



**University of
Zurich^{UZH}**

Flavor hierarchies, flavor anomalies, and the Higgs mass from a warped extra dimension

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Puzzles in the SM and Hints Toward New Physics

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi + |\mathcal{D}_\mu \phi|^2 - V(\phi) + \bar{\psi}_i y_{ij} \psi_j \phi + \text{h.c.}$$

 **Natural, Flavor Universal**

 **Flavor Puzzle**

- Has very hierarchical structure that does not seem accidental
- Violates LFU, quark number

Hints toward the structure of new physics?

 **Higgs Hierarchy Problem**

- Instability of the Higgs mass under quantum corrections

TeV-scale new physics?

Flavor could have a Multi-scale Explanation

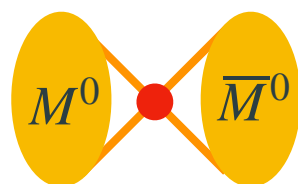
$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{Gauge}} + \underbrace{\mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \sum_{i,d} \frac{1}{\Lambda_i^{d-4}} C_i \mathcal{O}_i^d}_{\text{Non-trivial UV imprints}}$$

Non-trivial UV imprints

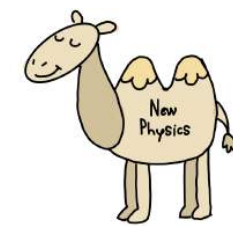
★ The SM Yukawas are very hierarchical because they originate from very different scales!

★ TeV-scale NP dominantly coupled to third and (to a lesser extent) second families
[$\approx U(2)^5$ protection from flavor constraints]

e.g. from $\frac{1}{\Lambda^2} (\psi_i \psi_j)^2$



★ Direct production of new states at the LHC is naturally more suppressed [NP scale can be lower]



$\nu \approx 246 \text{ GeV}$

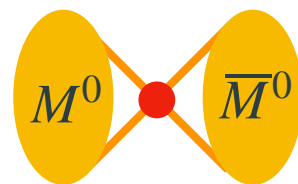
[Barbieri, [2103.15635](#)
Bordone et al., [1712.01368](#)
Panico, Pomarol, [1603.06609](#)
Dvali, Shifman, '00, ...]

Flavor could have a Multi-scale Explanation

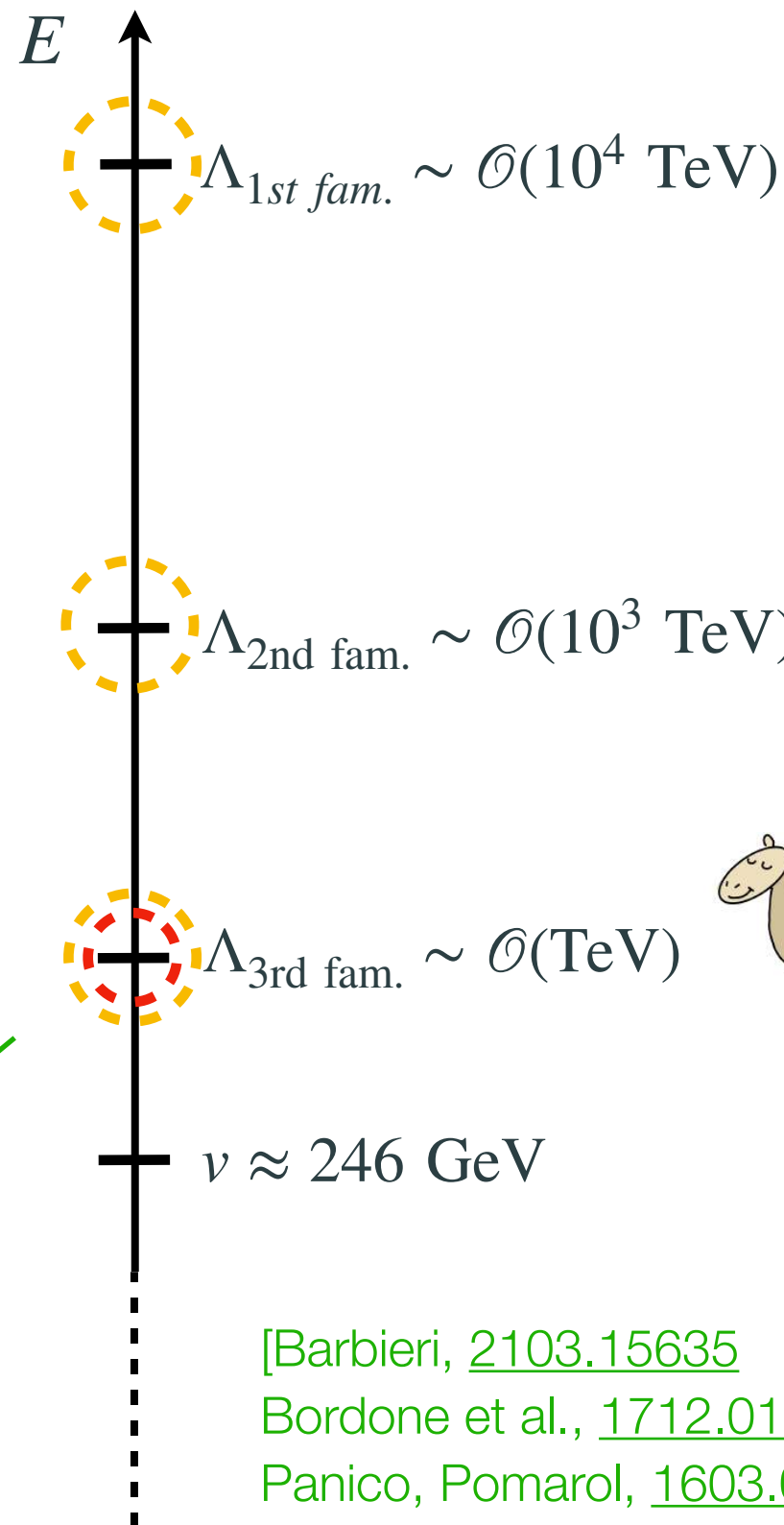
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Perhaps we are already seeing hints in B-meson decays?



[Barbieri, [2103.15635](#)
Bordone et al., [1712.01368](#)
Panico, Pomarol, [1603.06609](#)
Dvali, Shiftman, '00, ...]

Combined Explanation of the B-anomalies

	Model	$R_{K(*)}$	$R_{D(*)}$	$R_{K(*)}$ & $R_{D(*)}$
Scalars	$S_1 = (\mathbf{3}, \mathbf{1})_{-1/3}$	✗	✓	✗
	$R_2 = (\mathbf{3}, \mathbf{2})_{7/6}$	✗	✓	✗
	$\tilde{R}_2 = (\mathbf{3}, \mathbf{2})_{1/6}$	✗	✗	✗
	$S_3 = (\mathbf{3}, \mathbf{3})_{-1/3}$	✓	✗	✗
Vector	$U_1 = (\mathbf{3}, \mathbf{1})_{2/3}$	✓	✓	✓
	$U_3 = (\mathbf{3}, \mathbf{3})_{2/3}$	✓	✗	✗

[Angelescu, Bečirević, Faroughy, Sumensari, [1808.08179](#)]

★ $S_1 + S_3$

[Crivellin, Muller, Ota [1703.09226](#);
Buttazzo et al. [1706.07808](#);
Marzocca [1803.10972](#),...]

★ $S_3 + R_2$

[Bečirević et al., [1806.05689](#)]

★ $U_1 + UV$ completion

[di Luzio, Greljo, Nardecchia [1708.08450](#);
Calibbi, Crivellin, Li [1709.00692](#);
Bordone, Cornella, Fuentes-Martin, Isidori [1712.01368](#);
Barbieri, Tesi, [1712.06844](#); Greljo, BAS, [1802.04274](#)]

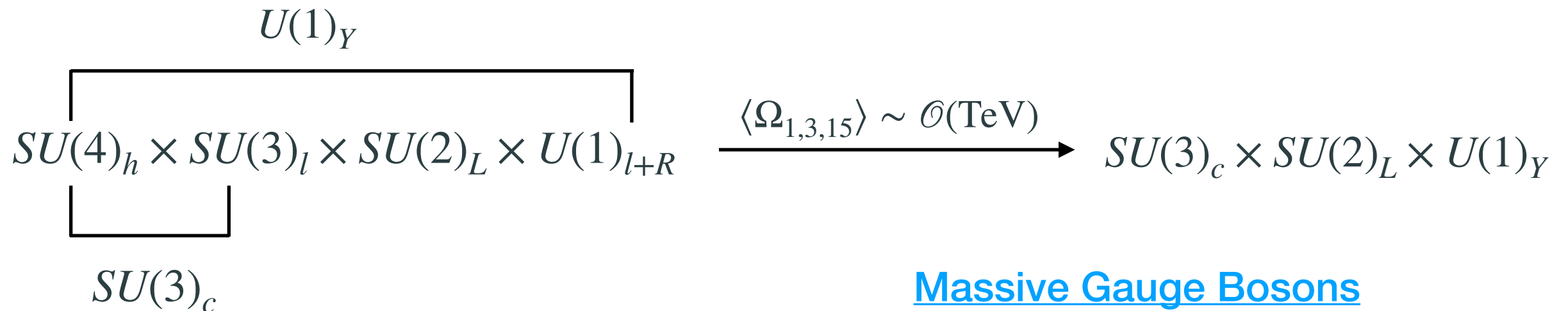
The U_1 is (one of) the most promising mediators to explain the B anomalies:

- ✓ No tree-level $b \rightarrow s\nu_{(\tau)}\nu_{(\tau)}$
- ✓ Being a vector, possibility to realize a $U(2)^5$ from a flavor non-universal gauge symmetry (possible connection to the SM flavor puzzle)
- ✓ Third-family quark-lepton unification: Hint towards Pati-Salam-like unification

Gauge UV Completion for the U_1 Leptoquark

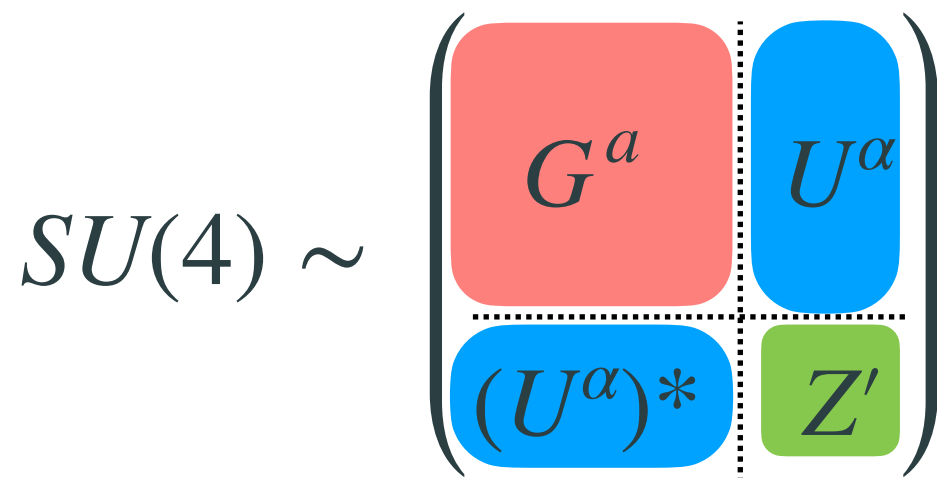
Based on “4321” gauge symmetry:

[di Luzio, Greljo, Nardecchia [1708.08450](#)
Bordone, Cornella, Fuentes-Martin, Isidori
[1712.01368](#), [1805.09328](#);
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Massive Gauge Bosons

$$U_1, G', Z'$$

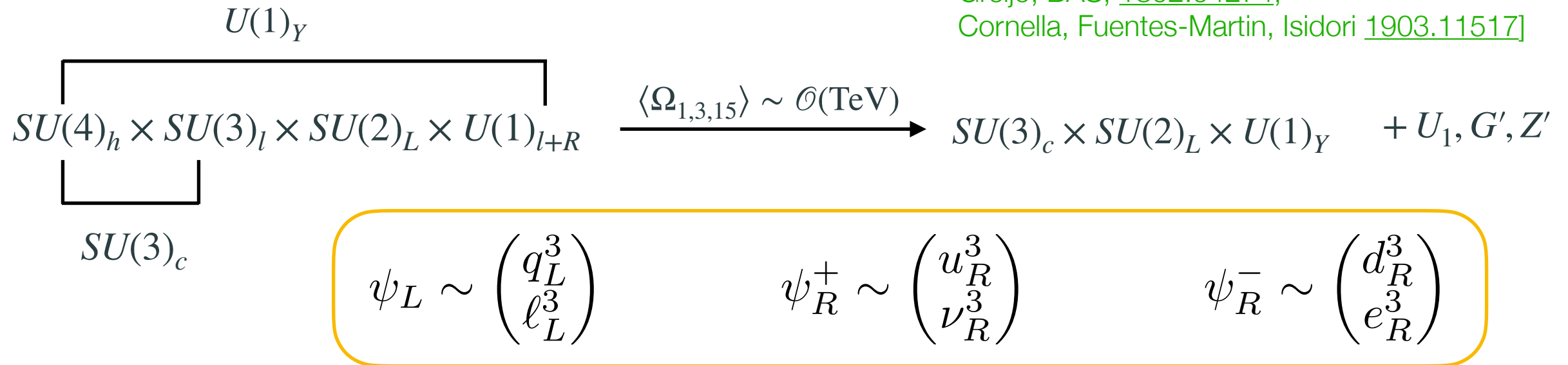


- Provides a UV completion for the U_1 vector LQ
- Necessarily comes with G', Z' [cannot have TeV-scale Pati-Salam unification: $SU(4) \supset SU(3)_c$]
- For $g_4 \gg g_{3,1}$, massive gauge bosons dominantly come from $SU(4)_h$

Third family quark-lepton unification at the TeV scale

Based on “4321” gauge symmetry:

[di Luzio, Greljo, Nardecchia [1708.08450](#)
 Bordone, Cornella, Fuentes-Martin, Isidori
[1712.01368](#), [1805.09328](#);
 Greljo, BAS, [1802.04274](#);
 Cornella, Fuentes-Martin, Isidori [1903.11517](#)]



- 3rd family charged under $SU(4)_h$
 \implies Direct NP couplings (L+R)
- Light families under 321 (SM-like)
- Accidental approximate $U(2)^5$ flavor symmetry: $\psi = (\psi_1 \psi_2 \psi_3)$

Field	$SU(4)_h$	$SU(3)_l$	$SU(2)_L$	$U(1)_{l+R}$
q_L^i	1	3	2	1/6
u_R^i	1	3	1	2/3
d_R^i	1	3	1	-1/3
ℓ_L^i	1	1	2	-1/2
e_R^i	1	1	1	-1
ψ_L	4	1	2	0
ψ_R^\pm	4	1	1	$\pm 1/2$
$\chi_{L,R}$	4	1	2	0
H	1	1	2	1/2
Ω_1	$\bar{4}$	1	1	-1/2
Ω_3	$\bar{4}$	3	1	1/6
Ω_{15}	15	1	1	0

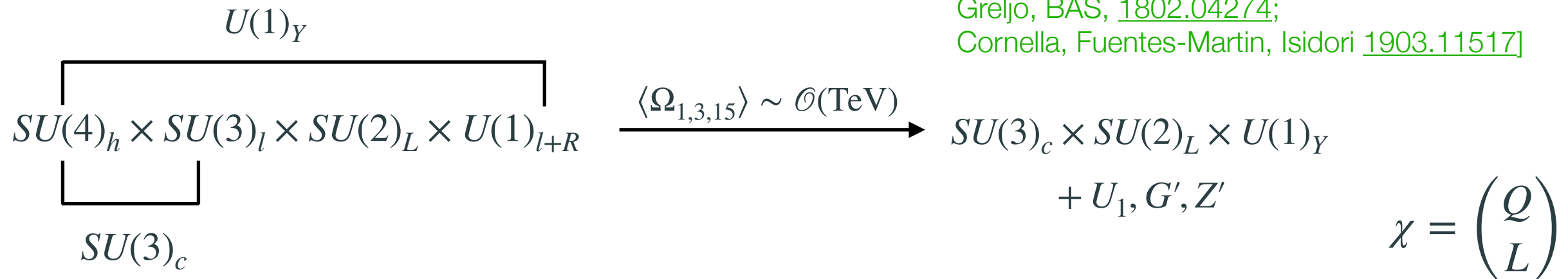
1st & 2nd families

3rd family

Third family quark-lepton unification at the TeV scale

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- CKM mixing and NP couplings to light families from the leading $\mathcal{O}(0.1)$ breaking of $U(2)_q \times U(2)_\ell$:

$$\mathcal{L} \supset -\bar{q}_L^i \lambda_q^i \Omega_3 \chi_R - \bar{\ell}_L^i \lambda_\ell^i \Omega_1 \chi_R$$

$$-y_+ \bar{\chi}_L \tilde{H} \psi_R^+ - y_- \bar{\chi}_L H \psi_R^-$$

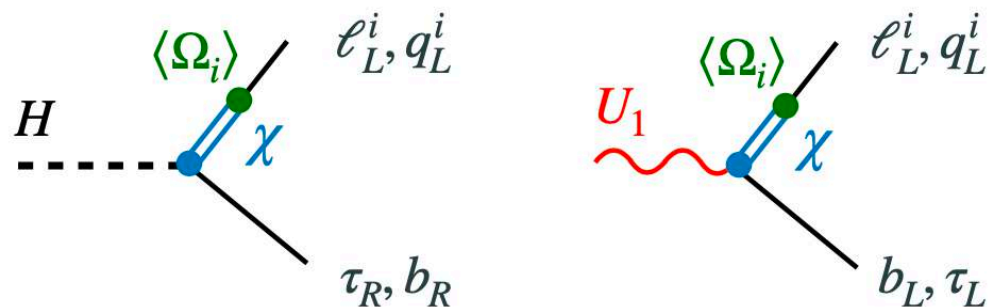
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ψ_L	4	1	2	0
ψ_R^\pm	4	1	1	$\pm 1/2$
$\chi_{L,R}$	4	1	2	0
H	1	1	2	1/2
Ω_1	$\bar{4}$	1	1	-1/2
Ω_3	$\bar{4}$	3	1	1/6
Ω_{15}	15	1	1	0

1st & 2nd families

3rd family

VL fermion

4321 SSB scalars



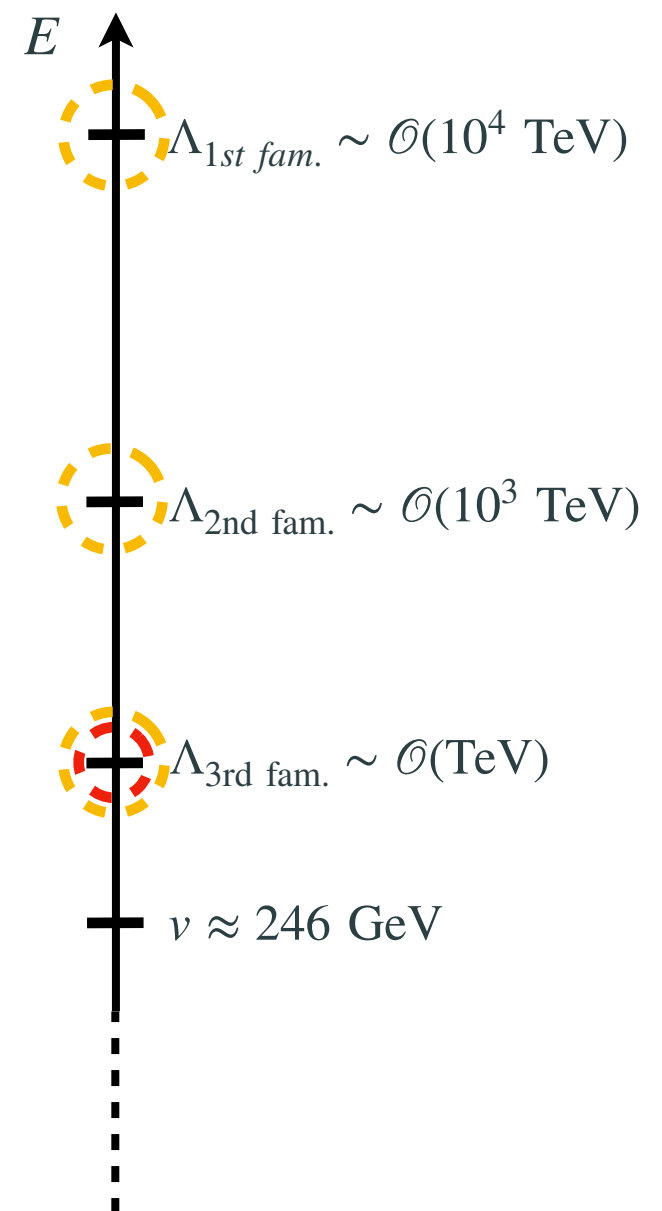
Why consider a 5D model?



4321 gauge models provide a nice combined explanation of the B-anomalies. Also fits well with the idea of 3rd family QL unification at the TeV scale: conceptually nice and allows us to realize a $U(2)^5$ flavor symmetry from the gauge symmetry.

But, 4321 doesn't say anything about:

1. EW hierarchy problem (IR problem)
2. Light Yukawa structure (UV problem)



Why consider a 5D model?



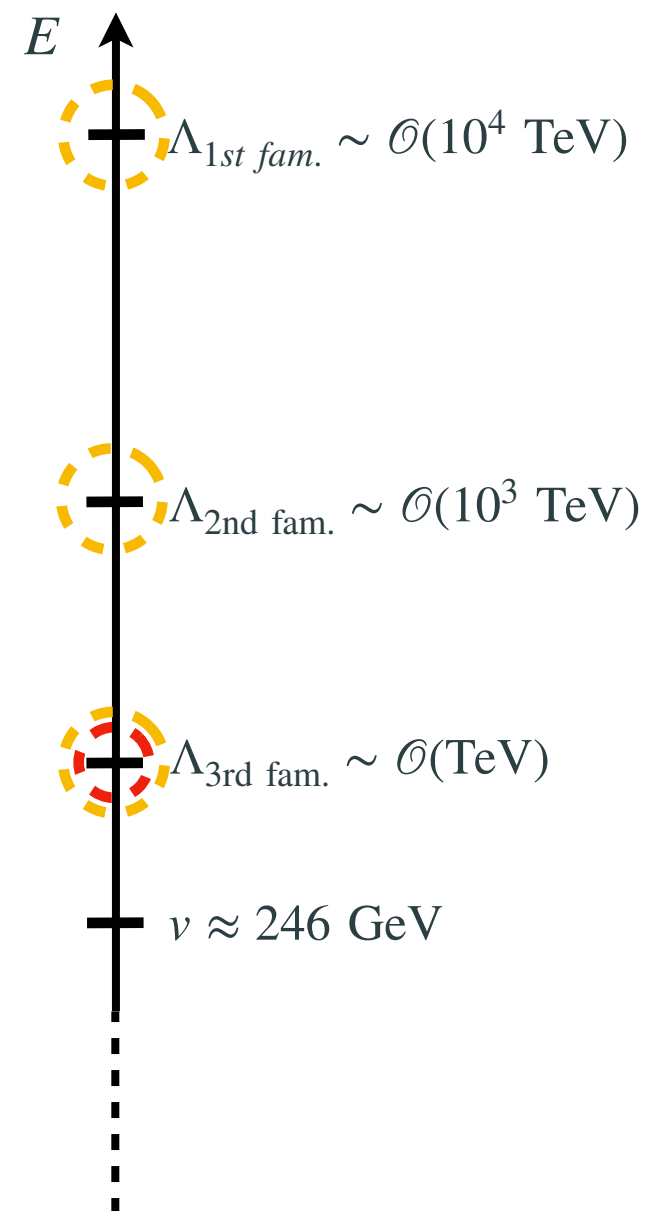
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Naturally combined in a warped 5D setup!

1. Warped geometry stabilizes large scale hierarchies.
2. Light family masses/mixing explained by fermion localization “in the UV”.



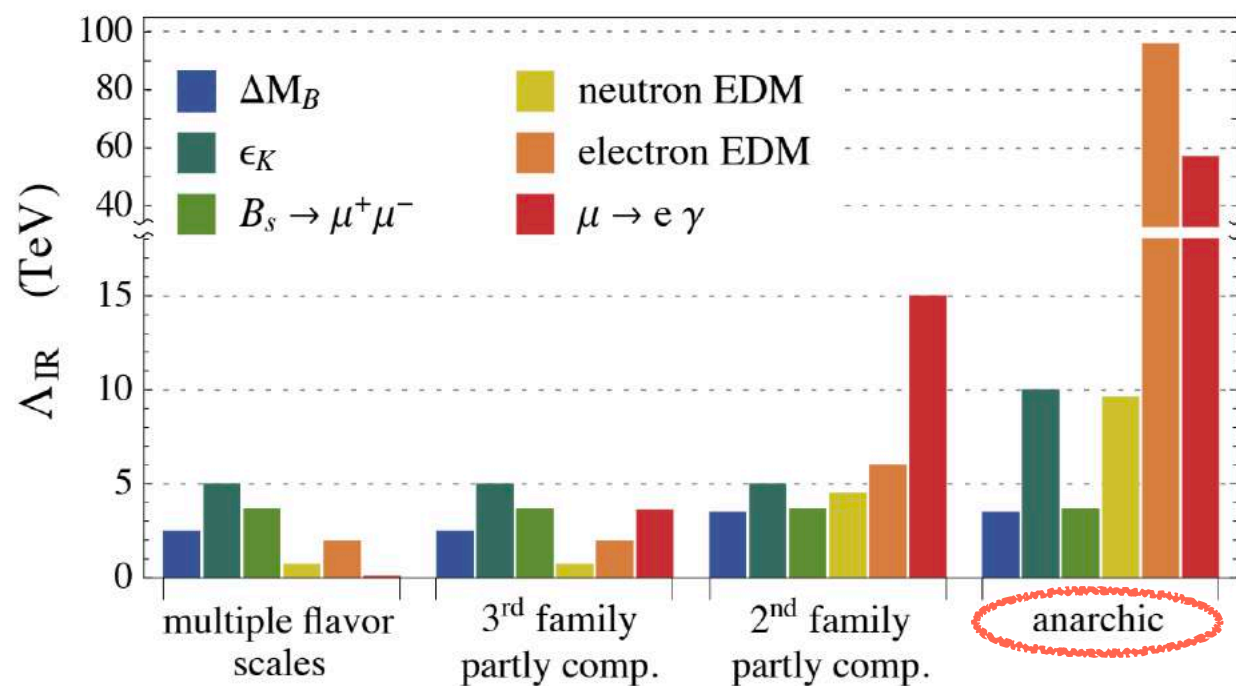
Flavor in Randall-Sundrum Models

- 5D Yukawa couplings are anarchical, Higgs in the IR
- Localize top in the IR, light families in the UV
- Light family Yukawas receive exponential suppression
- But, **RH** fields must reach the IR where **KK** modes peak

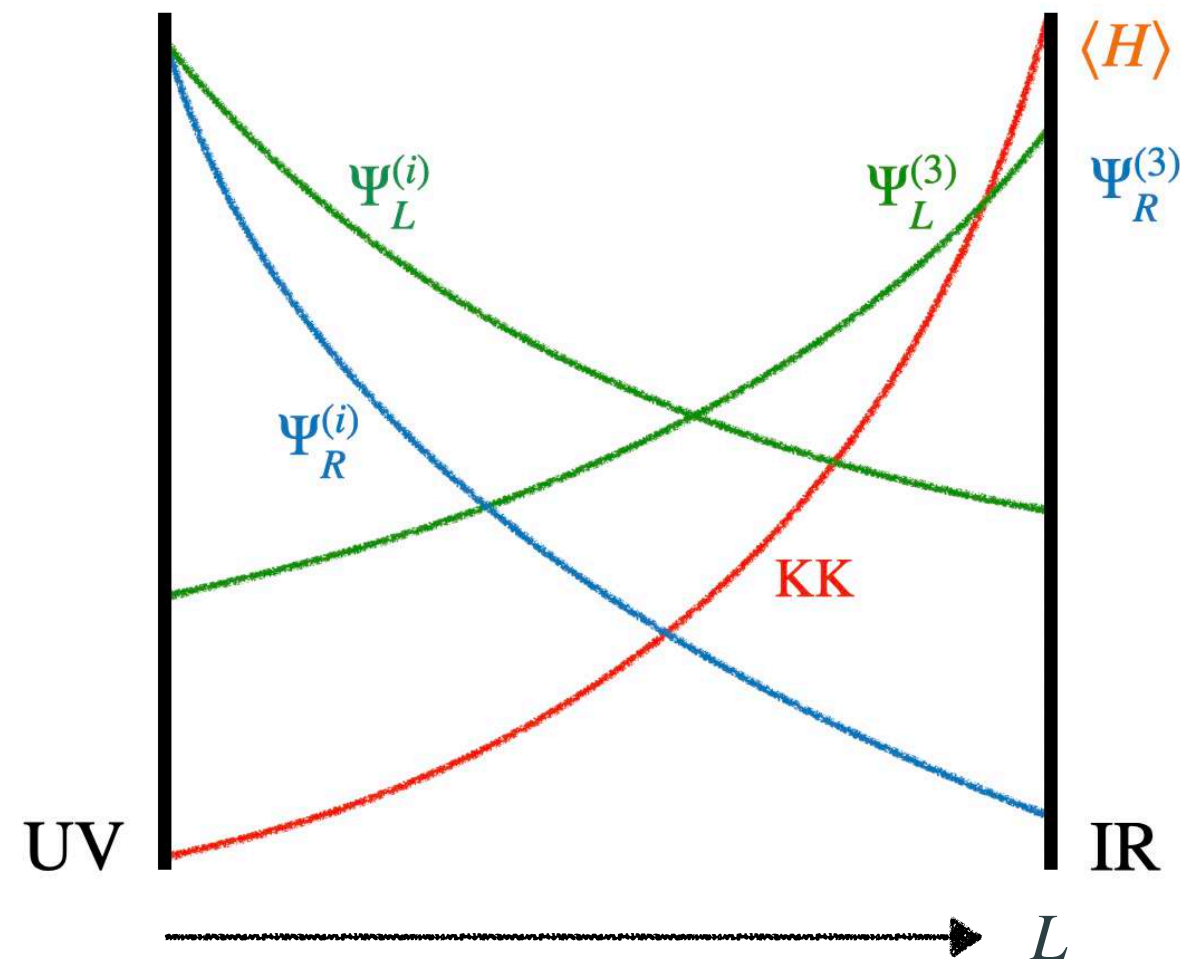
Dangerous dipoles (among others) generated at the IR scale

$$\sim \frac{g_*^2}{16\pi^2} \frac{m_e}{\Lambda_{\text{IR}}^2} \bar{e}_L \sigma_{\mu\nu} e_R F^{\mu\nu}$$

$g_* \sim (0,0,1) \implies U(2)$ flavor sym.



[Panico, Pomarol, [1603.06609](#)]



* $\Lambda_{\text{IR}}/\Lambda_{\text{UV}} = e^{-kL}$ Exponential hierarchies

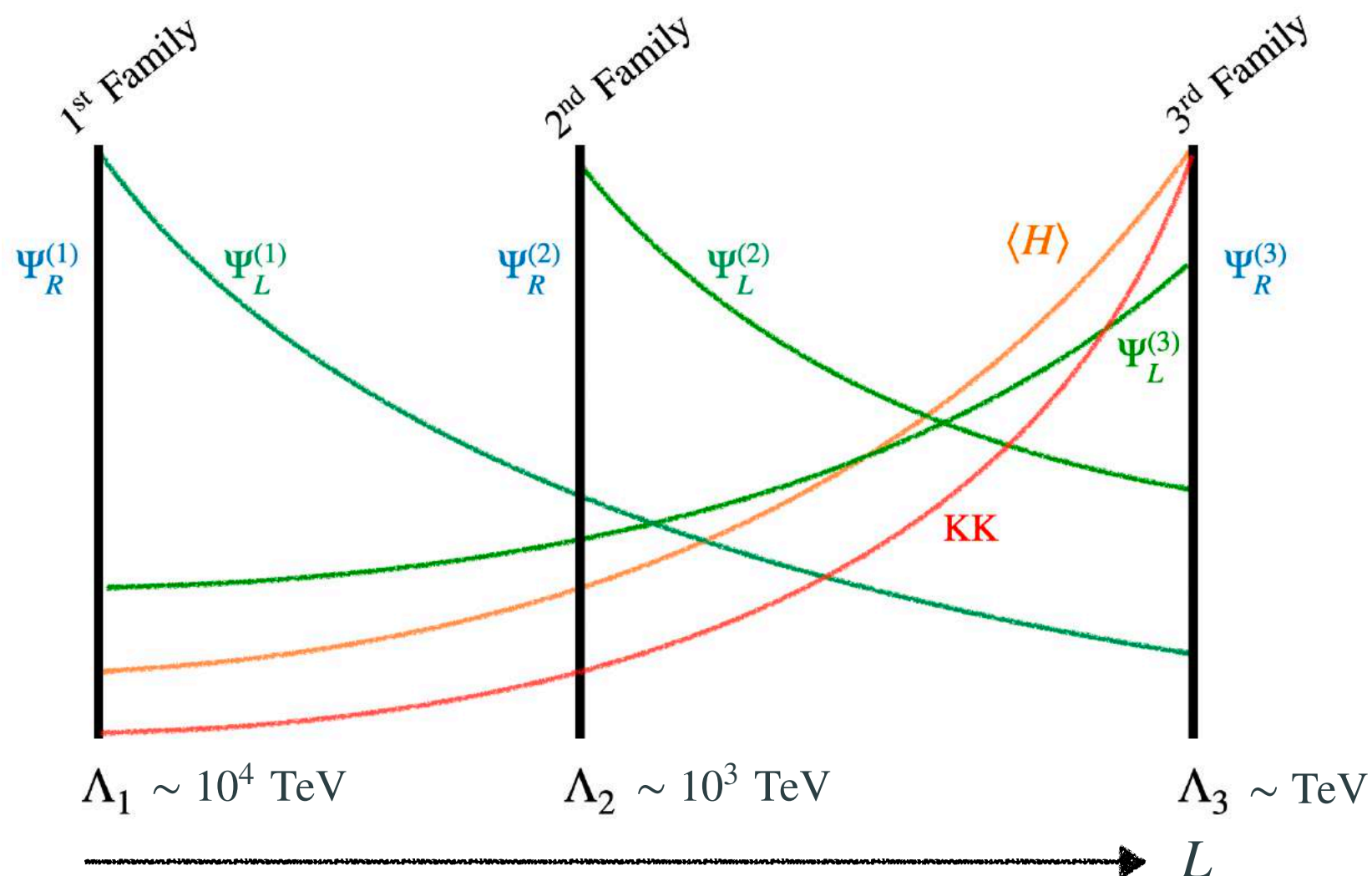
Benefits of a Multi-Scale Solution for Flavor

- Higgs profile extended into the bulk, **RH** fields localized in branes
- Light family Yukawas now done in the UV, smallness explained by the exponentially falling Higgs profile
- U(2) flavor symmetry with leading breaking in the **LH** sector.
- Dangerous operators involving **RH** fields naturally suppressed

[Dvali, Shifman, '00; Panico, Pomarol, 1603.06609]

Dangerous dipoles now suppressed by the UV scales

$$\sim \frac{g_*^2}{16\pi^2} \frac{m_e}{\Lambda_1^2} \bar{e}_L \sigma_{\mu\nu} e_R F^{\mu\nu}$$



Higgs VEV Profile

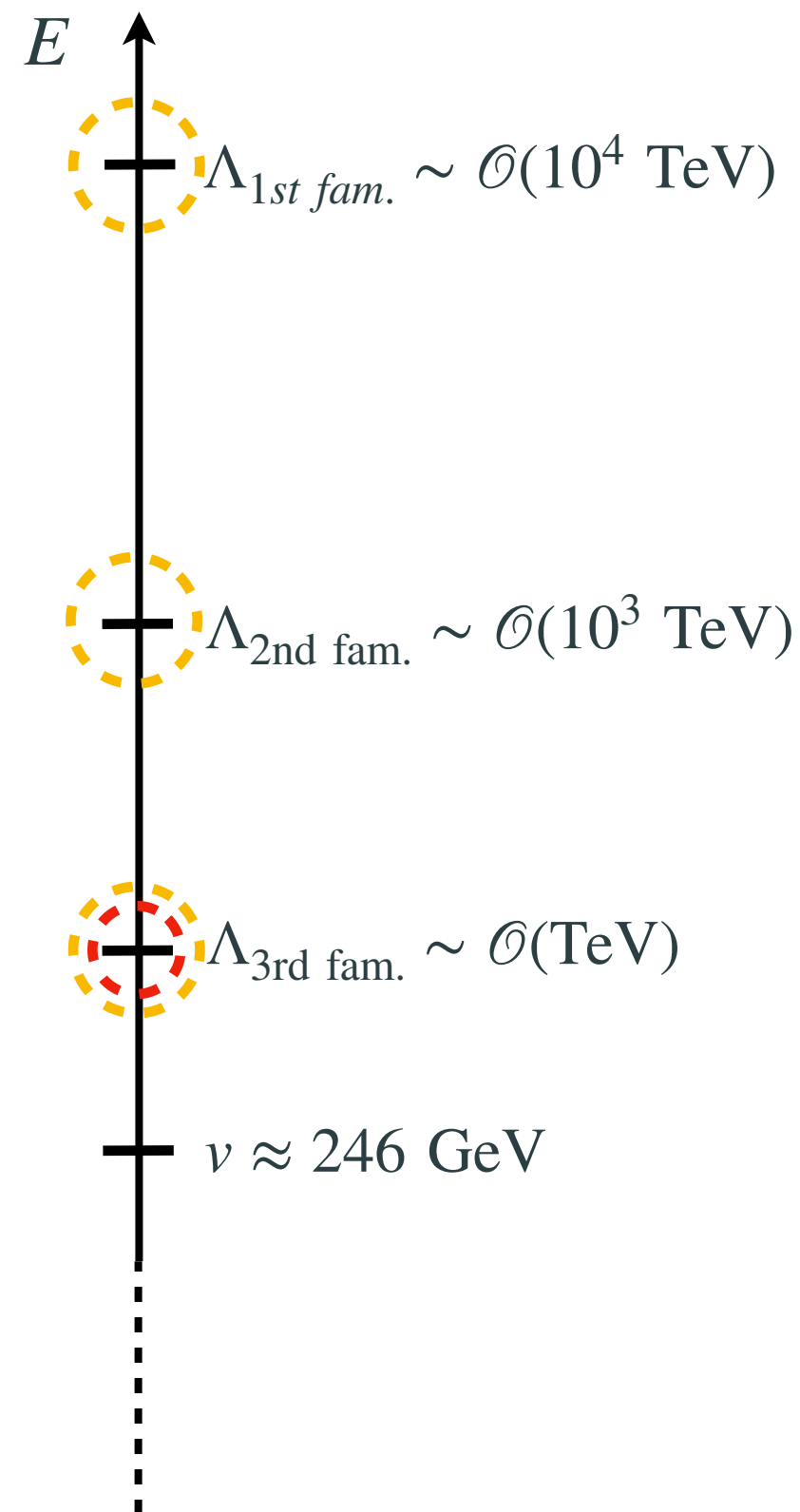
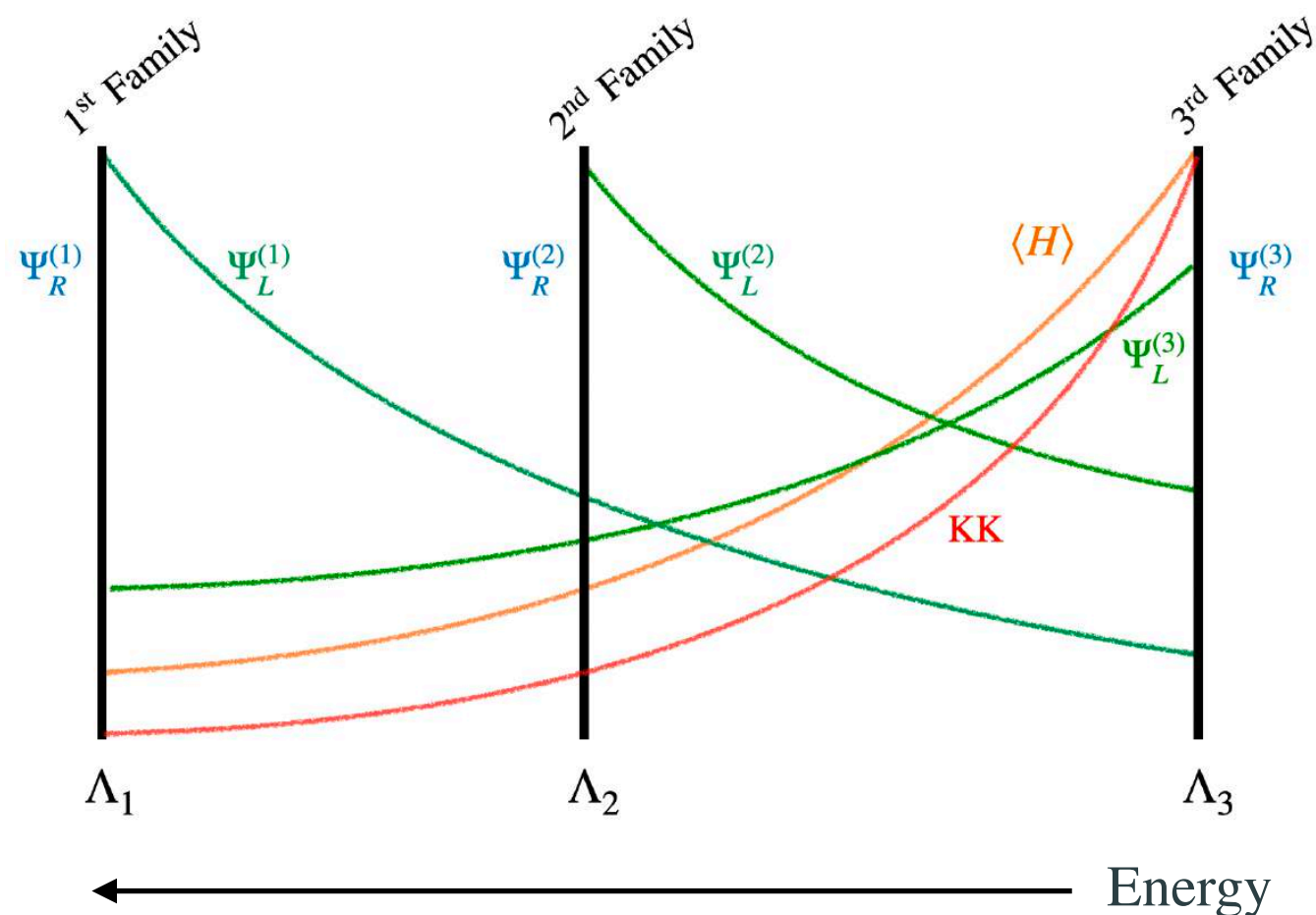
$$\langle H \rangle \sim v_{EW} e^{-k(L-y)}$$

*Fermion mass hierarchy fixes the total volume:

$$kL \approx \ln(m_t/m_u) \approx 10$$

[Fuentes-Martin, Isidori, Pagès, BAS, 2012.10492]

Roadmap to a Multi-scale Theory of Flavor



- ✓ 1. 4321 gauge symmetry at the **TeV scale** (3rd family QL unification, explanation of B-anomalies)
- ✓ 2. Flavor ↔ fermion (quasi)-localization in **three branes**
- 3. **Higgs as a pNGB** of the same dynamics responsible for breaking 4321 gauge symmetry

[Fuentes-Martin, Isidori, Pagès, BAS, 2012.10492
 Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, 2203.01952]

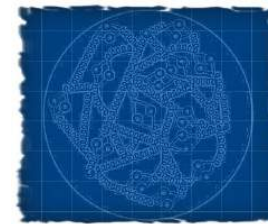
4321 + pNGB Higgs: The Composite Blueprint



Global symmetry	$\mathcal{G}_{\text{global}} = SU(4)_h \times SU(4)_l \times SO(5)$
Gauge symmetry	$\mathcal{G}_{\text{gauge}} = SU(4)_h \times SU(3)_l \times SU(2)_L \times U(1)_{l+R}$

(4321 gauged)

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“Minimal Composite Higgs (MCHM)”

$$SO(5) \rightarrow SO(4) \equiv SU(2)_L \times SU(2)_R + 4 \text{ NGBs}$$

[Agashe, Contino, Pomarol, [hep-ph/0412089](https://arxiv.org/abs/hep-ph/0412089)]

★ [4 NGBs \sim $\mathbf{4}$ or $(\mathbf{2}, \bar{\mathbf{2}}) \longleftrightarrow H$]

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Spontaneously broken by a condensate at some IR scale $f \sim \text{few TeV}$

Global SBB	$\mathcal{G}_{\text{IR}} = SU(4)_D \times SO(4)$	(Custodial sym.)
Gauge SSB	$\mathcal{G}_0 = \mathcal{G}_{\text{gauge}} \cap \mathcal{G}_{\text{IR}} = SU(3)_c \times SU(2)_L \times U(1)_Y$	(Standard Model)
Goldstones	15 + 4 (15 eaten + NGB Higgs)	$(U_1, G', Z' + H)$

[Fuentes-Martin, Stangl, [2004.11376](https://arxiv.org/abs/2004.11376)]

[Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](https://arxiv.org/abs/2203.01952)]

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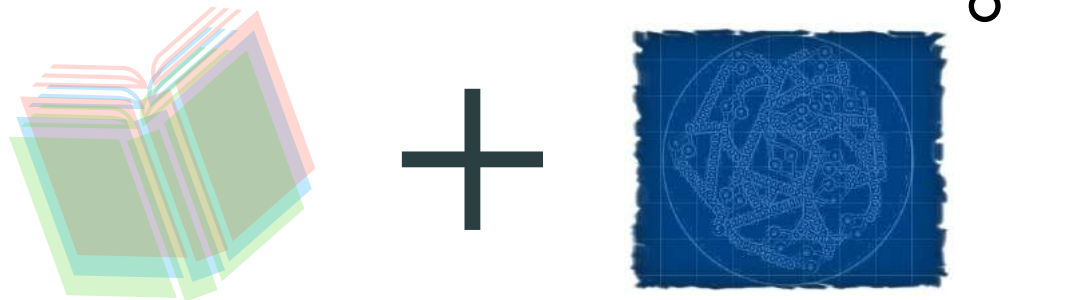
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SM Higgs emerges as a Nambu-Goldstone boson of the same (strong) dynamics breaking 4321 gauge symmetry!

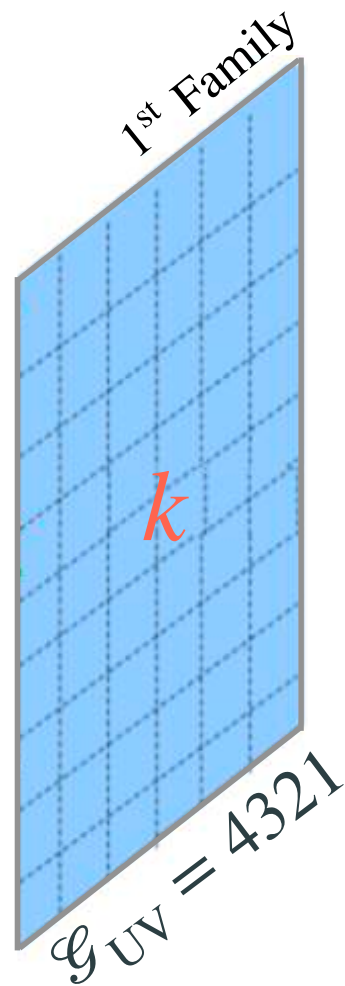
A Multi-Scale 5D Model

5D Model Builder's Toolbox

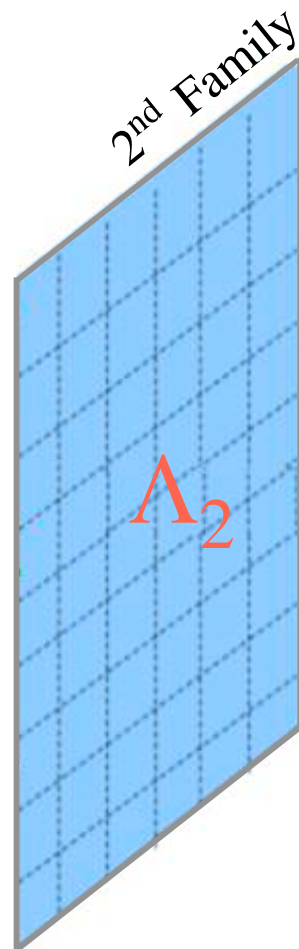


Spontaneous symmetry breakings

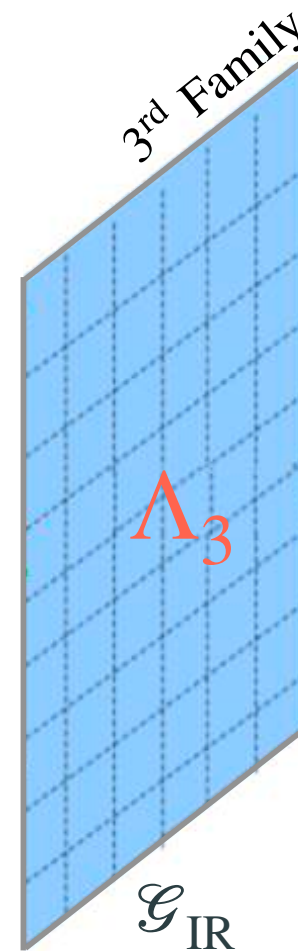
$$\mathcal{G}_{\text{bulk}}^{12} = SU(4)_h \times SU(4)_l \times SO(5)$$



$$\mathcal{G}_{\text{bulk}}^{12}$$



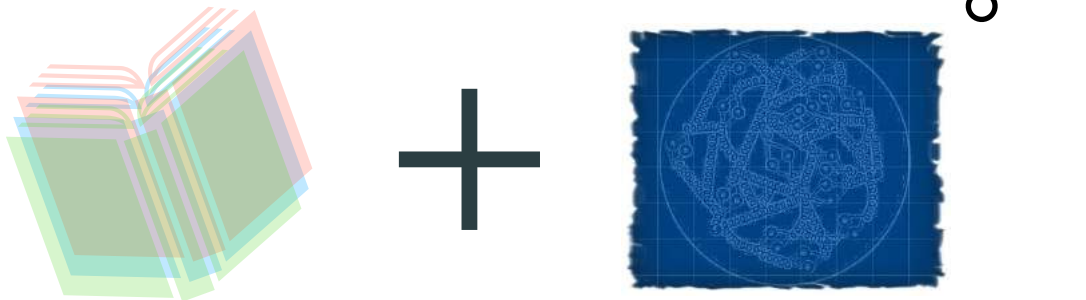
$$\mathcal{G}_{\text{bulk}}^{23}$$



\sim few TeV

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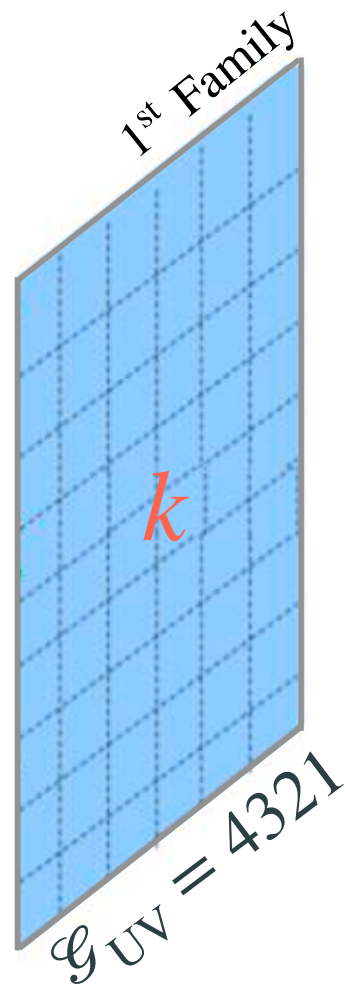


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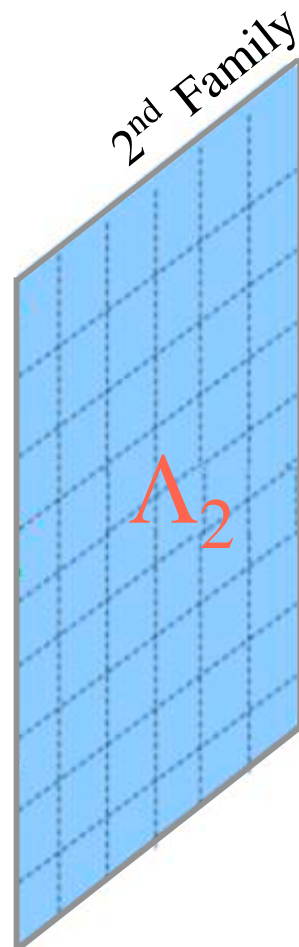
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↘ Λ_2 (6 broken)

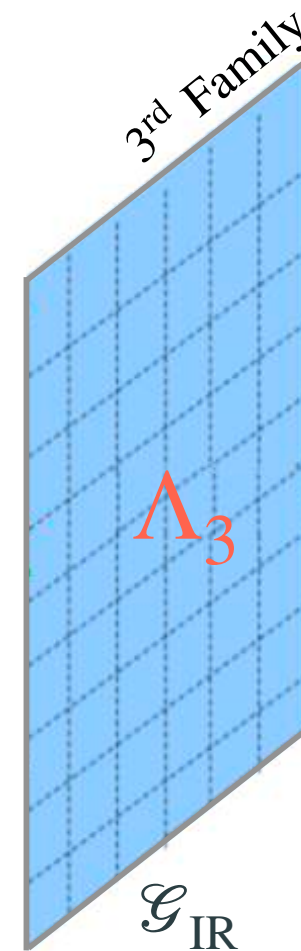
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$\mathcal{G}_{\text{bulk}}^{12}$



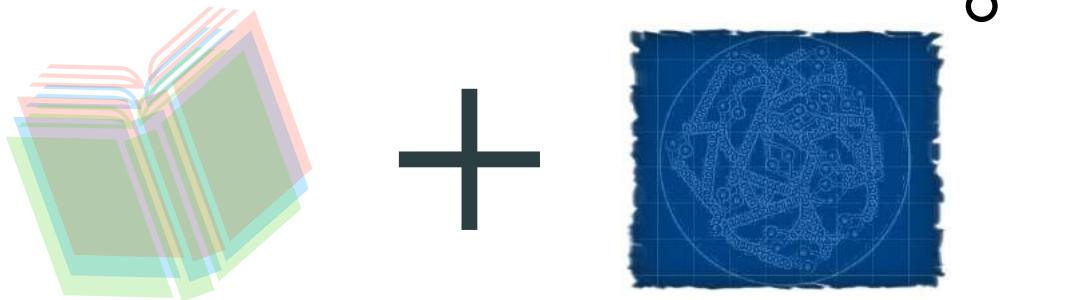
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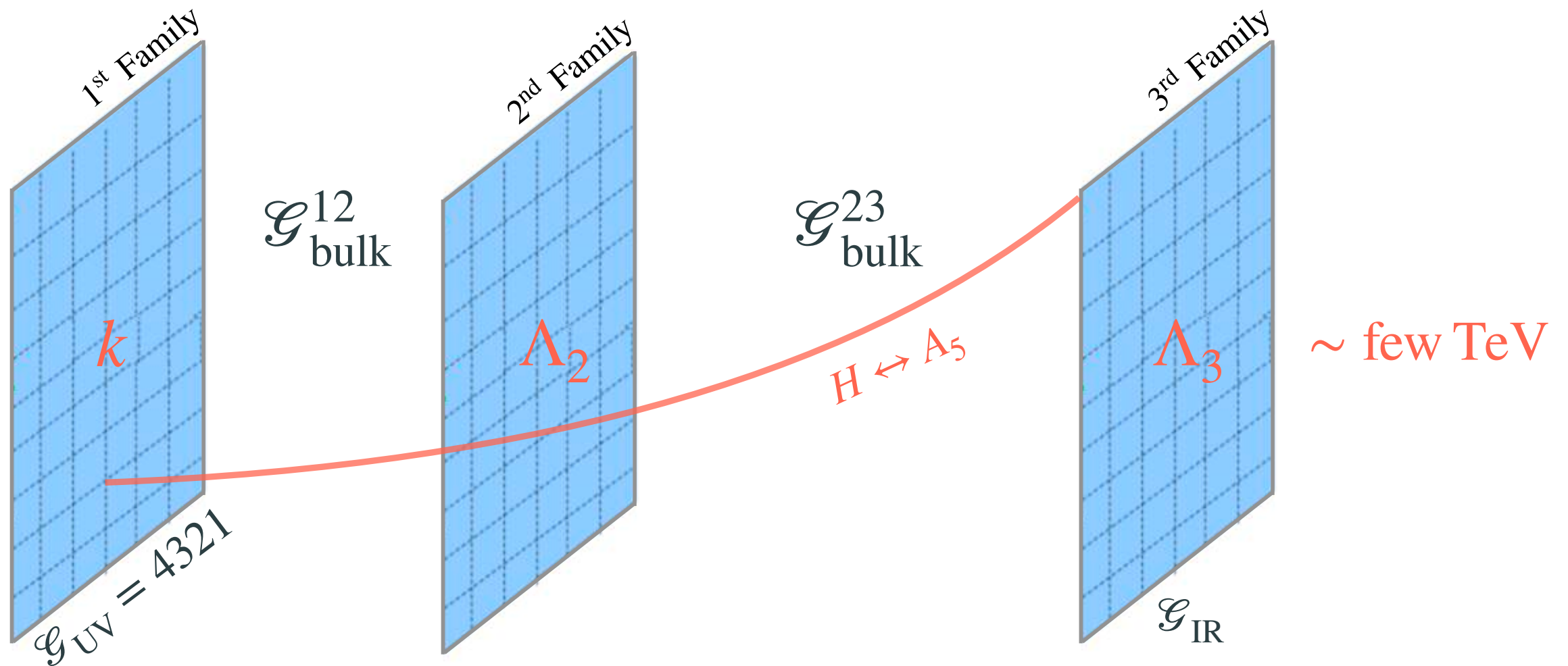
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↓ Λ_2 (6 broken)

$$\mathcal{G}_{\text{bulk}}^{23} = SU(4)_h \times SU(3)_l \times U(1)_l \times SO(5)$$

↓ $\Lambda_3 = \Lambda_{\text{IR}}$ (15 + 4 broken)

$$\mathcal{G}_{\text{IR}} = SU(3)_c \times U(1)_{B-L} \times SO(4)$$



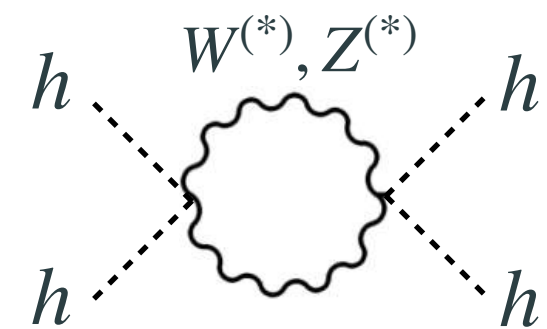
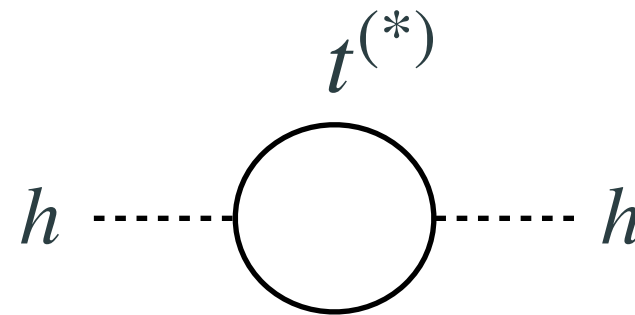
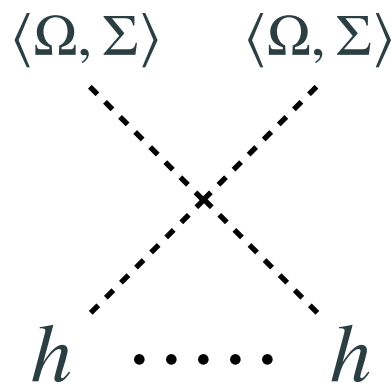
Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](#)

Higgs Potential

$$W(x) = e^{-i\theta(h(x)/f)}$$

- Potential is a function of the Wilson line.
- Tree-level contributions from the bulk scalars Σ, Ω that break $SO(5)$.
- 1-loop dominantly from the top and EW gauge bosons. Finite and fully calculable.

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
Ψ^3	4	1	4
Σ	1	1	5
Ω	1	4	4



$$V(h) \approx \alpha \cos\left(\frac{h}{f}\right) - \beta \sin^2\left(\frac{h}{f}\right)$$

\downarrow
 Ψ^3, Ω

\downarrow
 Ψ^3, Σ, W, Z

Higgs decay constant :

$$f = \frac{2\Lambda_{\text{IR}}}{g_*} \quad (M_{\text{KK}} = g_* f)$$

Little hierarchy and the B-anomalies

Our model connects the mass of the 4321 gauge bosons to fine-tuning in the Higgs potential.

$$\underline{U_1, G', Z' \text{ Masses :}} \quad M_{15} = \frac{M_{\text{KK}}}{\sqrt{2kL}} = \frac{g_* f}{\sqrt{2kL}} \quad (g_* \approx 2.5, kL \approx 10)$$

$$\text{Fine tuning : } \frac{v^2}{f^2}$$

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$$\text{Fine tuning : } \frac{v^2}{f^2}$$

Light U_1 leptoquark preferred to minimize fine-tuning!

*But also a light coloron- direct searches require $M_{15} \gtrsim 3.5 \text{ TeV}$ which translates to $f \gtrsim 6.4 \text{ TeV}$.

Little hierarchy and the B-anomalies

Our model connects the mass of the 4321 gauge bosons to fine-tuning in the Higgs potential.

$$\underline{U_1, G', Z' \text{ Masses :}} \quad M_{15} = \frac{M_{\text{KK}}}{\sqrt{2kL}} = \frac{g_* f}{\sqrt{2kL}} \quad (g_* \approx 2.5, kL \approx 10)$$

$$\text{Fine tuning : } \frac{v^2}{f^2}$$

Light U_1 leptoquark preferred to minimize fine-tuning!

*But also a light coloron- direct searches require $M_{15} \gtrsim 3.5 \text{ TeV}$ which translates to $f \gtrsim 6.4 \text{ TeV}$.

$$\text{Fine tuning : } \frac{v^2}{f^2} \approx 10^{-3}$$

(Could be improved slightly by splitting the 4321 gauge boson masses.)

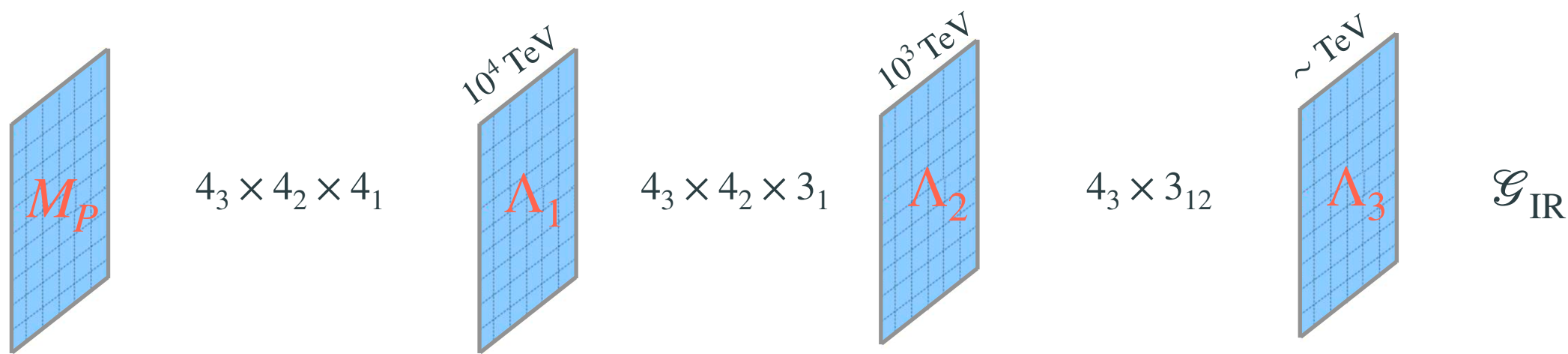
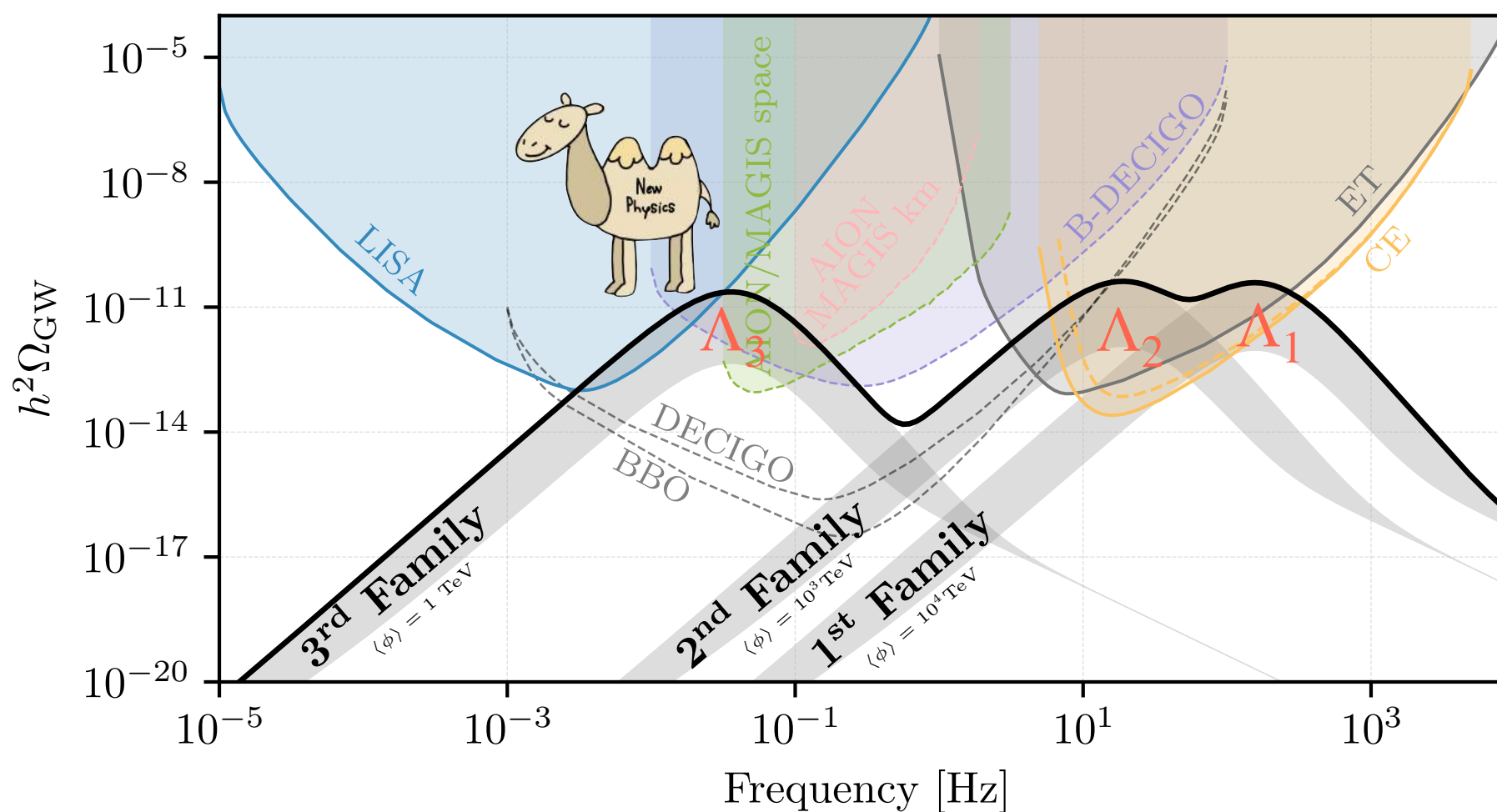
[Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](#)]

Conclusions

- ▶ We presented a model where **flavor hierarchies** naturally emerge **from a 3-brane structure** in a warped extra dimension, where each SM family is quasi-localized on a different brane.
- ▶ Our construction results in a $U(2)^n$ **flavor symmetry** with leading breaking in the left-handed sector.
- ▶ The **Higgs emerges as a pseudo-Nambu-Goldstone boson** from the same strong dynamics that breaks 4321 gauge symmetry.
- ▶ At low energies, **the model reduces to the 4321 model**, which is known to provide a **good explanation of the B -meson anomalies**.
- ▶ For more detail, ask me or see [2203.01952](#) and [2206.03096](#) .

Backup Slides

Extension to Planck and Cosmological Signatures



[Greljo, Opferkuch, BAS, [1910.02014](#)]

[Bordone, Cornella, Fuentes-Martin, Isidori [1712.01368](#)
Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](#)]

Higgs Potential

$$V(h) \approx \alpha \cos\left(\frac{h}{f}\right) - \beta \sin^2\left(\frac{h}{f}\right)$$

\downarrow
 Ψ^3, Ω

\downarrow
 Ψ^3, Σ, W, Z

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
Ψ^3	4	1	4
Σ	1	1	5
Ω	1	4	4

Higgs Mass :

$$m_h^2 \equiv 2\lambda \langle h \rangle^2 \approx \frac{2\beta \langle h \rangle^2}{f^4}$$

Higgs VEV ($\alpha \approx -2\beta$) :

$$\cos(\langle h \rangle / f) = -\frac{\alpha}{2\beta}$$

$$\lambda \approx \frac{1}{16\pi^2} \left[N_c y_t^4 \log \frac{\Lambda_{\text{IR}}^2}{m_t^2} - \frac{9}{32} \zeta(3) g_*^2 (3g_L^2 + g_Y^2) + \frac{\pi^2 g_*^4}{2(kL)^2} \frac{\langle \Sigma_{\text{IR}} \rangle^2}{\Lambda_{\text{IR}}^2} (\tilde{M}_{H'} - \tilde{M}_S) \right]$$



top loop



EW gauge loop



Σ tree

[Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](#)]

Higgs Potential

$$V(h) \approx \alpha \cos\left(\frac{h}{f}\right) - \beta \sin^2\left(\frac{h}{f}\right)$$

\downarrow
 Ψ^3, Ω

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 Ψ^3, Σ, W, Z

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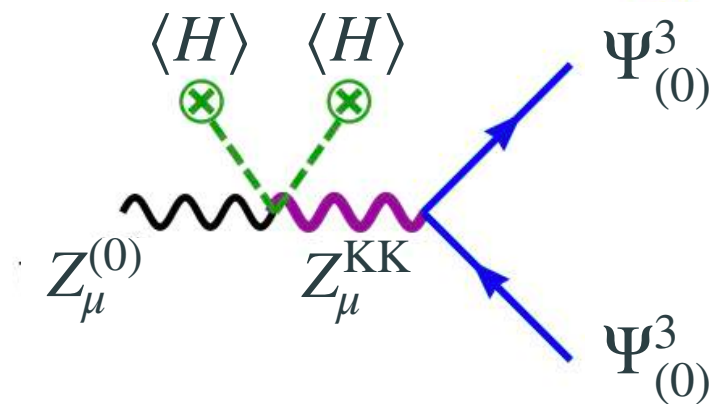
$$\cos(\langle h \rangle / f) = -\frac{\alpha}{2\beta}$$

$$\lambda \approx \frac{1}{16\pi^2} \left[N_c y_t^4 \log \frac{\Lambda_{\text{IR}}^2}{m_t^2} - \frac{9}{32} \zeta(3) g_*^2 (3g_L^2 + g_Y^2) + \frac{\pi^2 g_*^4}{2(kL)^2} \frac{\langle \Sigma_{\text{IR}} \rangle^2}{\Lambda_{\text{IR}}^2} (\tilde{M}_{H'} - \tilde{M}_S) \right]$$

Quartic of the right size for $g_* \approx 2.5$, also compatible with the top Yukawa.

Low-energy phenomenology, deviations from 4321

- Low energy pheno is the same as in the non-universal 4321 model.
- Strongest bound on the overall scale comes from coloron direct searches.
- Leading deviation in 3rd family EW vertex corrections from $\text{KK} \leftrightarrow \text{SM}$ mixing.



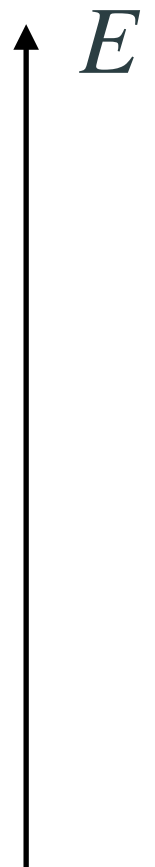
Benchmark Spectrum :

$$M_{\text{KK}} \approx 2\Lambda_{\text{IR}} = 16 \text{ TeV}$$

$$\Lambda_{\text{IR}} = 8 \text{ TeV}$$

$$f \approx \frac{M_{\text{KK}}}{g_*} \approx 6.4 \text{ TeV}$$

$$M_{15} \approx \frac{g_* f}{\sqrt{2kL}} \approx 3.6 \text{ TeV}$$



Leading effect in $Z \rightarrow \tau_L \tau_L$

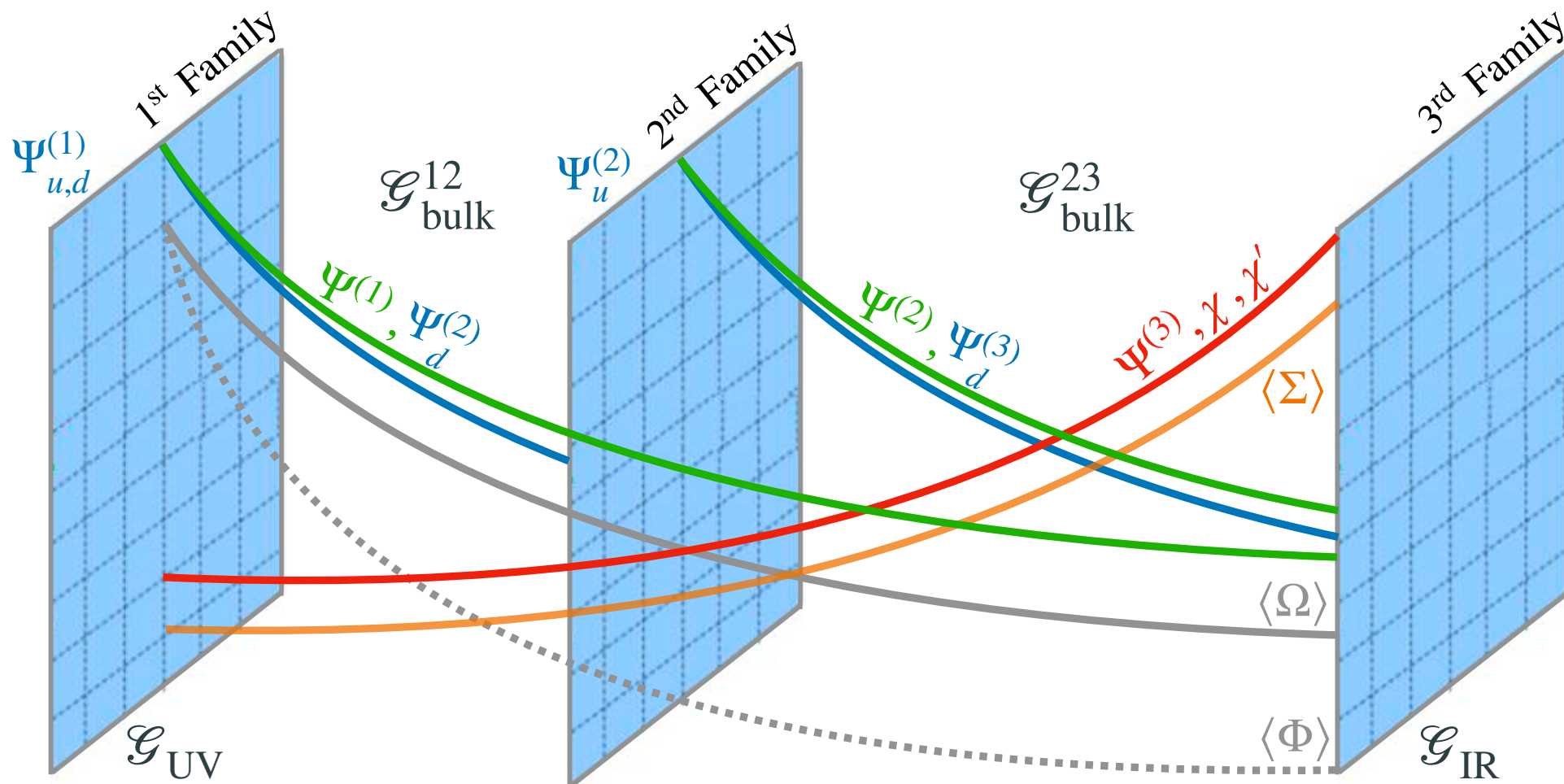
$$\frac{\delta g_{Z\Psi^3\Psi^3}}{g_{Z\Psi^3\Psi^3}} \approx -0.3 \frac{m_Z^2}{M_{\text{KK}}^2} \frac{g_*^2}{g_L^2} \approx -\frac{0.3}{4c_W^2} \frac{\langle h \rangle^2}{f^2} \lesssim 10^{-3} \quad [f > 2.5 \text{ TeV} \quad (M_{\text{KK}} > 6 \text{ TeV})]$$

[Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](#)]

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
$\Psi^3, \Psi_d^3, \mathcal{X}^{(l)}$	4	1	4
$\Psi^j, \Psi_{u,d}^j$	1	4	4
\mathcal{S}^i	1	1	1
Σ	1	1	5
Ω	1	4	4
Φ	1	1	1

$$\Psi^3 = \begin{bmatrix} \psi^3(+,+) \\ \psi_u^3(-,-) \\ \tilde{\psi}_d^3(+,-) \end{bmatrix}, \quad \Psi_d^3 = \begin{bmatrix} \tilde{\psi}^3(+,-) \\ \tilde{\psi}_u^3(+,-) \\ \psi_d^3(-,-) \end{bmatrix}, \quad \mathcal{X}^{(l)} = \begin{bmatrix} \chi^{(l)}(\pm,\pm) \\ \chi_u^{(l)}(\mp,\pm) \\ \chi_d^{(l)}(\mp,\pm) \end{bmatrix},$$

$$\Psi^j = \begin{bmatrix} \psi^j(+,+) \\ \tilde{\psi}_u^j(-,+) \\ \tilde{\psi}_d^j(-,+) \end{bmatrix}, \quad \Psi_u^j = \begin{bmatrix} \tilde{\psi}^j(+,-) \\ \psi_u^j(-,-) \\ \hat{\psi}_d^j(+,-) \end{bmatrix}, \quad \Psi_d^j = \begin{bmatrix} \hat{\psi}^j(+,-) \\ \hat{\psi}_u^j(+,-) \\ \psi_d^j(-,-) \end{bmatrix}.$$



IR Masses

$$\Psi^3 = \begin{bmatrix} \psi^3 (+, +) \\ \psi_u^3 (-, -) \\ \tilde{\psi}_d^3 (+, -) \end{bmatrix}, \quad \Psi_d^3 = \begin{bmatrix} \tilde{\psi}^3 (+, -) \\ \tilde{\psi}_u^3 (+, -) \\ \psi_d^3 (-, -) \end{bmatrix}, \quad \chi^{(\prime)} = \begin{bmatrix} \chi^{(\prime)} (\pm, \pm) \\ \chi_u^{(\prime)} (\mp, \pm) \\ \chi_d^{(\prime)} (\mp, \pm) \end{bmatrix},$$

$$\Psi^j = \begin{bmatrix} \psi^j (+, +) \\ \tilde{\psi}_u^j (-, +) \\ \tilde{\psi}_d^j (-, +) \end{bmatrix}, \quad \Psi_u^j = \begin{bmatrix} \tilde{\psi}^j (+, -) \\ \psi_u^j (-, -) \\ \hat{\psi}_d^j (+, -) \end{bmatrix}, \quad \Psi_d^j = \begin{bmatrix} \hat{\psi}^j (+, -) \\ \hat{\psi}_u^j (+, -) \\ \psi_d^j (-, -) \end{bmatrix}.$$

$$\mathcal{L}_{\text{IR}} \supset (\bar{\chi}_L \tilde{M}_\chi + \bar{\Psi}_L^3 \tilde{M}_\Psi + \bar{\Psi}_L^j \tilde{m}_\psi^j) \mathcal{P}_L \chi'_R,$$

$$\begin{aligned} \mathcal{L}_{\text{IR}} \supset & \bar{\Psi}_L^3 \tilde{M}_{\Psi_d}^L \mathcal{P}_L \Psi_{dR}^3 + \bar{\Psi}_L^j \tilde{m}_{\Psi_j}^R \mathcal{P}_R \Psi_R^3 + \bar{\chi}_L \tilde{M}_{\chi_u}^R \mathcal{P}_R \Psi_R^3 \\ & + \bar{\Psi}_L^j (\tilde{m}_{d_j}^L \mathcal{P}_L + \tilde{m}_{d_j}^R \mathcal{P}_R) \Psi_{dR}^3 + \bar{\chi}_L (\tilde{M}_{\chi_d}^L \mathcal{P}_L + \tilde{M}_{\chi_d}^R \mathcal{P}_R) \Psi_{dR}^3. \end{aligned}$$

Effective 4D “Holographic” Lagrangian

$$\mathcal{L}_{\text{IR}} \supset (\bar{\chi}_L \tilde{M}_\chi + \bar{\Psi}_L^3 \tilde{M}_\Psi + \bar{\Psi}_L^j \tilde{m}_\psi^j) \mathcal{P}_L \chi'_R,$$

$$\begin{aligned} \mathcal{L}_{\text{IR}} \supset & \bar{\Psi}_L^3 \tilde{M}_{\Psi d}^L \mathcal{P}_L \Psi_{dR}^3 + \bar{\Psi}_L^j \tilde{m}_{\Psi j}^R \mathcal{P}_R \Psi_R^3 + \bar{\chi}_L \tilde{M}_{\chi u}^R \mathcal{P}_R \Psi_R^3 \\ & + \bar{\Psi}_L^j (\tilde{m}_{dj}^L \mathcal{P}_L + \tilde{m}_{dj}^R \mathcal{P}_R) \Psi_{dR}^3 + \bar{\chi}_L (\tilde{M}_{\chi d}^L \mathcal{P}_L + \tilde{M}_{\chi d}^R \mathcal{P}_R) \Psi_{dR}^3. \end{aligned}$$



$$\begin{aligned} -\mathcal{L}_{4\text{D}} \supset & \frac{g_*}{2\sqrt{2}} \left[\bar{\psi}_L^3 - \bar{\chi}_L \tilde{M}_{\chi u}^R - c_j e^{-\frac{kz_j}{2}} \bar{\psi}_L^j \tilde{m}_{\Psi j}^R \right] \tilde{H} \psi_{uR}^3 + y_u^{ij} \bar{\psi}_L^i \tilde{H} \psi_{uR}^j + y_d^{ij} \bar{\psi}_L^i H \psi_{dR}^j \\ & + \frac{g_*}{2\sqrt{2}} c_2 e^{-\frac{kz_2}{2}} \left[\bar{\psi}_L^3 \tilde{M}_{\Psi d}^L + \bar{\chi}_L (\tilde{M}_{\chi d}^L - \tilde{M}_{\chi d}^R) + c_j e^{-\frac{kz_j}{2}} \bar{\psi}_L^j (\tilde{m}_{dj}^L - \tilde{m}_{dj}^R) \right] H \psi_{dR}^3 \\ & + \frac{\Lambda_{\text{IR}}}{\sqrt{kL}} \left[\bar{\psi}_L^3 \tilde{M}_\Psi + \bar{\chi}_L \tilde{M}_\chi + c_j e^{-\frac{kz_j}{2}} \bar{\psi}_L^j \tilde{m}_\psi^j \right] \chi'_R + \text{h.c.} \end{aligned}$$

Matching to 4321: $M_\chi = \tilde{M}_\chi \Lambda_{\text{IR}} / \sqrt{kL} \approx 2 \text{ TeV}, \quad y_t = g_* / 2\sqrt{2}, \quad y_+ = y_t \tilde{M}_{\chi u}^R, \quad \text{etc.}$

Holographic Dictionary: A 5D Model Builder's Guide



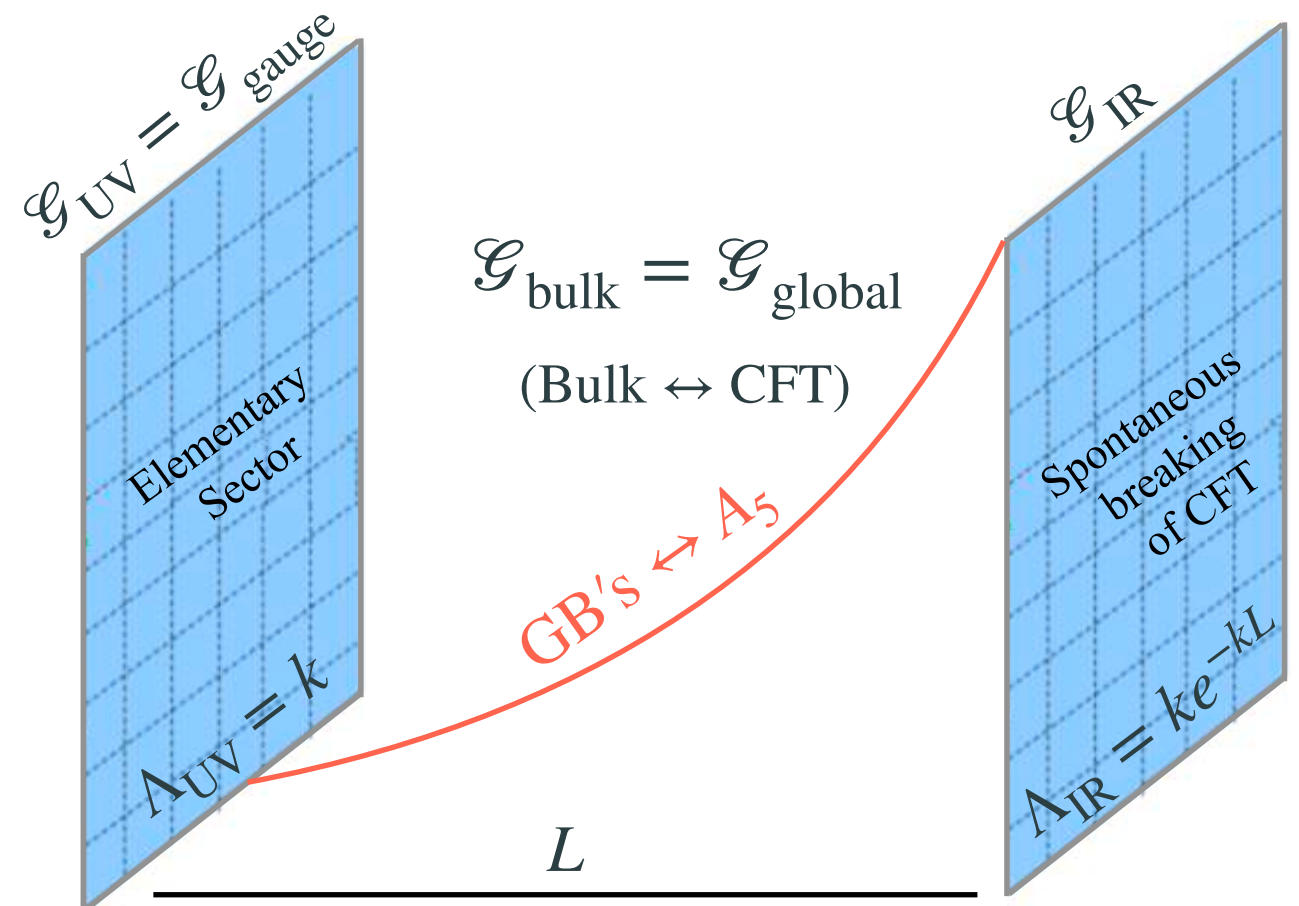
AdS/CFT correspondence relates a strongly coupled 4D CFT to a 5D theory with a warped fifth dimension.

Composite (4D CFT)

- Strongly coupled sector (CFT) with global symmetry $\mathcal{G}_{\text{global}}$
- Elementary sector with gauge symmetry $\mathcal{G}_{\text{gauge}} \subset \mathcal{G}_{\text{global}}$
- CFT becomes strongly coupled at some IR scale and spontaneously breaks $\mathcal{G}_{\text{global}} \rightarrow \mathcal{G}_{\text{IR}}$
- NGBs in the coset $\mathcal{G}_{\text{global}}/\mathcal{G}_{\text{IR}}$

Warped 5D (AdS₅)

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$



[Randall, Sundrum, [hep-ph/9905221](https://arxiv.org/abs/hep-ph/9905221)]

Gauge Sector

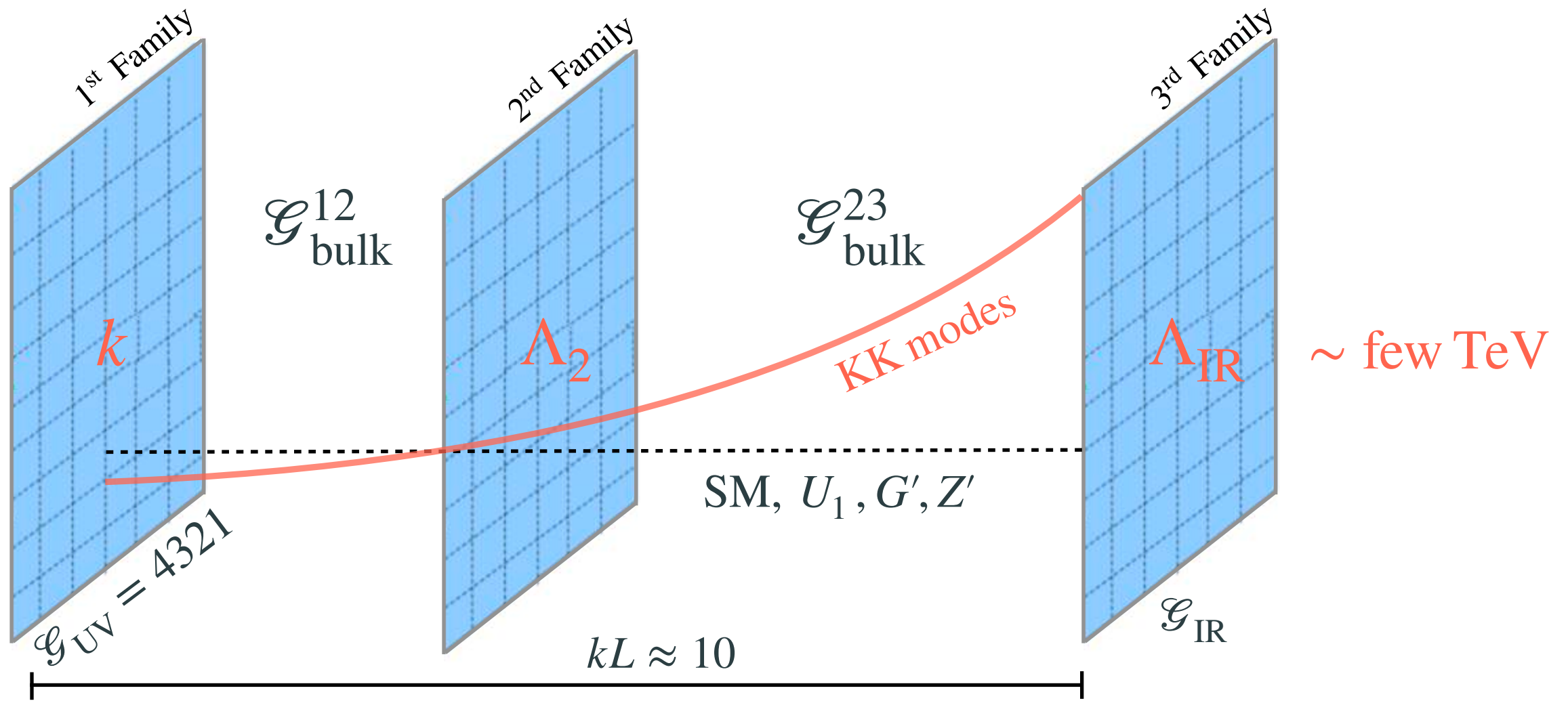
In breaking $\mathcal{G}_{\text{bulk}}^{23} \rightarrow \mathcal{G}_{\text{IR}}$, the 4321 gauge bosons acquire a mass of:

$$M_{15} = \sqrt{\frac{2}{kL}} \Lambda_{\text{IR}} = \frac{M_{\text{KK}}}{\sqrt{2kL}}$$

IR symmetry breaking

$$\mathcal{G}_{\text{bulk}}^{23} = SU(4)_h \times SU(3)_l \times U(1)_l \times SO(5)$$

\Downarrow $\Lambda_3 = \Lambda_{\text{IR}}$ (15 + 4 broken)

$$\mathcal{G}_{\text{IR}} = SU(3)_c \times U(1)_{B-L} \times SO(4)$$


[Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](#)]

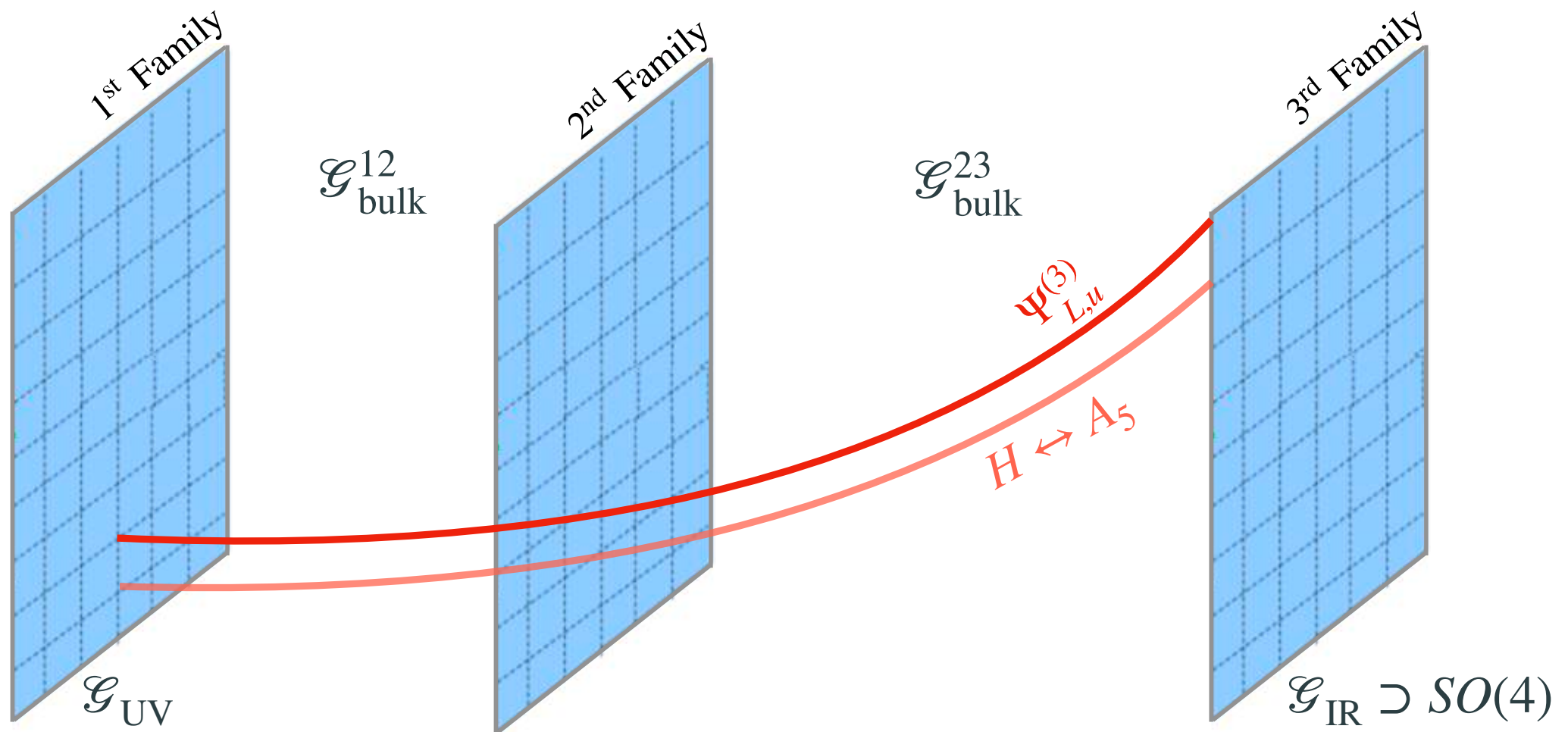
Gauge-Higgs Unification and the Top Yukawa

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
Ψ^3	4	1	4

$$\Psi^3 = \left[\begin{array}{l} \psi^3 (+, +) \\ \psi_u^3 (-, -) \\ \tilde{\psi}_d^3 (+, -) \end{array} \right] \left. \begin{array}{l} q_L \\ t_R \\ B_{L,R} \end{array} \right\} \begin{array}{l} SU(2)_L \\ \\ SU(2)_R \end{array}$$

Can remove H from the bulk due to the $SO(5)$ invariance:

$$W(x) = e^{-i\theta(x)}, \quad \theta(x) = g_5 \int_0^L dy A_5(x, y) = \frac{g_*}{\sqrt{2}} \frac{T^{\hat{a}} h^{\hat{a}}}{\Lambda_{\text{IR}}}$$



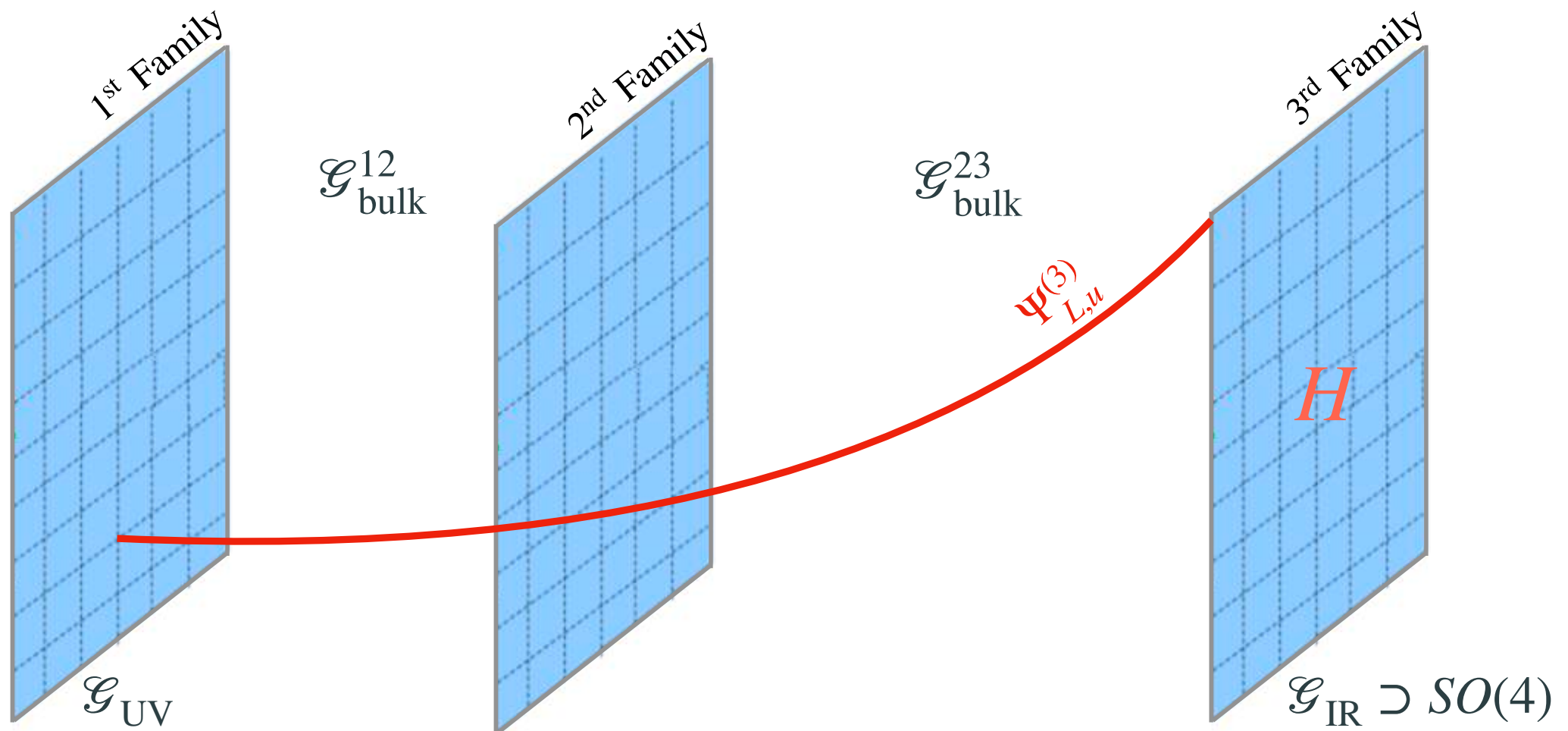
[Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](#)]

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Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
Ψ^3	4	1	4

$$\Psi^3 = e^{-i \frac{g_*}{\sqrt{2}} \frac{T^{\hat{a}} h^{\hat{a}}}{\Lambda_{\text{IR}}}} \begin{bmatrix} \psi^3 (+, +) \\ \psi_u^3 (-, -) \\ \tilde{\psi}_d^3 (+, -) \end{bmatrix} \left. \begin{array}{l} q_L \\ t_R \\ B_{L,R} \end{array} \right\} \begin{array}{l} SU(2)_L \\ \\ SU(2)_R \end{array}$$

$$\mathcal{L}_{4\text{D}} \supset -\frac{g_*}{2\sqrt{2}} \bar{\psi}_L^3 H \psi_{uR}^3 P(M_{\Psi^3}) \quad (g_*^2 = g_5^2 k) \quad \text{For } y_t : g_* \geq 2.2$$



[Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](#)]

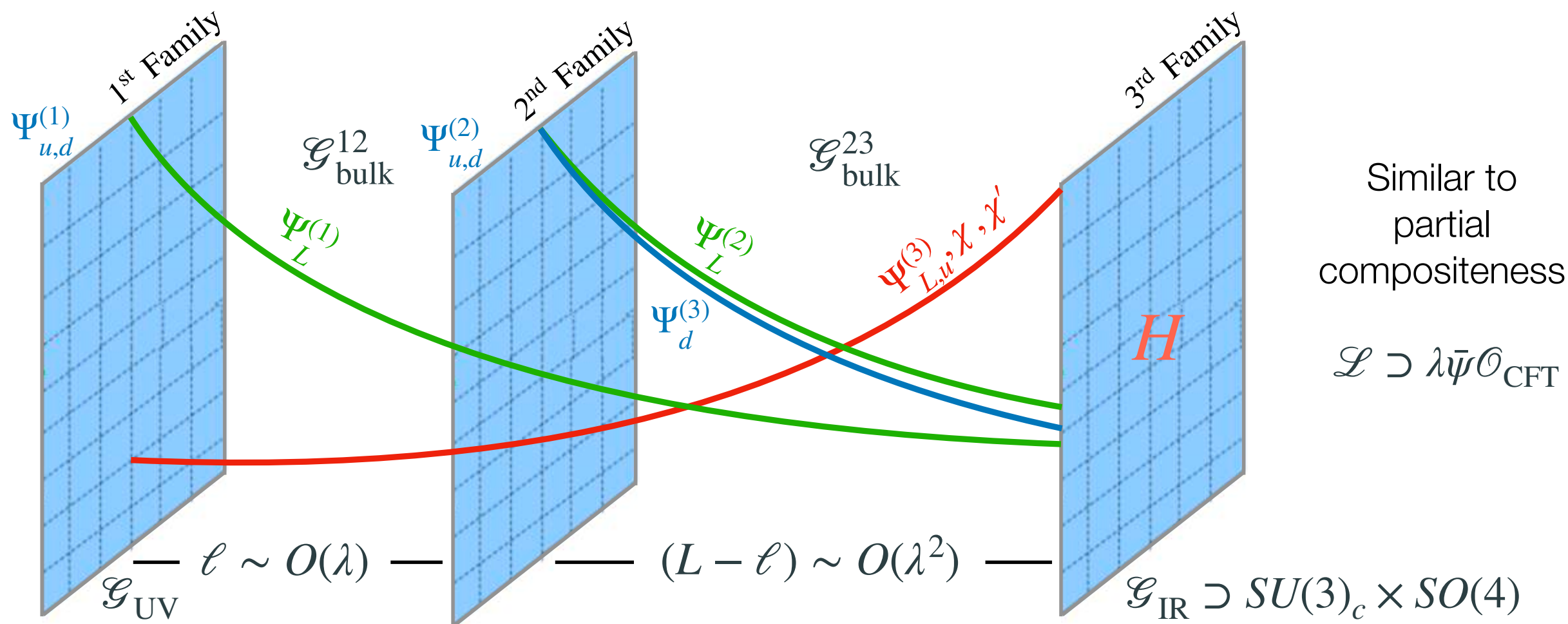
Light-Heavy Fermion Mixing in the IR

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
$\Psi^3, \Psi_d^3, \mathcal{X}^{(l)}$	4	1	4
$\Psi^j, \Psi_{u,d}^j$	1	4	4

- Mixing (Yukawa and VL) only occurs in the IR due to gauge symmetry (can be different for quarks and leptons).
- Results in a U(2) flavor symmetry with leading breaking in the **LH** sector.

$$y_{f_1 f_2} = \frac{g_*}{2\sqrt{2}} (\tilde{M}_{12}^L - \tilde{M}_{12}^R) P(M_{f_1}, M_{f_2})$$

$$\text{E.g. } P(M_{\Psi^j}, M_{\Psi^3}) \sim e^{-k(L-\ell_j)/2} \approx V_{j3}$$



[Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](#)]

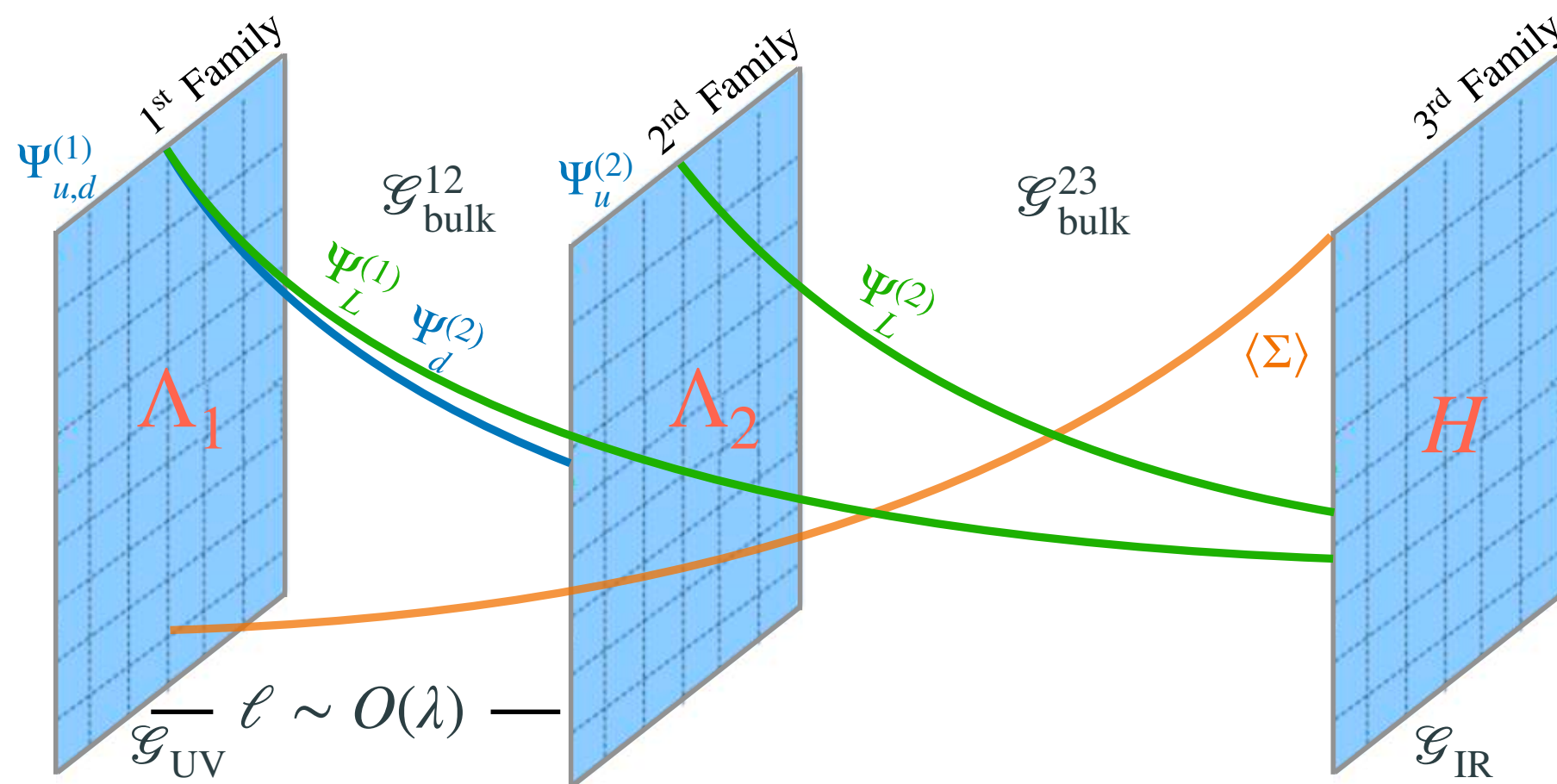
Light Family Yukawas in the UV

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
$\Psi^j, \Psi_{u,d}^j$	1	4	4
Σ	1	1	5

- Sigma $\Sigma^T \sim (H' \phi)$ takes a VEV along the singlet direction and propagates the breaking of $SO(5)$ into the bulk:

$$\mathcal{L}_{5D} \supset -Y_{u,d}^{ij} \bar{\Psi}^i \Sigma^a \Gamma^a P_R \Psi_{u,d}^j$$

$$y_{u,d}^{ij} \approx \frac{g_*}{2\sqrt{2}} \tilde{Y}_{u,d}^{ij} \frac{\langle \Sigma_{\text{IR}} \rangle}{\Lambda_{\text{IR}}} e^{-k(L-\ell_j)} e^{-k(c_i^{(1)} - \frac{1}{2})|y_i - \ell_j|} e^{k(c_j^{(1)} + \frac{1}{2})|y_j - \ell_j|}$$



Direct (irrelevant) coupling to a composite operator containing the Higgs

$$\mathcal{L} \sim \frac{1}{\Lambda} \bar{\psi}_L \mathcal{O}_H \psi_R$$

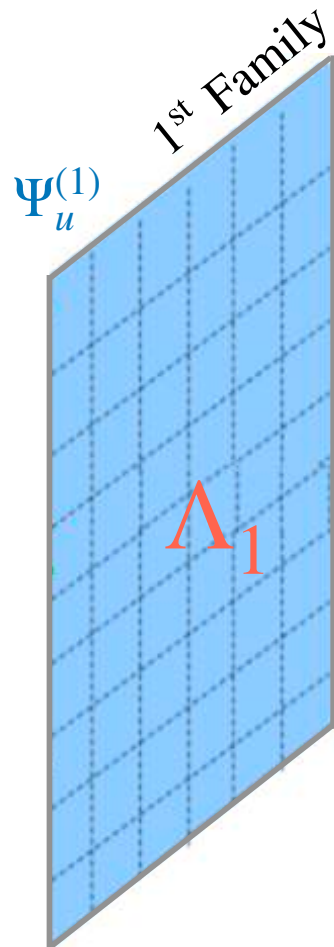
A Comment on Neutrino Masses

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
$\Psi^3, \Psi_d^3, \mathcal{X}^{(l)}$	4	1	4
$\Psi^j, \Psi_{u,d}^j$	1	4	4

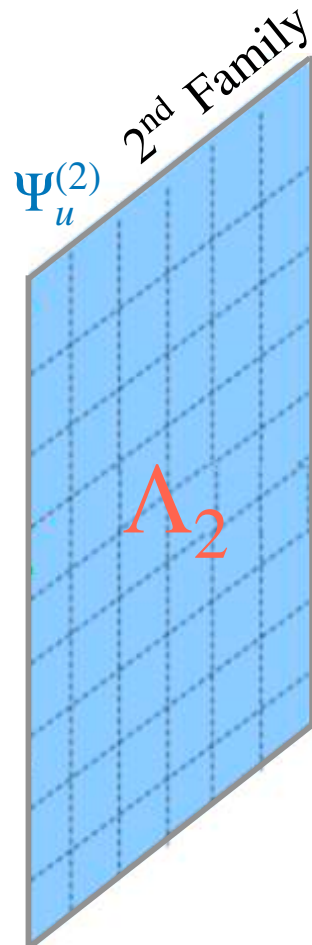
$$\Psi_u \sim \begin{pmatrix} u_R \\ \nu_R \end{pmatrix},$$

- Type 1 Seesaw would give:

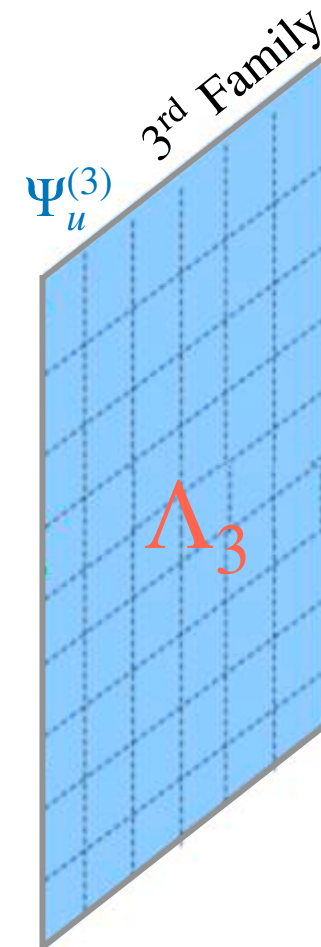
$$m_\nu^i \sim \frac{(M_u^i)^2}{M_R^i} \rightarrow \frac{(M_u^i)^2}{\Lambda_i}$$



10^4 TeV



10^3 TeV



\sim TeV

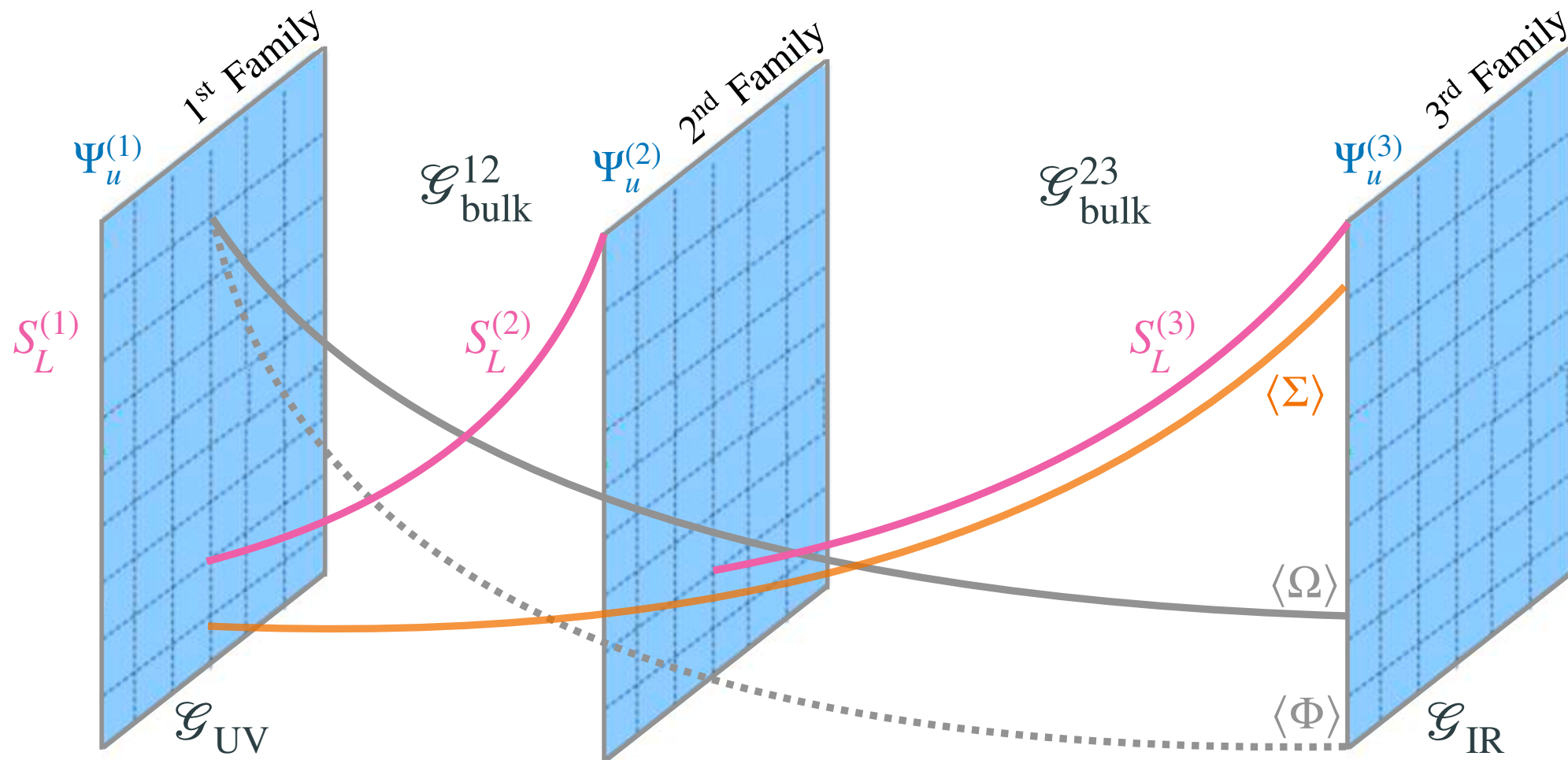
Neutrino Masses via an Inverse Seesaw Mechanism

ISS

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
$\Psi^3, \Psi_d^3, \chi^{(l)}$	4	1	4
$\Psi^j, \Psi_{u,d}^j$	1	4	4
\mathcal{S}^i	1	1	1
Ω	1	4	4
Φ	1	1	1

- Anarchic neutrino masses for:

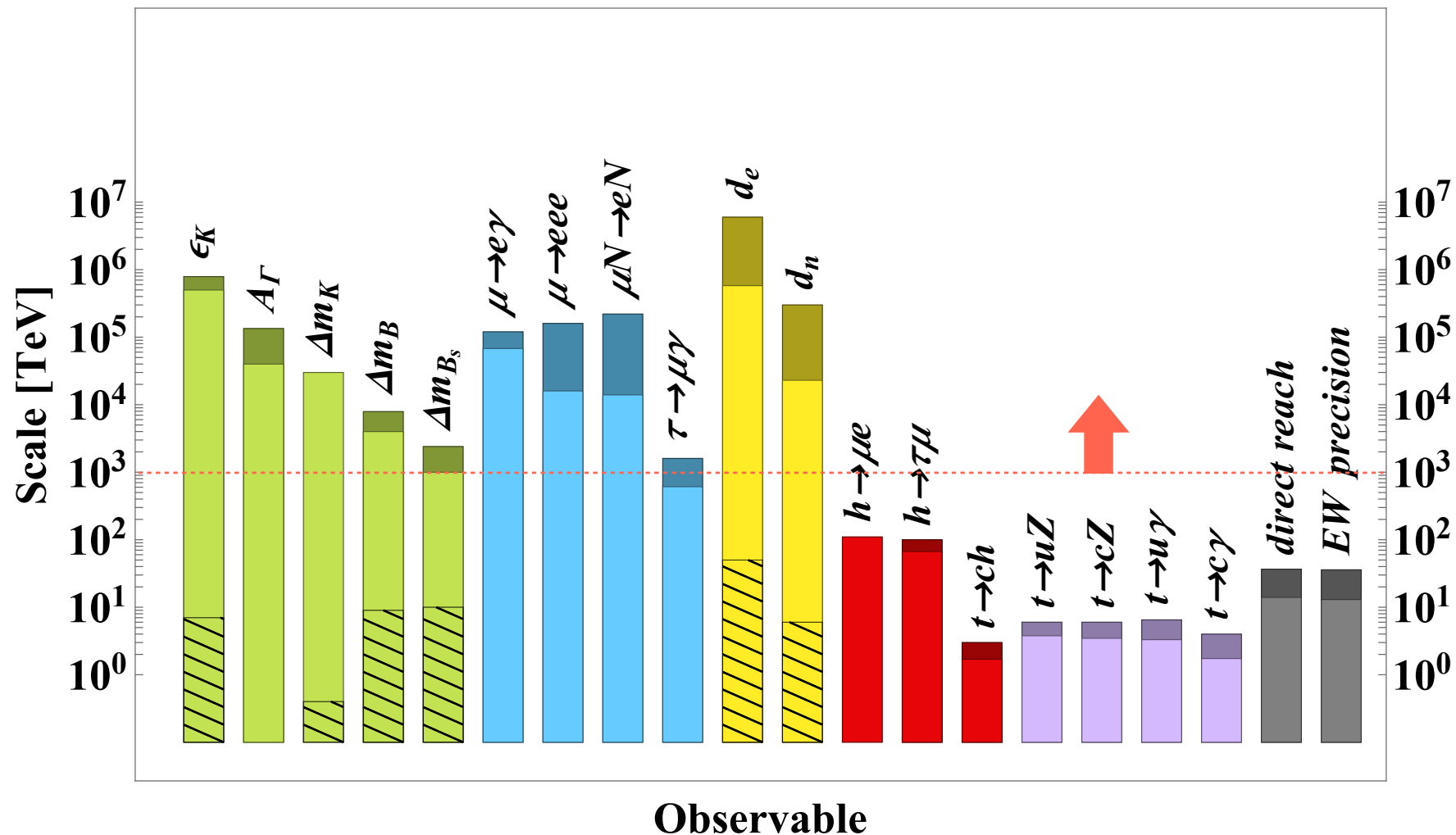
$$m_\nu^i = \left(\frac{M_u^i}{M_R^i} \right)^2 \mu_i \rightarrow \left(\frac{\langle \Sigma_i \rangle}{\langle \Omega_i \rangle} \right)^2 \langle \Phi_i \rangle \approx \text{const.}$$



[Fuentes-Martin, Isidori, Pages, BAS, [2012.10492](#)]

[Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](#)]

The Higgs Mass and the Flavor Puzzle

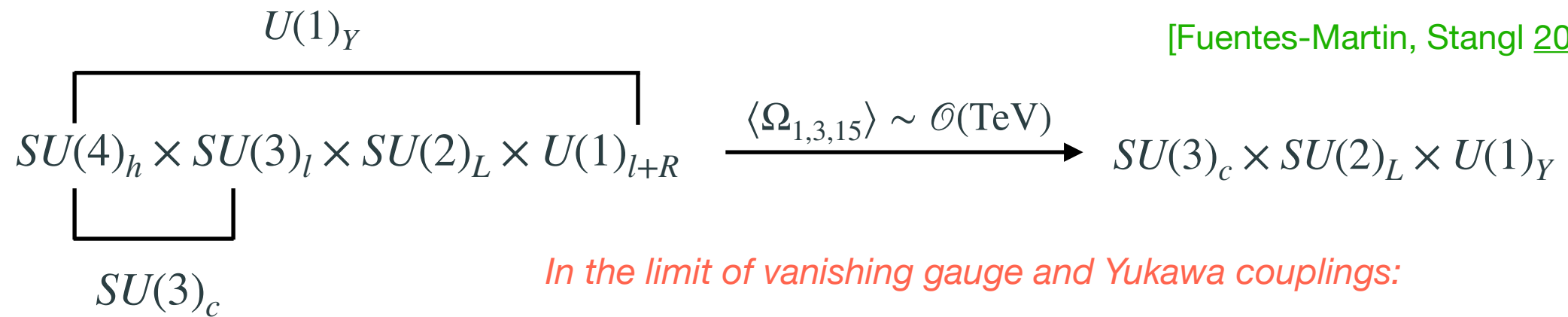


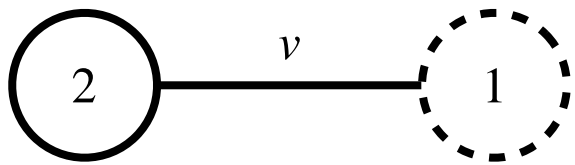
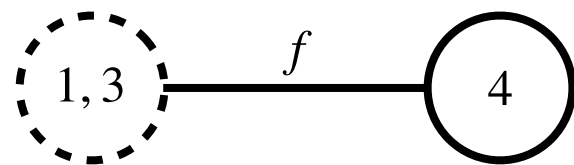
- Flavor bounds push the scale of flavor anarchic NP above 1000 TeV.
- NP that addresses the EW hierarchy problem should be light, but then it should have a very specific flavor structure to pass flavor bounds.
- Perhaps this same (flavored) NP sector is connected to the SM flavor puzzle...

[Physics Briefing Book 2020, [1910.11775](https://arxiv.org/abs/1910.11775)]

4321 symmetry breaking and EWSB: Parallels

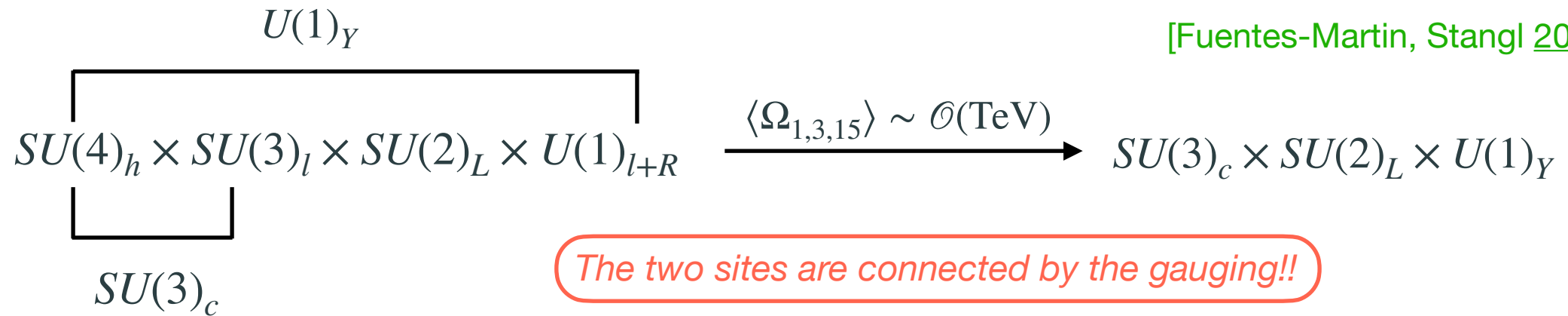
[Fuentes-Martin, Stangl 2004.11376]



	SM Higgs Sector	4321 Models
Global symmetry	$SU(2)_L \times SU(2)_R$	$SU(4)_l \times SU(4)_h$
Gauge symmetry	 <p style="text-align: center;">$SU(2)_L \times U(1)_R$</p> <p style="text-align: center;">Left-handed fermions Right-handed fermions</p>	 <p style="text-align: center;">$U(1)_l \times SU(3)_l \times SU(4)_h$</p> <p style="text-align: center;">Light fermions Heavy fermions</p>
Global SSB	$SU(2)_V$	$SU(4)_D$
Gauge SSB	$U(1)_V$	$U(1)_{B-L} \times SU(3)_c$
Goldstones	3 (3 eaten)	15 (15 eaten)

4321 symmetry breaking and EWSB: Parallels

[Fuentes-Martin, Stangl 2004.11376]



	SM Higgs Sector	4321 Models
Global symmetry	$SU(2)_L \times SU(2)_R$	$SU(4)_l \times SU(4)_h$
Gauge symmetry		
	$SU(2)_L$ × $U(1)_R$ × $U(1)_l \times SU(3)_l$ × $SU(4)_h$ Left-handed fermions Right-handed fermions Light fermions Heavy fermions	
Global SSB	$SU(2)_V$	$SU(4)_D$
Gauge SSB	$U(1)_{\text{em}} \times SU(3)_c$	
Goldstones	3 (W, Z)	15 (U_1, G', Z')

Higgs Potential

$$V(h) = \sum_r \frac{N_r}{16\pi^2} \int_0^\infty dp p^3 \log [\rho_r(-p^2)]$$

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
Ψ^3	4	1	4
Σ	1	1	5
Ω	1	4	4

$$V(h) \approx \alpha(h) \cos\left(\frac{h}{f}\right) - \beta(h) \sin^2\left(\frac{h}{f}\right)$$

VEV :

$$\alpha_\Omega \approx (\tilde{M}_\Omega^R - \tilde{M}_\Omega^L) \Lambda_{\text{IR}}^2 \langle \Omega_{\text{IR}} \rangle^2$$

$$\alpha_{\Psi^3}(h) \approx \frac{3N_c f^4}{32\pi^2} \zeta(3) y_t^2 g_*^2 - 2\beta_{\Psi^3}(h)$$

$$\cos(\langle h \rangle / f) = -\frac{\alpha}{2\beta}$$

Quartic :

$$\beta_\Sigma \approx \frac{1}{2} (\tilde{M}_{H'} - \tilde{M}_S) \frac{\Lambda_{\text{IR}}^2}{(kL)^2} \langle \Sigma_{\text{IR}} \rangle^2$$

$$\beta_{\Psi^3}(h) \approx \frac{N_c f^4}{16\pi^2} y_t^4 \left[\gamma + \log \frac{\Lambda_{\text{IR}}^2}{m_t^2(h)} \right]$$

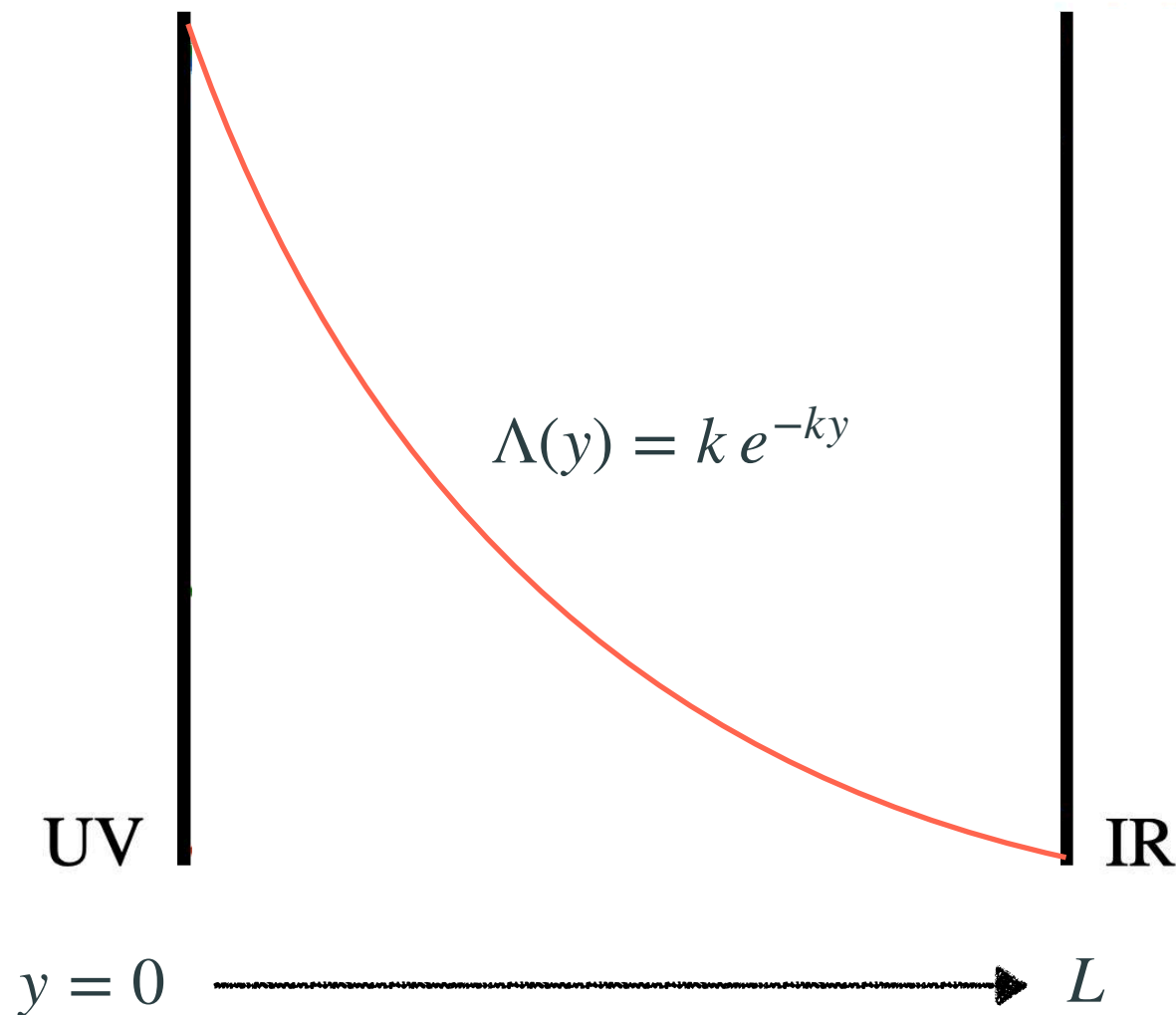
$$\beta_{\text{EW}} \approx -\frac{9f^4}{512\pi^2} g_*^2 \zeta(3) (3g_L^2 + g_Y^2)$$

[Fuentes-Martin, Isidori, Lizana, Selimovic, BAS, [2203.01952](#)]

5D Basics: Warping and Fermion Zero Modes

Warped Geometry (AdS₅)

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$



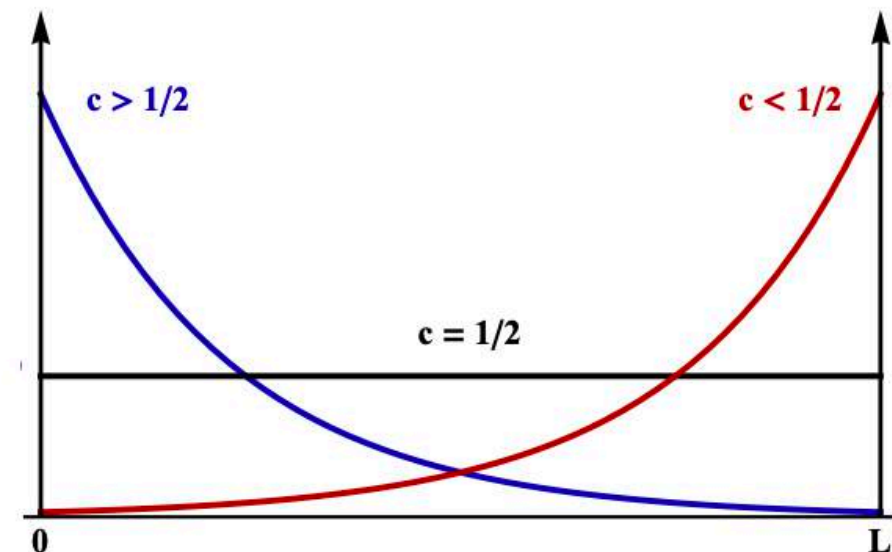
* $\Lambda_{\text{IR}} / \Lambda_{\text{UV}} = e^{-kL}$ Planck hierarchy
solved for $kL \approx 37$

KK Decomposition

$$X_{5\text{D}}(x^\mu, y) = \frac{1}{\sqrt{L}} \sum_{n=0}^{\infty} f_n(y) X_n(x^\mu)$$

For fermion zero modes:

$$f_{(0)}^{\Psi_{L,R}}(y) = N_{L,R} \exp \left[\left(\frac{1}{2} \mp c_{L,R} \right) ky \right], \quad c_{L,R} = \frac{M_{L,R}}{k}$$



*4D couplings determined by profile function overlap