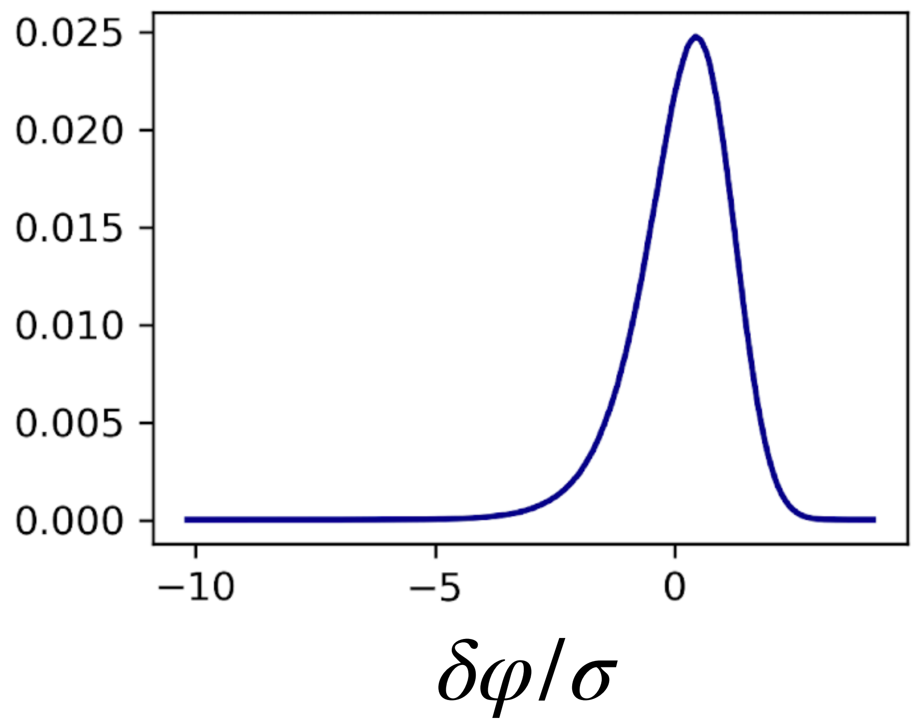


The linear case: what's new?

Define cumulants:

$$\kappa_n = \frac{\langle \delta\varphi^n \rangle_c}{\sigma^n}$$

κ_3 "skewness", κ_4 "kurtosis", etc.



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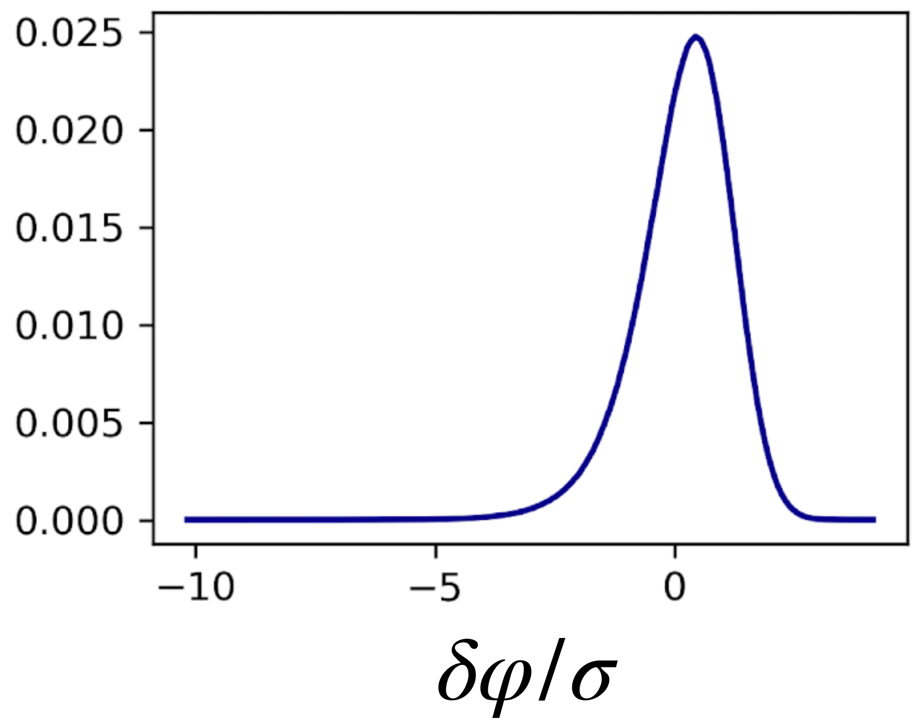

$$\ldots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3 > 1$$

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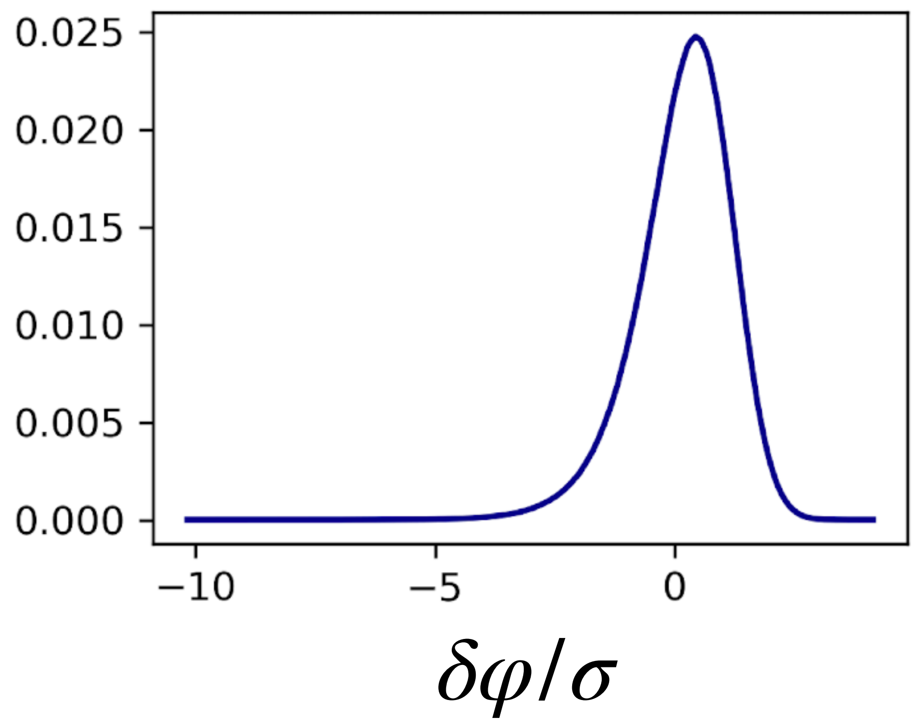

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Higher order polyspectra

Trispectrum

Bispectrum

Nonlinear case

Study transition linear $\longrightarrow$ nonlinear

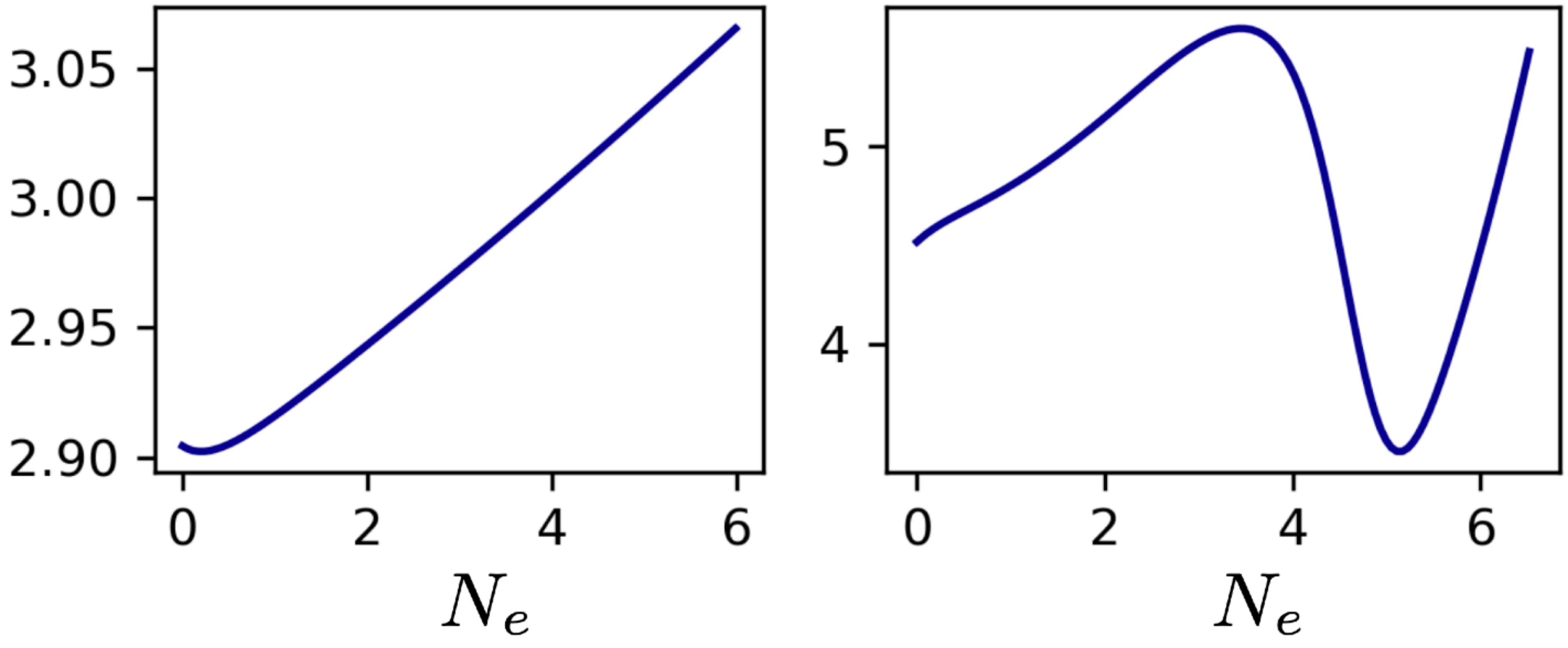
$$\xi = \frac{\alpha \dot{\phi}}{2fH}$$

Linear

(no backreaction)

Linear-nonlinear transition

(strong backreaction)

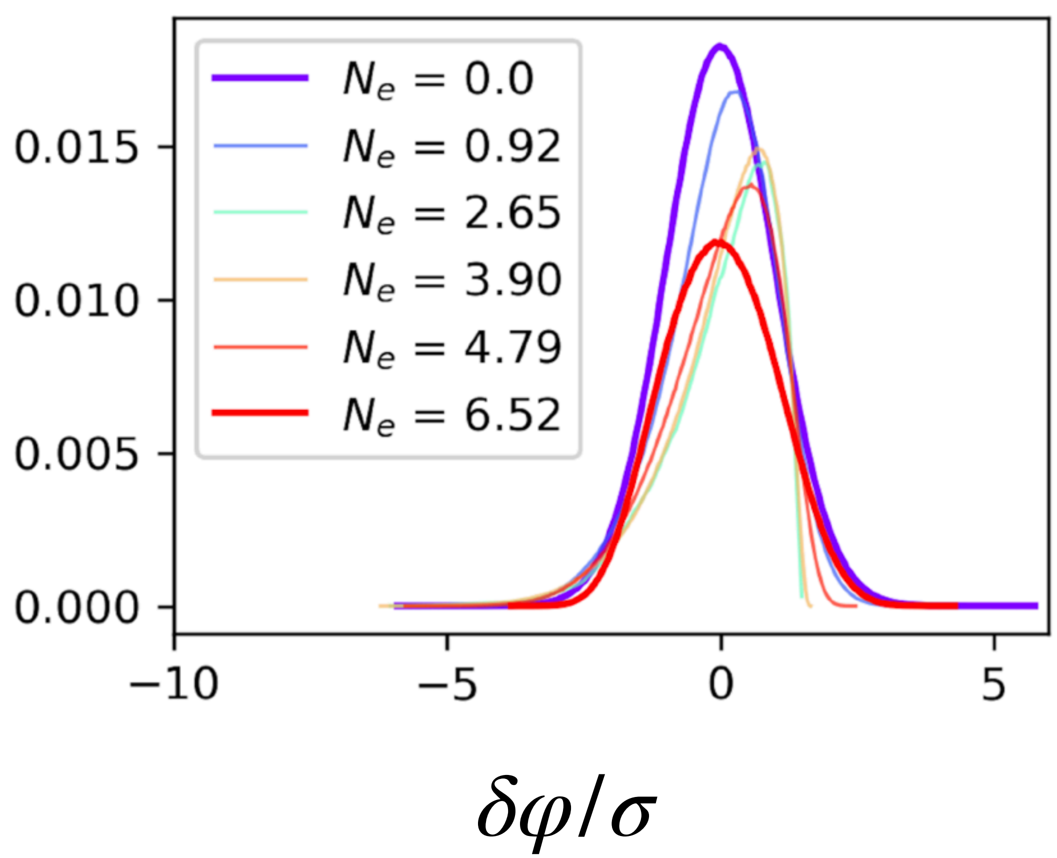


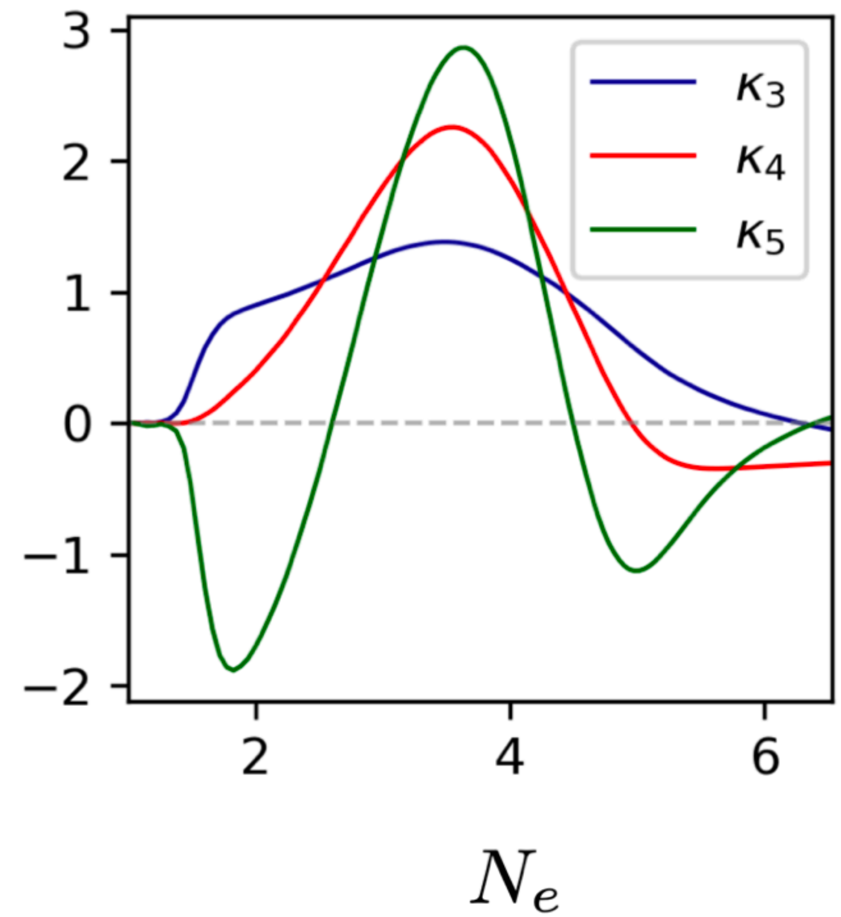
e-folds number (time)

Nonlinear case

Study transition linear ⟶ nonlinear

Non-Gaussianity is **suppressed** in the nonlinear regime.





Nonlinear case

A. Linde, S. Mooij, E. Pajer, arXiv:1212.1693

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etc…

Before our study, it was believed that:

Large ξ $\longrightarrow$ large non-Gaussianity

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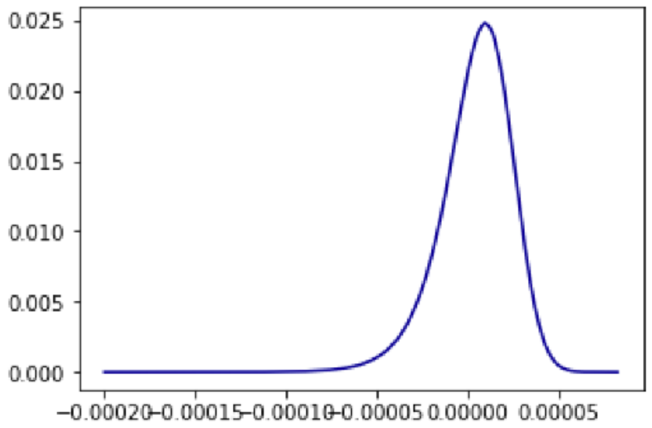
Very efficient production of Primordial BH



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Conclusions:

- First simulation of axion-gauge model during inflation

Results:

- Linear regime:

  Providing a full characterisation of non-Gaussianity.

$$\ldots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3 > 1$$

- Nonlinear regime:

  Perturbations become Gaussian.

  → Invalidate PBH bounds, allowing for observable GWs at interferometers scales