Using SELCIE to investigate screened scalar field models sourced by complex systems

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Is dark energy a scalar field?

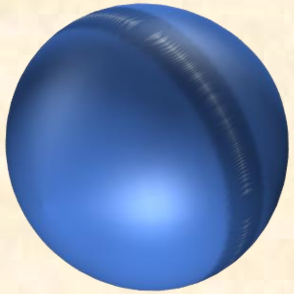
- Currently contributes ~70% of the energy content of the universe.
- Dominates in late time, leading to an accelerating expansion rate.
- One possible explanation is a scalar field (referred to as quintessence).
- Scalar fields coupled to matter are heavily constrained by fifth force experiments.

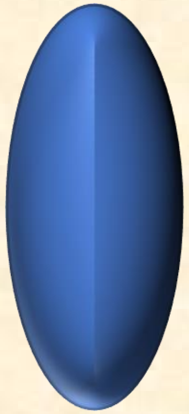
Chameleon Mechanism

- We will assume a field potential of the form: $$V(\phi) = \Lambda^4 \left(1 + \left(\frac{\Lambda}{\phi}\right)^n\right)$$
- The static field equation is therefore: $$\nabla^2 \phi = -\frac{n\Lambda^{n+4}}{\phi^{n+1}} + \frac{\beta\rho}{M_{pl}}$$
- The field value that minimises the potential is then: $$\phi_{min} = \left(\frac{n\Lambda^{n+4} M_{pl}}{\beta\rho}\right)^{1/(n+1)}$$
- The corresponding a Compton wavelength of: $$\lambda^2 = \frac{(n\Lambda^{n+4})^{1/(n+1)}}{(n+1)} \left(\frac{\beta\rho}{M_{pl}}\right)^{-\frac{n+2}{n+1}}$$
- We see as ρ increases λ deceases leading to the field being screened.

Approximate Analytic Solutions

- These solutions are for highly symmetrical systems such as:
  - Spheres - $\phi(r) \approx \phi_0 - \left(\frac{3}{4\pi M}\right)\left(\frac{\Delta R}{R}\right)\frac{M_c e^{-m_0(r-R)}}{r}.$
  - Cylinders - $\phi(r) \approx \phi_0 - \frac{\rho_c R^2}{2M}\left(1 - \frac{S^2}{R^2}\right) K_0(m_0 r).$
  - Ellipses - $\phi(\xi, \eta) \approx \phi_0 \left(1 - \frac{Q_0(\xi) - P_2(\eta) Q_2(\xi)}{Q_0(\xi_0)}\right).$





References:

- arXiv:astro-ph/0309411
- arXiv:1408.1409
- arXiv:1412.6373

What is SELCIE?

- SELCIE (Screening Equations Linearly Constructed and Iteratively Evaluated) is a python package designed to solve the chameleon field equations for arbitrary systems.

- It does this in two parts:
  - Mesh generation using the GMSH software.
  - Solves the field equations using FEniCS software.

Rescaled Chameleon equation

- We rescale the values in the equation in units of their vacuum values:

$$\hat{\rho} = \rho/\rho_0 \qquad \hat{\phi} = \phi/\phi_0 \qquad \hat{\nabla} = L\nabla \qquad \alpha = \frac{M_{pl}\phi_0}{\beta L^2 \rho_0}$$

- Rescaled field equation is: $\alpha \hat{\nabla}^2 \hat{\phi} = -\hat{\phi}^{-(n+1)} + \hat{\rho}$

- A nice way to interpret α is to consider its relation to the rescaled Compton wavelength:

$$\hat{\lambda}^2 = \frac{\alpha}{(n+1)} \hat{\rho}^{-\frac{(n+2)}{(n+1)}}$$

Finite Element Method (FEM)

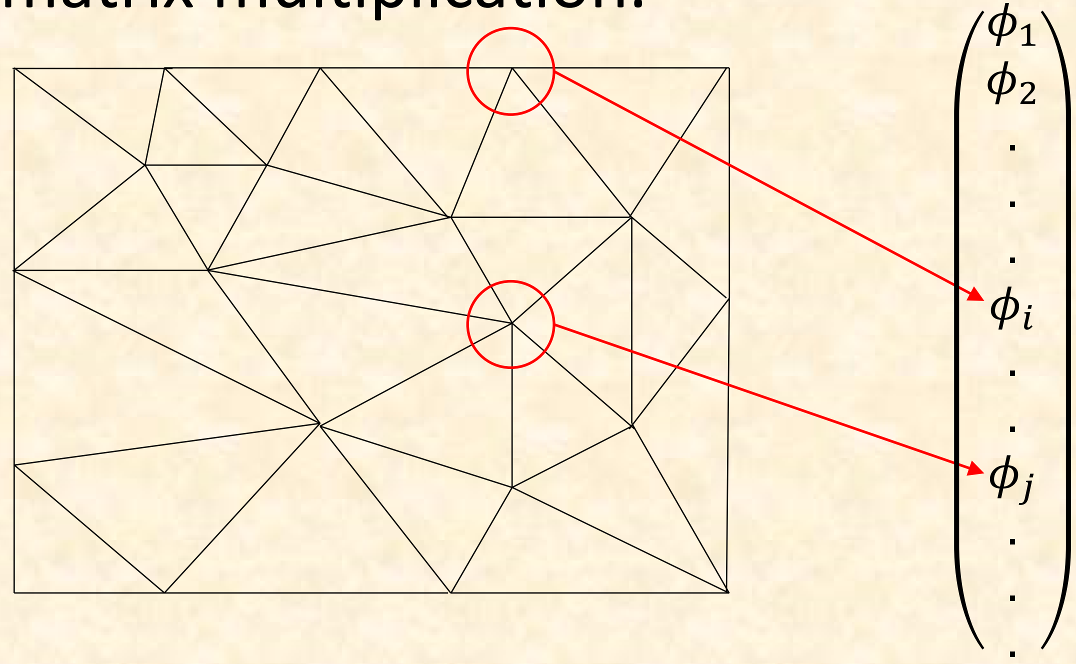
- To solve equations of the form $\nabla^2 u(x) = f(x)$, use Green's function (with zero field gradient at the boundary) and discretise the field. The problem can then described by a linear matrix multiplication.

$$\int_\Omega \nabla u \cdot \nabla v_j dx = \int_\Omega f(x) v_j dx$$

$$u(x) = \sum_i U_i e_i(x)$$

$$\sum_i \left(\int_\Omega \nabla e_i \cdot \nabla v_j dx \right) U_i = \int_\Omega f(x) v_j dx$$

$$MU = F$$



Solving the equations (Picard method)

- Expand the nonlinear part around $\hat{\phi}_k$:

$$\hat{\phi}^{-(n+1)} = \hat{\phi}_k^{-(n+1)} - (n+1)\hat{\phi}_k^{-(n+2)}(\phi - \phi_k) + \mathcal{O}(\phi - \phi_k)^2$$

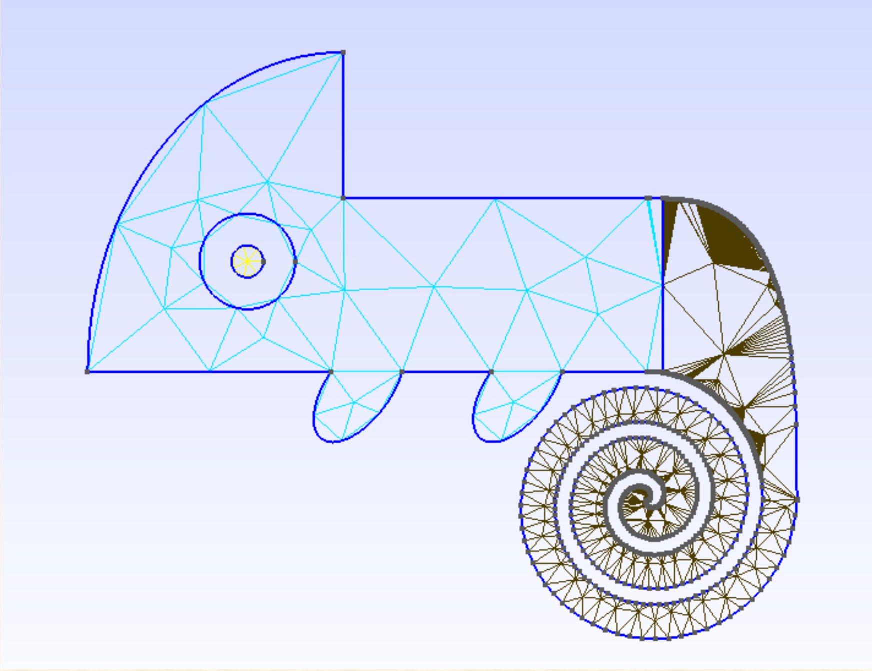
$$\hat{\phi}^{-(n+1)} \approx (n+2)\hat{\phi}_k^{-(n+1)} - (n+1)\hat{\phi}_k^{-(n+1)}\hat{\phi}$$

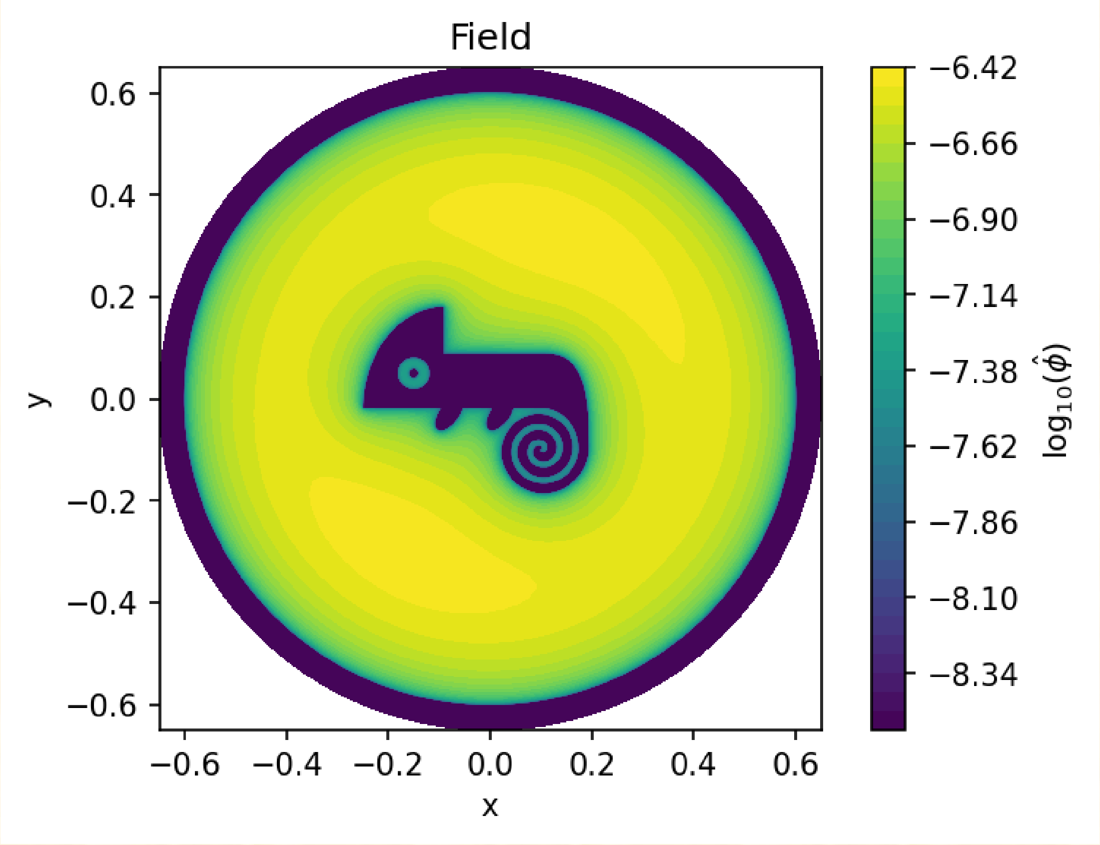
- We then solve for $\hat{\phi}$ around some $\hat{\phi}_k$.

- Perform update $\hat{\phi}_{k+1} = \omega\hat{\phi} + (1-\omega)\hat{\phi}_k$, where $0 < \omega \leq 1$.

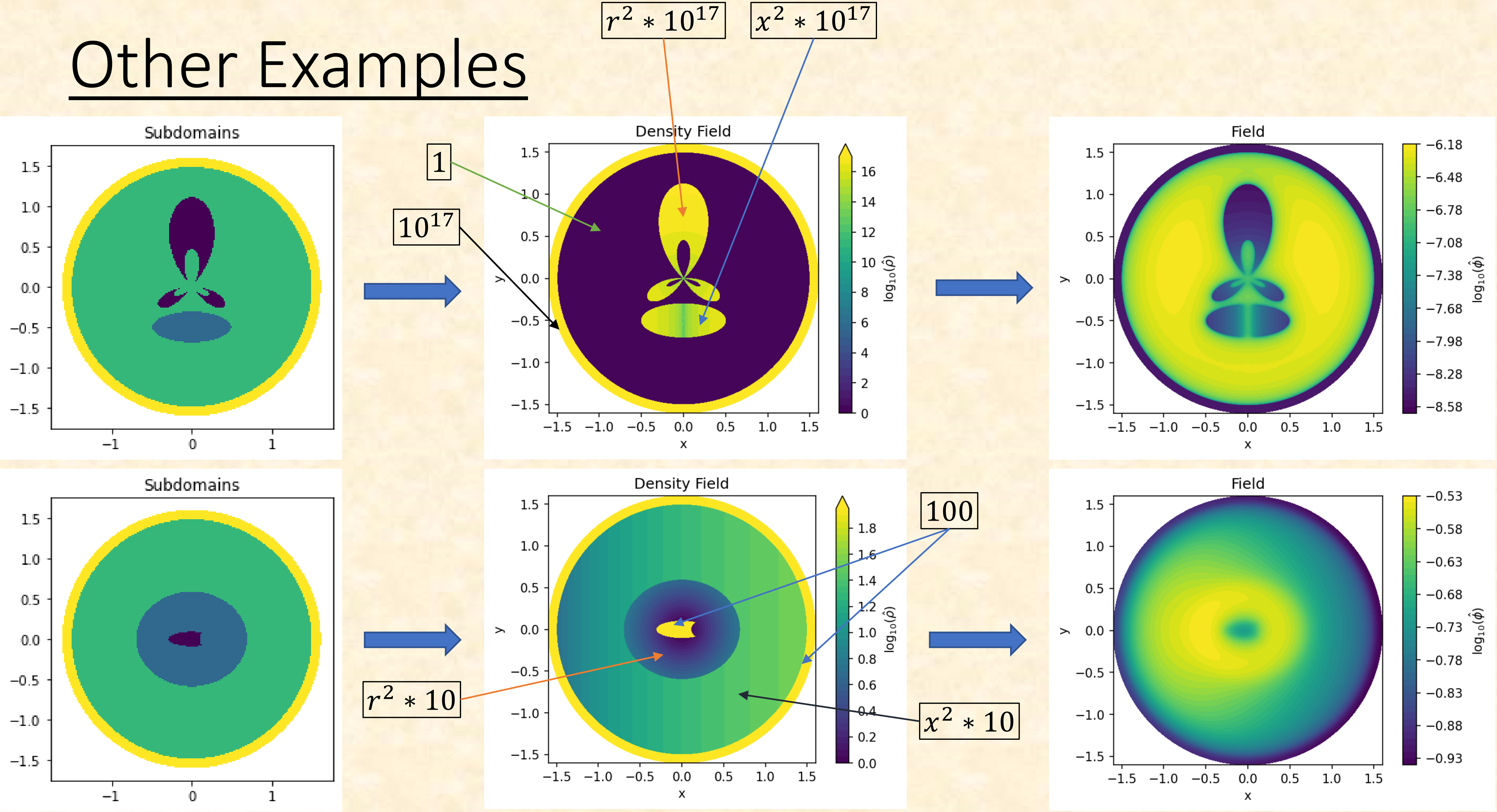
- Repeat from first step until convergence.

The chameleon of a chameleon





Other Examples



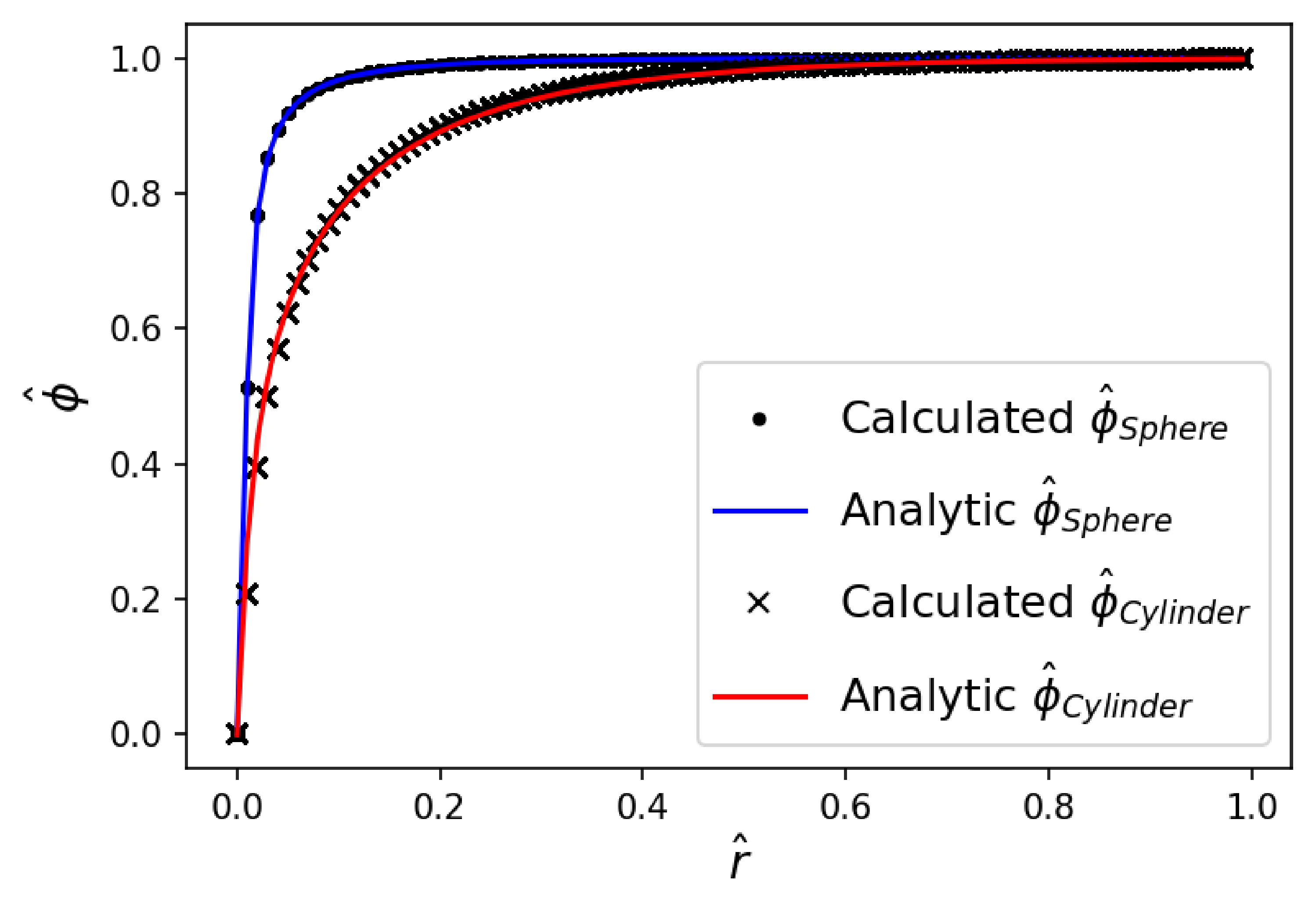
Test – Sphere & Cylinder

Sphere

$$\hat{\phi}(\hat{r}) \approx 1 - \frac{\hat{R}}{\hat{r}} e^{-(\hat{r}-\hat{R})\sqrt{\frac{(n+1)}{\alpha}}}$$

Cylinder

$$\hat{\phi}(\hat{r}) \approx 1 - \frac{\mathcal{K}_0\left(\hat{r}\sqrt{\frac{(n+1)}{\alpha}}\right)}{ln\left(\frac{4\alpha}{(n+1)\hat{R}}\right)}$$



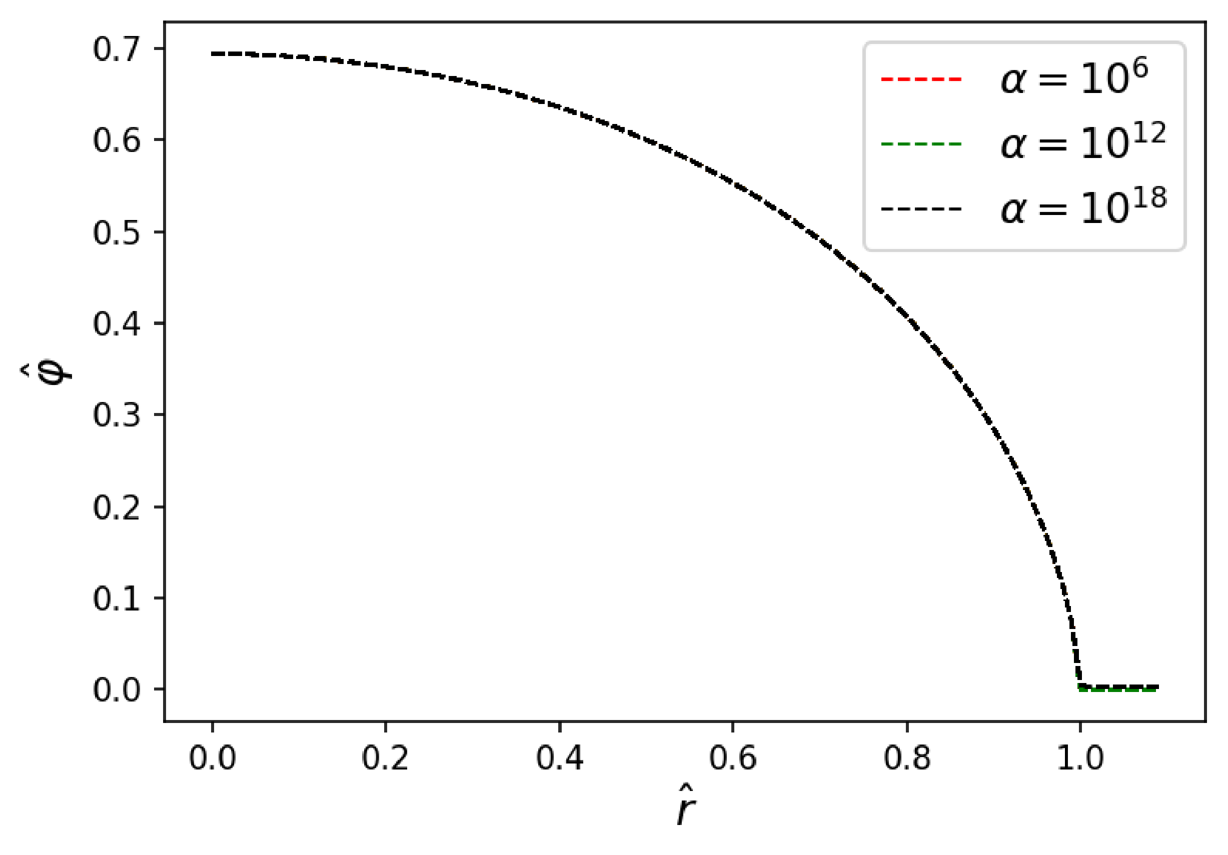
Test – Empty vacuum chamber

- If field's Compton wavelength is larger than the domain, then it will not reach its maximum inside an empty chamber.

- Therefore: $$\alpha\hat{\nabla}^2\hat{\phi} = -\hat{\phi}^{-(n+1)} + \hat{\rho} \approx -\hat{\phi}^{-(n+1)}$$

- E.O.M. is independent of α for:

$$\hat{\varphi} = \alpha^{1/(n+2)}\hat{\phi} \;\Rightarrow\; \hat{\nabla}^2\hat{\varphi} \approx -\hat{\varphi}^{-(n+1)}$$

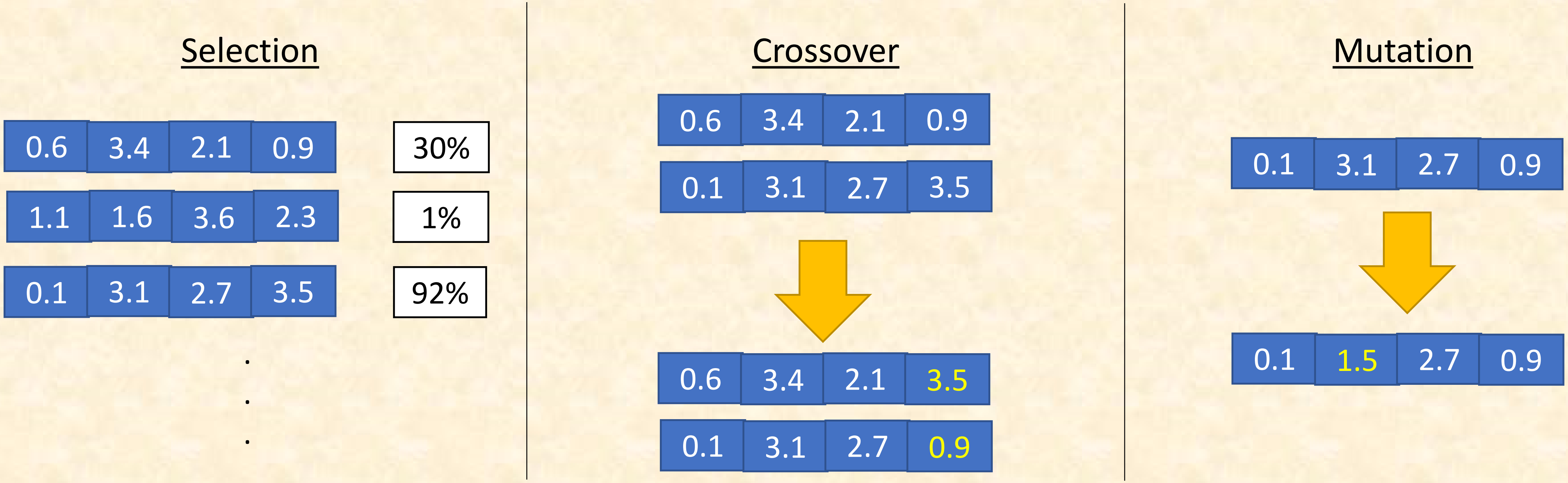


Ref: [1408.1409.pdf (arxiv.org)]

Genetic Algorithm

- A minimising/maximising algorithm based on organic evolution.

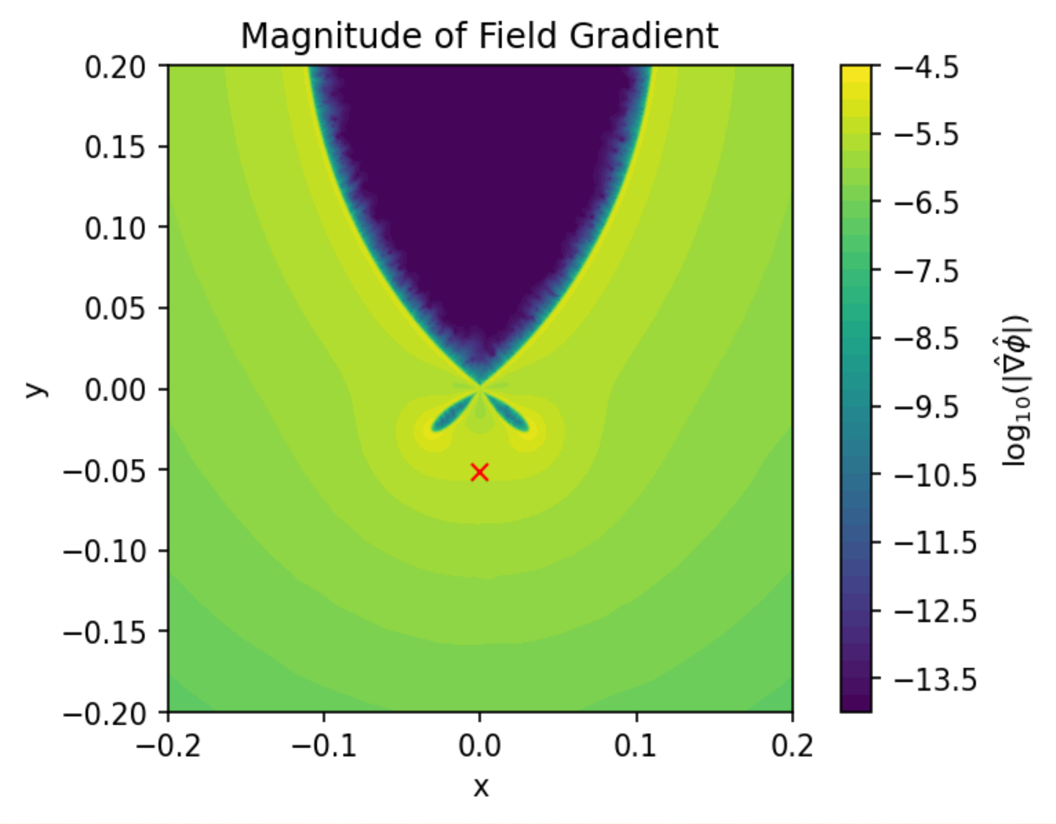
- Consists of 3 part:

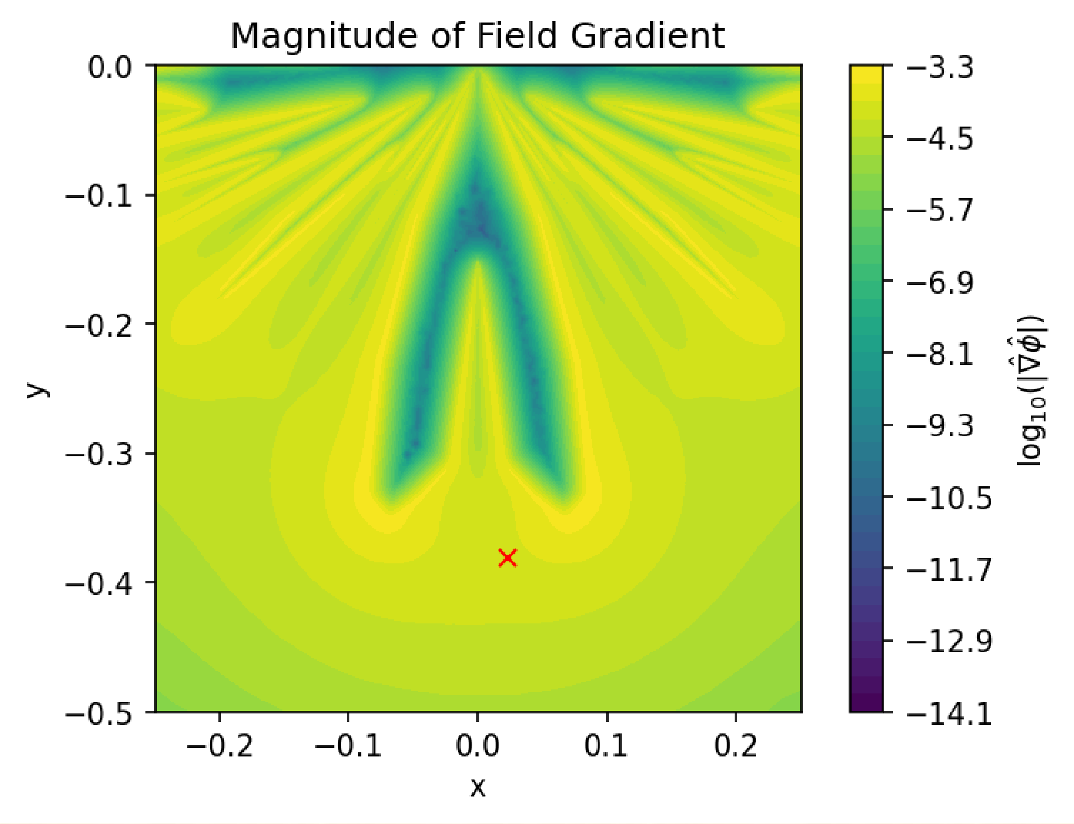


Some Results

$$R(\theta) = \sum_{n=0}^{N} a_n P_n(\cos(\theta))$$

$$R(\theta) = \sum_{n=0}^{N} R_n e_i(\theta)$$





Current/Future works

- Investigate general trends between classes of shapes.

- Introduce Neumann boundary conditions

- Developing a symmetron version (possibility of making methodology work for other models).

- Working to add time-dependence/dynamic meshes.

Thank you for listening

ArXiv: arXiv:2110.11917, arXiv:2206.06480 , arXiv:2108.10364
Github: GitHub - C-Briddon/SELCIE