Background Field Method and Initial time-singularity for Coherent States

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based on L.Berezhiani, G.C., M. Zantedeschi; arXiv:2108.13235

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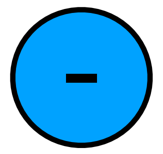

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= Low level of ‘quantumness’

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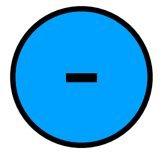

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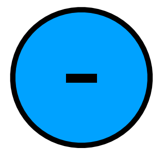

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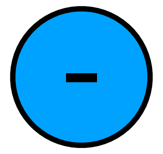

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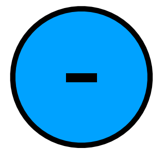

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Corrections to Classical Evolution could be relevant for BH, De Sitter and Inflation: *Depletion of De Sitter, Non-Thermal corrections to Hawking radiation,…*

(See Dvali, Gomez, arXiv:1312.4795; Dvali, Gomez, Zell, arXiv:1701.08776)

Semiclassical Approximation is the first approach to compute quantum corrections to classical systems:

$$\hat{\phi}(t_0, x) = \underbrace{\phi_{cl}(t_0, x)}_{\text{Classical configuration: vacuum}} + \underbrace{\delta\hat{\phi}_{cl}(t_0, x)}_{\text{Fluctuations defined on the 'new' vacuum}}$$

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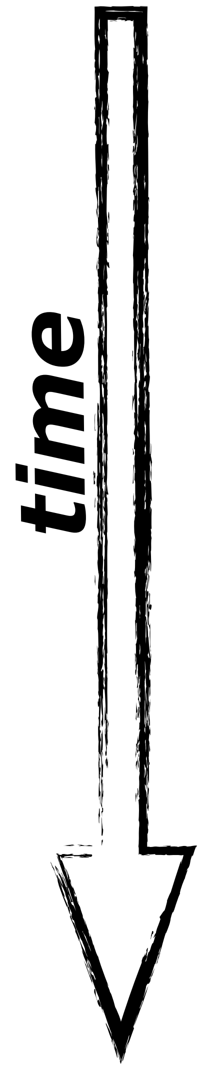
Classical configuration: vacuum

Fluctuations defined on the 'new' vacuum

1) The original classical background is not a reliable vacuum anymore

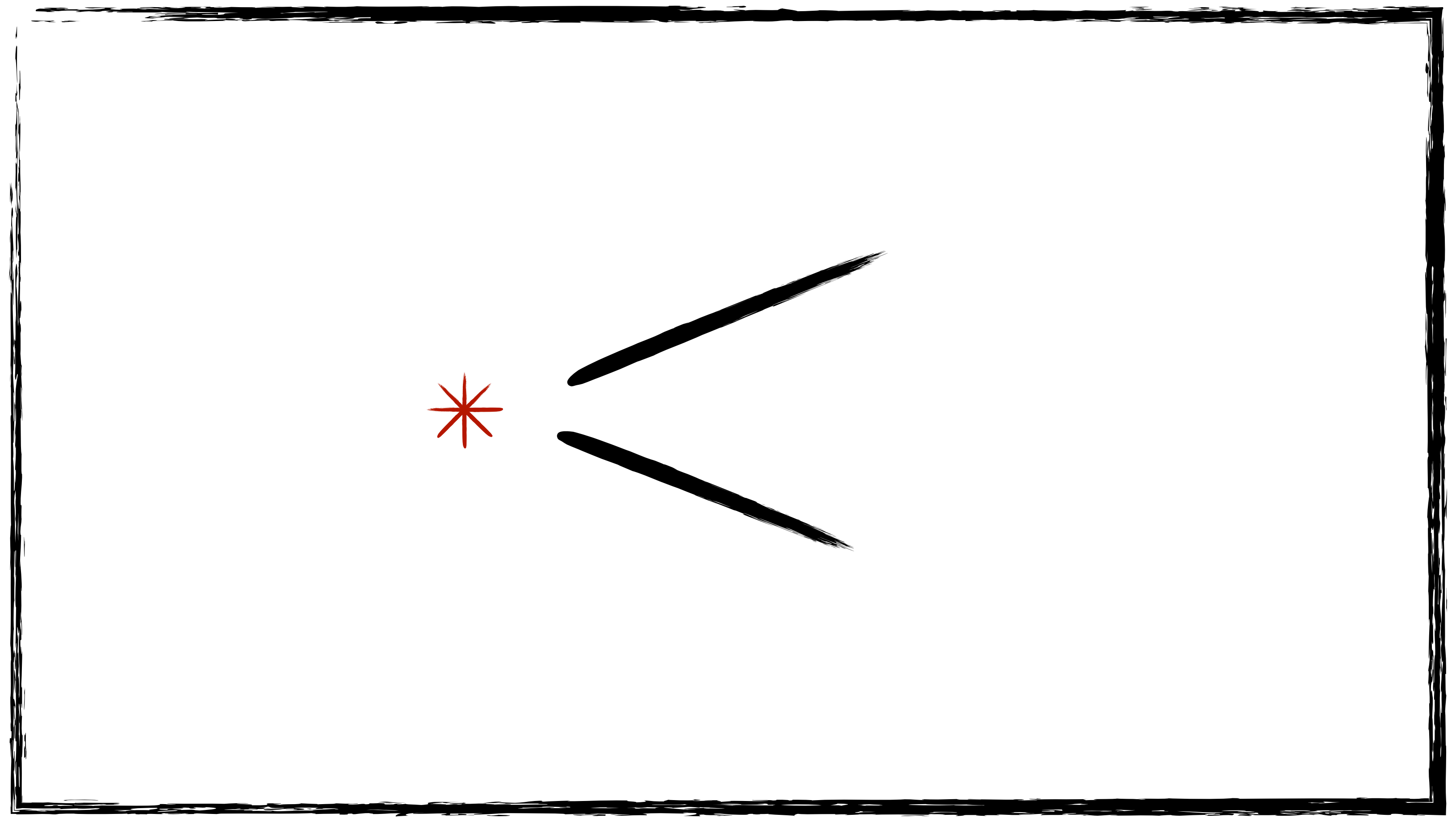
2) Higher-order correlators start to dominate the dynamics (loss of coherence).



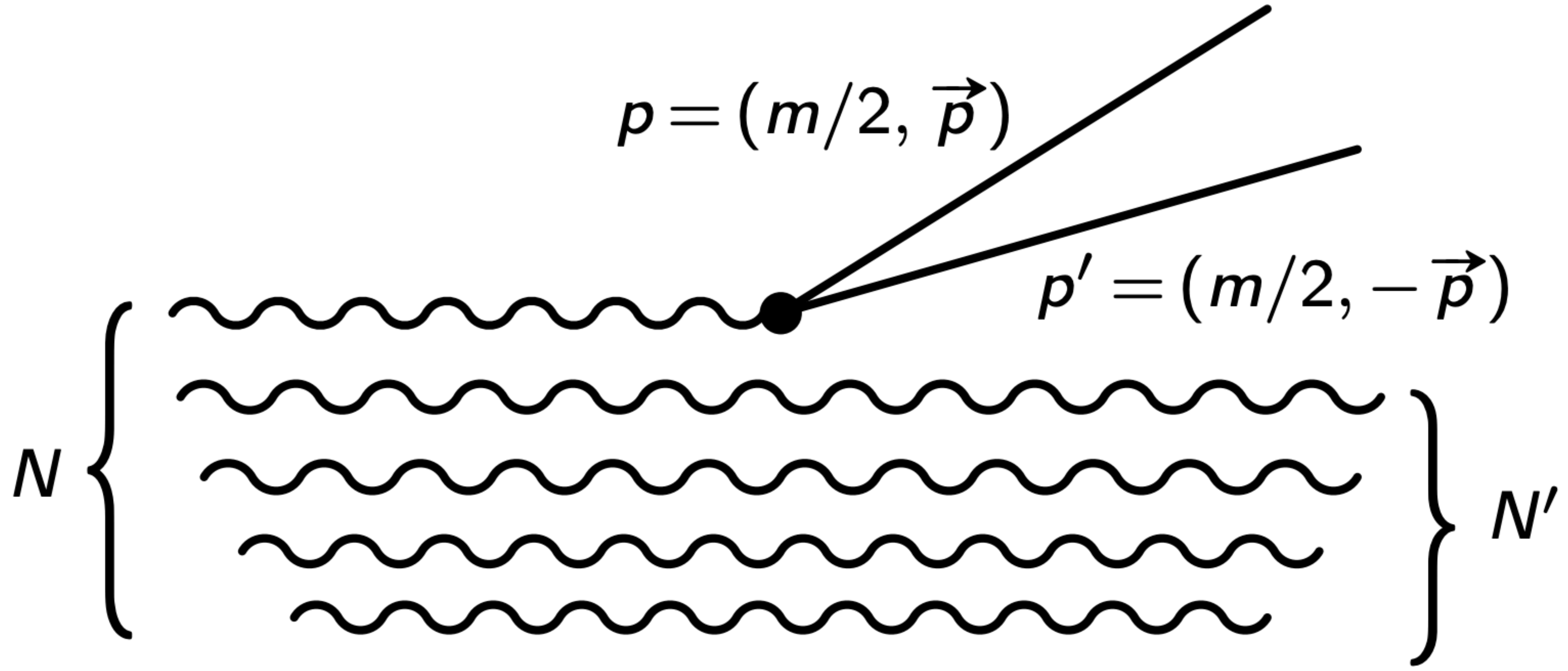
Inconsistencies arising at higher order, which you cannot see in a semiclassical analysis

$$\hat{\phi}(t > t_{qb}) = ?$$

Going beyond *the semiclassical approximation means to 'resolve' the structure of the 'vacuum'*



Background: Vacuum



Background: Coherent state of zero momentum of-shell particles built on the true vacuum

(Fig. from Dvali, Gomez, Zell, arXiv:1701.08776)

Example: Perturbations in de Sitter are constituents of the ground state that annihilate into excited particles. *(Depletion of de Sitter background)*

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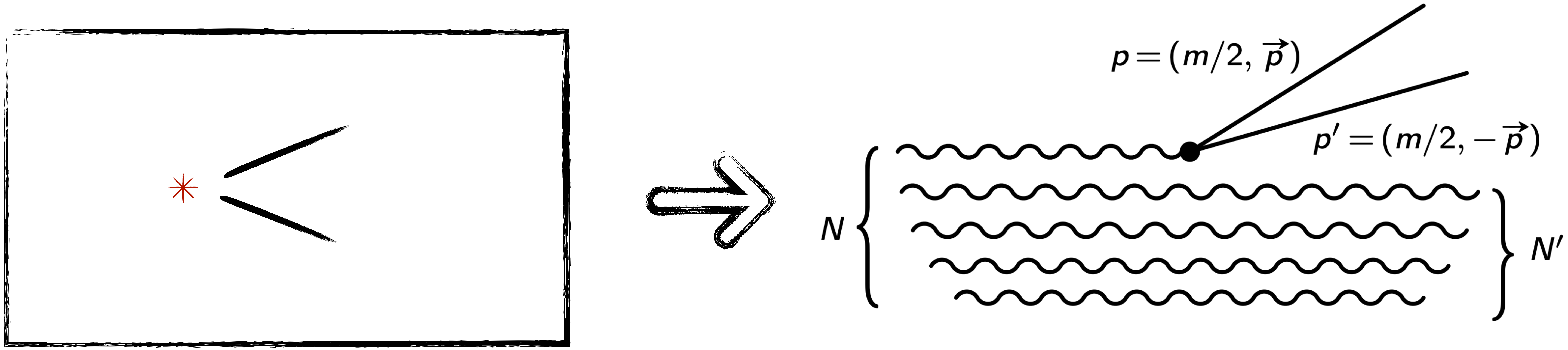

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Estimates obtained in S-matrix: Can we rigorously compute them?

$$|C\rangle = e^{-i\int d^3x\left(\phi_0\hat{\pi}(x,t_0)+\pi_0\hat{\phi}(x,t_0)\right)}|\Omega\rangle$$

'Def': State of Minimal indetermination and Eigenstate of the annihilation operator

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Why Coherent States:

1) Tool to understand quantum corrections to classical dynamics
2) They provide a full set of initial conditions for our system.

C-numbers

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interacting vacuum of the theory

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Initial conditions from the state:

$$\langle C|\hat{\phi}(x,0)|C\rangle = \phi_0 \qquad \langle C|\hat{\phi}(x,0)\hat{\phi}(y,0)|C\rangle = \phi_0^2 + \langle\Omega|\hat{\phi}(x,0)\hat{\phi}(y,0)|\Omega\rangle$$

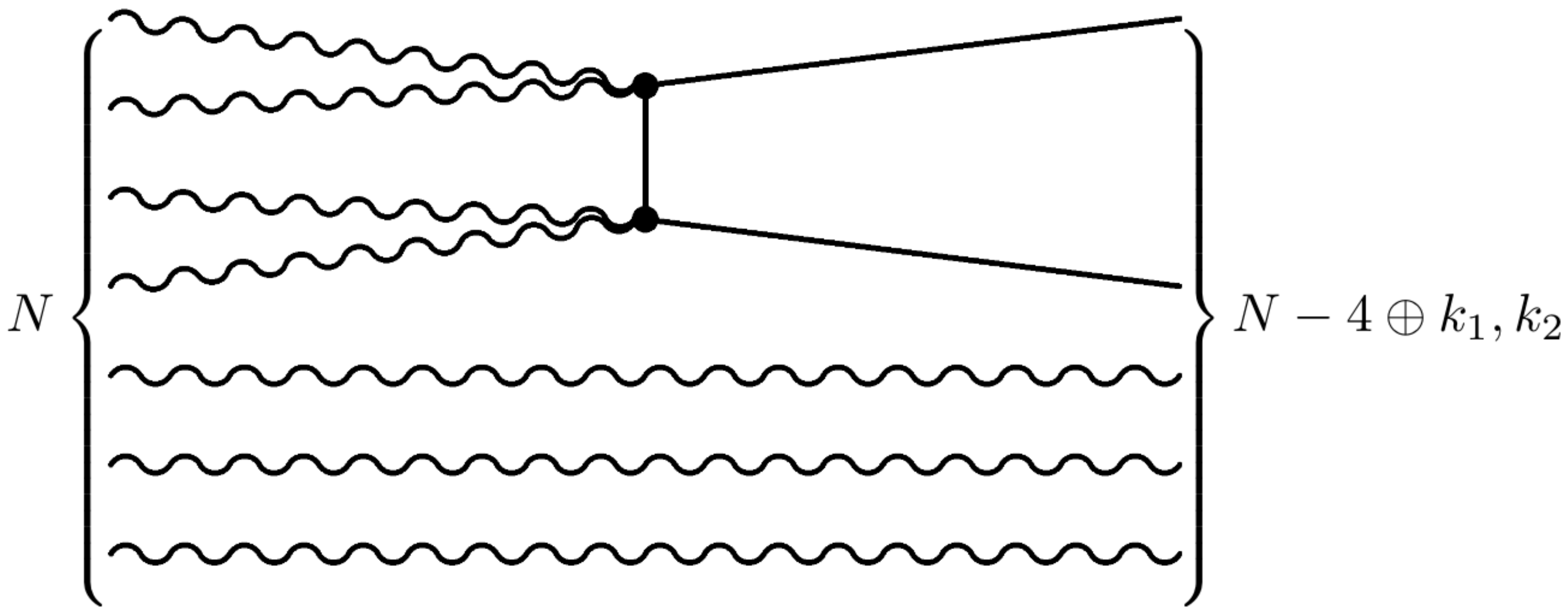
$$\langle C|\hat{\pi}(x,0)|C\rangle = \pi_0 \qquad \langle C|\hat{\pi}(x,0)\hat{\pi}(y,0)|C\rangle = \pi_0^2 + \langle\Omega|\hat{\pi}(x,0)\hat{\pi}(y,0)|\Omega\rangle$$

$|C\rangle$ is a Condensate of Scalar Bosons with n≥2 n-point functions *initialized in the interacting vacuum.*

Message: by defining the state, we have a clear view of quantum effects and their consistency

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4 \quad + \quad |C\rangle = e^{-i\int d^3x\phi_0\hat{\pi}(x,t_0)}|\Omega\rangle$$

An example of a theory where we know what processes between constituents we should expect.



1) Dynamics of the State (Back-reaction)

2) Consistency of the State

1) Dynamics: back-reaction as depletion of the Coherent state

$$\langle C|\hat{\phi}(x,t)|C\rangle = \Phi(t)$$

Back-ground field method to compute the time-evolution:

$$\Box\hat{\phi} + m^2\hat{\phi} + \frac{\lambda}{3!}\hat{\phi}^3 = 0 \quad + \quad \hat{\phi} = \Phi + \hat{\psi}, \quad \langle C|\hat{\psi}(x,t)|C\rangle = 0$$

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$$\begin{cases} \ddot{\Phi} + m^2\Phi + \frac{\lambda}{3!}\Phi^3 + \underbrace{\frac{\lambda}{2}\langle C|\hat{\psi}(x,t)^2|C\rangle}_{\text{Quantum source}} + \mathcal{O}(\hbar^2) = 0 \\ \left(\Box_{x,t} + m^2 + \underbrace{\frac{\lambda}{2}\Phi(t)^2}_{\text{Classical solution}}\right)\langle C|\hat{\psi}(x,t)\hat{\psi}(t,y')|C\rangle + \mathcal{O}(\hbar^2) = 0 \end{cases}$$

The state gives the initial conditions to solve the system

We solve the second equation perturbatively in λ and plug in the first equation

$$\begin{cases} \left(\partial_t^2 + m_{\rm ph}^2\right)\Phi(t) + \frac{\lambda}{3!}\Phi^3(t) = \underbrace{\lambda^2\Phi(t)S_1(t)}_{\text{Quantum } \lambda^2 \text{ source}} + \underbrace{\lambda^3\Phi(t)S_2(t)}_{\text{Quantum } \lambda^3 \text{ source}} + \underbrace{\lambda^4\Phi(t)S_3(t)}_{\text{Quantum } \lambda^4 \text{ source}} + \dots \\ S_n(t_0) = \int_0^{t_0} dt_1 \dots \int_0^{t_{n-1}} dt_n \Phi^2(t_1)\dots\Phi^2(t_n) D_n(t, t_1, \dots, t_n) \end{cases}$$

1) 1-point and 2-point functions contain all $\hbar$ the information on the system

2) Semiclassical modes gives the 1-loop back reaction

3) Re-sum Classical non-linearities but not quantum non-linearities

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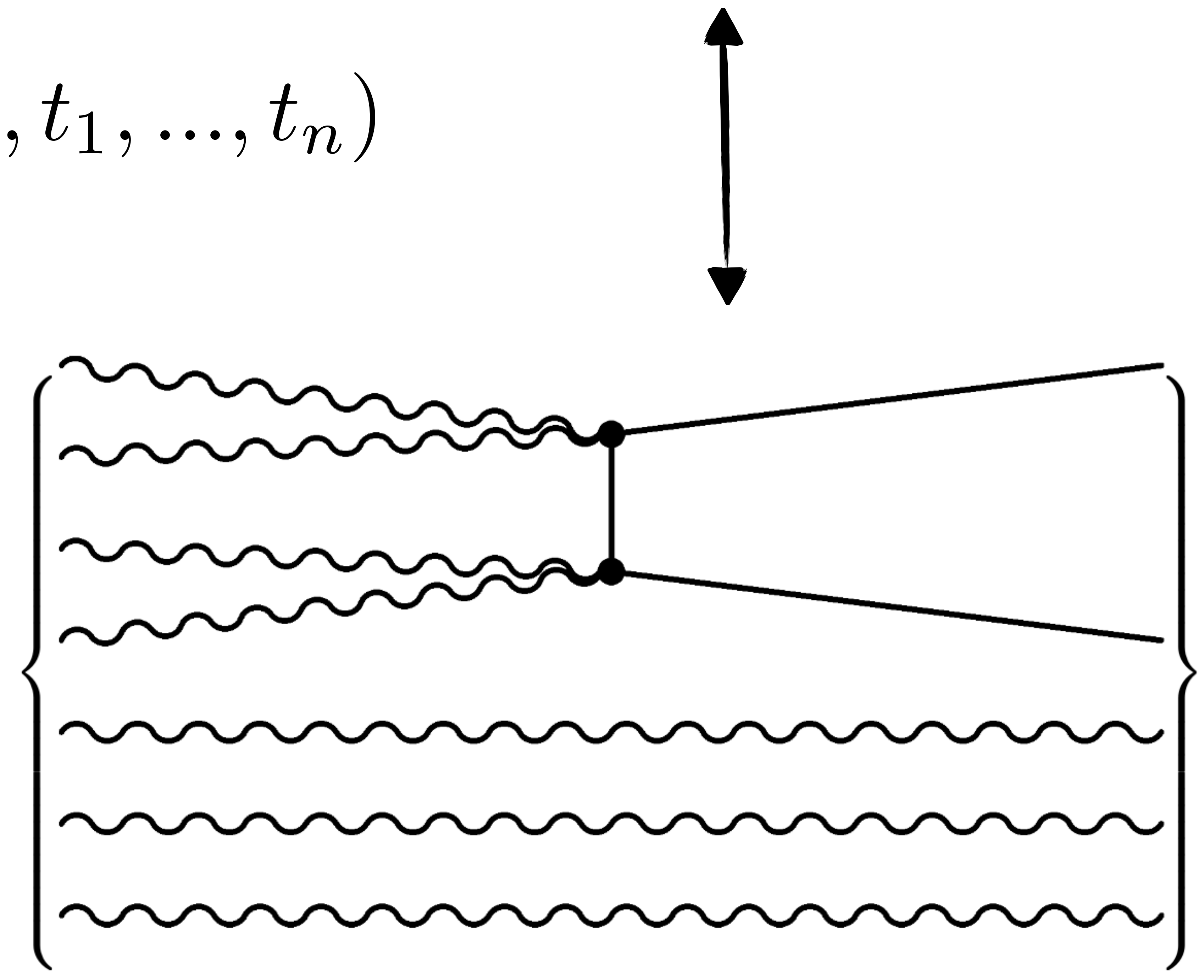

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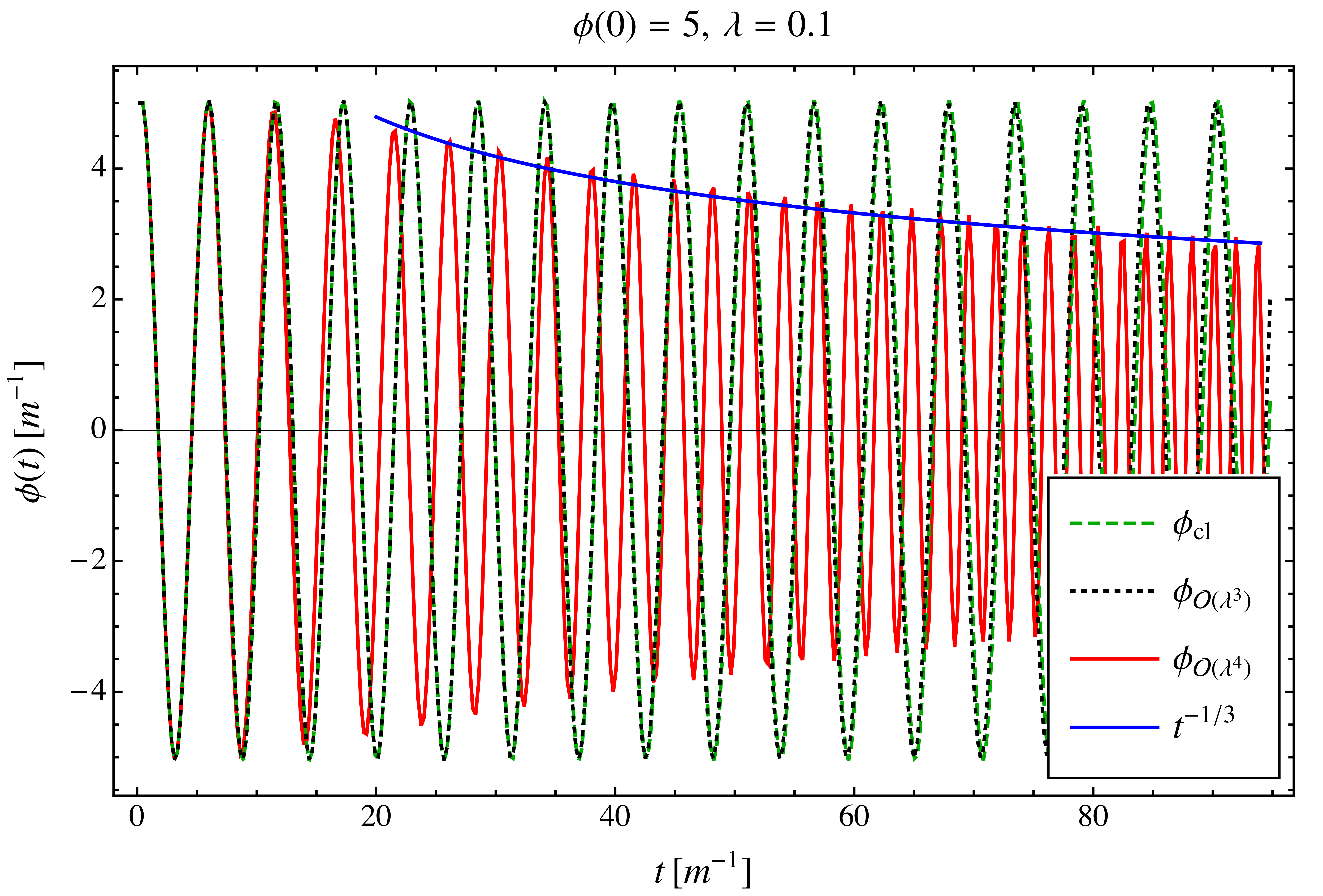
$$\Gamma \sim \frac{\lambda^4 \phi_0^6}{m^5}\hbar$$



$\phi(0) = 5,\ \lambda = 0.1$
$\phi(t)\ [m^{-1}]$
$t\ [m^{-1}]$
4
2
0
−2
−4
0
20
40
60
80
$\phi_{\rm cl}$
$\phi_{\mathcal{O}(\lambda^3)}$
$\phi_{\mathcal{O}(\lambda^4)}$
$t^{-1/3}$

2.a) Consistency of the state from Initial conditions

$$|C\rangle = e^{-i\int d^3x\left(\phi_0\hat{\pi}(x,t_0)+\pi_0\hat{\phi}(x,t_0)\right)}|\Omega\rangle$$

$$\langle C|\hat{\phi}(x,t_0)|C\rangle = \phi_0(x)$$

finite

$$\langle C|\hat{\pi}(x,t_0)|C\rangle = \pi_0(x)$$

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$$\langle C|\hat{H}|C\rangle = \int d^3x\, Z\left[\frac{1}{2}\dot{\phi}_{cl}^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}\left(m^2 + \frac{\lambda Z}{2}\langle\Omega|\hat{\phi}^2|\Omega\rangle\right) + \frac{\lambda Z}{4!}\phi_{cl}^4\right]$$

Strongly depend on the state

(Berezhiani, Zantedeschi, arXiv:2011.11229)

1) At 1-loop, energy is finite only if the bare coupling is finite: no ***logΛ*** divergence.
2) Standard perturbative prescriptions are inadequate to renormalize the theory
3) The total energy is a conserved quantity, statements at all time.

2.b) Consistency of the state from time-evolution

The perturbative dynamics has divergences:

$$\ddot{\Phi} + m^2\Phi + \frac{\lambda}{3!}\Phi^3 + \frac{\lambda}{2}\langle C|\hat{\psi}(x,t)^2|C\rangle + \mathcal{O}(\hbar^2) = 0$$

Divergent: logΛ and Λ^2 terms

logΛ divergence vanishes at t=0, Λ^2 is mass renormalization

(Similar to the energy: bare coupling as to be finite, mass needs to be renormalized, *in t=0*)

but, we have to renormalize to have a finite evolution: S-matrix prescriptions for t>0 and t<0:

$$\lambda_{\text{ph}} = \lambda - 3\lambda^2 \int \frac{d^3p}{(2\pi)^3(2E_p)^3}$$

Renormalization moves some divergences in the field acceleration at t=0

1)The one-point function is finite at t=0:

$$\Phi(0) = \phi_{cl}$$ ✓

2) The second time derivative of the 1-point function is divergent at t=0:

$$\lim_{t\to 0} \ddot{\Phi} \sim \log mt$$ ✗

3) The total energy (conserved quantity) inherits the same divergences:

$$\langle C|\hat{H}|C\rangle$$ ✗

because we had to 'remove' divergences which were not there

Implications for the state?

All perturbative divergences re-sum in a full quantum computation **OR** e have to dismiss 'unsqueezed' coherent states as members of a physical Hilbert space

Conclusions

Coherent states are an interesting tool to parametrize highly occupied systems.

- We found that within a quartic theory, the one-point function of the field operator is depleted according to S-matrix arguments.

- We discussed possible pathologies of the state itself that emerges at $\hbar$

Next Steps?

- Choose different Coherent states as the initial state of the system:

$$|C\rangle = e^{-i\int d^3x\phi_0\hat{\pi}(x,t_0)+\frac{1}{2}\int d^3x d^3y \hat{\pi}(x,t_0)G(x-y)\hat{\pi}(y,t_0)}|\Omega\rangle$$

- Apply to Cosmological Systems. De Sitter? Black Holes?

Comments on higher order corrections

1) The problem with divergences at initial times is ‘enhanced’ at 2-loop loops: 'sunset diagram’ divergence vanishes at t=0. Mass renormalization becomes problematic too.

$$\langle C|\hat{H}|C\rangle = \int d^3x\, Z\left[\frac{1}{2}\dot{\phi}_{cl}^2 + \frac{1}{2}\left(\nabla\phi\right)^2 + \frac{1}{2}\left(m^2 + \frac{\lambda Z}{2}\langle\Omega|\hat{\phi}^2|\Omega\rangle\right) + \frac{\lambda Z}{4!}\phi_{cl}^4\right]$$

2) At $\hbar^2$, equation for the two-point function acquires ‘non-trivial’ corrections:

$$\left(-\Box_{x,t} + m^2 + \underbrace{\frac{\lambda}{2}\Phi^2(t)}_{\text{1-loop one-point function}}\right)\langle C|\hat{\psi}(x,t)\hat{\psi}(y,t')|C\rangle$$

1-loop one-point function

$$\underbrace{+\frac{\lambda}{2}\Phi(t)\langle C|\hat{\psi}^2(x,t)\hat{\psi}(y,t')|C\rangle + \frac{\lambda}{3!}\Phi(t)\langle C|\hat{\psi}^3(x,t)\hat{\psi}(y,t')|C\rangle}_{\text{Non-trivial evolution due to modes interactions}} = 0$$

Non-trivial evolution due to modes interactions