Deconfinement Phase Transition in Thermal QCD at Intermediate Coupling from M Theory

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(UV)-Complete Top-Down String Dual of Large N Thermal QCD-like Theories at Intermediate Gauge/'t Hooft Coupling and Its M Theory Uplift

- Type IIB string dual of Large N thermal QCD-like theories have been worked out by M. Mia, K. Dasgupta, C. Gale and S. Jeon[2009] whose brane setup is given in the table below.

S. No.	Branes	World Volume
1.	$N\ D3$	$\mathbb{R}^{1,3}(t, x^{1,2,3}) \times \{r = 0\}$
2.	$M\ D5$	$\mathbb{R}^{1,3}(t, x^{1,2,3}) \times \{r = 0\} \times S^2(\theta_1, \phi_1) \times \mathrm{NP}_{S^2_a(\theta_2,\phi_2)}$
3.	$M\ \overline{D5}$	$\mathbb{R}^{1,3}(t, x^{1,2,3}) \times \{r = 0\} \times S^2(\theta_1, \phi_1) \times \mathrm{SP}_{S^2_a(\theta_2,\phi_2)}$
4.	$N_f\ D7$	$\mathbb{R}^{1,3}(t, x^{1,2,3}) \times \mathbb{R}_+(r \in [\lvert\mu_{\rm Ouyang}\rvert^{\frac{2}{3}}, r_{\rm UV}]) \times S^3(\theta_1, \phi_1, \psi) \times \mathrm{NP}_{S^2_a(\theta_2,\phi_2)}$
5.	$N_f\ \overline{D7}$	$\mathbb{R}^{1,3}(t, x^{1,2,3}) \times \mathbb{R}_+(r \in [\mathcal{R}_{D5/\overline{D5}} + \epsilon, r_{\rm UV}]) \times S^3(\theta_1, \phi_1, \psi) \times \mathrm{SP}_{S^2_a(\theta_2,\phi_2)}$

- In the UV($r > \mathcal{R}_{D5/\overline{D5}}$), Color gauge group is $SU(N + M_{D5}) \times SU(N + M_{\overline{D5}})$ and flavor gauge group is $SU(N_f) \times SU(N_f)$. In the IR($r < \mathcal{R}_{D5/\overline{D5}}$), $SU(N + M_{D5}) \times SU(N + M_{\overline{D5}})$ changes to $SU(N + M_{D5}) \times SU(N)$ because $\overline{D5}$ are not present in the IR.
- Pair of gauge couplings flow oppositely for $SU(N + M)$ and $SU(N)$. Higher rank gauge coupling flow towards the strong coupling and vice-versa.
- The flux of NS-NS B through the vanishing S^2, apart from N_f via the dilaton being dependent on the same, is the reason for introduction of non-conformality. This is why M $\overline{D5}$-branes were introduced in the UV to cancel the net M $D5$-branes charge in the UV.
- N_f flavor D7-branes were introduced via Ouyang embedding P. Ouyang[2003].
- N_f flavor D7-branes enters the RG flow of the gauge couplings via the dilaton, therefore N_f $\overline{D7}$-branes were introduced(in the UV, UV-IR interpolating reason) to cancel the net N_f $D7$-branes charges in the UV and UV-IR interpolating reason.
- In the IR, at the end of Seiberg-like duality cascade, the number of colors N_c gets identified with M, which in the 'MQGP limit' can be tuned to equal 3. ▸ Color-flavor enhancement of the KS-like length scale in the IR ensures supergravity can still be trusted K.Sil, A.Misra [2015].

Gravitational Dual

- Finite temperature and finite separation between $D5$ and $\overline{D5}$-branes (denoted by $\mathcal{R}_{D5/\overline{D5}}$) on the gauge theory side correspond to introduction of black hole (for high temperatures, i.e., $T > T_c$ on the gauge theory side) and resolution of the two cycle in the gravity dual side.
- IR confinement on the gauge theory side corresponds to deformation of three cycle in the gravity dual side.
- Hence, gravitational dual of type IIB brane construct is warped resolved deformed conifold. Backreaction are included in warp factor and fluxes.

and $\mathcal{O}(l_p^6)$ correction to the same

- Type IIA SYZ mirror (involving a warped resolved (deformed) conifold in the (gravity dual/) brane construct) of the type IIB gravitational dual of M. Mia et al [2009] (involving a warped resolved deformed conifold), and its M theory uplift (involving a seven-fold with G_2 structure) at finite gauge/string coupling(MQGP limit) have been worked out by M. Dhuria, A. Misra[2013];K. Sil, A. Misra[2015].
- **MQGP Limit:** $g_s \sim \frac{1}{\mathcal{O}(1)}$, $M, N_f \equiv \mathcal{O}(1)$, $g_s N >> 1$, $g_s N_f < 1$, $g_s M < 1$, $\frac{g_s M^2}{N} \ll 1$.

- The T^3-valued (x, y, z) (used for effecting SYZ mirror via a triple T-dual in M.Dhuria, A.Misra [2013]; K.Sil, A.Misra [2015]) are defined via (based on A. Knauf's thesis [2006] and papers therein):

$$\phi_1 = \phi_{10} + \frac{x}{\sqrt{h_2}\,[h(r_0, \theta_{10,20})]^{\frac{1}{4}} \sin\theta_{10}\ r_0},$$
$$\phi_2 = \phi_{20} + \frac{y}{\sqrt{h_4}\,[h(r_0, \theta_{10,20})]^{\frac{1}{4}} \sin\theta_{20}\ r_0}$$
$$\psi = \psi_0 + \frac{z}{\sqrt{h_1}\,[h(r_0, \theta_{10,20})]^{\frac{1}{4}}\ r_0},$$

the squashing factors $h_{1,2,4}$ defined in M.Mia et al [2009], and one works up to linear order in (x, y, z). Up to linear order in r, i.e., in the IR, it can be shown Dasgupta et al [2006] that $\theta_{10,20}$ can be promoted to global coordinates $\theta_{1,2}$ in all the results in the paper.

- For $D5$-branes wrapping the resolved S^2 of a resolved conifold geometry Zayas, Tseytlin [2000]], which breaks SUSY globally, as in Becker et al [2004], to begin with, SYZ is implemented wherein the pair of S^2s are replaced by a pair of T^2s in the delocalized limit, and the correct T-duality coordinates are identified.
- Upon uplifting the mirror to M theory, it is found that a G_2-structure can be chosen which is free of the delocalization, implying that descending back to type IIA theory is also free of delocalization K. Dasgupta et al [2004]. For the SYZ mirror of the resolved warped deformed conifold which figures in the gravitational dual of large-N thermal QCD-like theories of M.Mia et al [2009] that gets uplifted to M-theory with G_2 structure worked out in M.Dhuria, A.Misra [2013]; K.Sil, A. Misra [2015], the idea is exactly the same.
- Higher derivative correction to the M theory uplift of M. Dhuria, A. Misra[2013]; K. Sil, A. Misra[2015] have been worked out by V. Yadav, A. Misra[2020] by incorporating $\mathcal{O}(R^4)$ terms in the $D = 11$ SUGRA action. HD correction on the gravity side corresponds to the intermediate gauge/'t Hooft coupling in gauge theory side (relevant to QGP - Natsuume [2007]).

$\mathcal{O}(l_p^6)$-Corrections to the MQGP Background

- The ▸ $\mathcal{N}=1, D=11$ supergravity action inclusive of the leading quantum corrections at $\mathcal{O}(l_p^6)$ terms, is given by Tseytlin [2000]:

$$\mathcal{S}_{D=11} = \frac{1}{2\kappa_{11}^2}\left[\int_{M_{11}}\sqrt{G}R + \int_{\partial M_{11}}\sqrt{h}K - \frac{1}{2}\int_{M_{11}}\sqrt{G}G_4^2 - \frac{1}{6}\int_{M_{11}} C_3\wedge G_4\wedge G_4\right.$$
$$\left.+\frac{\left(4\pi\kappa_{11}^2\right)^{\frac{2}{3}}}{(2\pi)^4 3^2.2^{13}}\left(\int_{\mathcal{M}} d^{11}x\sqrt{G^{\mathcal{M}}}\left(J_0 - \frac{1}{2}E_8\right) + \int C_3\wedge X_8 + \int t_8 t_8 G^2 R^3 + \cdot\right)\right] - \mathcal{S}^{\rm ct},$$

where:

$$J_0 = 3\cdot 2^8\left(R^{HMNK}R_{PMNQ}R_H{}^{RSP}R^Q{}_{RSK} + \frac{1}{2}R^{HKMN}R_{PQMN}R_H{}^{RSP}R^Q{}_{RSK}\right)$$
$$E_8 = \frac{1}{3!}\epsilon^{ABCM_1N_1\ldots M_4N_4}\epsilon_{ABCM_1'N_1'\ldots M_4'N_4'}R^{M_1'N_1'}{}_{M_1N_1}\ldots R^{M_4'N_4'}{}_{M_4N_4},$$
$$X_8 = \frac{1}{192(2\pi)^4}\left[{\rm tr}R^4 - \frac{1}{4}({\rm tr}R^2)^2\right],$$
$$\kappa_{11}^2 = \frac{(2\pi)^8 l_p^9}{2},$$

- $$t_8^{N_1\ldots N_8} =$$
$$\frac{1}{16}\Big(-2\left(G^{N_1N_3}G^{N_2N_4}G^{N_5N_7}G^{N_6N_8}+G^{N_1N_5}G^{N_2N_6}G^{N_3N_7}G^{N_4N_8}+G^{N_1N_7}G^{N_2N_8}G^{N_3N_5}G^{N_4N_6}\right)$$
$$+8\left(G^{N_2N_3}G^{N_4N_5}G^{N_6N_7}G^{N_8N_1}+G^{N_2N_5}G^{N_6N_3}G^{N_4N_7}G^{N_8N_1}+G^{N_2N_5}G^{N_6N_7}G^{N_8N_3}G^{N_4N_1}\right)$$
$$-(N_1\leftrightarrow N_2)-(N_3\leftrightarrow N_4)-(N_5\leftrightarrow N_6)-(N_7\leftrightarrow N_8)\Big)\,.$$

- Also, from J.Liu, R.Minasian [2013]:

$$t_8t_8G^2R^3 = t_8^{M_1\ldots M_8}t^8_{N_1\ldots N_8}G_{M_1}{}^{N_1PQ}G_{M_2}{}^{N_2}{}_{PQ}R_{M_3M_4}{}^{N_3N_4}R_{M_5M_6}{}^{N_5N_6}R_{M_7M_8}{}^{N_7N_8}\,.$$

- In the MQGP limit: $|J_0| > |E_8| > |t_8^2G^2R^3|$ V.Yadav, A.Misra [2020].

- The EOMS are:

$$R_{MN} - \frac{1}{2} g_{MN} R - \frac{1}{12}\left(G_{MPQR}G_N{}^{PQR} - \frac{1}{8} G_{PQRS}G^{PQRS}\right)$$
$$= -\gamma l_p^6 \left[\frac{g_{MN}}{2}\left(J_0 - \frac{1}{2}E_8\right) + \frac{\delta}{\delta g^{MN}}\left(J_0 - \frac{1}{2}E_8\right)\right],$$
$$d * G = \frac{1}{2} G \wedge G + 2\kappa_{11}^2 \left(\frac{2\pi^2}{\kappa_{11}^2}\right)^{\frac{1}{3}} X_8,$$

where $\gamma \equiv \frac{(4\pi)^{\frac{2}{3}}}{(2\pi)^4 3^2 2^{13}}$.

- Consider the following ansatz:

$$G_{MN} = G_{MN}^{(0)} + l_p^6 G_{MN}^{(1)} \equiv G_{MN}^{(0)}\left(1 + \beta f_{MN}\right) = G_{MN}^{(0)} + \beta \mathcal{F}_{MN},$$
$$C_{MNP} = C_{MNP}^{(0)} + l_p^6 C_{MNP}^{(1)},$$

$(M, N) = t, x^{1,2,3}, r, \theta_{1,2}, \phi_{1,2}, \psi, x^{11}$. Utilizing $X_8 = 0$ M.Dhuria, A.Misra [2013], it was shown in V.Yadav, A.Misra [2020] that one can self-consistently set $C_{MNP}^{(1)} = 0$.

Deconfinement Phase Transition in thermal QCD-like theories from Witten's Prescription E. Witten [1998], C.Herzog [2006]

- To compute the deconfinement temperature from gauge/gravity duality one has to know what is gravity dual of corresponding gauge theory.
- Obtain the metric for thermal(relavant to $T < T_c$) and blackhole(relavant to $T > T_c$) backgrounds.
- Let $\beta_{\rm th}$ and $\beta_{\rm BH}$ are periodicities for thermal circle in thermal and blackhole background and compute action densities for both backgrounds.Then use the following relation,

$$\beta_{\rm BH}\tilde{S}_{\rm BH} = \beta_{\rm Th}\tilde{S}_{\rm th}\Big|_{\beta_{\rm BH}\sqrt{G_{tt}^{\rm BH}}=\beta_{\rm Th}\sqrt{G_{tt}^{\rm Th}}},$$

  where $\tilde{S}_{\rm BH/th}$ excludes the coordinate integral of x^0.
- From the previous step we obtain relation between black hole horizon radius r_h and IR cut off for thermal background $r0$.
- Deconfinement temperature on gauge theory side is given by the following expression K. Sil, A. Misra[2015]:

$$T_c = \frac{r_h}{\pi L^2}.$$

Deconfinement Temperature from $\mathcal{M}$ Theory Dual Inclusive of $\mathcal{O}(R^4)$ Corrections

- If $\beta_{\rm BH}$ and $\beta_{\rm Th}$ are respectively the periodicitiees of the thermal circle in the black and thermal $\mathcal{M}$-theory backgrounds then at $r = \mathcal{R}_{\rm UV}$, $\beta_{\rm BH}\sqrt{G^{\rm BH}_{tt}} = \beta_{\rm Th}\sqrt{G^{\rm Th}_{tt}}$. Now at $T = T_c$ E. Witten[1998],

$$\beta_{\rm BH}\int_{M_{11}}\left(S^{\rm BH}_{\rm EH} + S^{\rm BH}_{\rm GHY}\delta(r-\mathcal{R}_{\rm UV}) + S^{\rm BH}_{\mathcal{O}(R^4)}\right) = \beta_{\rm Th}\int_{\tilde{M}_{11}}\left(S^{\rm Th}_{\rm EH} + S^{\rm Th}_{\rm GHY}\delta(r-\mathcal{R}_{\rm UV}) + S^{\rm Th}_{\mathcal{O}(R^4)}\right)$$

where $\int$ excludes the coordinate integral w.r.t. x^0.

- Since $\beta_{\rm BH} = \left(\sqrt{1-\frac{r_h^4}{\mathcal{R}^4_{\rm UV}}}\right)^{-1}\beta_{\rm Th}$, therefore,

$$\left(\sqrt{1-\frac{r_h^4}{\mathcal{R}^4_{\rm UV}}}\right)^{-1}\int_{M_{11}}\left(S^{\rm BH}_{\rm EH} + S^{\rm BH}_{\rm GHY}\delta(r-\mathcal{R}_{\rm UV}) + S^{\rm BH}_{\mathcal{O}(R^4)}\right)$$
$$= \int_{\tilde{M}_{11}}\left(S^{\rm Th}_{\rm EH} + S^{\rm Th}_{\rm GHY}\delta(r-\mathcal{R}_{\rm UV}) + S^{\rm Th}_{\mathcal{O}(R^4)}\right).$$

- On-shell action corresponding to eleven dimensional SUGRA action is,

$$S_{D=11}^{\rm on-shell} = -\frac{1}{2}\Bigg[-2S_{\rm EH}^{(0)} + 2S_{\rm GHY}^{(0)}$$
$$+\beta\left(\frac{20}{11}S_{\rm EH}^{(1)} - 2\int_{M_{11}}\sqrt{-g^{(1)}}R^{(0)} + 2S_{\rm GHY}^{(1)} - \frac{2}{11}\int_{M_{11}}\sqrt{-g^{(0)}}g_{(0)}^{MN}\frac{\delta J_0}{\delta g_{(0)}^{MN}}\right)\Bigg].$$

- Writing $g_{MN} = g_{MN}^{\rm MQGP}(1+\beta f_{MN})$, $g_{MN}^{\rm MQGP}$ being the MQGP metric worked out at $\mathcal{O}(\beta^0)$ in M.Dhuria, A.Misra [2013]; K.Sil, A.Misra[2015], and f_{MN} are the $\mathcal{O}(\beta)$-corrections; $f_{MN} \approx 0$ in the UV GY, V. Yadav, A. Misra [2020].

- Partitioning r into the IR ($r \in [r_h, \mathcal{R}_{D5/\overline{D5}}^{\rm bh} = \sqrt{3}a^{\rm bh}]$) and the UV ($r \in [\mathcal{R}_{D5/\overline{D5}}^{\rm bh}, \mathcal{R}_{\rm UV}^{\rm bh}]$), utilizing the results of V. Yadav, A. Misra [2020] as regards $\mathcal{O}(R^4)$ corrections to the $\mathcal{M}$-theory uplift of large-N thermal QCD-like cousins as worked out in M.Dhuria, A.Misra [2013]; K.Sil, A.Misra [2015], and realizing the dominant contributions to the EH/GHY/$\sqrt{-G}J_0$/$\sqrt{-G}G^{MN}\frac{\delta J_0}{\delta G^{MN}}$ arise from the small-$\theta_{1,2}$ values, introduce polar angular cut-offs $\epsilon_{1,2}$: $\theta_1 \in [\epsilon_1, \pi - \epsilon_1]$ and $\theta_2 \in [\epsilon_2, \pi - \epsilon_2]$.

- For blackhole background UV finite on-shell action, which includes terms LO in N, $\log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}_{D5/\overline{D5}}}\right)$ and $\frac{r_h}{\mathcal{R}_{D5/\overline{D5}}}$, in the neighborhood of the $(\theta_1,\theta_2)=(\epsilon_1,\epsilon_2)$-branch (near which there is a decoupling of $M_5(t,x^{1,2,3},r)$ and $M_6(\theta_1,\theta_2,\phi_1,\phi_2,\psi,x^{11})$ K.Sil, A.Misra [2015]), is (every terms in the on-shell actions appear as $\frac{\log(\epsilon_2)}{\log(\epsilon_1)}$, we have written the final results after setting $\epsilon_1=\epsilon_2$ to ensure holographic IR regularization in the theory):

$$\left(1+\frac{r_h^4}{2\mathcal{R}_{\rm UV}^4}\right)S^{BH}_{D=11,\ {\rm on-shell UV-finite}}\sim\frac{2\kappa^{\rm bh}_{\rm GHY}M_{\rm UV}{r_h}^4\log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm bh}_{D5/\overline{D5}}}\right)}{{g_s}^{3/2}N^{1/2}}$$
$$+\left[-2\mathcal{C}_{\theta_1 x}\kappa^{\rm IR}_{\sqrt{G^{(1)}}R^{(0)}}+\frac{20\left(-\mathcal{C}^{\rm bh}_{zz}+2\mathcal{C}^{\rm bh}_{\theta_1 z}-3\mathcal{C}^{\rm bh}_{\theta_1 x}\right)\kappa^{\beta,\ \rm IR}_{\rm EH}}{11}\right]$$
$$\times\frac{b^2{g_s}^{3/2}M{N_f}^3{r_h}^4\log^3(N)\log\left(\frac{r_h}{\mathcal{R}_{D5/\overline{D5}}}\right)\log\left(1-\frac{r_h}{\mathcal{R}_{D5/\overline{D5}}}\right)}{N^{1/2}{\mathcal{R}_{D5/\overline{D5}}}^4}\beta,$$

where $\mathcal{C}_{MN}$ are constants of integration appearing (roughly) in the solutions of the EOMs of f_{MN}.

- For thermal background:

$$S^{\rm thermal}_{D=11} \sim \frac{2\kappa^{\rm th,\ \beta^0}_{\rm GHY} M_{\rm UV} {r_0}^4 \log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}\right)}{{g_s^{\rm UV}}^{3/2} N^{1/2} {\mathcal{R}^{\rm th}_{D5/\overline{D5}}}^4} + \frac{2{g_s}^{3/2}\kappa^{\rm th,\ \beta^0}_{\rm EH,IR} M {N_f}^3 {r_0}^2 \log^2(N)}{N^{1/2}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}^2}$$
$$\times \log\left(\frac{r_0}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}\right) - \frac{20\beta\kappa^{\rm IR,\ \beta}_{\rm EH,th} {r_0}^3 N^{1/2} f_{x^{10}x^{10}}(r_0)}{11{g_s}^{3/2} M {N_f}^{5/3} {\mathcal{R}^{\rm th}_{D5/\overline{D5}}}^3 \log^{\frac{2}{3}}(N) \log\left(\frac{r_0}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}\right)}\beta.$$

- On equating $O(\beta^0)$ terms for blackhole and thermal background, we need to solve the following equation:

$$\frac{2\kappa^{\rm bh}_{\rm GHY} M_{\rm UV} {r_h}^4 \log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm bh}_{D5/\overline{D5}}}\right)}{{g_s}^{3/2} N^{1/2}} = \frac{2\kappa^{\rm th,\ \beta^0}_{\rm GHY} M_{\rm UV} {r_0}^4 \log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}\right)}{{g_s^{\rm UV}}^{3/2} N^{1/2} {\mathcal{R}^{\rm th}_{D5/\overline{D5}}}^4 \log}$$
$$+\frac{2{g_s}^{3/2}\kappa^{\rm th,\ \beta^0}_{\rm EH,IR} M {N_f}^3 {r_0}^2 \log^2(N) \log\left(\frac{r_0}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}\right)}{N^{1/2}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}^2}.$$

- Now, one can show that near, e.g., $(\theta_1, \theta_2) \sim (\epsilon_1, \epsilon_2)$, $\frac{\kappa^{\rm th,\ \beta^0}_{\rm GHY}}{\kappa^{\rm th,\ \beta^0}_{\rm EH,IR}} \sim 10^5$, and hence, one can drop the $\kappa^{\rm th,\ \beta^0}_{\rm EH,IR}$ term. Therefore,

$$r_h = \frac{\sqrt[4]{\frac{\kappa^{\rm th,\ \beta^0}_{\rm GHY}}{\kappa^{\rm bh,\ \beta^0}_{\rm GHY}}}\, r_0 \mathcal{R}^{\rm bh}_{D5/\overline{D5}} \sqrt[4]{\frac{\log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}\right)}{\log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm bh}_{D5/\overline{D5}}}\right)}}}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}.$$

- Identifying $\frac{r_0}{L^2}$ with $\frac{m^{0^{++}}}{4}$ K. Sil, V. Yadav and A. Misra[2017], where $m^{0^{++}}$ is the mass of 0^{++} glueball and using $T_c = \frac{r_h}{\pi L^2}$ K. Sil, A. Misra[2015], Deconfinement temperature is given by the following expression:

$$T_c = \frac{\sqrt[4]{\frac{\kappa^{\rm th,\ \beta^0}_{\rm GHY}}{\kappa^{\rm bh,\ \beta^0}_{\rm GHY}}}\, m^{0^{++}} \mathcal{R}^{\rm bh}_{D5/\overline{D5}} \sqrt[4]{\frac{\log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}\right)}{\log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm bh}_{D5/\overline{D5}}}\right)}}}{4\pi \mathcal{R}^{\rm th}_{D5/\overline{D5}}}.$$

- Now equating $O(\beta)$ term for blackhole and thermal background obtains:

$$f_{x^{10}x^{10}}(r_0)$$

$$\sim -\frac{b^2 {g_s}^3 M^2 {N_f}^{14/3}\left(\frac{r_h}{\mathcal{R}^{\rm bh}_{D5/\overline{D5}}}\right)^4 \log^3(N)\log\left(\frac{r_0}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}\right)\log\left(\frac{r_h}{\mathcal{R}^{\rm bh}_{D5/\overline{D5}}}\right)\log\left(1-\frac{r_h}{\mathcal{R}^{\rm bh}_{D5/\overline{D5}}}\right)}{\kappa^{\beta,\ \rm IR}_{\rm EH,th} N\left(\frac{r_0}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}\right)^3}$$

$$\times\left(-11\mathcal{C}_{\theta_1 x}\kappa^{\rm IR}_{\sqrt{G^{(1)}}R^{(0)}}\log^3(N)-10\kappa^{\beta,\ \rm IR}_{\rm EH,bh}\left(-\mathcal{C}^{\rm bh}_{zz}+2\mathcal{C}^{\rm bh}_{\theta_1 z}-3\mathcal{C}^{\rm bh}_{\theta_1 x}\right)\right),$$

with the understanding that one substitutes r_h in terms of r_0 as obtained in the previous slide. Above equation relates $\mathcal{O}(R^4)$ corrections to the thermal background along M theory circle and combination of integrations constant appearing in the $\mathcal{O}(R^4)$ corrections to the black hole background along the compact part of non-compact four cycle in type IIB setup around which flavor branes are wrapping. This relation is valid in the IR. Which is a version of "UV-IR mixing".

- The aforementioned combination of integrations constant appearing in the $\mathcal{O}(R^4)$ corrections to the black hole background encodes information about the flavor branes in parent type IIB dual. We refer this as " Flavor Memory" effect in the context of M theory dual.

Effect of vorticity on the Deconfinement Temperature GY[2022]

- We can introduce rotation on the QGP side by the following Lorentz tranformatons on the gravity dual side M. Bravo Gaete, L. Guajardo, and M. Hassaine [2017]; C. Erices Gaete, and C. Martinez [2018].

$$t \to \frac{1}{\sqrt{1-l^2\omega^2}}\left(t + l^2\omega\phi\right);\ \phi \to \frac{1}{\sqrt{1-l^2\omega^2}}\left(\phi + \omega t\right).$$

- We obtained the M theory metric in canonical form as given below.

$$ds^2_{11}|_{BH} = e^{-\frac{2\phi^{\rm IIA}}{3}}\left[\frac{1}{\sqrt{h(r,\theta_{1,2})}}\left(-\mathcal{Y}_1(r)dt^2 + \mathcal{Y}_2(r)\left(d\phi + \mathcal{Y}_3(r)dt\right)^2 + \left(dx^1\right)^2 + \left(dx^2\right)^2\right)\right.$$
$$\left. + \sqrt{h(r,\theta_{1,2})}\left(\frac{dr^2}{g(r)} + ds^2_{\rm IIA}(r,\theta_{1,2},\phi_{1,2},\psi)\right)\right] + e^{\frac{4\phi^{\rm IIA}}{3}}\left(dx^{11} + A^{F_1^{\rm IIB}+F_3^{\rm IIB}+F_5^{\rm IIB}}_{\rm IIA}\right)^2$$

where,

$$\mathcal{Y}_1(r) = \frac{g(r)\left(1-l^2\omega^2\right)}{\left(1-g(r)l^2\omega^2\right)},$$

$$\mathcal{Y}_2(r) = \frac{l^2\left(1-g(r)l^2\omega^2\right)}{\left(1-l^2\omega^2\right)},$$

$$\mathcal{Y}_3(r) = \frac{\omega\left(1-g(r)\right)}{\left(1-g(r)l^2\omega^2\right)}.$$

- Hawking temperature of the black hole turns out to be X. Chen et al [2021]:

$$T = \left|\frac{\kappa}{2\pi}\right| = \left|\frac{\lim_{r\to r_h} -\frac{1}{2}\sqrt{\frac{G^{rr}}{-\hat{G}_{tt}}}\hat{G}_{tt},r}{2\pi}\right|$$
$$= \frac{r_h}{\sqrt{3}\pi^{3/2}\sqrt{N}\sqrt{g_s}}\left(\frac{1}{\gamma} + \frac{\beta}{2}\left(-\mathcal{C}_{zz}^{\rm BH} + 2\mathcal{C}_{\theta_1 z}^{\rm BH} - 3\mathcal{C}_{\theta_1 x}^{\rm BH}\right)\right)$$

where $\hat{G}_{tt} = -\mathcal{Y}_1(r)$, $\hat{G}_{tt},r$ implies derivative of $\hat{G}_{tt}$ with respect to r, and $\gamma = \frac{1}{\sqrt{1-l^2\omega^2}}$. Since $L^4 = 4\pi g_s N$, therefore Hawking temperature of the rotating cylindrical black hole turns out to be:

$$T \sim \left(\frac{r_h}{\pi L^2}\right)\left(\frac{1}{\gamma} + \frac{\beta}{2}\left(-\mathcal{C}_{zz}^{\rm BH} + 2\mathcal{C}_{\theta_1 z}^{\rm BH} - 3\mathcal{C}_{\theta_1 x}^{\rm BH}\right)\right),$$
$$\sim \left(\frac{r_h}{\pi L^2}\right)\left(\sqrt{1-l^2\omega^2} + \frac{\beta}{2}\left(-\mathcal{C}_{zz}^{\rm BH} + 2\mathcal{C}_{\theta_1 z}^{\rm BH} - 3\mathcal{C}_{\theta_1 x}^{\rm BH}\right)\right),$$
$$= T(0)\left(\sqrt{1-l^2\omega^2} + \frac{\beta}{2}\left(-\mathcal{C}_{zz}^{\rm BH} + 2\mathcal{C}_{\theta_1 z}^{\rm BH} - 3\mathcal{C}_{\theta_1 x}^{\rm BH}\right)\right).$$

where $T(0)$ was calculated in K. Sil and A. Misra [2015].$\mathcal{O}(\beta)$ correction to the Hawking temperature had been calculated in small frequency limit. Since $\mathcal{C}_{zz}^{\rm BH} = 2\mathcal{C}_{\theta_1 z}^{\rm BH}$ and $\mathcal{C}_{\theta_1 x}^{\rm BH} \ll 1$ V. Yadav and A. Misra [2020], therefore no $\mathcal{O}(\beta)$ correction to the Hawking temperature.

- At $\mathcal{O}(\beta^0)$, UV-finite on-shell action for the $\mathcal{M}$-theory rotating cylindrical black hole and thermal background uplift are:

$$\left(1+\frac{r_h^4}{2\mathcal{R}_{\rm UV}^4}\right) S_{D=11}^{\rm BH} \sim \lambda_{\rm EH,IR}^{\rm BH} \frac{\epsilon\gamma^8\omega^2 M N_f^3 g_s^{3/2} r_h^4 \log^3(N) \log\left(\frac{r_h}{\mathcal{R}_{D5/\overline{D5}}^{\rm BH}}\right) \log\left(1-\frac{r_h}{\mathcal{R}_{D5/\overline{D5}}^{\rm BH}}\right)}{{\mathcal{R}_{D5/\overline{D5}}^{\rm BH}}^4 N^{1/2}}$$

$$+\lambda_{\rm EH,UV}^{\rm BH} \frac{\gamma^8 l M^{\rm UV} r_h^4 \log^2\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}_{D5/\overline{D5}}^{\rm BH}}\right)}{N^{1/2} {g_s^{\rm UV}}^{3/2} {\mathcal{R}_{D5/\overline{D5}}^{\rm BH}}^4} + \lambda_{\rm GHY}^{\rm BH} \frac{l M^{\rm UV} r_h^4 \log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}_{D5/\overline{D5}}^{\rm BH}}\right)}{N^{1/2} {\mathcal{R}_{D5/\overline{D5}}^{\rm BH}}^4 {g_s^{\rm UV}}^{3/2}}$$

where $\lambda_{\rm EH,IR}^{\rm BH}$, $\lambda_{\rm EH,UV}^{\rm BH}$ and $\lambda_{\rm GHY}^{\rm BH}$ are the numerical prefactors.

- At $\mathcal{O}(\beta^0)$, UV-finite on-shell action for the $\mathcal{M}$-theory cylindrical thermal background uplift is:

$$S_{D=11}^{\rm thermal} \sim \frac{\lambda_{\rm GHY}^{\rm th} M_{\rm UV} l {r_0}^4 \log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}_{D5/\overline{D5}}^{\rm th}}\right)}{{g_s^{\rm UV}}^{3/2} N^{1/2} {\mathcal{R}_{D5/\overline{D5}}^{\rm th}}^4} + \frac{{g_s}^{3/2} \lambda_{\rm EH,IR}^{\rm th} M {N_f}^3 l {r_0}^2 \log^2(N) \log\left(\frac{r_0}{\mathcal{R}_{D5/\overline{D5}}^{\rm th}}\right)}{N^{1/2} {\mathcal{R}_{D5/\overline{D5}}^{\rm th}}^2}$$

$$+\frac{\lambda_{\rm EH,\ UV}^{\rm th} M_{\rm UV} N_f^{\rm UV} l \left(-\frac{121 {r_0}^4}{16 {\mathcal{R}_{D5/\overline{D5}}^{\rm th}}^4} - \frac{6 {r_0}^2}{{\mathcal{R}_{D5/\overline{D5}}^{\rm th}}^2} + 2\right)}{{g_s^{\rm UV}}^{1/2} N^{\frac{1}{2}}},$$

where $\lambda_{\rm GHY}^{\rm th}$, $\lambda_{\rm EH,IR}^{\rm th}$ and $\lambda_{\rm EH,\ UV}^{\rm th}$ are the numerical prefactors.

- At the UV-cutoff GY,VY,AM[2021]:

$$\left(1+\frac{r_h^4}{2\mathcal{R}_{\rm UV}^4}\right)S^{\rm BH}_{D=11}=S^{\rm thermal}_{D=11}.$$

Since $\omega^2<1$, $\lambda^{\rm BH}_{\rm GHY}\sim\mathcal{O}(10^3)\lambda^{\rm BH}_{\rm EH,IR}$ and $\lambda^{\rm BH}_{\rm GHY}\sim\mathcal{O}(10)\lambda^{\rm BH}_{\rm EH,UV}$, and $\lambda^{\rm th}_{\rm GHY}\sim\mathcal{O}(10^3)\lambda^{\rm th}_{\rm EH,IR}$, $\lambda^{\rm th}_{\rm GHY}\sim\mathcal{O}(10^2)\lambda^{\rm th}_{\rm EH,\ UV}$, therefore, one is required to solve the following equation:

$$\lambda^{\rm BH}_{\rm GHY}\frac{lM^{\rm UV}r_h^4\log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm BH}_{D5/\overline{D5}}}\right)}{N^{1/2}{\mathcal{R}^{\rm BH}_{D5/\overline{D5}}}^4{g_s^{\rm UV}}^{3/2}}-\frac{\lambda^{\rm th}_{\rm GHY}M_{\rm UV}l{r_0}^4\log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}\right)}{{g_s^{\rm UV}}^{3/2}N^{1/2}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}^4}=0.$$

- Solution to the above equation is:

$$r_h=\frac{\sqrt[4]{\frac{\lambda^{\rm th}_{\rm GHY}}{\lambda^{\rm BH}_{\rm GHY}}}\,r_0\mathcal{R}^{\rm BH}_{D5/\overline{D5}}\sqrt[4]{\frac{\log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}\right)}{\log\left(\frac{\mathcal{R}_{\rm UV}}{\mathcal{R}^{\rm BH}_{D5/\overline{D5}}}\right)}}}{\mathcal{R}^{\rm th}_{D5/\overline{D5}}}.$$

- Therefore, deconfinement temperature of the thermal QCD-like theories in the presence vorticity is:

$$T_c(\gamma) = \frac{r_h}{\pi L^2}\sqrt{1 - l^2\omega^2} = T_c(0)\sqrt{1 - l^2\omega^2},$$

where $T_c(0)$ was calculated in GY,VY,AM[2021].

- Plot of ratio of deconfinement temperature in the presence and absence of vorticity with angular velocity of rotating QGP is shown below:

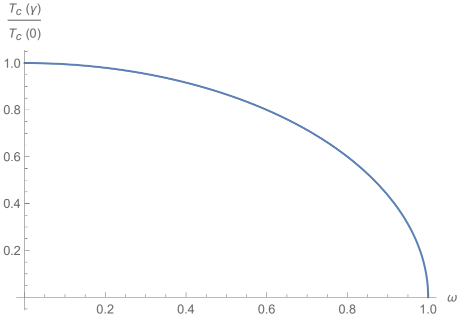

Figure: Plot of $\frac{T_c(\gamma)}{T_c(0)}$ versus ω.

- Unwarped metric in the $t-\phi$ subspace can be rewritten as:

$$ds^2 = -\left(1 - \frac{r_h^4}{r^4\gamma^2}\right) dT^2 + l^2 d\Phi^2,$$

where,

$$dT = dt - \frac{l^2 r_h^4 \omega d\phi}{r^4 - r_h^4},$$
$$d\Phi = \frac{l^2 r_h^4 \omega dt}{r^4 - r_h^4} + d\phi.$$

- The $\mathcal{O}(\beta)$ corrected metric in the diagonal basis can be written as:

$$G_{MN}^{\mathcal{M}} = G_{MN}^{\rm MQGP}\left(1 + \beta f_{MN}\right),$$

where f_{MN} are given in VY,AM[2020].

- In the small ω limit $\gamma = \frac{1}{\sqrt{1-l^2\omega^2}} = 1$, therefore results from $O(\beta)$ contributions are same as GY,VY,AM [2021].

- We found that $\mathcal{O}(\beta)$ terms in the on-shell actions of the black hole and thermal backgrounds are similar for both the cases, $\omega = 0$ and $\omega \neq 0$, therefore again we observe the “UV-IR mixing" even in the presence of rotating quark gluon plasma.

Conjectural $SL(2,\mathbb{Z})$ Completion of Type IIB Action and Non-Renormalization Beyond 1 Loop at $\mathcal{O}(R^4)$ M.B. Green and M. Gutperle[1997]

- Complete effective R^4 action in the Einstein's frame,

$$S_{R^4} = (\alpha')^{-1}\left[a\zeta(3)\tau_2^{3/2} + b\tau_2^{-1/2} + ce^{2\pi i\tau} + \cdots\right] R^4 \equiv (\alpha')^{-1} f(\tau,\bar{\tau}) R^4,$$

  where $\tau = \tau_1 + i\tau_2 = C_0 + ie^{-\phi}$.
- Complete expression for S_{R^4} must be invariant under $SL(2,\mathbb{Z})$ transformations: $\tau \to (a\tau+b)(c\tau+d)^{-1}$ $(a,b,c,d \in \mathbb{Z} : ad - bc = 1)$, which provides very strong constraints on its structure.
- There is a simple function proposed by the authors that satisfies all these criteria, namely,

$$f(\tau,\bar{\tau}) = \sum_{(p,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|p+n\tau|^3}$$

$$= 2\zeta(3)\tau_2^{3/2} + \frac{\tau_2^{3/2}}{\Gamma(3/2)} \sum_{n\neq 0,p} \int_0^\infty dy y^{1/2} \exp\left\{-y|p+n\tau|^2\right\}.$$

- Using the Poisson resummation formula, one obtains,

$$f(\tau,\bar{\tau}) = 2\zeta(3)\tau_2^{3/2} + \frac{2\pi^2}{3}\tau_2^{-1/2} + 2\tau_2^{3/2}\sum_{m,n\neq 0}\int_0^\infty dy\,\exp\left(-\frac{\pi^2 m^2}{y} + 2\pi i m n\tau_1 - y n^2\tau_2^2\right),$$

$$= 2\zeta(3)\tau_2^{3/2} + \frac{2\pi^2}{3}\tau_2^{-1/2} + 8\pi\tau_2^{1/2}\sum_{m\neq 0 n\geq 1}\left|\frac{m}{n}\right| e^{2\pi i m n\tau_1} K_1(2\pi|mn|\tau_2)$$

$$= 2\zeta(3)\tau_2^{3/2} + \frac{2\pi^2}{3}\tau_2^{-1/2}$$

$$+4\pi^{3/2}\sum_{m,n\geq 1}\left(\frac{m}{n^3}\right)^{1/2}\left(e^{2\pi i m n\tau} + e^{-2\pi i m n\bar{\tau}}\right)\left(1 + \sum_{k=1}^{\infty}(4\pi m n\tau_2)^{-k}\frac{\Gamma(k-1/2)}{\Gamma(-k-1/2)}\right)$$

  which includes a perturbative expansion in $\frac{1}{\tau_2}$ of the non-perturbative instanton contribution of charge *mn*.

- The first term corresponds to tree-level perturbative contribution, second term corresponds to one-loop perturbative term and the third represents the infinite sequence of perturbative corrections around the instantons of charge *mn*. Therefore (assuming the conjectured $SL(2,\mathbb{Z})$-completion to be correct) in the zero-instanton sector, there are no perturbative corrections figuring in the action up to $O(R^4)$, beyond one loop (as the non-perturbative contributions begin at winding number (*m*)/instanton number 1).

Consistency with MχPT

- The thermal supergravity background dual to type IIB (solitonic) $D3$-branes at low temperatures, includes $\mathbb{R}^2 \times S^1\left(\frac{1}{M_{\rm KK}}\right)$, where $M_{\rm KK} = \frac{2r_0}{L^2}\left(1 + \mathcal{O}\left(\frac{g_s M^2}{N}\right)\right)$. Solutions to the EOMs of $\mathcal{O}(R^4)$ corrections to the thermal SUGRA background was obtained by taking $\tilde{g}(r) \to 1$ in GY,V. Yadav, A.Misra[2021] and solutions to the same was obtained in V.Yadav, G.Yadav, A.Misra[2020] by taking $\tilde{g}(r) \equiv 1 - \frac{r_0^4}{r^4}$. By taking the $M_{\rm KK} \to 0$ limit (to recover a boundary four-dimensional QCD-like theory after compactifying on the base of a non-Kähler warped resolved deformed conifold) of the results of V. Yadav and A. Misra; GY, V.Yadav, A.Misra [2020] and comparing with the results obtained by setting $\tilde{g}(r) \equiv 1 - \frac{r_0^4}{r^4}$ to unity ▸ $\mathcal{O}(R^4)$metric perturbations. We obtained:

$$\mathcal{C}_{zz}^{\rm th} \sim \frac{\left(\frac{1}{N}\right)^{3/4} {r_0}^2 \Sigma_1}{\epsilon^5 {g_s}^{7/2} \log N^2 M {N_f}^3 \alpha_{\theta_2}^3 \log(r_0)};\ \mathcal{C}_{\theta_1 z}^{\rm th} \sim \frac{\left(\frac{1}{N}\right)^{3/4} {r_0}^2 \Sigma_1}{2\epsilon^5 {g_s}^{7/2} \log N^2 M {N_f}^3 \alpha_{\theta_2}^3 \log(r_0)}$$

$$\mathcal{C}_{\theta_1 x}^{\rm th} \sim \frac{\left(\frac{1}{N}\right)^{7/6} \Sigma_1}{\sqrt{6}\pi^3 \epsilon^{11} {g_s}^{9/4} \log N^4 {N_f}^3 {r_0}^5 \alpha_{\theta_1}^7 \alpha_{\theta_2}^6},$$

where $\mathcal{C}_{MN}^{\rm th}$ are the integration constants appearing in the $\mathcal{O}(R^4)$ corrections to the thermal gravitational metric.

- This thus confirms that $\mathcal{C}^{\rm th}_{zz} - 2\mathcal{C}^{\rm th}_{\theta_1 z} + 2\mathcal{C}^{\rm th}_{\theta_1 x} = 2\mathcal{C}^{\rm th}_{\theta_1 x} < 0$ ($\Sigma_1 \equiv 19683\sqrt{6}\alpha_{\theta_1}^6 + 6642\alpha_{\theta_2}^2\alpha_{\theta_1}^3 - 40\sqrt{6}\alpha_{\theta_2}^4 < 0$), which in V. Yadav, G. Yadav and A. Misra[2020] was argued by requiring compatibility with phenomenological value of the 1-loop renormalized LEC appearing in the $\mathcal{O}(p^4)$ $SU(3)$ χPT $\left(\nabla_\mu U^\dagger \nabla_\mu U\right)^2$, where $\nabla_\mu U \equiv \partial_\mu U - i\mathcal{L}_\mu U + iU\mathcal{R}_\mu, U = e^{\frac{2i\pi}{F_\pi}}$, (the lightest pseudo-scalar meson field) $\pi = \frac{1}{\sqrt{2}}\left(\begin{array}{ccc} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{array}\right)$, and $\mathcal{L}_\mu \equiv \mathcal{V}_\mu - \mathcal{A}_\mu, \mathcal{R}_\mu \equiv \mathcal{V}_\mu + \mathcal{A}_\mu$ constructed from the external vector $\mathcal{V}_\mu$ and axial-vector $\mathcal{A}_\mu$ fields.

Deconfinement Phase Transition in thermal QCD-like theories from Entanglement Entropy

- We will deconfinement phase transition in thermal QCD-like theories similar to Klebanov, Kutasov, Murugan [2007] by computing the entanglement entropies for the connected and disconnected surfaces.
- Entanglement entropies for the connected and disconnected surfaces are (r^* is the turning point of the Ryu-Takayanagi surface):

$$\frac{S_{\rm disconnected}-S_{\rm UV}^{\rm disconnected}}{2\mathcal{V}_1}\sim -{g_s}^2 M\sqrt[2]{\frac{1}{N}}{N_f}^{8/3}r_0^4\log^2(r_0)(\log N-3\log(r_0))^{5/3};$$

$$\frac{S_{\rm connected}-S_{\rm UV}^{\rm connected}}{2\mathcal{V}_1}\approx -{g_s}^{2/3}M_{\rm UV}{N_f^{\rm UV}}^{4/3}{r^*}^4\log^{\frac{4}{3}}(N)\log(r^*)N^{1/10},$$

where,

$$S_{\rm UV}^{\rm disconnected}\sim\frac{{g_s}^{2/3}M_{\rm UV}N_f^{\rm UV\ \frac{4}{3}}\mathcal{R}_{\rm UV}^4\left(\log N-3\log\mathcal{R}_{\rm UV}\right)^{\frac{4}{3}}\log\mathcal{R}_{\rm UV}}{N^{\frac{1}{2}}},$$

$$S_{\rm UV}^{\rm connected}\sim {g_s}^{2/3}N^{3/20}M_{\rm UV}N_f^{\rm UV\ \frac{4}{3}}\mathcal{R}_{\rm UV}^4\left(\log N\right)^{\frac{4}{3}}\log\mathcal{R}_{\rm UV}.$$

- *As l increases, i.e., r^* decreases and reaches $\mathcal{R}^{\rm th}_{D5/\overline{D5}}$, $S_{\rm connected}$ changes from being negative to vanishing and $S_{\rm disconnected}$ stays negative implying disconnected region has lesser entropy.*
- At $r^* = r_{\rm criticial}$, $S^{\rm UV-Finite}_{\rm connected} = S^{\rm UV-Finite}_{\rm disconnected}$ and $r_{\rm critical}$ is given by:

$$r_{\rm critical} \sim \frac{\sqrt[3]{g_s} M^{1/4} N_f{}^{2/3} \log^{\frac{7}{6}}(N)}{N^{3/20}} r_0.$$

- For $N = 100, M = N_f = 3, g_s = 0.1 - 1$, $r_{\rm criticial} \sim 3.8 r_0$ and $l(r = r_{\rm criticial}) \sim 5.4$.
- For $N = 100, M_{\rm UV} = N_f^{\rm UV} = 0.01, M = N_f = 3$, and $r_0 = N^{-\frac{f_{r_0}}{3}}$, $f_{r_0} \approx 1$ (VY,GY,AM[2020]), we obtained the following plot for the entanglement entropies for the connected and disconnected surface versus l. This graph depicts that $l < l_{\rm crit}$ and $l > l_{\rm crit}$ correspond to confined and deconfined phases of thermal QCD-like theories:

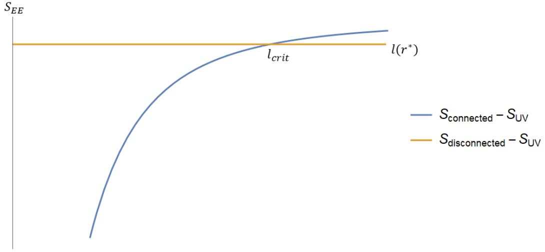

Figure: Plot $S_{\rm connected}$ (blue) and $S_{\rm disconnected}$ (orange) versus $l(r^*)$

Summary

- UV-IR mixing: By matching the actions at the deconfinement temperature of the $\mathcal{M}$ theory uplifts of the thermal and black-hole backgrounds at the UV cut-off, one sees that one obtains a relationship in the IR between the $\mathcal{O}(R^4)$ corrections to the $\mathcal{M}$-theory metric along the $\mathcal{M}$ theory circle in the thermal background and the $\mathcal{O}(R^4)$ correction to a specific combination of the $\mathcal{M}$-theory metric components along the compact part of the four-cycle “wrapped" by the flavor $D7$-branes of the type IIB (warped resolved deformed) conifold geometry.
- We found that deconfinement temperature of thermal QCD-like theories is decreasing as the rotation of the QGP is increasing.
- Non-renormalization of T_c: The LO result for T_c also holds even after inclusion of the $\mathcal{O}(R^4)$ corrections. The dominant contribution from the $\mathcal{O}(R^4)$ terms in the large-N limit arises from the $t_8 t_8 R^4$ terms, which from a type IIB perspective in the zero-instanton sector, correspond to the tree-level contribution at $\mathcal{O}\left((\alpha')^3\right)$ as well as one-loop contribution to four-graviton scattering amplitude. As from the type IIB perspective, the $SL(2,\mathbb{Z})$ completion of these R^4 terms M.B. Green and M. Gutperle[1997] suggests that they are not renormalized perturbatively beyond one loop in the zero-instanton sector, this therefore suggests the non-renormalization of T_c at all loops in $\mathcal{M}$ theory at $\mathcal{O}(R^4)$.

- MχPT-T_c Connection:
  - As shown in V. Yadav, G. Yadav and A. Misra[2020], matching the phenomenological value of the 1-loop renormalized coupling constant correpsonding to the $\mathcal{O}(p^4)$ $SU(3)$ χPT Lagrangian term "$\left(\nabla_\mu U^\dagger \nabla^\mu U\right)^2$" with the value obtained from the type IIA dual of thermal QCD-like theories inclusive of the aforementioned $\mathcal{O}(R^4)$ corrections, required $\mathcal{C}^{\rm th}_{zz} - 2\mathcal{C}^{\rm th}_{\theta_1 z} + 2\mathcal{C}^{\rm th}_{\theta_1 x} < 0$.
  - By taking the $M_{\rm KK} \rightarrow 0$ limit of $\mathbb{R}^2 \times S^1(\frac{1}{M_{\rm KK}})$ (to recover a four-dimensional QCD-like theory at $\partial M_5(S^1(x^0) \times \mathbb{R}^2 \times S^1(\frac{1}{M_{\rm KK}}) \times \mathbb{R}_{\geq 0})$, M_5 obtained after integrating out the base $M_6(\theta_1, \theta_2, x, y, z, x^{11})$ of the seven-fold $M_7 = \mathbb{R}_{\geq 0} \times M_6$ of G_2-structure), remarkably, we not only verify $\mathcal{C}^{\rm th}_{zz} - 2\mathcal{C}^{\rm th}_{\theta_1 z} + 2\mathcal{C}^{\rm th}_{\theta_1 x} < 0$, but in fact obtain the values of $\mathcal{C}^{\rm th}_{zz}, \mathcal{C}^{\rm th}_{\theta_1 z}, \mathcal{C}^{\rm th}_{\theta_1 x}$.

- Flavor Memory Effect: For the blackhole background, a specific combination of integration constants is appearing in $\mathcal{M}$ theory metric perturbations. These integration constants are along compact part of the non-compact four cycle wrapped by the flavor $D7$ brane in parent type IIB dual. We refer to this as the "Flavor Memory" effect in the $\mathcal{M}$ theory uplift.
- There is phase transition at a critical value of l i.e. l_{crit} similar to Klebanov, Kutasov, Murugan [2007]. In our case this is confinement-deconfinement phase transition in thermal QCD-like theories.
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Thank you.