27th PASCOS @ MPIK, 07/27/2022

Decay of dressed particles in preheating

Yusuke Yamada

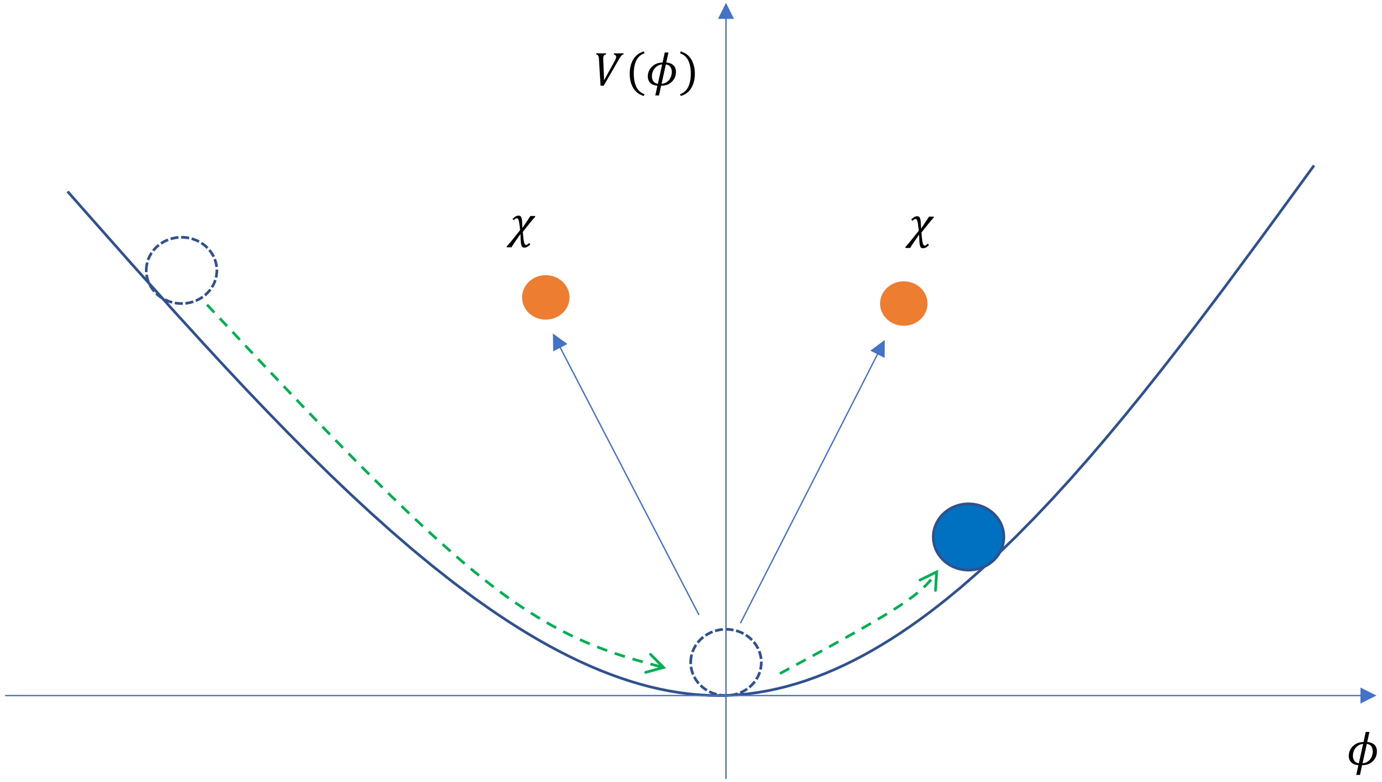
(Waseda Institute for Advanced Study)

in collaboration with Hidetoshi Taya (RIKEN, iTHEMS)

based on [arXiv: 2207.03831]

Introduction

after the end of inflation,
the background inflaton field $\phi(t)$ creates coupled particles



$$\mathcal{L}_\chi = -\frac{1}{2}(\partial\chi)^2 - \frac{1}{2}\left(m_\chi^2 + \zeta\phi^2\right)\chi^2$$

$$M_{\chi,\mathrm{eff}}^2(t) = m_\chi^2 + \zeta\phi^2(t) + \mathcal{O}(H^2)$$
$$\to m_\chi^2 \quad (\text{as } \phi(t) \to 0)$$

$$n_k^\chi \sim \exp\left[-\frac{\pi\left(k^2 + m_\chi^2\right)}{\sqrt{\zeta}|\dot{\phi}(0)|}\right]$$

non-perturbative in ζ !

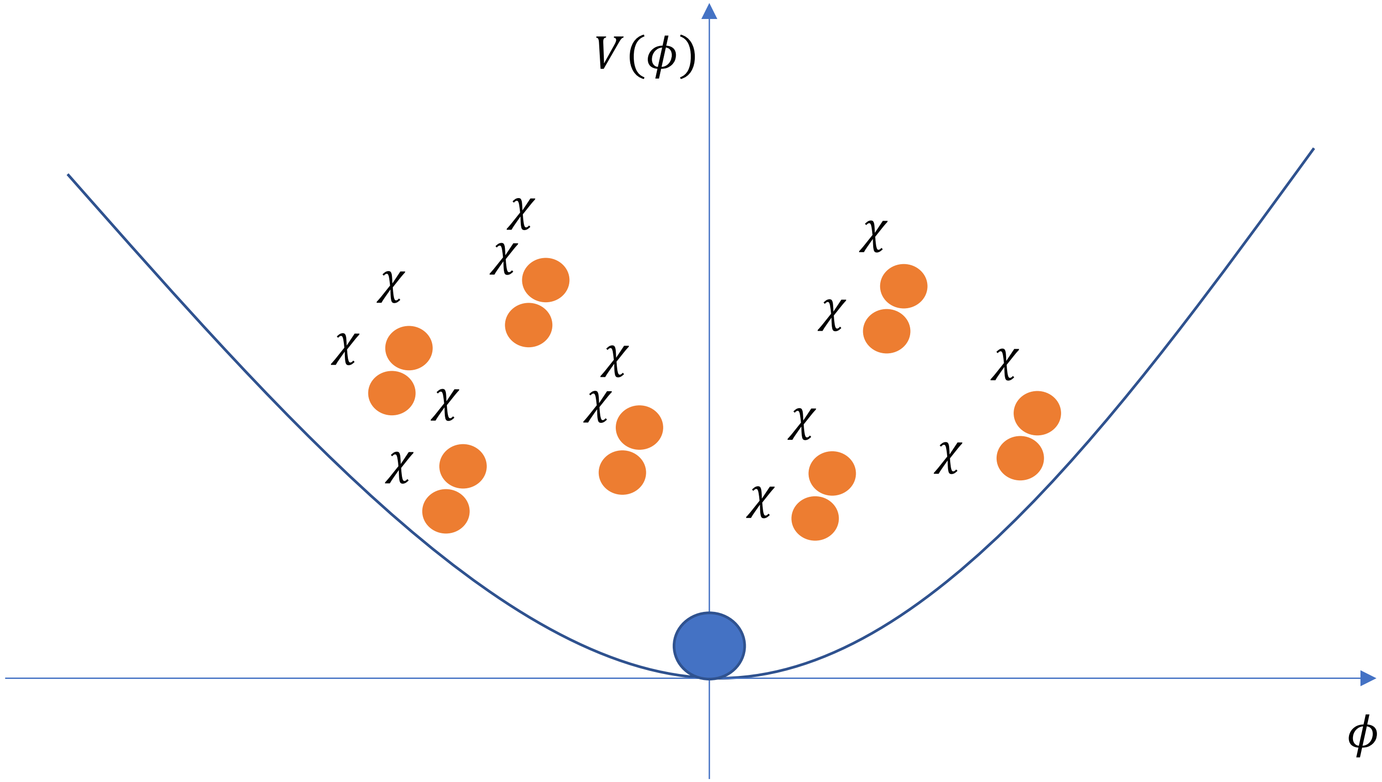
Kofman, Linde, Starobinsky (1997)

At $\phi(\mathrm{t}) = 0$, χ-production from vacuum!→ **“preheating”**

What is the fate of χ?

Introduction

repetition of the process→ huge number of χ → back-reaction to ϕ



$$\mathcal{L}_\chi = -\frac{1}{2}(\partial\chi)^2 - \frac{1}{2}\left(m_\chi^2 + \zeta\phi^2\right)\chi^2$$

$$M_{\chi,\mathrm{eff}}^2(t) = m_\chi^2 + \zeta\phi^2(t) + \mathcal{O}(H^2)$$
$$\to m_\chi^2$$

$$n_k^\chi \sim \exp\left[-\frac{\pi\left(k^2 + m_\chi^2\right)}{\sqrt{\zeta}\left|\dot{\phi}(0)\right|}\right]$$

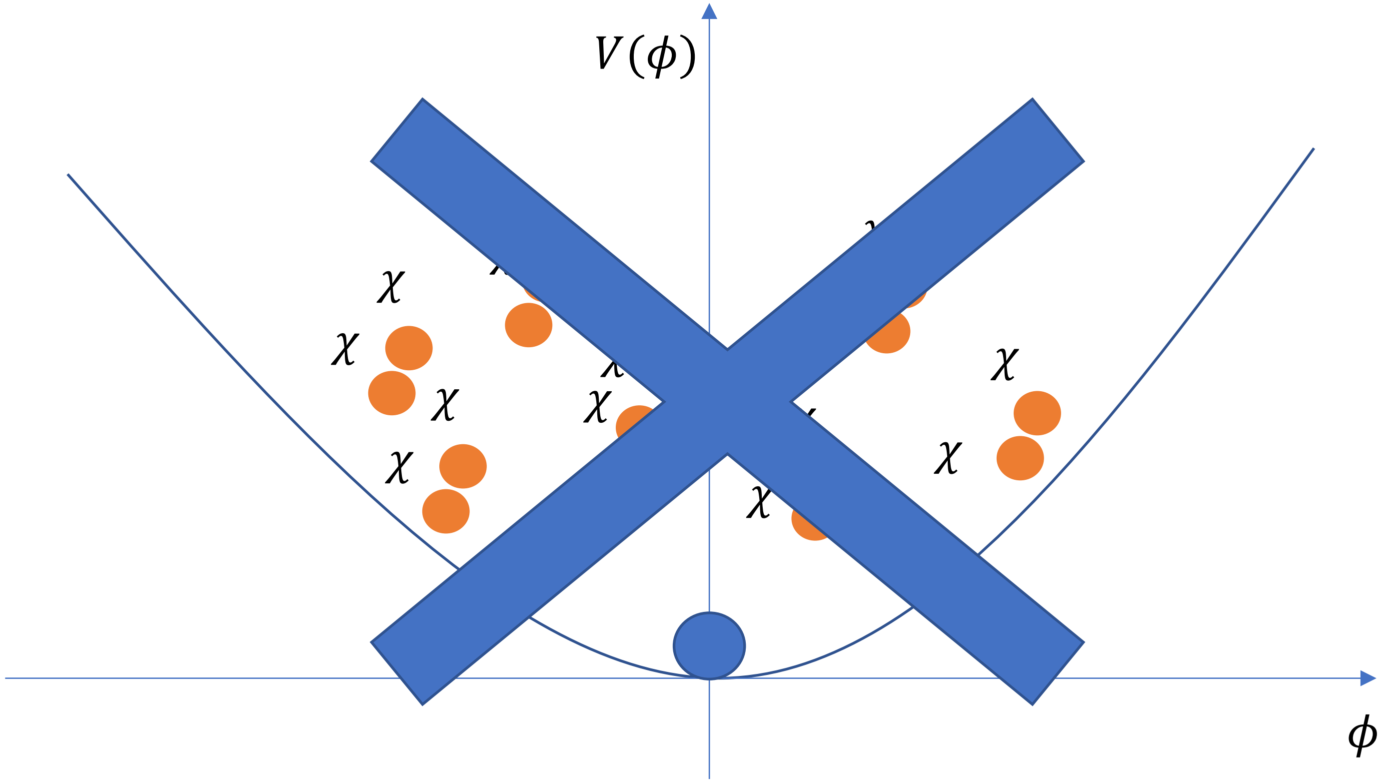
Kofman, Linde, Starobinsky (1997)

In this regime, lattice simulations are necessary

Khlebnikov, Tkachev (1996)(1997)
Prokopec, Roos (1997)
….etc.

Introduction

repetition of the process→ huge number of χ → back-reaction to ϕ



$$\mathcal{L}_\chi = -\frac{1}{2}(\partial\chi)^2 - \frac{1}{2}\left(m_\chi^2 + \zeta\phi^2\right)\chi^2$$

$$M_{\chi,\mathrm{eff}}^2(t) = m_\chi^2 + \zeta\phi^2(t) + \mathcal{O}(H^2)$$
$$\to m_\chi^2$$

$$n_k^\chi \sim \exp\left[-\frac{\pi\left(k^2 + m_\chi^2\right)}{\sqrt{\zeta}\left|\dot{\phi}(0)\right|}\right]$$

Kofman, Linde, Starobinsky (1997)

We will NOT focus on this regime!

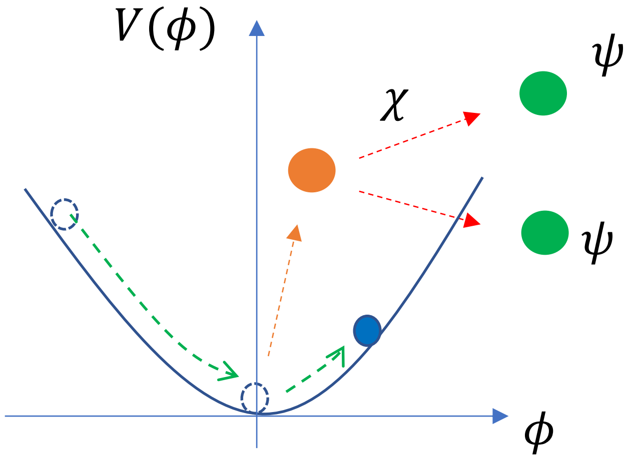
In this regime, lattice simulations are necessary

Khlebnikov, Tkachev (1996)(1997)
Prokopec, Roos (1997)
⋯.etc.

Introduction

What if χ is coupled to other fields?

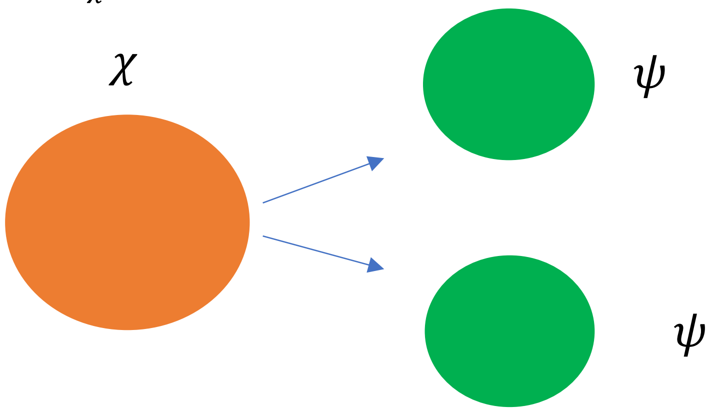
$$\mathcal{L}_\psi = -\frac{1}{2}(\partial\psi)^2 - \frac{1}{2}m_\psi^2\psi^2 - \frac{1}{2}\lambda\,\chi\,\psi^2$$



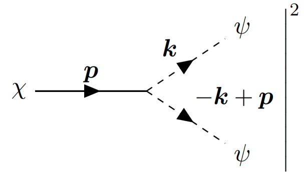
“instant preheating”

Felder, Kofman, Linde (1998), (1999)

$$m_{\chi,\mathrm{eff}}^2 = m_\chi^2 + \lambda\phi^2(t) + \cdots$$



but its QFT description is nontrivial



$$\left|\ \cdots\ \right|^2 = \lambda^2\frac{\pi}{8}\frac{\delta\left(\omega_{\boldsymbol{p}}^\chi - \Omega_{\boldsymbol{k}}^\psi - \Omega_{-\boldsymbol{k}+\boldsymbol{p}}^\psi\right)}{\omega_{\boldsymbol{p}}^\chi\Omega_{\boldsymbol{k}}^\psi\Omega_{-\boldsymbol{k}+\boldsymbol{p}}^\psi} \quad \Rightarrow \quad \Gamma(\mathrm{t}) = \lambda^2\frac{\pi}{8}\frac{\delta\left(\omega_{\boldsymbol{p}}^\chi(t) - \Omega_{\boldsymbol{k}}^\psi - \Omega_{-\boldsymbol{k}+\boldsymbol{p}}^\psi\right)}{\omega_{\boldsymbol{p}}^\chi(t)\Omega_{\boldsymbol{k}}^\psi\Omega_{-\boldsymbol{k}+\boldsymbol{p}}^\psi} \quad (???)$$

We will give a fully QFT description of this process!

Strategy

quantum fields dressed by backgrounds

→ **Furry perturbation theory** = interaction picture for dressed particles

Furry (1951)

"free" part: $S_0 = \frac{1}{2}\int dt\, d^3x[\dot{\chi}^2 - a^{-2}(\partial_i\chi)^2 - M_\chi^2(t)\chi^2 + \dot{\psi}^2 - a^{-2}(\partial_i\psi)^2 - M_\psi^2(t)\psi^2]$

interaction part: $S_{\rm int} = -\frac{\lambda}{2}\int dt d^3x\left[a^{-3/2}\,\chi\,\psi^2\right]$

(after $\chi \to a^{3/2}\chi\,, \psi \to a^{3/2}\psi$)

"dressed" masses

$$M_\chi^2(t) \approx m_\chi^2 + \zeta\phi^2(t) - \frac{9}{4}H^2(t) - \frac{3}{2}\dot{H}(t)$$
$$M_\psi^2(t) \approx m_\psi^2 - \frac{9}{4}H^2(t) - \frac{3}{2}\dot{H}(t)$$

treat S_0 non-perturbatively but $S_{\rm int}$ perturbatively

step 1: Solve free dynamics of "dressed" fields

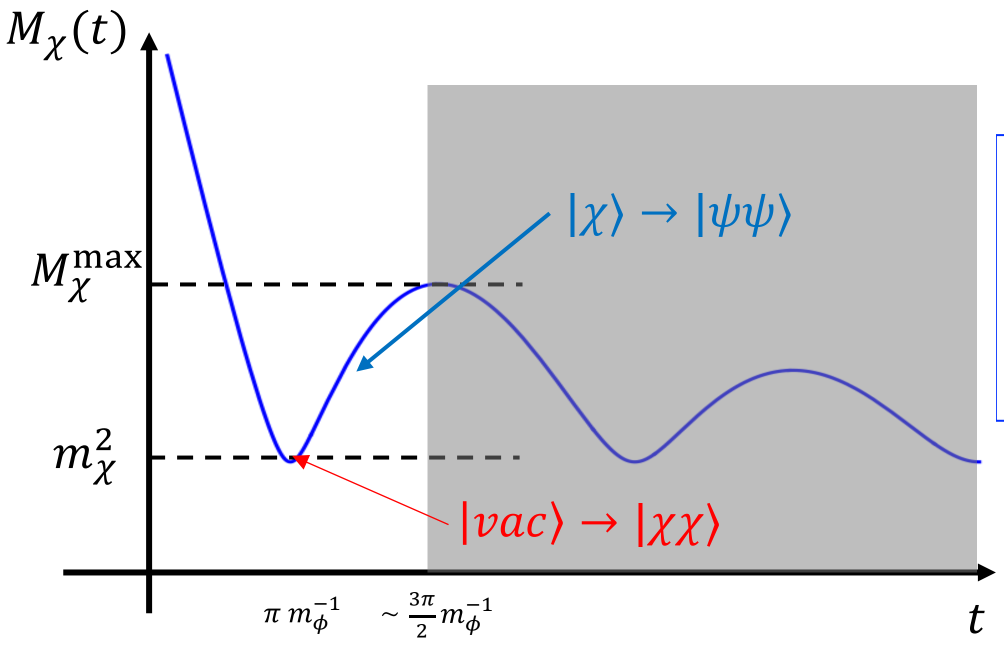
step 2: Evolve operators by $\mathcal{H}_{\rm int}$ perturbatively

this approach is applicable to QFT in various backgrounds

e.g. strong field QED, inflation, preheating…etc.

Strategy

To solve free dynamics, we use the following approximation:



1. focus only on the scattering process within $t \sim \left[0\,,\ \frac{3\pi}{2} m_\phi^{-1}\right]$ which dominates χ & ψ-production

2. $H(t) \ll m_\chi\,, m_\psi,\ a(t) \sim const.$ (neglect Hubble terms⋯etc.)

$$M_\chi^2(t) \approx m_\chi^2 + \zeta\phi^2(t) - \frac{9}{4}H^2(t) - \frac{3}{2}\dot{H}(t)$$
$$M_\psi^2(t) \approx m_\psi^2 - \frac{9}{4}H^2(t) - \frac{3}{2}\dot{H}(t)$$

→

$$M_\chi^2 \approx m_\chi^2 + \zeta\, A^2 m_\phi^2 \left(t - \pi m_\phi^{-1}\right)^2$$
$$M_\psi^2 \approx m_\psi^2$$

step 1: “free” field dynamics

dressed free field χ

$$\hat{\chi}^{(0)}(t,\boldsymbol{x}) = \int \frac{d^3p}{(2\pi)^3}\left[e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\hat{a}_{\boldsymbol{p}}\,\chi_{\boldsymbol{p}}^{(0)}(t) + e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}\hat{a}_{\boldsymbol{p}}^{\dagger}\,\chi_{\boldsymbol{p}}^{(0)*}(t)\right]$$

$$\ddot{\chi}_{\boldsymbol{p}}^{(0)}(t) + \left(\mu_{\boldsymbol{p}}^2 + \zeta A^2 m_\phi^4 (t-t_0)^2\right)\chi_{\boldsymbol{p}}^{(0)} = 0$$

exact sol: $\chi_{\boldsymbol{p}}^{(0)} = \frac{1}{\left(2\sqrt{\zeta}Am_\phi^2\right)^{1/4}} e^{-\frac{\pi\mu_{\boldsymbol{p}}^2}{8\sqrt{\zeta}Am_\phi^2}} D_{\frac{i}{2}\frac{\pi\mu_{\boldsymbol{p}}^2}{\sqrt{\zeta}Am_\phi^2}-\frac{1}{2}}\left(-e^{-\frac{i\pi}{4}}\sqrt{\frac{2\sqrt{\zeta}A}{\pi}}\,m_\phi^2(t-t_0)\right)$

χ-production time: $t_0 = \pi m_\phi^{-1}$ bare energy: $\mu_{\boldsymbol{p}}^2 \equiv \boldsymbol{p}^2 + m_\chi^2$

most importantly,

$\chi_{\boldsymbol{p}}^{(0)}(t) \to f_{\boldsymbol{p}}^{WKB}$ as $t \to -\infty$

Kofman, Linde, Starobinsky (1997)

$\chi_{\boldsymbol{p}}^{(0)}(t) \to \alpha_{\boldsymbol{p}} f_{\boldsymbol{p}}^{WKB} + \beta_{\boldsymbol{p}} f_{\boldsymbol{p}}^{WKB\,*}$ as $t \to +\infty$

where $f_{\boldsymbol{p}}^{WKB} = \frac{1}{\sqrt{2\omega_{\boldsymbol{p}}(t)}} e^{-i\int^t dt'\omega_{\boldsymbol{p}}(t')}$ $\quad |\alpha_{\boldsymbol{p}}|^2 - |\beta_{\boldsymbol{p}}|^2 = 1$

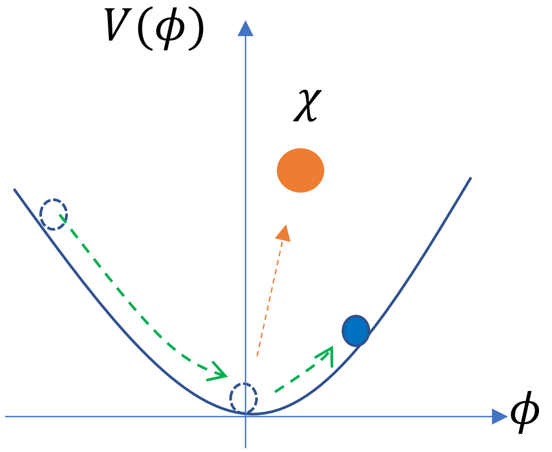
free field ψ

trivial mode function:

$$\psi_{\boldsymbol{k}}^{(0)} = \frac{1}{\sqrt{2\Omega_{\boldsymbol{k}}}} e^{-i\Omega_{\boldsymbol{k}} t}$$

$$\Omega_k = \sqrt{\boldsymbol{k}^2 + m_\psi^2}$$

→ no pair ψ-production from vacuum



pair production of the parent particle χ :

$$n_{\boldsymbol{p}}^{\chi} \sim |\beta_{\boldsymbol{p}}|^2 = \exp\left[-\frac{\pi\mu_{\boldsymbol{p}}^2}{\sqrt{\zeta}Am_\phi^2}\right]$$

step 2: perturbative time evolution

perturbative evolution of operators

$$\begin{pmatrix}\hat{\chi}(t,\boldsymbol{x})\\ \hat{\psi}(t,\boldsymbol{x})\end{pmatrix} = U^\dagger(t,t_{\rm in})\begin{pmatrix}\hat{\chi}^{(0)}(t_{\rm in},\boldsymbol{x})\\ \hat{\psi}^{(0)}(t_{\rm in},\boldsymbol{x})\end{pmatrix}U(t,t_{\rm in}) \quad \text{where } U(t,t_{\rm in}) = T\exp\left[-i\int_{t_{\rm in}}^{t}\mathcal{H}_{\rm int}dt\right] \qquad \mathcal{H}_{\rm int} = \frac{\lambda}{2}\int d^3\boldsymbol{x}\, a^{-3/2}\,\hat{\chi}^{(0)}\left(\psi^{(0)}\right)^2$$

1st order pert. ⇒

$$\hat{\psi}(t,\boldsymbol{x}) = \hat{\psi}^{(0)}(t,\boldsymbol{x}) - i\lambda\int_{t_{\rm in}}^{t} dt'\, d^3\boldsymbol{x}' a^{-\frac{3}{2}}(t')\left[\hat{\psi}^{(0)}(t,\boldsymbol{x}),\hat{\psi}^{(0)}(t',\boldsymbol{x}')\right]\hat{\chi}^{(0)}(t',\boldsymbol{x}')\hat{\psi}^{(0)}(t',\boldsymbol{x}') + \mathcal{O}(\lambda^2)$$

we can obtain $\hat{\chi}$ as well

out annihilation operator of ψ:

$$\hat{b}_{\boldsymbol{k}}^{out} = \lim_{t\to+\infty} +i\int d^3\boldsymbol{x}\left(\frac{e^{i\boldsymbol{k}\cdot\boldsymbol{x}}}{(2\pi)^{\frac{3}{2}}}\frac{e^{-i\Omega_{\boldsymbol{k}}t}}{\sqrt{2\Omega_{\boldsymbol{k}}}}\right)^* \overleftrightarrow{\partial_t}\ \hat{\psi}(t,\boldsymbol{x})$$

⇒

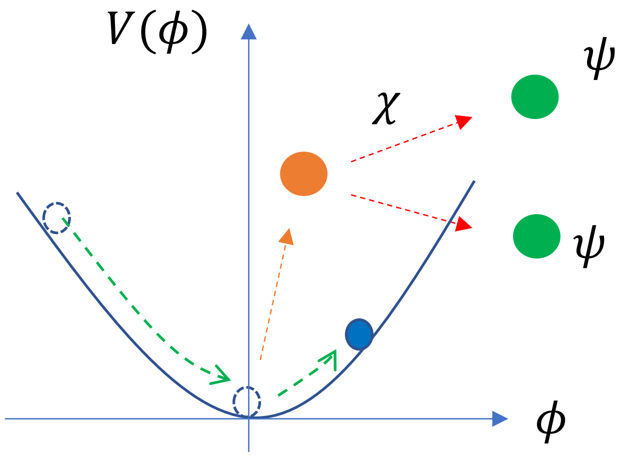
$$n_{\boldsymbol{k}}^{\psi,\rm out} = \frac{(2\pi)^3}{V}\langle 0_{in}|(b_k^{out})^\dagger b_k^{out}|0_{in}\rangle = \lambda^2\int\frac{d^3p}{(2\pi)^3}\left|\int_{t_{\rm in}}^{\infty}dt'\ a^{-\frac{3}{2}}(t')\,\chi_{\boldsymbol{p}}^{(0)}(t')\psi_{-\boldsymbol{k}-\boldsymbol{p}}^{(0)}(t')\psi_{\boldsymbol{k}}^{(0)}(t')\right| + \mathcal{O}(\lambda^3)$$

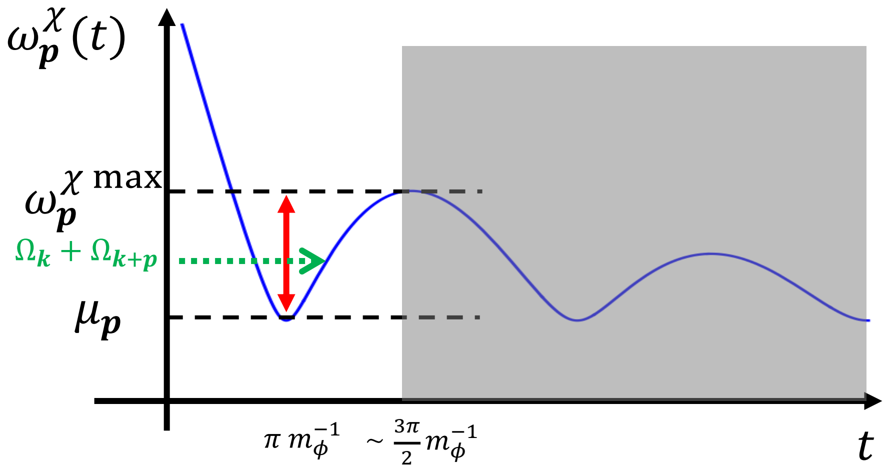
⇒

$$n_k^{\psi} = \lambda^2\frac{\pi}{(2C)^{3/2}}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{e^{-\frac{3\pi}{4}\frac{\mu_{\boldsymbol{p}}^2}{C}}}{\Omega_k\Omega_{k+p}}\left|D_{\frac{i}{2}\frac{\mu_{\boldsymbol{p}}^2}{C}-\frac{1}{2}}\left(e^{\frac{i\pi}{4}}\sqrt{\frac{2}{C}}\left(\Omega_k+\Omega_{k+p}\right)\right)\right|^2$$

where $C \equiv \frac{\sqrt{\zeta}Am_\phi^2}{\pi}$

result 1: kinematically allowed decay





kinematic condition:
$\Omega_{\boldsymbol{k}} + \Omega_{-\boldsymbol{k}+\boldsymbol{p}} = \omega_p(t)$ at some t

for the modes satisfying the kinematic condition $\mu_p < \Omega_{\boldsymbol{k}} + \Omega_{-\boldsymbol{k}+\boldsymbol{p}} < \omega_p^{\chi\,\max}$

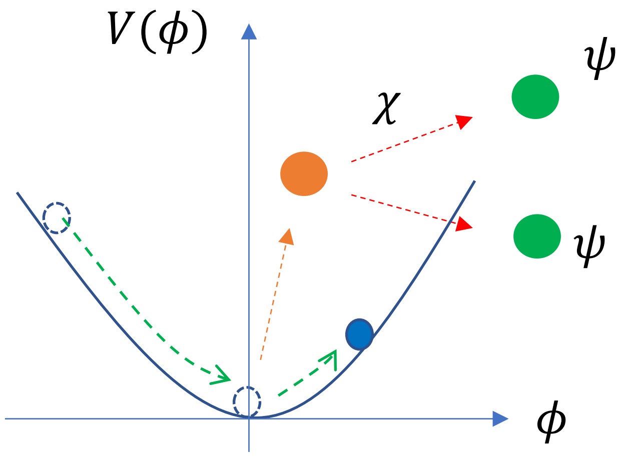
approximation of the exact formula:

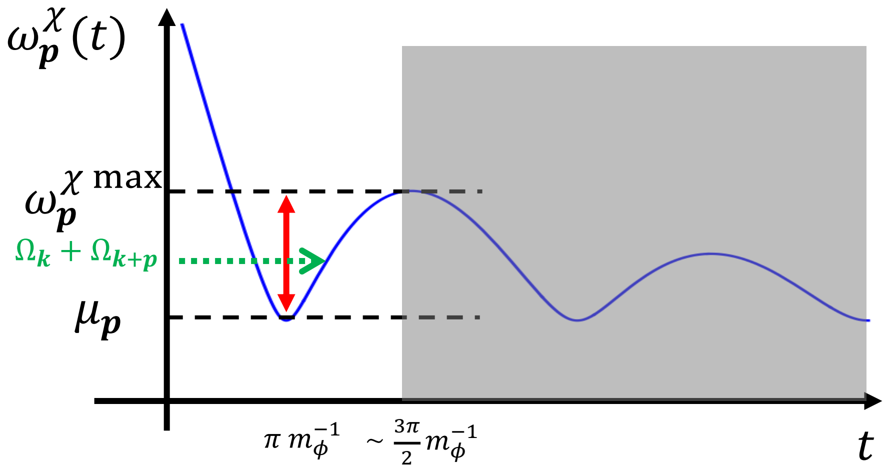
$$n_{\boldsymbol{k}}^{\psi} = \lambda^2 \frac{\pi}{(2C)^{3/2}} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{e^{-\frac{3\pi}{4}\frac{\mu_{\boldsymbol{p}}^2}{C}}}{\Omega_{\boldsymbol{k}}\Omega_{\boldsymbol{k}+\boldsymbol{p}}} \left| D_{\frac{i\mu_{\boldsymbol{p}}^2}{2C}-\frac{1}{2}}\left(e^{\frac{i\pi}{4}} \sqrt{\frac{2}{C}} (\Omega_{\boldsymbol{k}} + \Omega_{\boldsymbol{k}+\boldsymbol{p}}) \right) \right|^2 \approx \lambda^2 \frac{\pi}{4C} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{\exp\left[-\pi \frac{\mu_{\boldsymbol{p}}^2}{C}\right]}{\Omega_{\boldsymbol{k}}\Omega_{\boldsymbol{k}+\boldsymbol{p}} \sqrt{(\Omega_{\boldsymbol{k}} + \Omega_{\boldsymbol{k}+\boldsymbol{p}})^2 - \mu_{\boldsymbol{p}}^2}}$$

(by using integral. rep of $D_\nu(z)$& saddle point method)

How can we interpret this result?

result 1: kinematically allowed decay

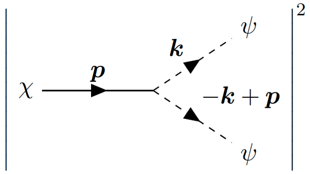




kinematic condition:
$\Omega_{\boldsymbol{k}} + \Omega_{-\boldsymbol{k}+\boldsymbol{p}} = \omega_p(t)$ at some t

phenomenological approach:

decay probability (without background field):



$$\left| \cdots \right|^2 = \lambda^2 \frac{\pi}{8} \frac{\delta\left(\omega_{\boldsymbol{p}} - \Omega_{\boldsymbol{k}} - \Omega_{-\boldsymbol{k}+\boldsymbol{p}}\right)}{\omega_{\boldsymbol{p}} \Omega_{\boldsymbol{k}} \Omega_{-\boldsymbol{k}+\boldsymbol{p}}}$$

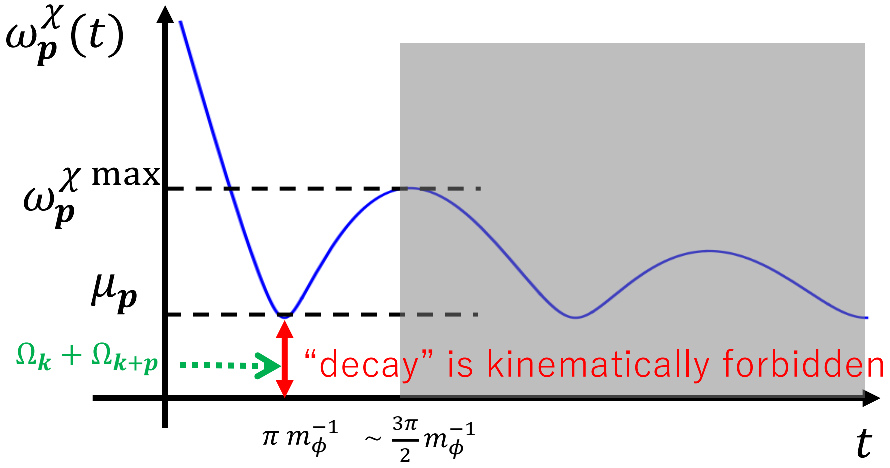
decay probability with time-dependent background : $P_{\boldsymbol{k},\boldsymbol{p}}^{\text{tree}} = \int dt\, \lambda^2 \frac{\pi}{8} \frac{\delta\left(\omega_{\boldsymbol{p}}(t) - \Omega_{\boldsymbol{k}} - \Omega_{-\boldsymbol{k}+\boldsymbol{p}}\right)}{\omega_{\boldsymbol{p}}(t) \Omega_{\boldsymbol{k}} \Omega_{-\boldsymbol{k}+\boldsymbol{p}}}$

$$n_{\boldsymbol{k}}^{\psi,\text{pert}} = 2 \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} n_{\boldsymbol{p}}^{\chi} P_{\boldsymbol{k},\boldsymbol{p}}^{\text{tree}} = \lambda^2 \frac{\pi}{4C} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{\exp\left[-\pi \frac{\mu_{\boldsymbol{p}}^2}{C}\right]}{\Omega_{\boldsymbol{k}} \Omega_{\boldsymbol{k}+\boldsymbol{p}} \sqrt{\left(\Omega_{\boldsymbol{k}} + \Omega_{\boldsymbol{k}+\boldsymbol{p}}\right)^2 - \mu_{\boldsymbol{p}}^2}} \quad \left(= n_{\boldsymbol{k}}^{\psi,\text{exact}}\right)$$

Furry perturbation theory reproduces a phenomenological expectation!

result 2: kinematically forbidden particle production

What about the modes **NOT** satisfying the kinematic condition?

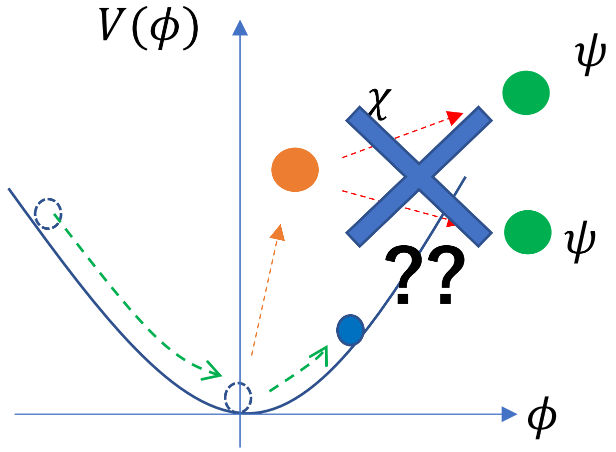


if $\mu_{\boldsymbol{p}} > \Omega_{\boldsymbol{k}} + \Omega_{\boldsymbol{k}+\boldsymbol{p}}$,

no instantaneous energy conservation

→ **no perturbative “decay”**

→ **no particle number for such modes**



such a perturbative expectation is completely **wrong**!

result 2: kinematically forbidden particle production

exact formula shows ***the kinematically-forbidden particle production*** :

$$n_k^\psi \approx \frac{\lambda^2 \pi}{4C} \int \frac{d^3 p}{(2\pi)^3} \frac{\exp\left[-\pi \frac{\mu_{\boldsymbol{p}}^2}{C} \left(1 - F(x)\right)\right]}{\Omega_{\boldsymbol{k}} \Omega_{\boldsymbol{k}+\boldsymbol{p}} \sqrt{\mu_{\boldsymbol{p}}^2 - \left(\Omega_{\boldsymbol{k}} + \Omega_{\boldsymbol{k}+\boldsymbol{p}}\right)^2}}$$

$$x \equiv \frac{\Omega_{\boldsymbol{k}} + \Omega_{\boldsymbol{k}+\boldsymbol{p}}}{\mu_{\boldsymbol{p}}} \quad \left(\text{for} \ \ \mu_{\boldsymbol{p}} > \Omega_{\boldsymbol{k}} + \Omega_{\boldsymbol{k}+\boldsymbol{p}}\right)$$

$$F(0) = \frac{1}{2} \geq F(x) \geq F(1) = 0$$

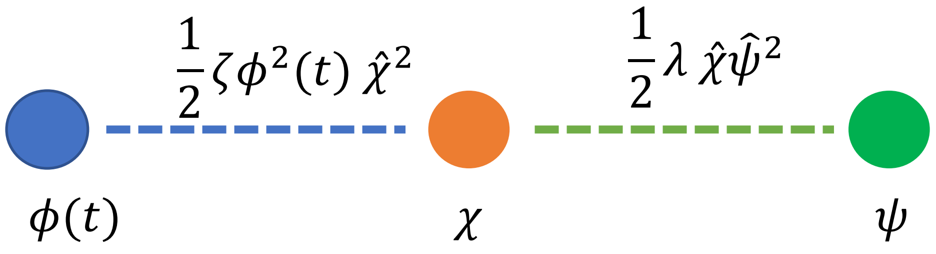
since $n_{\boldsymbol{p}}^\chi = \exp\left[-\pi \frac{\mu_{\boldsymbol{p}}^2}{C}\right]$, $n^\psi \gg n^\chi$ is possible in this regime

note that for perturbative (kinematically allowed) decay, $n_\psi < 2 n_\chi$

- not understood from the standard decay $|\chi\rangle \to |\psi\psi\rangle$
- not restricted to $\rho_\psi < \rho_\chi$ as energy conservation does not hold

result 2: kinematically forbidden particle production

non-trivial consequence of the kinematically-forbidden production for $m_\chi \gg m_\psi$:



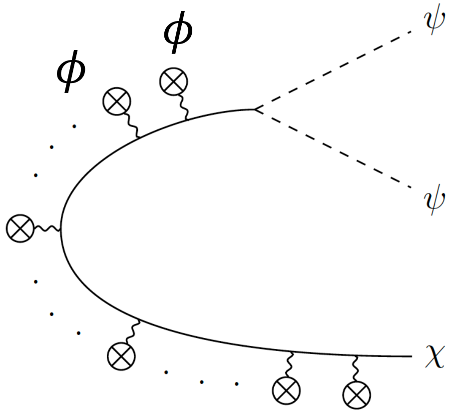
e.g. $(m_\chi, m_\psi) = (1.3\times10^{15}, 5\times10^{13})\mathrm{GeV}$

$\zeta = 1.0\times10^{-4}, \lambda = m_\chi, m_\phi = 10^{13}\mathrm{GeV}$

$$\Rightarrow \frac{\rho_\psi}{\rho_\chi} = 2\times10^3$$

(but backreaction is negligible as $\frac{\rho_\psi}{\rho_\phi} = 2.6\times10^{-19}$)

ψ is ***indirectly*** coupled to inflaton but produced ***more*** !

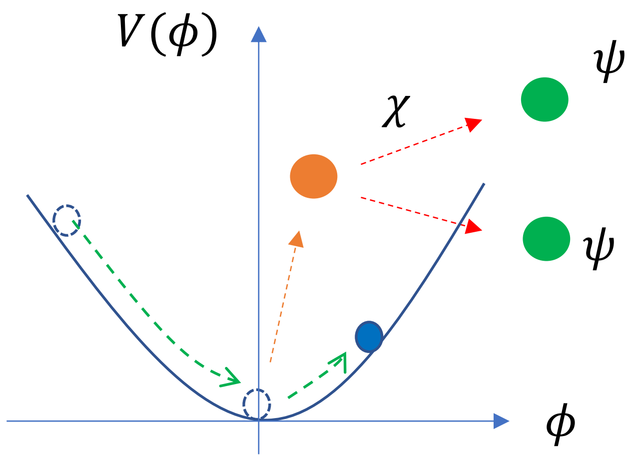


$$n_k^\psi \approx \frac{\lambda^2\pi}{4C}\int\frac{d^3p}{(2\pi)^3}\frac{\exp\left[-\pi\frac{\mu_{\boldsymbol{p}}^2}{C}\left(1-F(x)\right)\right]}{\Omega_{\boldsymbol{k}}\Omega_{\boldsymbol{k+p}}\sqrt{\mu_{\boldsymbol{p}}^2-\left(\Omega_{\boldsymbol{k}}+\Omega_{\boldsymbol{k+p}}\right)^2}}$$

the effect of multiple scattering with ϕ

not captured by perturbative scattering calculations

Summary



QFT approach to scattering during preheating
→ Furry picture perturbation theory
(perturbative processes of dressed free fields)

this approach is applicable to QFT in various backgrounds
(strong field QED, preheating, inflation, ⋯etc.)

- we confirmed the expected behavior of kinematically allowed decay
- we found a new non-perturbative particle production= **kinematically forbidden production**

furthermore, for light daughter particles, $n^{\psi}_{\mathrm{forbidden}} \gg n^{\psi}_{\mathrm{allowed}}$

For a more systematic approach to general backgrounds,
semiclassical methods (such as WKB) may be useful

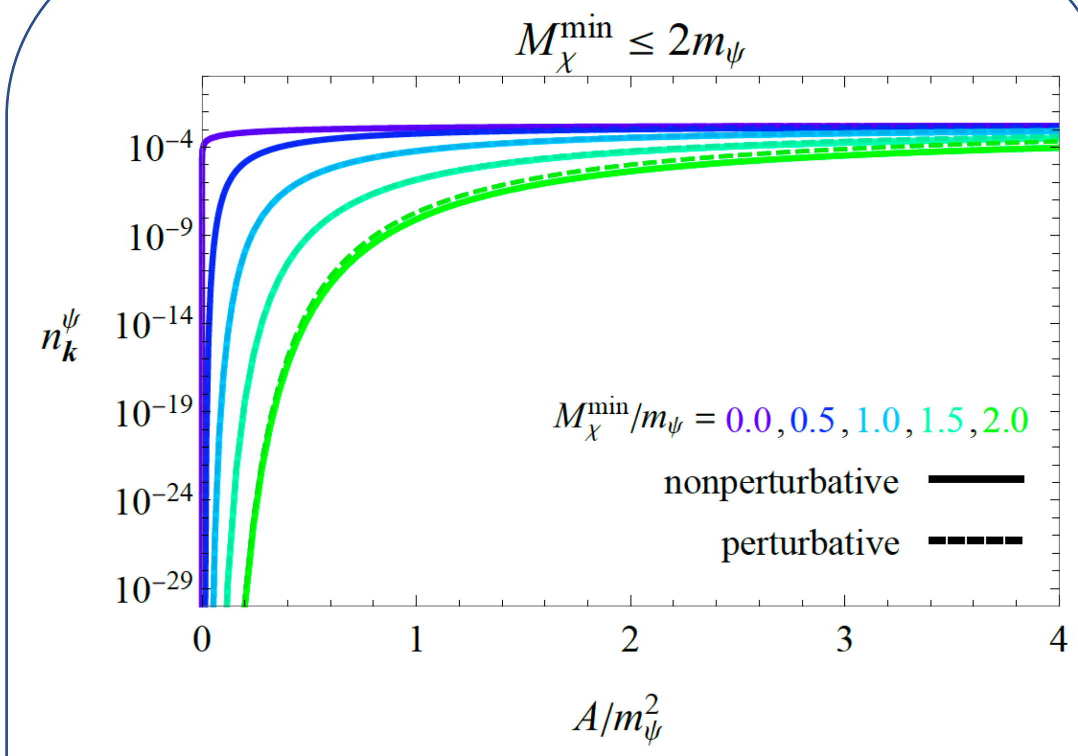
(including Stokes phenomena ~ pair creation from vacuum)

H. Taya, YY
work in progress

backup slide

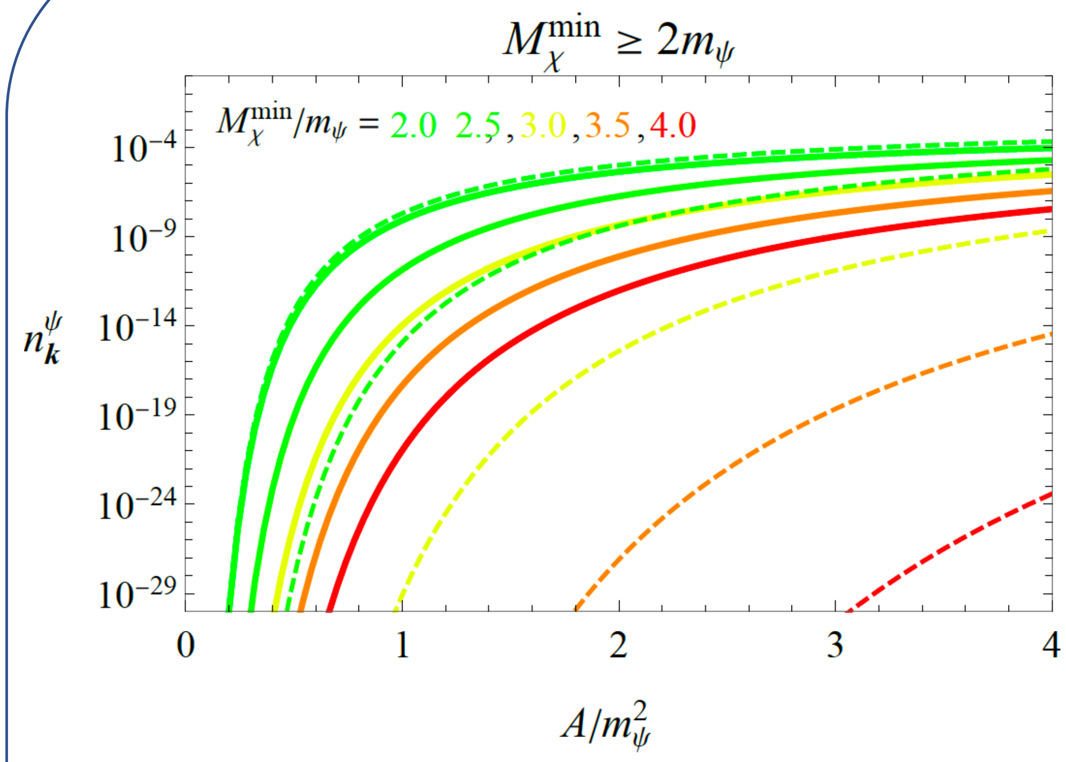
numerical comparison

a heavy daughter particle case



perturbative decay(=kinematically allowed decay)
dominates the particle production
→ exact result ~ phenomenological expectation

a light daughter particle case



kinematically-forbidden production dominates
→ phenomenological (perturbative) result
shows exponentially large discrepancy