Minimal SU(6) Gauge-Higgs Grand Unification
or
How to Unify Gauge Interactions with Their Breaking Sector

Andrei Angelescu
Max-Planck-Institut für Kernphysik

Based on 2104.07366 [PRD] & work in progress,
in collaboration with Andreas Bally, Simone Blasi, Florian Goertz, & Sascha Weber

PASCOS 2002, Heidelberg

July 21st, 2021

Unification in Physics

Unification $\longleftrightarrow$ deeper understanding of Nature:

- Maxwell $\rightarrow$ electromagnetism
- Einstein $\rightarrow$ spacetime
- Glashow, Salam, Weinberg $\rightarrow$ Standard Model of Electroweak (EW) interactions

- Georgi, Glashow $\rightarrow$ strong + EW interactions $\rightarrow$ $SU(5)$ grand unified theory (GUT)

- Strong + EW interactions + Higgs sector? $\rightarrow$ go to 5D (warped) spacetime!

GAUGE–HIGGS GRAND UNIFICATION

$\Rightarrow A_M = (A_\mu, A_5)$, $\{G^a_\mu, W^i_\mu, B_\mu\} \subset A_\mu$, $H \subset A_5$

Unification in Physics

Unification $\longleftrightarrow$ deeper understanding of Nature:

- Maxwell $\rightarrow$ electromagnetism
- Einstein $\rightarrow$ spacetime
- Glashow, Salam, Weinberg $\rightarrow$ Standard Model of Electroweak (EW) interactions

- Georgi, Glashow $\rightarrow$ strong + EW interactions $\rightarrow$ $SU(5)$ grand unified theory (GUT)

- Strong + EW interactions + Higgs sector? $\rightarrow$ go to 5D (warped) spacetime!

GAUGE–HIGGS GRAND UNIFICATION

$\Rightarrow A_M = (A_\mu, A_5)$, $\{G^a_\mu, W^i_\mu, B_\mu\} \subset A_\mu$, $H \subset A_5$

Unification in Physics

Unification $\longleftrightarrow$ deeper understanding of Nature:

- Maxwell $\rightarrow$ electromagnetism
- Einstein $\rightarrow$ spacetime
- Glashow, Salam, Weinberg $\rightarrow$ Standard Model of Electroweak (EW) interactions

- Georgi, Glashow $\rightarrow$ strong + EW interactions $\rightarrow$ $SU(5)$ grand unified theory (GUT)

- Strong + EW interactions + Higgs sector? $\rightarrow$ go to 5D (warped) spacetime!

GAUGE–HIGGS GRAND UNIFICATION

$\Rightarrow A_M = (A_\mu, A_5)$, $\{G^a_\mu, W^i_\mu, B_\mu\} \subset A_\mu$, $H \subset A_5$

Unification in Physics

Unification $\longleftrightarrow$ deeper understanding of Nature:

- Maxwell $\rightarrow$ electromagnetism
- Einstein $\rightarrow$ spacetime
- Glashow, Salam, Weinberg $\rightarrow$ Standard Model of Electroweak (EW) interactions

- Georgi, Glashow $\rightarrow$ strong + EW interactions $\rightarrow$ $SU(5)$ grand unified theory (GUT)

- Strong + EW interactions + Higgs sector? $\rightarrow$ go to 5D (warped) spacetime!

Gauge–Higgs Grand Unification

$$\Rightarrow A_M = (A_\mu, A_5),\ \{G^a_\mu, W^i_\mu, B_\mu\} \subset A_\mu,\ H \subset A_5$$

Hierachy Problems: Gauge, Flavour, Doublet–Triplet Splitting

- UV model → Higgs + heavy VL fermions:

$$-\mathcal{L}_{\rm UV} \supset m_L^2|H|^2 + M^2(\overline{L}L + \overline{E}E) + \left(y\overline{L}HE + \text{h.c.}\right)$$

- Match on an EFT (the SM **is** an EFT!):

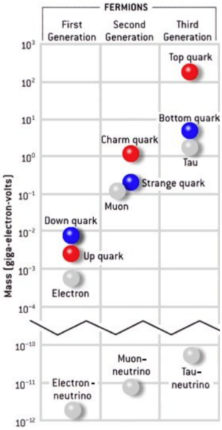
$$-\mathcal{L}_{\rm EFT} \supset m^2|H|^2$$

- One–loop matching at $\mu = M$ (avoid large logs):

$$m^2(M) = m_L^2(M) + \frac{y^2}{16\pi^2}M^2$$

- But if $\frac{y^2}{16\pi^2}M^2 \gg m^2 \Rightarrow$ very precise cancellation (fine tuning) to get $m^2 \simeq (100\,\text{GeV})^2$:

$$m_L^2 \simeq -\frac{y^2}{16\pi^2}M^2 \gg m^2$$

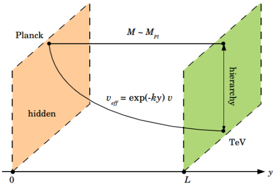


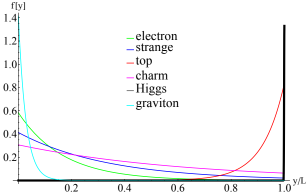
- Doublet–triplet splitting: $\mathbf{5}_{SU(5)} \to H(1,2)_{1/2} \oplus S_1(3,1)_{-1/3}$
  $\to m_h \simeq 100$ GeV but $m_{S_1} \geq 10^{12}$ GeV (proton decay) ⇒ "tree–level hierarchy problem"!

Contents

Overview

The Randall-Sundrum (RS) Model [Randall, Sundrum '99]





- Setup $\rightarrow$ two 4D worlds ("branes") situated at the endpoints of the ED:
  - $y = 0 \rightarrow$ Planck/UV/hidden brane;
  - $y = \pi R \equiv L \rightarrow$ TeV/IR/visible brane;
- Between the two branes $\rightarrow$ bulk with anti-de-Sitter (AdS_5) warped geometry:

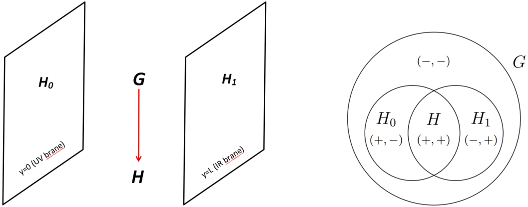
$$\mathrm{d}s^2 = \mathrm{e}^{-2ky}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}y^2 \Rightarrow g_{MN} = (\mathrm{e}^{-2ky}\eta_{\mu\nu}, -1);$$

$$k \sim M_{\mathrm{Pl}}, \quad kL \simeq \mathcal{O}(30) \Rightarrow k\mathrm{e}^{-kL} = \mathcal{O}(1 \div 10\,\mathrm{TeV});$$

- Gauge hierarchy problem solution? $\rightarrow$ Higgs localized close to/on the IR brane

$$\Lambda_{\mathrm{EW}} \sim k\mathrm{e}^{-kL} \equiv m_{KK}$$

Gauge–Higgs (Grand) Unification: General Aspects [Manton '79; Fairlie '79, Hosotani '83]

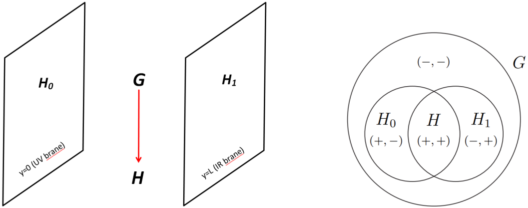


- Simple idea → Higgs embedded in 5th component of the 5D gauge field, A_5
- Gauge symmetry forbids Higgs potential at tree level → Higgs naturally light!
- Bulk gauge group G → broken on the branes through BCs:

$$y = 0: \quad G \to H_0, \qquad y = L: \quad G \to H_1$$

- At low energies → unbroken gauge group $H = H_0 \cap H_1$
- Higgs embedded into $G/H_0 \cap G/H_1$

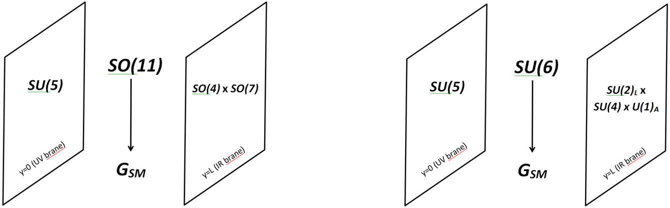
Gauge–Higgs (Grand) Unification: General Aspects



$$G \supset G_{GUT} = SU(5), SO(10), \ldots$$
$$H \equiv H_0 \cap H_1 = G_{SM}$$
$$\text{Higgs} \supset G/H_0 \cap G/H_1$$

- Unification of gauge interactions with their breaking sector
- Warped geometry explains hierarchies $\rightarrow$ gauge, flavour
- No doublet–triplet splitting problem
- Proton decay forbidden at all orders in perturbation theory!

Existing GHGUT Models



- SU(6) (mostly flat extra dimension)

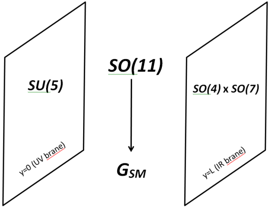
SUSY models	Hall, Nomura, Smith arXiv:0107331 Burdman, Nomura arXiv:0210257 Haba, Hosotani, Kawamura, Yamashita arXiv:0401183
non- SUSY models	Lim, Maru arXiv:0706.1397 Maru, Yagatai arXiv:1903.08359 arXiv: 1911.03465

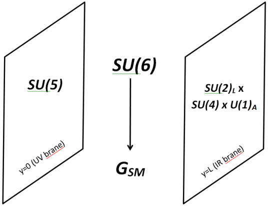
- SO(11) (warped extra dimension)

Hosotani, Yamatsu arXiv: 1504.03817

Furui, Hosotani, Yamatsu arXiv: 1606.07222

Existing GHGUT Models





Severe pheno challenges:

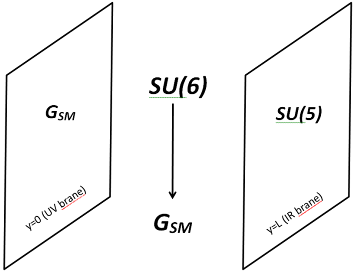
- too light exotic fermions due to large irreps of bigger symmetry
- difficult to obtain correct EWSB, m_h too low
- massless/degenerate SM fermions, no flavour

Solutions?

- go to 6D (Hosotani, Yamatsu, 1706.03503, 1710.04811)
- introduce new BSM 5D multiplets, additional brane fermions, ... (Maru, Yatagai, 1903.08359, 1911.03465)

Overview

Minimal SU(6) GHGUT [AA, Bally, Blasi, Goertz '21]



- Our approach → relax breaking pattern (and $SU(5)$ on IR brane, not on UV brane)
- No light exotics, realistic SM fermion masses + flavour
- Extra scalars besides the Higgs: $\dim(SU(6)/SU(5)) = 35 - 24 = 11$ real scalar dof's

Minimal SU(6) GHGUT: Gauge Sector

- Symmetry breaking pattern:

$$A^a_\mu T^a \sim \left(\begin{array}{ccc|cc|c} (++) & (++) & (++) & (-+) & (-+) & (--) \\ (++) & (++) & (++) & (-+) & (-+) & (--) \\ (++) & (++) & (++) & (-+) & (-+) & (--) \\ \hline (-+) & (-+) & (-+) & (++) & (++) & (--) \\ (-+) & (-+) & (-+) & (++) & (++) & (--) \\ \hline (--) & (--) & (--) & (--) & (--) & (--) \end{array}\right), \quad A^a_5 T^a : (+ \leftrightarrow -)$$

- Branching rule for $SU(6)$ adjoint:

$$\begin{aligned} \mathbf{35} \to & (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus \left[(\mathbf{3},\mathbf{2})_{-5/6} \oplus (\mathbf{3}^*,\mathbf{2})_{5/6}\right] \\ & \oplus \left[(\mathbf{3},\mathbf{1})_{-1/3} \oplus (\mathbf{3}^*,\mathbf{1})_{1/3}\right] \oplus \left[(\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2}\right] \oplus (\mathbf{1},\mathbf{1})_0 \\ & \sim G^a_\mu \oplus W^i_\mu \oplus B_\mu \oplus V_\mu(\mathbf{3},\mathbf{2})_{-5/6} \oplus S_1(\mathbf{3},\mathbf{1})_{-1/3} \oplus H \oplus s \end{aligned}$$

- Extended pNGB sector → Higgs + leptoquark (LQ) + singlet
  ⇒ no doublet–triplet splitting! ($\mathbf{5}_{SU(5)}$ = [LQ, Higgs])
- $\mathcal{O}(m_{KK})$ masses for (1st KK mode of) GUT gauge bosons $X, Y \supset V_\mu(\mathbf{3},\mathbf{2})_{-5/6}$
  ⇒ GUT without a desert! [Pomarol '00]

Minimal SU(6) GHGUT: Fermion Sector

- Fermion content for each generation:

$$\mathbf{20}_L \to \begin{cases} \mathbf{10}_L \to q'(\mathbf{3},\mathbf{2})^{+,-}_{1/6} \oplus \tilde{u}'^c(\mathbf{3}^*,\mathbf{1})^{+,-}_{-2/3} \oplus e'^c(\mathbf{1},\mathbf{1})^{+,-}_{1} \\ \mathbf{10}^*_L \to \tilde{q}^c(\mathbf{3}^*,\mathbf{2})^{+,-}_{-1/6} \oplus u(\mathbf{3},\mathbf{1})^{-,-}_{2/3} \oplus \tilde{e}(\mathbf{1},\mathbf{1})^{+,-}_{-1} \end{cases}$$

$$\mathbf{15}_L \to \begin{cases} \mathbf{10}_L \to q(\mathbf{3},\mathbf{2})^{+,+}_{1/6} \oplus \tilde{u}^c(\mathbf{3}^*,\mathbf{1})^{-,+}_{-2/3} \oplus e^c(\mathbf{1},\mathbf{1})^{+,+}_{1} \\ \mathbf{5}_L \to d'(\mathbf{3},\mathbf{1})^{-,+}_{-1/3} \oplus l'^c(\mathbf{1},\mathbf{2})^{-,+}_{1/2} \end{cases}$$

$$\mathbf{6}_L \to \begin{cases} \mathbf{5}_L \to d(\mathbf{3},\mathbf{1})^{-,-}_{-1/3} \oplus l^c(\mathbf{1},\mathbf{2})^{-,-}_{1/2} \\ \mathbf{1}_L \to \nu^c(\mathbf{1},\mathbf{1})^{+,+}_{0} \end{cases}$$

$$\mathbf{1}_L \to \nu'^c(\mathbf{1},\mathbf{1})^{+,-}_{0}$$

$$\begin{array}{lcccc} \mathbf{20} \to & \mathbf{10} & \oplus & \mathbf{10}^* & \\ & & \Updownarrow M_{q/e}, M_{\tilde{u}} & & \\ \mathbf{15} \to & \mathbf{10} & \oplus & \mathbf{5} & \\ & & & \Updownarrow M_{d/l} & \\ \mathbf{6} \to & & & \mathbf{5} & \oplus\ \mathbf{1} \\ & & & & \Updownarrow M_{\nu} \\ \mathbf{1} \to & & & & \mathbf{1} \end{array}$$

- All SM fermion masses (ν's also) reproduced with $\mathcal{O}(1)$ fermion bulk mass parameters (c's) and brane masses (M's)
- No light exotics
- All brane masses allowed by symmetry are necessary:
  + $M_{q/e}, M_{d/l}, M_{\nu}$ to get correct SM fermion masses w/o light exotics
  + $M_{\tilde{u}}$ to get correct EWSB and Higgs mass

Proton Decay: 4D GUT vs 5D GHGUT

- Ordinary 4D $SU(5)$ GUT $\rightarrow$ can write Yukawa term using $\overline{\psi}$ and ψ^c
  $\Rightarrow$ q_L and u_R^c fit in one $SU(5)$ multiplet and EWSB generates the u mass:

$$\mathbf{10}_L \rightarrow q_L(\mathbf{3},\mathbf{2})_{1/6} \oplus u_R^c(\mathbf{3}^*,\mathbf{1})_{-2/3} \oplus e_R^c(\mathbf{1},\mathbf{1})_1$$

$$\mathcal{L}_{4d} \supset i\,\overline{\mathbf{10}}_L \gamma^\mu D_\mu \mathbf{10}_L - y\,\overline{\mathbf{10}}_L \mathbf{10}_L^c \mathbf{5}_H + \text{h.c.} \supset g_{SU(5)}\,\overline{u_R^c}\gamma^\mu V_\mu q_L - y\,\bar{q}_L \tilde{H} u_R + \text{h.c.}$$

- GUT V_μ boson has diquark couplings $\Rightarrow$ proton decay!
- GHGUT $\rightarrow$ Higgs included in covariant derivative $\rightarrow$ only $\overline{\psi}H\psi$ Yukawas
  $\Rightarrow$ no EWSB mass for u if q_L and u_R^c in the same $SU(5)$ multiplet!

$$\mathbf{20}_L \rightarrow \begin{cases} \mathbf{10}_L \rightarrow q'(\mathbf{3},\mathbf{2})^{+,-}_{1/6} \oplus \tilde{u}'^c(\mathbf{3}^*,\mathbf{1})^{+,-}_{-2/3} \oplus e'^c(\mathbf{1},\mathbf{1})^{+,-}_1 \\ \mathbf{10}^*_L \rightarrow \tilde{q}^c(\mathbf{3}^*,\mathbf{2})^{+,-}_{-1/6} \oplus u(\mathbf{3},\mathbf{1})^{-,-}_{2/3} \oplus \tilde{e}(\mathbf{1},\mathbf{1})^{+,-}_{-1} \end{cases}$$

$$\mathbf{15}_L \supset \quad \mathbf{10}_L \rightarrow q(\mathbf{3},\mathbf{2})^{+,+}_{1/6} \oplus \tilde{u}^c(\mathbf{3}^*,\mathbf{1})^{-,+}_{-2/3} \oplus e^c(\mathbf{1},\mathbf{1})^{+,+}_1$$

- GUT V_μ boson and $SU(5)$ Higgs partner do not have diquark couplings
- In fact, baryon no. $\rightarrow$ accidental symmetry in GHGUT
  $\Rightarrow$ proton stable at all orders in perturbation theory!

Proton Decay: 4D GUT vs 5D GHGUT

- Ordinary 4D $SU(5)$ GUT $\to$ can write Yukawa term using $\overline{\psi}$ and ψ^c
  $\Rightarrow$ q_L and u_R^c fit in one $SU(5)$ multiplet and EWSB generates the u mass:

$$\mathbf{10}_L \to q_L(\mathbf{3},\mathbf{2})_{1/6} \oplus u_R^c(\mathbf{3}^*,\mathbf{1})_{-2/3} \oplus e_R^c(\mathbf{1},\mathbf{1})_1$$

$$\mathcal{L}_{4d} \supset i\,\overline{\mathbf{10}}_L\gamma^\mu D_\mu \mathbf{10}_L - y\,\overline{\mathbf{10}}_L\mathbf{10}_L^c\mathbf{5}_H + \text{h.c.} \supset g_{SU(5)}\,\overline{u_R^c}\gamma^\mu V_\mu q_L - y\bar{q}_L\tilde{H}u_R + \text{h.c.}$$

- GUT V_μ boson has diquark couplings $\Rightarrow$ proton decay!
- GHGUT $\to$ Higgs included in covariant derivative $\to$ only $\overline{\psi}H\psi$ Yukawas
  $\Rightarrow$ no EWSB mass for u if q_L and u_R^c in the same $SU(5)$ multiplet!

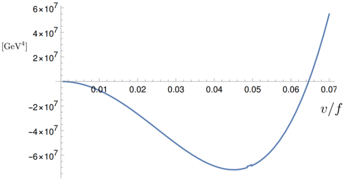
$$\mathbf{20}_L \to \begin{cases} \mathbf{10}_L \to q'(\mathbf{3},\mathbf{2})^{+,-}_{1/6} \oplus \tilde{u}'^c(\mathbf{3}^*,\mathbf{1})^{+,-}_{-2/3} \oplus e'^c(\mathbf{1},\mathbf{1})^{+,-}_1 \\ \mathbf{10}_L^* \to \tilde{q}^c(\mathbf{3}^*,\mathbf{2})^{+,-}_{-1/6} \oplus u(\mathbf{3},\mathbf{1})^{-,-}_{2/3} \oplus \tilde{e}(\mathbf{1},\mathbf{1})^{+,-}_{-1} \end{cases}$$

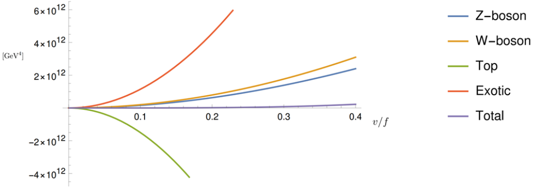
$$\mathbf{15}_L \supset \quad \mathbf{10}_L \to q(\mathbf{3},\mathbf{2})^{+,+}_{1/6} \oplus \tilde{u}^c(\mathbf{3}^*,\mathbf{1})^{-,+}_{-2/3} \oplus e^c(\mathbf{1},\mathbf{1})^{+,+}_1$$

- GUT V_μ boson and $SU(5)$ Higgs partner do not have diquark couplings
- In fact, baryon no. $\to$ accidental symmetry in GHGUT
  $\Rightarrow$ proton stable at all orders in perturbation theory!

pNGB Effective Potential

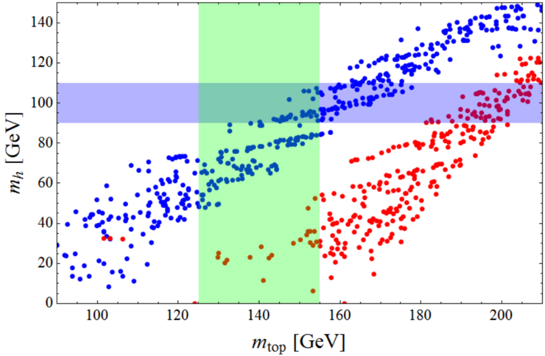
$$m_{KK} = 10\,\text{TeV}, \quad \frac{v_{SM}}{f} \simeq 0.05 \;\Rightarrow\; f \simeq 5\,\text{TeV}$$





- Exotic contribution crucial $\rightarrow$ counterbalances top contribution
- $\mathcal{O}(0.1\,\%)$ fine tuning to get correct vev, but then $m_h = \mathcal{O}(100)$ GeV w/o any extra tuning!

Scalar Masses



$$m_{S_1}^2 \simeq \frac{3\zeta(3)\left(3g_s^2 - 2y_t^2\right)}{16\pi^2} m_{KK}^2 \simeq (0.2\, m_{KK})^2 \simeq 2\,\text{TeV}, \qquad m_s < 0.05\, m_{KK} \simeq 500\,\text{GeV}$$

Bound on m_{KK} pushed higher by $pp \to S_1^* S_1 \Rightarrow m_{KK} \gtrsim 7 - 8$ TeV

Overview

Summary and Conclusions

- GHGUT → appealing model–building direction:
  + unification of gauge interactions and EWSB
  + GUTs w/o perturbative proton decay and doublet–triplet splitting problem
  + solution to gauge and flavour hierarchies
  + Higgs mass calculable → microscopic understanding of EWSB
- Viable $SU(6)$ GUT by introducing more breaking on the boundaries:
  + minimal fermionic content ⇒ correct SM spectrum + flavour hierarchies, no light exotics
  + correct EWSB ⇒ correct m_h follows naturally (no extra tuning needed)
  + new scalars → $S_1(\mathbf{3},\mathbf{1})_{-1/3}$ LQ w/ Yukawas predicted by $SU(5)$ + scalar singlet
  + m_{KK} pushed upwards by bounds on m_{S_1} ⇒ competitive with flavour constraints
- Lots of directions to explore!

  → baryogenesis, Higgs physics, running of gauge couplings (see Sascha Weber's talk), ...

THANK YOU FOR YOUR ATTENTION!

Summary and Conclusions

- GHGUT → appealing model–building direction:
  - \+ unification of gauge interactions and EWSB
  - \+ GUTs w/o perturbative proton decay and doublet–triplet splitting problem
  - \+ solution to gauge and flavour hierarchies
  - \+ Higgs mass calculable → microscopic understanding of EWSB
- Viable $SU(6)$ GUT by introducing more breaking on the boundaries:
  - \+ minimal fermionic content ⇒ correct SM spectrum + flavour hierarchies, no light exotics
  - \+ correct EWSB ⇒ correct m_h follows naturally (no extra tuning needed)
  - \+ new scalars → $S_1(\mathbf{3},\mathbf{1})_{-1/3}$ LQ w/ Yukawas predicted by $SU(5)$ + scalar singlet
  - \+ m_{KK} pushed upwards by bounds on m_{S_1} ⇒ competitive with flavour constraints
- Lots of directions to explore!

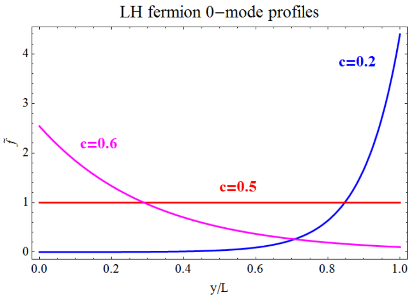
  → baryogenesis, Higgs physics, running of gauge couplings (see Sascha Weber's talk), ...

Thank you for your attention!

Backup

Solving the Flavour Hierarchy Problem [Gherghetta, Pomarol '00; Grossman, Neubert '00]

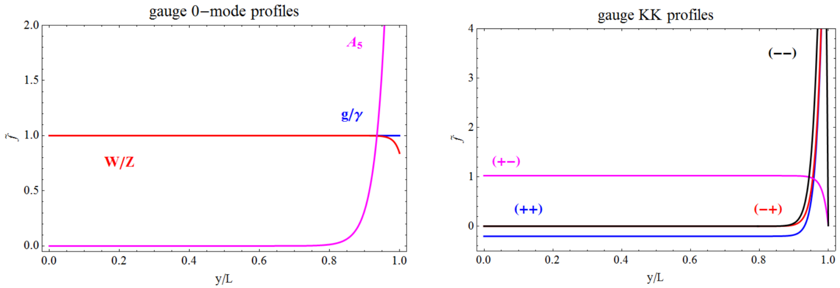
- SM Yukawas $\sim$ overlap of fermion profiles with IR–localized Higgs
- Flavour hierarchy explained geometrically ($c \to -c$ for RH):



Exponential dependence on c of Yukawas $\Rightarrow$

SM FLAVOUR HIERARCHY FROM $\mathcal{O}(1)$ PARAMETERS!

Gauge Bosons in Warped Space [Gherghetta, Pomarol '00; Grossman, Neubert '00]

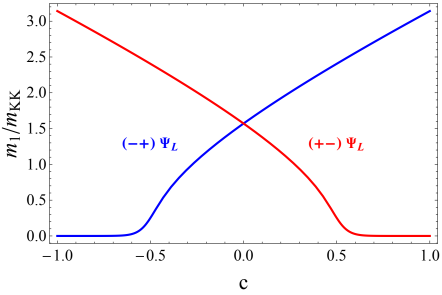


Masses of 1st KK modes of gauge bosons ($m_{KK} = k\, e^{-kL}$):

$$m_{++} \simeq 2.45\, m_{KK}\,, \quad m_{-+} \simeq 2.40\, m_{KK}\,, \quad m_{+-} \simeq 0.25\, m_{KK}\,, \quad m_{--} \simeq 3.83\, m_{KK}$$

Exotic Fermions from Warped Space

- $(-+)$ BCs forbid 0–mode, but 1st KK mass exponentially suppressed for certain values of c $\Rightarrow$ light exotics!



$$m_1^{(-+)} \sim e^{(c+\frac{1}{2})kL} m_{KK}, \quad c < -0.5$$

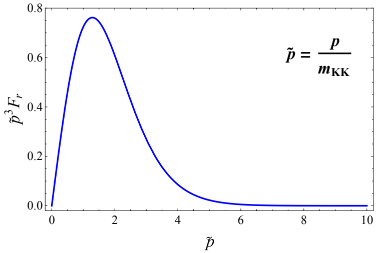
- Light exotics $\rightarrow$ generic problem in GHGUTs b/c large fermion irreps

Gauge–Higgs (Grand) Unification: Higgs Potential [Hosotani '83]

- A_5 appears through $F_{\mu 5} \supset \partial_\mu A_5 \Rightarrow$ shift symmetry!
  $\rightarrow$ explicitly broken by $\overline{\Psi}\gamma_5 A_5 \Psi$ and $f_{abc} f_{ade} \eta^{\mu\nu} A^a_\mu A^d_\nu A^b_5 A^e_5$
- $\Rightarrow$ Higgs potential generated at one loop $\rightarrow$ Hosotani mechanism
- Higgs $\sim$ pseudo Nambu–Goldstone boson (pNGB) arising from $SU(3) \rightarrow G_{\mathrm{EW}}$;
- Typical contribution (of species r) to the Higgs potential:

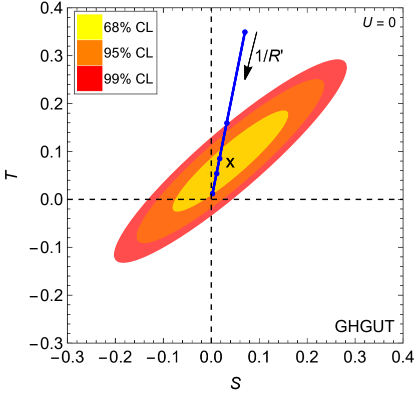
$$V_r(h) = \frac{N_r}{16\pi^2} \int_0^\infty \mathrm{d}p\, p^3 \log\left[1 + F_r(p^2) \sin^2\left(\frac{h}{f}\right)\right], \quad f = \frac{2}{g_{5D}^2 k} m_{KK} = \frac{m_{KK}}{g_*}$$

- Analogy with QCD: $f \leftrightarrow f_\pi \simeq 130$ MeV, $m_{KK} = g_* f \leftrightarrow m_\rho \simeq 770$ MeV



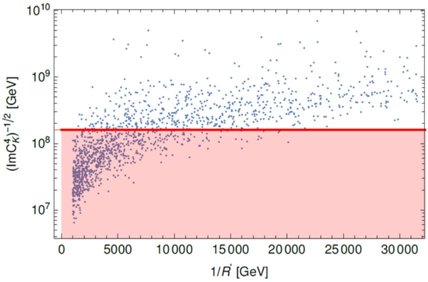
Electroweak Precision Tests

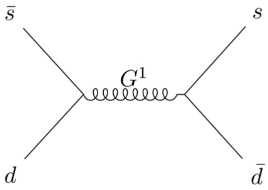
$$\begin{cases} S = \dfrac{3\pi}{2}\dfrac{v^2}{m_{KK}^2} \simeq 0.003 \\ T = \dfrac{\alpha_{\rm em}^{-1}}{10}\dfrac{v^2}{f^2} \simeq 0.03 \end{cases} \Rightarrow m_{KK} = g_* f \gtrsim 5 \text{ TeV at } 2\sigma$$



Meson Mixing

Meson Mixing





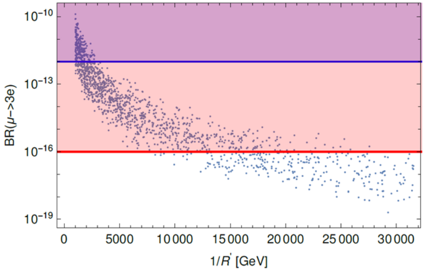
$$\mathcal{H} \supset C_4^K Q_4^{ds} \sim \left(\frac{g^{ds}}{m_{G^1}}\right)^2 (\bar{d}_L^\alpha s_{R,\alpha})(\bar{d}_R^\beta s_{L,\beta})$$

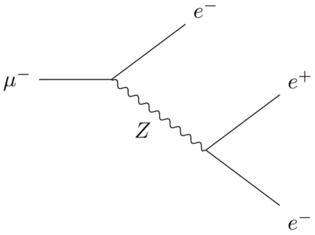
$\mu \to eee$

Tree-level flavor violation: $\mu^- \to e^- e^+ e^-$

SINDRUM (1988) $\mathrm{Br}(\mu \to 3e) < 10^{-12}$

Mu3e (2026(?)) $\mathrm{Br}(\mu \to 3e) < 10^{-16}$



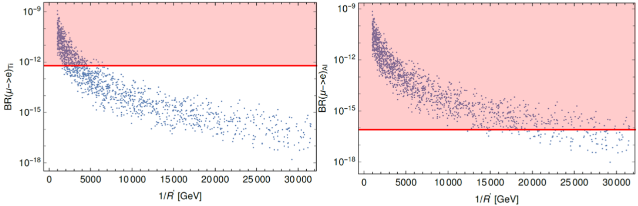


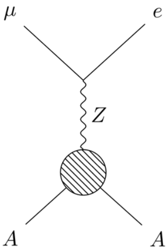
$\mu \to e$ Conversion in Nuclei

Tree-level flavor violation: $\mu \to e$ conversion

SINDRUM II (1998) $\mathrm{Br}(\mu \to e)_{\mathrm{Ti}} < 6.1 \times 10^{-13}$

Mu2e (20??) $\mathrm{Br}(\mu \to e)_{\mathrm{Al}} < 8 \times 10^{-17}$



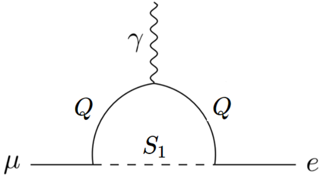


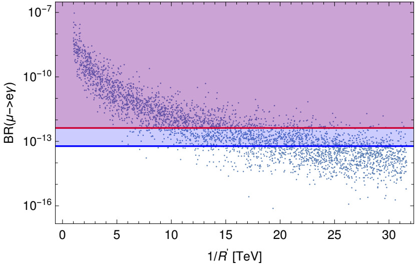
$\mu \to e\gamma$

- Random scan over m_{KK}, brane mass matrices:

$$0.2 < \left|\left(M_{q/e}\right)_{ij}\right|, \left|\left(M_{d/l}\right)_{ij}\right| < 2$$

- LQ contribution dominates $\mu \to e\gamma$!
- $m_{KK} \gtrsim 7$ TeV; other onstraints less stringent





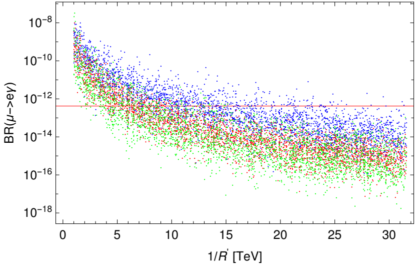


Figure: Left: current (red) and future (blue) constraints (MEG) on $\mu \to e\gamma$.
Right: breakdown of contributions to $\mu \to e\gamma$: LQ (blue), Higgs (red), Z (green).

Scalar Leptoquark Yukawas

$$q_{L_i} = \begin{pmatrix} V^\dagger u_L \\ d_L \end{pmatrix}_i, \quad \ell^c_{L_i} = \begin{pmatrix} e^c_L \\ -U^* \nu^c_L \end{pmatrix}_i, \quad V \equiv V_{CKM}, \quad U \equiv U_{PMNS}, \quad W \equiv U_{d_R} U^T_{e_L}$$

- Higgs + scalar LQ come from the same $SU(5)$ multiplet $\Rightarrow$ Yukawas highly correlated:

$$-\mathcal{L}_{S_1} = S_1 \left\{ \overline{q}_{L_i} [y_d\, W]_{ij} \ell^c_{L_j} + \overline{d}_{R_i} \, [W\, U^*\, y_\nu]_{ij}\, \nu^c_{R_j} + \overline{u}_{R_i} \left[y_u\, V\, y_d^{-1}\, W\, y_e \right]_{ij} e^c_{R_j} \right\} + \text{h.c.}$$

- S_1 couples almost exclusively to $t_R \tau^c_R$:

$$\left[y_u\, V\, y_d^{-1}\, W\, y_e \right] \sim \begin{pmatrix} 10^{-6} & 10^{-4} & 10^{-4} \\ 10^{-4} & 10^{-3} & 10^{-2} \\ 10^{-3} & 10^{-2} & 1 \end{pmatrix}$$

$$[y_d\, W] \sim \begin{pmatrix} 10^{-5} & 10^{-6} & 10^{-6} \\ 10^{-4} & 10^{-4} & 10^{-4} \\ 10^{-3} & 10^{-2} & 10^{-2} \end{pmatrix}$$

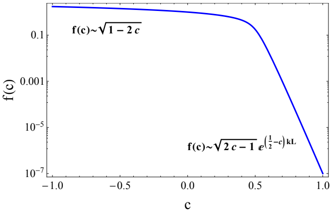
- Generates $C_9^{\mu\mu} = -C_{10}^{\mu\mu}$ at one loop, but $y_R^{t\mu} \sim 10^{-2}$ too small to explain $b \to s\mu\mu$

CKM and PMNS Matrices

- Quark mass matrices in flavour basis:

$$m_u \simeq \frac{g_* v}{2\sqrt{2}} f_{c_{15}} M^\dagger_{q/e} f_{-c_{20}}, \quad m_d \simeq -\frac{g_* v}{2\sqrt{2}} f_{c_{15}} M^\dagger_{d/l} f_{-c_6},$$

$$f_c = \mathrm{diag}\left[f(c_1), f(c_2), f(c_3)\right], \quad f(c) = \sqrt{\frac{1-2c}{1-e^{(2c-1)kL}}}$$



- Unitary matrices diagonalizing $m_{u,d}$ carry specific hierarchies [Csaki+ '08, Goertz+ '08]:

$$U_{u_L}, U_{d_L} \sim \begin{pmatrix} 1 & \frac{f(c_{15,1})}{f(c_{15,2})} & \frac{f(c_{15,1})}{f(c_{15,3})} \\ \frac{f(c_{15,1})}{f(c_{15,2})} & 1 & \frac{f(c_{15,2})}{f(c_{15,3})} \\ \frac{f(c_{15,1})}{f(c_{15,3})} & \frac{f(c_{15,2})}{f(c_{15,3})} & 1 \end{pmatrix}, \quad \begin{matrix} U_{u_R} \sim U_{u_L}\big|_{c_{15_i} \leftrightarrow -c_{20_i}} \\ U_{d_R} \sim U_{u_L}\big|_{c_{15_i} \leftrightarrow -c_{6_i}} \end{matrix}$$

- CKM inherits hierarchies from U_{u_L}, U_{d_L}:

$$\frac{f(c_{15,1})}{f(c_{15,2})} \sim \lambda, \ \frac{f(c_{15,2})}{f(c_{15,3})} \sim \lambda^2 \ \Rightarrow \ V_{CKM} \equiv U_{u_L} U^\dagger_{d_L} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

CKM and PMNS Matrices

- What about the PMNS matrix?

$$U_{PMNS} \equiv U_{e_L} U_{\nu_L}^\dagger, \quad U_{e_L}, U_{\nu_L} \sim \begin{pmatrix} 1 & \frac{f(-c_{6,1})}{f(-c_{6,2})} & \frac{f(-c_{6,1})}{f(-c_{6,3})} \\ \frac{f(-c_{6,1})}{f(-c_{6,2})} & 1 & \frac{f(-c_{6,2})}{f(-c_{6,3})} \\ \frac{f(-c_{6,1})}{f(-c_{6,3})} & \frac{f(-c_{6,2})}{f(-c_{6,3})} & 1 \end{pmatrix}$$

- All relevant c–paramters fixed [Csaki+ '08, Goertz+ '08]

$$\frac{m_c}{m_t} \sim \lambda^2 \frac{f(-c_{20,2})}{f(-c_{20,3})}, \quad \frac{m_u}{m_t} \sim \lambda^3 \frac{f(-c_{20,1})}{f(-c_{20,3})}, \quad \frac{m_b}{m_t} \sim \frac{f(-c_{6,3})}{f(-c_{20,3})},$$

$$\frac{m_s}{m_b} \sim \lambda^2 \frac{f(-c_{6,2})}{f(-c_{6,3})}, \quad \frac{m_d}{m_b} \sim \lambda^3 \frac{f(-c_{6,1})}{f(-c_{6,3})}$$

- $\Rightarrow$ no large hierarchies in U_{e_L}, U_{ν_L}:

$$\frac{f(-c_{6,1})}{f(-c_{6,2})} \sim 0.25, \quad \frac{f(-c_{6,2})}{f(-c_{6,3})} \sim 0.5$$

- Fix fermion masses and $V_{CKM} \Rightarrow$ no large hierarchy in U_{PMNS}!