String Theory on Time-Dependent Backgrounds: a Big Bang type singularity

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Mostly based on [AA, Riccardo Finotello, Igor Pesando (2020)] and ongoing work

Talk Outline

- Introduction and Motivations
- The Null Boost Orbifold
- Failure of Perturbation Theory
- Background *B*-field and Seiberg-Witten map
- Conclusions and Outlooks

The Cosmological Point of View

- The **inflationary scenario** is the current paradigm of early universe cosmology, as it is a successful solution to several problems (flatness, horizon, structure formation…), but it doesn't address the singularities matter

- **Bouncing cosmologies**, like the **Big Crunch/Big Bang** scenario or the Ekpyrotic (cyclic) scenario, are in a certain way more ambitious as they admit cosmological singularities

- There are also non trivial conceptual issues: is the Big Bang the beginning of time? If there is a bounce, how are the information before and after the singularity related? How do we define observables and the S-matrix?

- Besides the "origin of the universe" argument, **spacetime singularities** are anyway a prediction of General Relativity where a theory of quantum gravity is needed

The String Theory Point of View

- The study of string theory on **time-dependent backgrounds**, despite being widely addressed in the literature, has never proved completely successful
- One of the most challenging tasks in this context concerns precisely the stringy description of **cosmological singularities**, which is ultimately expected from a theory of quantum gravity.
- We want to construct toy models capable of reproducing a spacelike or lightlike singularity which appears in a certain region of space at a specific value of the time coordinate and then disappears

The Null Boost Orbifold (NBO)

The NBO is defined on $(\mathbb{M}^3/\Gamma) \otimes \mathbb{R}^{D-3}$ where Γ is the subgroup of $ISO(2,1)$ whose generator in lightcone coordinates is the Poincaré Killing vector $k = 2\pi i \Delta J_{+2}$.

Spacetime points are therefore identified as

$$x = \begin{pmatrix} x^- \\ x^2 \\ x^+ \\ \vec{x} \end{pmatrix} \sim e^{nk} x = \begin{pmatrix} x^- \\ x^2 + n(2\pi\Delta)x^- \\ x^+ + n(2\pi\Delta)x^2 + \frac{1}{2}n^2(2\pi\Delta)^2 x^- \\ \vec{x} \end{pmatrix}, \quad n \in \mathbb{Z}.$$

Under the change of coordinates

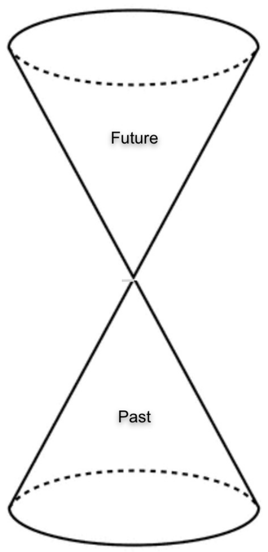
$$\begin{cases} x^- &= u \\ x^2 &= \Delta u z \\ x^+ &= v + \frac{1}{2}\Delta^2 u z^2 \end{cases},$$

the new metric reads

$$ds^2 = -2\, du\, dv + (\Delta u)^2 (dz)^2 + d\vec{x}^{\,2},$$

while the new identifications are

$$(u, v, z, \vec{x}) \sim (u, v, z + 2\pi n, \vec{x}).$$



Scattering Amplitudes and Divergences: Closed Strings

Among the various issues which we can encounter when dealing with strings on time-dependent backgrounds, the most urgent of this model concerns **interactions**.

It was shown that the four tachyons **closed string** $2 \rightarrow 2$ tree-level scattering amplitude, the analogous of **Virasoro-Shapiro** amplitude in flat spacetime, exhibits unusual divergences in a specific kinematical regime (the Regge limit of high energy and small fixed angle):

$$A_{4T}^{closed} \sim \int^{q\sim\infty} \frac{dq}{|q|} q^{4-\alpha' \vec{p}^{\,2}_{\perp t}} ,$$

which is divergent when $\alpha' \vec{p}^{\,2}_{\perp t} < 4$ where $\vec{p}_{\perp t}$ is the orbifold transverse momentum in the t channel.

This pathological behaviour has been interpreted in the literature as "*the result of a large **gravitational backreaction** of the incoming matter into the singularity due to the exchange of a single graviton*".

[*H.Liu, G.Moore, N.Seiberg (2002) - M.Berkooz, B.Craps, D.Kutasov, G.Rajesh (2005) - L.Cornalba, M.S.Costa (2005)*]

Scattering Amplitudes and Divergences: Open Strings

What had gone unnoticed is that if we perform an analogous computation for the four tachyons **open string "Veneziano"** amplitude we find:

$$A_{4T}^{open} \sim \int^{q\sim\infty} \frac{dq}{|q|} q^{1-\alpha' \vec{p}^{\,2}_{\perp t}} \, \mathrm{tr}\left(\{T_1, T_2\}\{T_3, T_4\}\right) ,$$

[*AA, R.Finotello, I.Pesando (2020)*]

which is divergent when $\alpha' \vec{p}^{\,2}_{\perp t} < 1$.

Our claim is therefore that **string perturbation theory fails**, but the nature of the divergences is not strictly related to a gravitational backreaction, since here we are dealing with open string at tree level.

To make this even more evident, we showed that also the three point function of two tachyons and the first massive level, when some physical polarization are chosen, reads:

$$A_{TTM}^{open} \sim \int_{u\sim 0} du \, \frac{1}{|u|^{5/2}} \mathrm{tr}\left(\{T_1, T_2\}, T_3\right) .$$

[*AA, R.Finotello, I.Pesando (2020)*]

The origin of the divergences

To understand the origin of this quite weird phenomenon it seems necessary to look at what happens if we try to define a quantum field theory on the NBO.
It has been suggested very recently [*I.Pesando (2022)*] that the root of these divergences lies in the **non-existence of the perturbative expansion** through the singularity of the usual time evolution operator in the interaction picture.
This results in the **breakdown of the standard Feynman diagrammatic approach** and, as a consequence, of the particle interpretation of interactions.

As a very trivial example, it's easy to show that in a scalar QFT with action defined on the NBO

$$S = \int_\Omega d^D x \sqrt{-\det g}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda_N}{N!}\phi^N\right) \ ,$$

the N-point scalar tree level vertex behaves like

$$I_N \sim \int_{u\sim 0} du|u|^{-\frac{N}{2}+1} \ ,$$

which is clearly not integrable for $N \geq 4$.

What can we do now?

All of this begs a fundamental question: **does the theory exist?**

For similar models it has been shown [*I.Pesando (2022)*] that at the quantum mechanical level **the full Hamiltonian theory is well defined**, at least in the minisuperspace approximation.

From the string theory point of view there are two roads:

- we move on to a non perturbative treatment, like the (Lightcone) String Field Theory
- we find a way to recover a meaningful perturbation theory on this background

Let's try the second one!

Nonlinear sigma model

Since strings feel not only the metric $g_{\mu\nu}$ but also the Kalb-Ramond $B_{\mu\nu}$, the introduction of a background field of this kind could provide additional degrees of freedom.

Let's consider the nonlinear sigma model of the bosonic open string

$$S = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \, \left(g_{\mu\nu} \partial_a x^\mu \partial^a x^\nu - 2\pi i \alpha' \epsilon^{ab} B_{\mu\nu} \partial_a x^\mu \partial_b x^\nu \right),$$

and apply the Seiberg-Witten map

$$\frac{1}{(g + 2\pi\alpha' B)_{\mu\nu}} = (G + \frac{\theta}{2\pi\alpha'})^{\mu\nu}$$

for the following rescaled closed string metric and orbifold invariant B-field

$$ds^2 = -2\beta\gamma \, du dv + \Delta^2 \beta^2 u^2 dz^2 + d\vec{x}^{\,2} \qquad\qquad B = 2b\Delta\beta^2 u \, du \wedge dz$$

Noncommutative Open String Theory (NCOS)

We get the open string metric and antisymmetric θ matrix

$$G^{\mu\nu}\partial_\mu\partial_\nu = -\frac{2}{\beta\gamma}\partial_u\partial_v - \frac{4b^2\pi^2\alpha'^2}{\gamma^2}\partial_v^2 + \frac{1}{\Delta^2\beta^2u^2}\partial_z^2 \qquad \theta = \frac{8b\pi^2\alpha'^2}{\Delta\beta\gamma u}\,\partial_v \wedge \partial_z$$

In the decoupling zero slope limit $\alpha' \to 0$ where $\beta \sim (\alpha')^{\frac{1}{2}}\,,\quad \gamma \sim (\alpha')^{\frac{1}{2}}\,,\quad \Delta \sim (\alpha')^0\,,\quad b \sim (\alpha')^{-1}$
we have that $G^{\mu\nu} \sim (\alpha')^{-1}$ while θ and B stay finite and g vanishes.

Looking at the boundary propagator on the D2-brane wrapping u, v and z

$$\langle x^\mu(\tau)x^\nu(\tau')\rangle = -\alpha' G^{\mu\nu}\log(\tau-\tau')^2 + \frac{i}{2}\theta^{\mu\nu}\epsilon(\tau-\tau')\,,$$

we are left with a **Noncommutative Open String Theory** (**NCOS**) where the modes of closed strings decouple but the massive modes from the open string sector do not!

To sum up

- Time-dependent orbifolds like the NBO are useful toy models to reproduce and study cosmological singularities

- The origin of the unexpected divergences which appear in string scattering amplitudes can be traced back to the non-existence of a standard Feynman perturbation theory of the underlying effective QFT, rather than to gravitational backreaction

- String Theory allows us to introduce additional degrees of freedom and we expect these to cure the scattering amplitudes divergences and “resolve” the singularity

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Thank you for your attention!

Time-dependent orbifolds: the geometry

First formulated by Horowitz and Steif in 1990, then heavily studied by Seiberg, Moore, Craps, Costa, Komargodski et al. in the early 2000s

Construction procedure

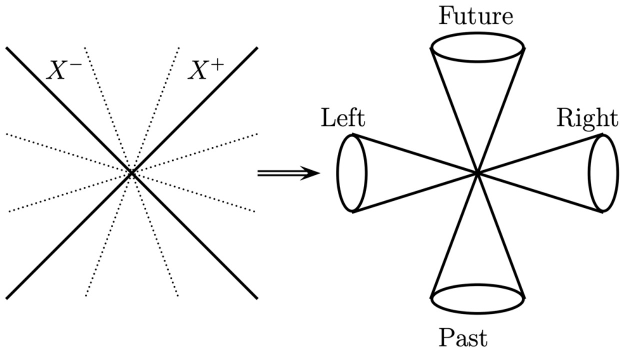
- Consider the spacetime $(\mathbb{M}^3/\Gamma) \times \mathbb{R}^{D-3}$ Γ subgroup of $\mathrm{ISO}(2,1)$
- Perform the identifications $x \sim e^{nk}x, \ n \in \mathbb{Z}$ $k = 2\pi i(\alpha^{\mu}P_{\mu} + \beta^{\mu\nu}J_{\mu\nu})$
- Work in light cone coordinates $x = (x^+, x^-, x^2, \vec{x})$
- Exclude pure translations and Lorentz generators not involving time
- Choose two parameters (Δ and R) to describe the inequivalent conjugacy classes

Time-dependent orbifolds: the geometry

Up to ISO(2,1) conjugations, there exist only four different $\mathbb{M}^3$ time-dependent orbifolds

Orbifold	Generator
Boost (BO)	$2\pi i \Delta J_{+-}$
Null-Boost (NBO)	$2\pi i \Delta J_{+2}$
Shifted-Boost (SBO)	$2\pi i (\Delta J_{+-} + R P_2)$
O-Plane (OPO)	$2\pi i (\Delta J_{+2} + R P_-)$

[*L.Cornalba, M.S.Costa (2005)*]



The Boost Orbifold

All these models possess unphysical regions containing closed timelike curves which we will exclude from our study

Time-dependent orbifolds: scattering amplitudes

Review of Seiberg et al. computation

- Define wave functions (vertex operators) as superpositions of plane waves invariant under the group orbifold action

$$x^{\pm} \sim e^{\pm 2\pi n\Delta}\, x^{\pm},\ n \in \mathbb{Z} \quad \Longrightarrow \quad \Psi_{p^+,p^-,l,\vec{p}}(x) \propto e^{i\vec{p}\cdot\vec{x}} \int_{-\infty}^{+\infty} dq\, e^{i(p^+ x^- e^{-q} + p^- x^+ e^{q})} e^{i\frac{l}{\Delta}q},\ l \in \mathbb{Z}$$

- Compute the 4 point function of **closed string** tachyons ($2 \to 2$ tree-level scattering amplitude) starting from the Virasoro-Shapiro amplitude and using the **inheritance principle**
- Study the orbifold amplitude in the Regge limit of high energy s and small fixed angle t, which corresponds to large values of the only remaining integration variable

$$s = -(p_1 + p_2)^2 \overset{q\to\infty}{\sim} q$$
$$t = -(p_1 - p_3)^2 \overset{q\to\infty}{\sim} \text{const}$$

Scalar QED on the NBO

$$S_{sQED} = \int_\Omega \mathrm{d}^D x\, \sqrt{-\det g}\left(-(D^\mu\phi)^* D_\mu\phi - M^2\phi^*\phi - \frac{1}{4}f^{\mu\nu}\, f_{\mu\nu} - \frac{g_4}{4}|\phi|^4\right)$$

with $\quad D_\mu\phi = (\partial_\mu - i\, e\, a_\mu)\phi \qquad f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$

First we derive the eigenfunctions of the scalar d'Alembertian, which near the singularity $u = 0$ reads

$$\phi(u) \propto \frac{1}{\sqrt{|u|}} e^{i\frac{C}{u}}$$

with $\boldsymbol{C \sim l^2}$, where l is the discrete momentum associated to the compact direction z

Now if we compute the interaction contact term of N scalar $\boldsymbol{\phi_i}$ and consider the specific case where all $l_i = 0$, we obtain

$$\mathcal{A}_N \propto \int_{u\sim 0} du\, |u|^{-\frac{N}{2}+1}$$

This result is technically not integrable for $N \geq 4$.
The absence of a continuous exponential term (the C_i's have isolated zeros due to the discrete momenta) does not allow for a distributional interpretation of A_N near the singularity.