One-Loop Calculation of the Charged Wino Decay Rate

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Work in progress

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Backgournd; Tree-level calculation

□ Pure Wino case

- ✓ Main channel: $\widetilde{w}^- \rightarrow \widetilde{w}^0 + \pi^-$
- ✓ Wino-Pion interaction: charged current interaction

$$\mathcal{L}_{\rm int} = -2\sqrt{2} f_\pi G_F V_{ud} D_\mu \pi^- \times \bar{\psi}_- \gamma^\mu \psi_0 + \text{h.c.}$$

$$\Rightarrow \quad \Gamma_{\rm tree} = \frac{4}{\pi} f_\pi^2 G_F^2 V_{ud}^2 \delta m^3 \left(1 - \frac{m_\pi^2}{\delta m^2}\right)^{1/2}$$

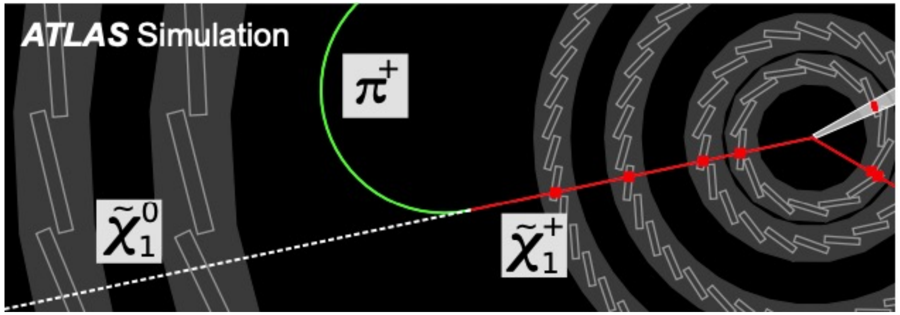
□ Mass difference ($m_{\widetilde{w}^-} - m_{\widetilde{w}^0}$) from EW correction

$$\delta m \simeq 165 \text{ MeV} \Rightarrow \tau_{\rm tree} \simeq 0.2 \text{ ns} \simeq 6 \text{ cm}$$

- ✓ Long-lived -> disappearing charged track search

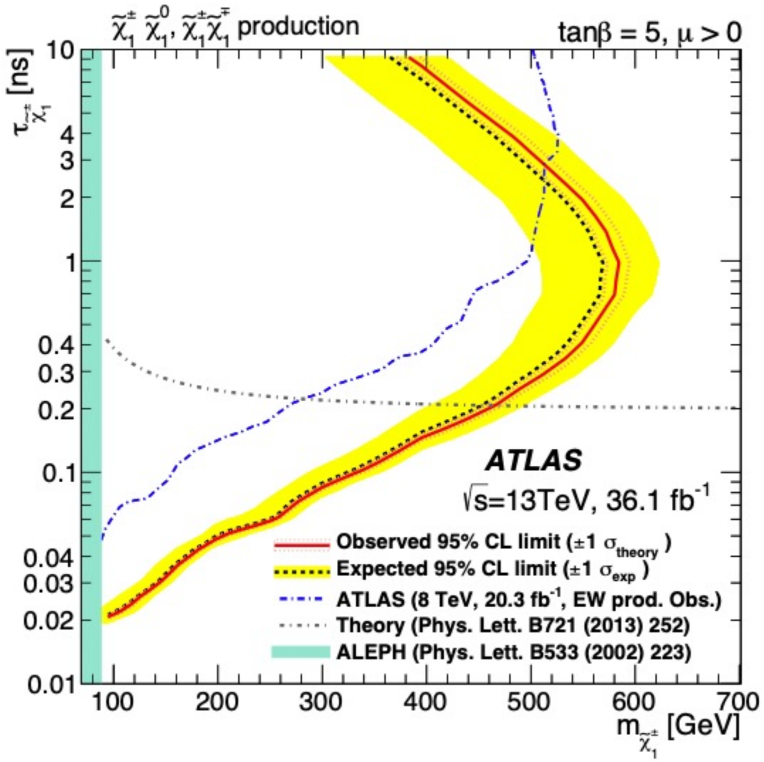
Background; Experiment

- Constraint on Wino mass is strongly dependent on the charged wino decay rate



The ATLAS collaboration, JHEP06 (2018)022
Search for the disappearing track from $\hat{\chi}^+ \to \hat{\chi}^0 + \pi^+$

$$\Gamma_{\rm tree} = \frac{4}{\pi} f_\pi^2 G_F^2 V_{ud}^2 \delta m^3 \left(1 - \frac{m_\pi}{\delta m}\right)^{1/2} \simeq 0.2\ {\rm ns}$$



- Question
  - ✓ How large does loop correction contribute?
    - ➢ suppressed by δm^3 as well?
    - ➢ large contribution like $\log m_{\tilde{W}}/m_\pi$?

Method; ChPT & 2-Step matching

- □ Decay process involved with pion -> ChPT
  - ✓ To be predictive @1-loop, matching to UV theories is necessary

EW theory w/ wino

$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{quark} + \mathcal{L}_{wino}$

↕ match with free-quark decay rate $\Gamma(\widetilde{w}^{-} \to \widetilde{w}^{0} + \bar{u} + d)$@1-loop

Four-Fermi theory w/ wino

$\mathcal{L} = \mathcal{L}_{quark-kin} + \mathcal{L}_{wino-kin} + \mathcal{L}_{4Fermi} + \mathcal{L}_{C.T.}$

↕ match with spurion correlator (later) @1-loop

ChPT w/ wino

$\mathcal{L} = \mathcal{L}_{wino-kin} + \mathcal{L}_{p^2} + \underline{\mathcal{L}_{C.T.}}$

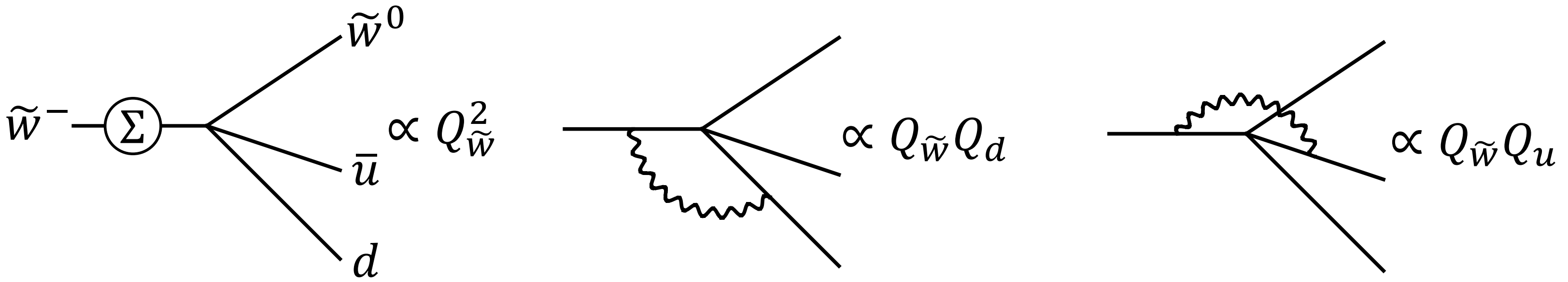
Already calculated →
- ✓ Loop correction w/o photon ($\mathcal{L}_L$, later)
- ✓ Loop correction w/ photon and w/o wino ($\mathcal{L}_K$)

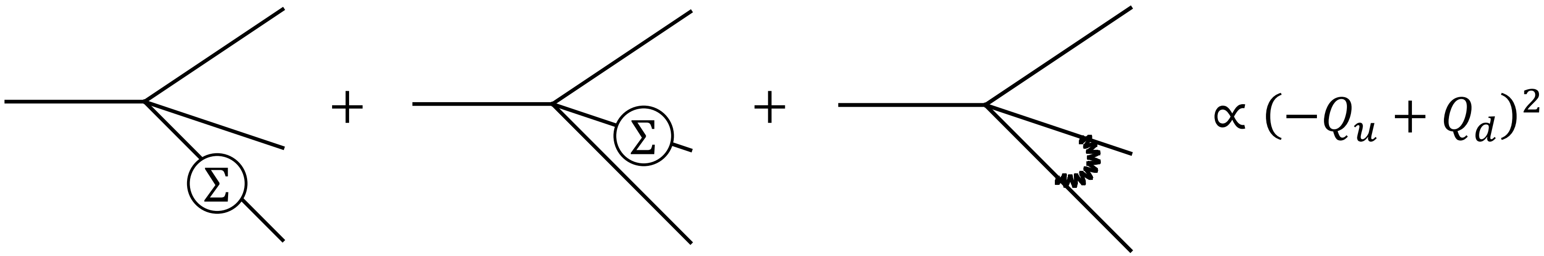
This work →
- ✓ Loop correction w/ photon and wino ($\mathcal{L}_Y$)

Matching; EW theory and Four-Fermi

□ Free-quark decay @1-loop (Four-Fermi)

- ✓ Decay rate as a function of charges; $Q_{\tilde{w}}, Q_u, Q_d$



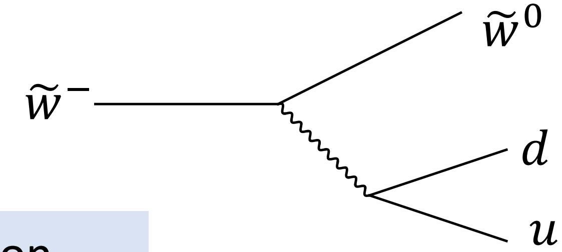


□ Counterterms

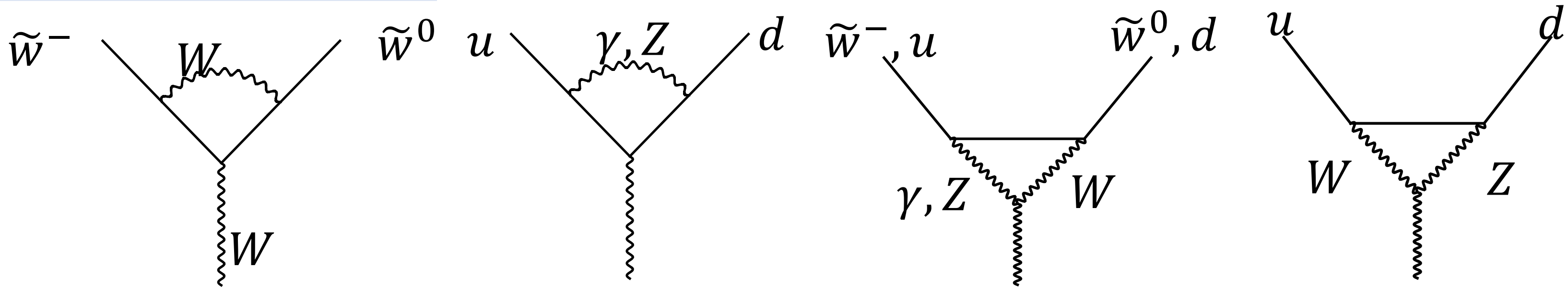
$$\mathcal{L}_{\rm C.T.} = -2G_F(\bar{u}\gamma^\mu(1-\gamma_5)d\ \bar{\psi}_-\gamma_\mu\psi_0 + h.c.)$$
$$\times e^2[f_{\tilde{w}\tilde{w}}Q_{\tilde{w}}^2 + f_{ud}(-Q_u+Q_d)^2 + f_{\tilde{w}u}Q_{\tilde{w}}Q_u + f_{\tilde{w}d}Q_{\tilde{w}}Q_d]$$

Matching; EW theory and Four-Fermi

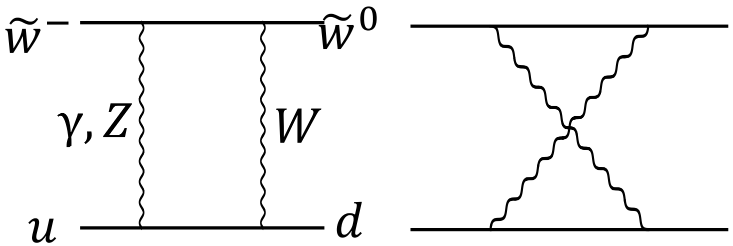
□ Free-quark decay @1-loop (EW theory)



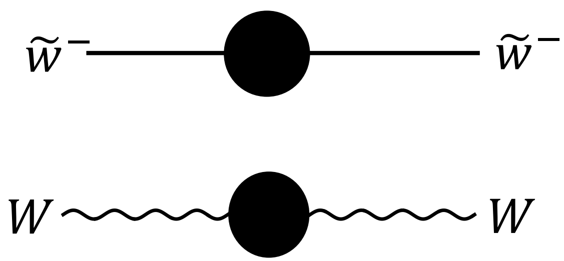
Vertex correction



Box diagrams



Wave function renormalization



□ Compare with Four-Fermi result

⇒Determine f_{ii}'s (coeff's of counterterms)

Matching; Four-Fermi and ChPT

- Replace EM and weak charge matrices of quarks by spurion sources $\mathbf{q}_i$.

  ✓ e.g. $\bar{q} Q_q \not{A} q \to \bar{q}_L \mathbf{q}_L \not{A} q_L + \bar{q}_R \mathbf{q}_R \not{A} q_R$

  $\bar{q}_L Q_W \gamma_\mu q_L \times J^\mu_{\tilde{w}} \to \bar{q}_L \mathbf{q}_W \gamma_\mu q_L \times J^\mu_{\tilde{w}}$

- Counter terms
  - ✓ w/o photon $\mathcal{L}_L = L_1 \times (\text{pion term}) + \cdots$
  - ✓ w/ photon w/o wino $\mathcal{L}_K = K_1 \times (\text{pion \& spurion}) + \cdots$
  - ✓ w/ photon & wino $\mathcal{L}_Y = Y_1 \times (\text{pion, spurion \& wino}) + \cdots$

⇒Matching to determine Y_i $(i = 1 \ldots 6)$

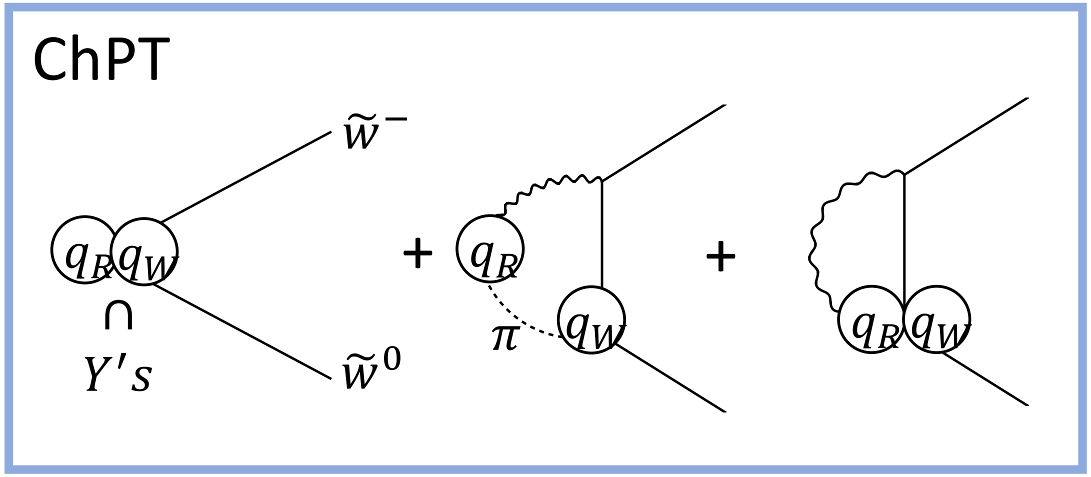
Matching; Four-Fermi and ChPT

□ Four-Fermi theory rewritten with spurions

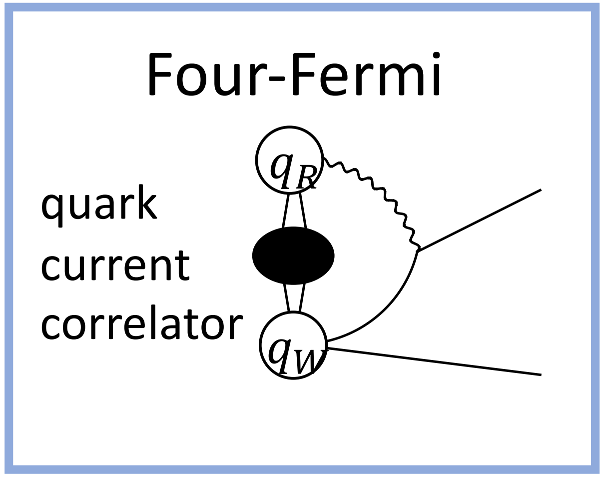
$$\mathcal{L} \supset \bar{q}_L i(\partial\!\!\!/ - ie\mathbf{q}_L A\!\!\!/)q_L + \bar{q}_R i(\partial\!\!\!/ - ie\mathbf{q}_R A\!\!\!/)q_R$$
$$+ \frac{1}{2}G_F(\bar{q}_L \mathbf{q}_W \gamma^\mu q_L \times 2\sqrt{2}\bar{\psi}_- \gamma_\mu \psi_0 + \mathrm{h.c.})$$

□ Spurion correlators

✓ e.g. $\int \mathrm{d}^4x\, e^{irx} \langle \psi_-(p)\bar{\psi}_0(q) | \frac{\delta^2}{\delta \mathbf{q}_R^a(x) \delta \mathbf{q}_W^b(0)} e^{i\int \mathcal{L}(\mathbf{q})} |0\rangle$



$\Leftrightarrow$



$\Rightarrow$ determine $Y's$

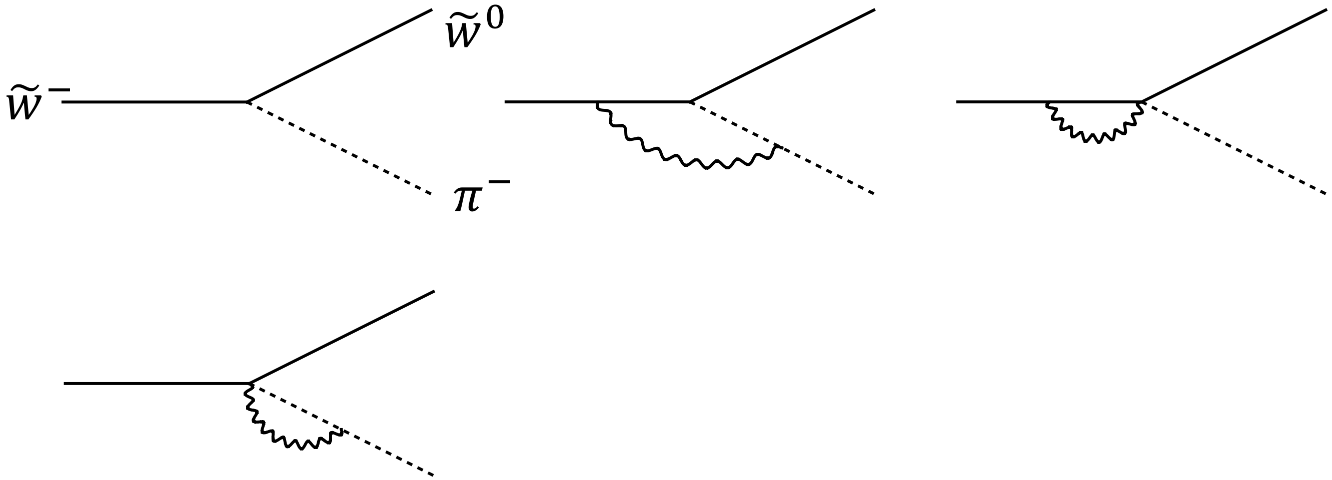
In this way matching is completed !

Decay Rate; Virtual and Real Photon correction

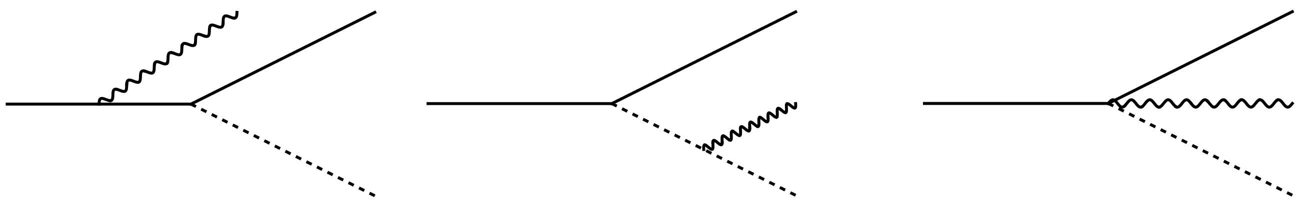
□ Decay rate @1-loop -> IR divergence

⇒Evaluate $\Gamma(\tilde{w}^- \to \tilde{w}^0 \pi^- (\gamma))$

Virtual



Real



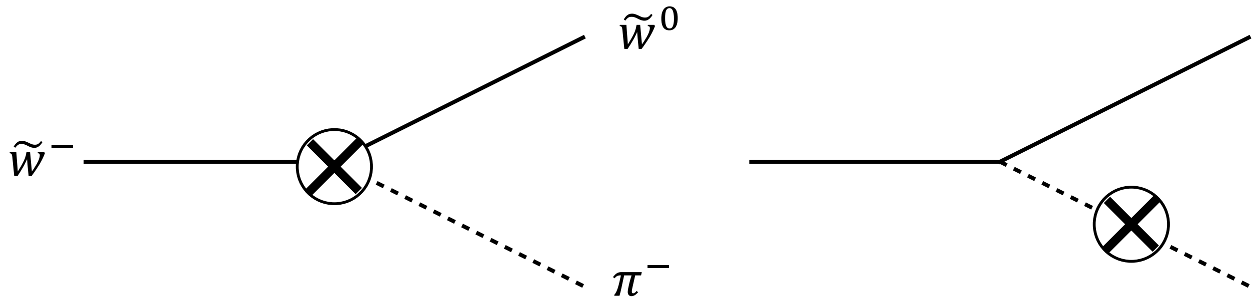
⇒ Checked with existing result (e.g. τ lepton decay)

Decay Rate; Counterterm contributions

□ Counterterm contributes to decay rate

$$\mathcal{L}_K \supset \frac{4}{9}e^2[6K_1 + 6K_2 + 5(K_5 + K_6)]\underline{|D_\mu \pi|^2}$$
$$- \frac{4\sqrt{2}}{9}e^2 F_0 G_F V_{ud}[12K_1 + 9K_{12} + 2(6K_2 + 5K_5 + 5K_6)]\underline{(D_\mu \pi)\bar{\psi}_- \gamma^\mu \psi_0} + h.c.$$
$$\mathcal{L}_Y \supset \frac{4\sqrt{2}}{3}e^2 F_0 G_F V_{ud}[Y_1 + \hat{Y}_1 + 3(Y_2 + \hat{Y}_2)]\underline{(D_\mu \pi)\bar{\psi}_- \gamma_\mu \psi_0}$$
$$+ i4\sqrt{2}e^2 Y_3 F_0 G_F V_{ud} m_{\tilde{w}} \underline{\pi \bar{\psi}_- \psi_0} + h.c.$$

✓ Contribute to decay rate via vertex & wavefnct. correction



Result(Current status)

□ We have finished almost all calculation

✓ Calculate $\Gamma\left(\widetilde{w}^- \to \widetilde{w}^0 \pi^-(\gamma)\right)$ with UV & IR convergent result.

✓ No large correction like

➢ $m_{\widetilde{w}}/\delta m$

➢ $\log m_{\widetilde{w}}/m_\pi$ <- To be checked more carefully

✓ Non-leading parts of some counterterms are left.

□ It is expected that $\frac{\delta\Gamma}{\Gamma_{\text{tree}}} = \mathcal{O}(1)\%$

□ Please wait for final result…

Backup

Chiral Perturbation Theory (ChPT)

□ Leading term

$$\mathcal{L}_{p^2} \supset \frac{f_\pi^2}{2} \mathrm{Tr}(u_\mu u^\mu)$$

$$u_\mu = i[u_R^\dagger(\partial_\mu - ir_\mu)u_R - u_L^\dagger(\partial_\mu - il_\mu)u_L]$$

$$r_\mu = v_\mu + a_\mu - eQ_R A_\mu$$

$$l_\mu = v_\mu - a_\mu - eQ_L A_\mu + G_F[Q_W J_{\tilde{w}} + Q_W^\dagger J_{\tilde{w}}^\dagger]$$

-> the current interaction

Counterterm: $\mathcal{L}_K$

$$\mathcal{L}_K = e^2 {F_0}^2 \Big\{ \frac{1}{2} K_1 \, \langle (\mathcal{Q}_L)^2 + (\mathcal{Q}_R)^2 \rangle \langle u_\mu u^\mu \rangle + K_2 \, \langle \mathcal{Q}_L \mathcal{Q}_R \rangle \langle u_\mu u^\mu \rangle$$

$$- K_3 \, [\langle \mathcal{Q}_L u_\mu \rangle \langle \mathcal{Q}_L u^\mu \rangle + \langle \mathcal{Q}_R u_\mu \rangle \langle \mathcal{Q}_R u^\mu \rangle] + K_4 \, \langle \mathcal{Q}_L u_\mu \rangle \, \langle \mathcal{Q}_R u^\mu \rangle$$

$$+ K_5 \, \langle [(\mathcal{Q}_L)^2 + (\mathcal{Q}_R)^2] u_\mu u^\mu \rangle + K_6 \, \langle (\mathcal{Q}_L \mathcal{Q}_R + \mathcal{Q}_R \mathcal{Q}_L) u_\mu u^\mu \rangle$$

$$+ \frac{1}{2} K_7 \, \langle (\mathcal{Q}_L)^2 + (\mathcal{Q}_R)^2 \rangle \, \langle \chi_+ \rangle + K_8 \, \langle \mathcal{Q}_L \mathcal{Q}_R \rangle \, \langle \chi_+ \rangle$$

$$+ K_9 \, \langle [(\mathcal{Q}_L)^2 + (\mathcal{Q}_R)^2] \chi_+ \rangle + K_{10} \, \langle (\mathcal{Q}_L \mathcal{Q}_R + \mathcal{Q}_R \mathcal{Q}_L) \chi_+ \rangle$$

$$- K_{11} \, \langle (\mathcal{Q}_L \mathcal{Q}_R - \mathcal{Q}_R \mathcal{Q}_L) \chi_- \rangle$$

$$- i K_{12} \, \langle [(\mathcal{Q}_{L\mu} \mathcal{Q}_L - \mathcal{Q}_L \mathcal{Q}_{L\mu}) - (\mathcal{Q}_{R\mu} \mathcal{Q}_R - \mathcal{Q}_R \mathcal{Q}_{R\mu}) u^\mu \rangle$$

$$+ K_{13} \, \langle \mathcal{Q}_{L\mu} \mathcal{Q}_R^\mu \rangle + K_{14} \, \langle (\mathcal{Q}_{L\mu} \mathcal{Q}_L^\mu) + (\mathcal{Q}_{R\mu} \mathcal{Q}_R^\mu) \rangle \Big\} \; .$$

Counterterm: $\mathcal{L}_Y$

$$\begin{aligned}\mathcal{L}_{\mathrm{Y}} = e^2\Big\{\sqrt{2}F_0^2 G_F\Big[&Y_1\bar{\psi}_-\gamma_\mu\psi_0\langle u^\mu\{\mathcal{Q}_R,\mathcal{Q}_W\}\rangle + \hat{Y}_1\bar{\psi}_-\gamma_\mu\psi_0\langle u^\mu\{\mathcal{Q}_L,\mathcal{Q}_W\}\rangle\\
&+ Y_2\bar{\psi}_0\gamma_\mu\psi_0\langle u^\mu[\mathcal{Q}_R,\mathcal{Q}_W]\rangle + \hat{Y}_2\bar{\psi}_-\gamma_\mu\psi_0\langle u^\mu[\mathcal{Q}_L,\mathcal{Q}_W]\rangle\\
&+ Y_3 m_{\tilde{w}}\bar{\psi}_-\psi_-\langle\mathcal{Q}_R\mathcal{Q}_W\rangle\\
&+ iY_4\bar{\psi}_-\gamma_\mu\psi_0\langle\mathcal{Q}_L^\mu\mathcal{Q}_W\rangle + iY_5\bar{\psi}_-\gamma_\mu\psi_0\langle\mathcal{Q}_R^\mu\mathcal{Q}_W\rangle + h.c.\Big]\\
&+ \hat{Y}_6\bar{\psi}_-(i\partial\!\!\!/ - eA\!\!\!/)\psi_- + \hat{Y}_7 m_{\tilde{w}}\bar{\psi}_-\psi_-\Big\}\ .\end{aligned}$$

Four-Fermi counteterm w/ spurion

$$\mathcal{L}_{\text{C.T.}} = -2G_F(\bar{u}\gamma^\mu(1-\gamma_5)d\,\bar{\psi}_-\gamma_\mu\psi_0 + h.c.)$$
$$\times e^2[f_{\tilde{w}\tilde{w}}Q_{\tilde{w}}^2 + f_{ud}(-Q_u+Q_d)^2 + f_{\tilde{w}u}Q_{\tilde{w}}Q_u + f_{\tilde{w}d}Q_{\tilde{w}}Q_d]$$

↕

Same terms when charges are set physical value

$$\begin{aligned}\mathcal{L}_{C.T.} = &-2e^2Q_0^2 f_{00}\bar{\psi}_-(i\partial\!\!\!/ + eQ_0 A\!\!\!/ - m_{\tilde{w}})\psi_- \\ &- ie^2 f_{23}(\bar{q}_L[\mathbf{q}_L, D^\mu\mathbf{q}_L]\gamma_\mu q_L + L \leftrightarrow R) \\ &- \sqrt{2}e^2 Q_0 G_F\left[\bar{\psi}_-\gamma_\mu\psi_0 \times (f_{02}\bar{q}_L\gamma^\mu\mathbf{q}_W\mathbf{q}_L q_L + f_{03}\bar{q}_L\gamma^\mu\mathbf{q}_L\mathbf{q}_W q_L) + h.c.\right]\end{aligned}$$

Definition of Spurion correlators

- 4-Fermi theory rewritten with spurions

$$\mathcal{L} \supset \bar{q}_L i(\partial\!\!\!/ - ie\mathbf{q}_L A\!\!\!/)q_L + \bar{q}_R i(\partial\!\!\!/ - ie\mathbf{q}_R A\!\!\!/)q_R$$
$$+ \frac{1}{2}G_F(\bar{q}_L \mathbf{q}_W \gamma^\mu q_L \times 2\sqrt{2}\bar{\psi}_- \gamma_\mu \psi_0 + \text{h.c.})$$

- Spurion correlator

$$i\int \mathrm{d}^4x\, e^{irx}\, \langle \psi_-(p)\bar{\psi}_0(q)| \frac{\delta^2 W}{\delta \mathbf{q}_R^a(x)\delta \mathbf{q}_W^b(0)} |0\rangle =: \delta^{ab} G_{RW}(p,q,r)$$

$$\int \mathrm{d}^4x\, \langle \psi_-(p)\bar{\psi}_0(q)| \frac{\delta^2 W}{\delta \mathbf{q}_V^b(x)\delta \mathbf{q}_W^c(0)} |\pi^a(r)\rangle =: if^{abc}F_{VW}(p,q,r) + d^{abc}D_{VW}(p,q,r)$$

$$\int \mathrm{d}^4x\, \langle \psi_-(p)\bar{\psi}_0(q)| \frac{\delta^2 W}{\delta \mathbf{q}_A^b(x)\delta \mathbf{q}_W^c(0)} |\pi^a(r)\rangle =: if^{abc}F_{AW}(p,q,r) + d^{abc}D_{AW}(p,q,r)$$

✓ $G_{RW}, F_{V(A)W}, D_{V(A)W}$ are defined in both theories

⇒determine Y_i

Result

$$\frac{\delta\Gamma_{\tilde{w}}}{\Gamma_{\tilde{w}}} = \frac{\alpha}{4\pi}\left[3(\frac{1}{\bar{\epsilon}} + \log\frac{\mu^2}{m_\pi^2}) - 6\frac{m_{\tilde{w}}}{\delta m}\left(\frac{1}{\bar{\epsilon}} + \log\frac{\mu^2}{m_\pi^2} + \frac{4}{3}\right) - 3\log\frac{\delta m^2}{m_{\tilde{w}}^2} + f_{\tilde{w}}(z)\right]$$

$$+ e^2\left[\frac{8}{3}(K_1 + K_2) + \frac{20}{9} + 4K_{12} - \hat{Y}_6 - \frac{4}{3}(Y_1 + \hat{Y}_1) - 4(Y_2 + \hat{Y}_2) + \frac{m_{\tilde{w}}}{\delta m}Y_3\right]$$

$$z = \frac{m_\pi}{\delta m}$$

$$K_1 + K_2 = -\frac{3}{8}\left(\frac{1}{\bar{\epsilon}} + \log\frac{\mu^2}{m_\pi^2}\right) + \text{finite}$$

$$K_5 + K_6 = \frac{9}{8}\left(\frac{1}{\bar{\epsilon}} + \log\frac{\mu^2}{m_\pi^2}\right) + \text{finite}$$

$$K_{12} = -\frac{1}{24}\left(\frac{1}{\bar{\epsilon}} + \log\frac{\mu^2}{m_\pi^2}\right) + \text{finite}$$

$$Y_1 + \hat{Y}_1 = \mathcal{O}\left(m_{\tilde{w}}^{-1}\right)$$

$$Y_2 + \hat{Y}_2 = \frac{5}{64\pi^2}\left[\frac{1}{\bar{\epsilon}}\log\frac{\mu^2}{\mu_0^2} - \frac{17}{10} + \frac{1}{5}\log\frac{m_{\tilde{w}}^2}{M_V^2 + M_A^2} + f_{VW}\left(\frac{M_V}{M_A}\right)\right] - \frac{1}{4}(f_{02}^r - f_{03}^r)$$

$$Y_3 = \frac{1}{16\pi^2}\left(\frac{3}{2}\frac{1}{\bar{\epsilon}} + 2 + \frac{3}{2}\log\frac{\mu^2}{m_{\tilde{w}}^2} + \frac{1}{2}\frac{\delta m}{m_{\tilde{w}}}\log\frac{m_{\tilde{w}}^2}{4\delta m^2}\right) - \frac{1}{16\pi}\frac{1}{m_{\tilde{w}}}\frac{M_V M_A}{M_V + M_A} + ?$$

$$Y_6 = -\frac{\alpha}{4\pi}\left(\frac{1}{\bar{\epsilon}} + \log\frac{\mu^2}{m_{\tilde{w}}^2} + 4 + 2\log\frac{m_\gamma^2}{m_{\tilde{w}}^2}\right)$$

Result

$$
\begin{aligned}
f_{\tilde{w}}(z) =& 5 + \frac{5}{2}\log z - 2\log\left[4(1-z^2)\right] \\
&+ \frac{1}{\sqrt{1-z^2}}\Bigg[-\frac{2\pi^2}{3} - \frac{1}{2}\log^2[2] + \log z + 6\log^2 z \\
&- \frac{1}{2}\log[2]\log\left[1-z^2\right] - \log[z]\log\left[4(1-z^2)\right] - \frac{1}{8}\log^2\left[1-z^2\right] \\
&+ \frac{1}{2}\left(-2 + 8\log[2] - 20\log[z] + 3\log\left[1-z^2\right]\right)\log\left[1-\sqrt{1-z^2}\right] \\
&+ \frac{7}{2}\log^2\left[1-\sqrt{1-z^2}\right] - \mathrm{Li}_2\left[\frac{1}{2} - \frac{1}{2\sqrt{1-z^2}}\right] + 3\mathrm{Li}_2\left[\frac{1-\sqrt{1-z^2}}{1+\sqrt{1-z^2}}\right]\Bigg]
\end{aligned}
$$

$$
z = \frac{m_\pi}{m_\delta}
$$

Minimal consistent resonance model

□ Calculation of quark current correlators

(S. Weinberg, Phys. Rev. Lett. **18** (1967) 507.)

$$i\int d^4x\, \mathrm{e}^{ikx}\langle 0|V^b_\sigma(x)V^c_\lambda(0) - A^b_\sigma(x)A^c_\lambda(0)|0\rangle \equiv \delta^{bc}\Pi^{\sigma\lambda}_{VV-AA}(k)$$

$$\Pi^{\rho\sigma}_{VV-AA}(k) \equiv F_0^2(k^\rho k^\sigma - k^2 g^{\rho\sigma})\,\Pi_{VV-AA}(k^2)$$

$$\Pi_{VV-AA}(k^2) = \frac{M_A^2 M_V^2}{k^2(k^2 - M_V^2)(k^2 - M_A^2)}$$

✓ Consistent with QCD asymptotic behavior

✓ Contain the effects of light resonances ρ and a_1