Reconstruction of the UV graviton scattering amplitude from IR singularities

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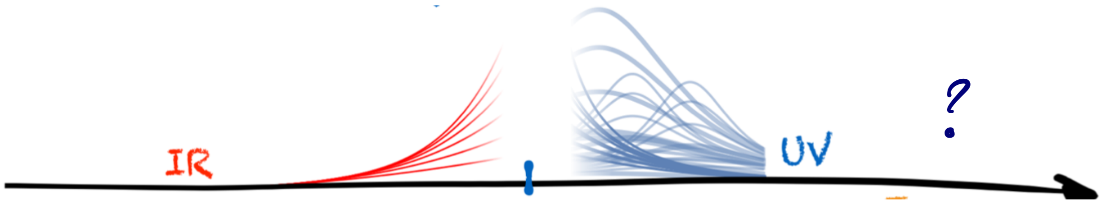
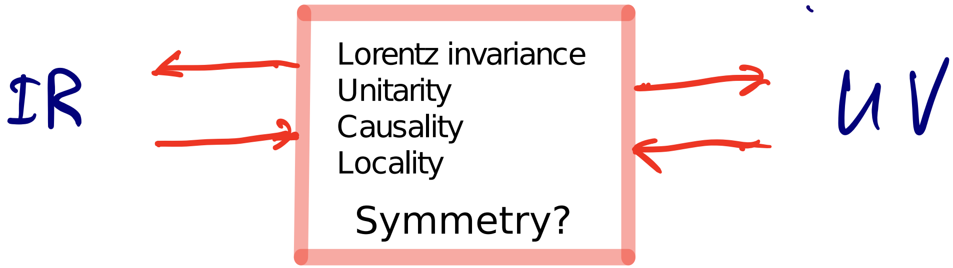
UV - IR connections

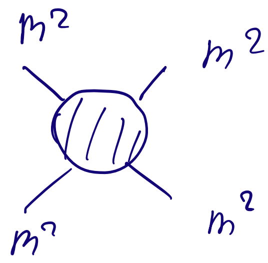




- Assumptions about UV constraints on IR (positivity bounds)
- IR results may require special UV properties for consistency
- The symmetry working in UV and IR can constrain the structure of IR EFT (the case of scale and conformal symmetries was studied in Shaposhnikov, AT'22)

A 'good' UV completion

2 → 2 amplitude



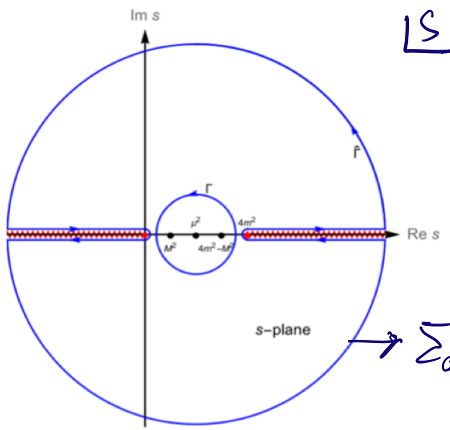
What do we mean by 'good'?

- Lorenz-invariant $\Rightarrow \mathcal{A} = \mathcal{A}(s, t, u)$
- unitary $\Rightarrow Im\, \mathcal{A} > 0$
- satisfying causality $\Rightarrow \mathcal{A}(s, t, u)$ is analytic everywhere except real axes
- local $\Rightarrow$ polynomial boundedness (Froissart-Martin bound)

$$A(s) < s \log^2 s$$

What is positive in positivity bounds?

Example: forward limit $t = 0$



ΣS

$\rightarrow \Sigma_\infty = 0$

Amplitude has singularities only on the real axis:

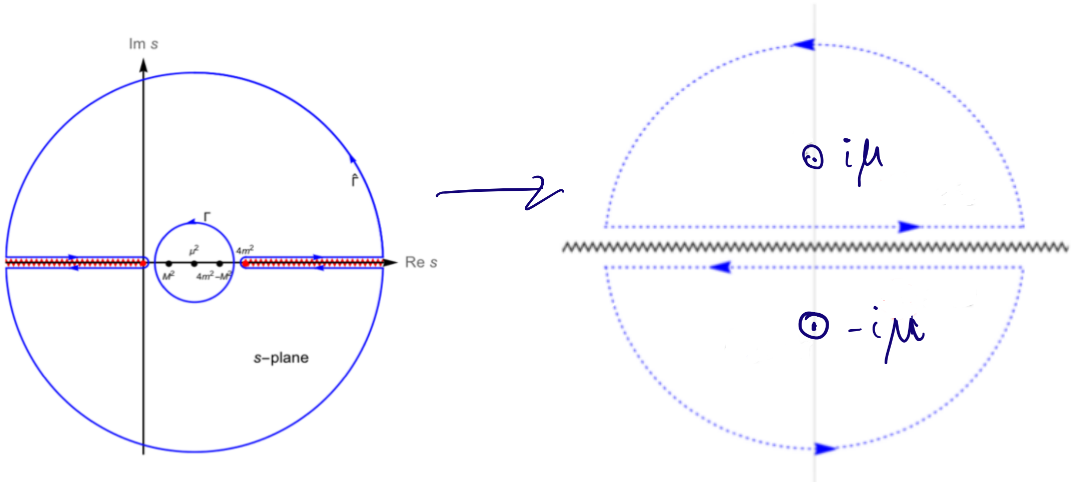
- poles for $0 < s < 4m^2$
- cuts for $s < 0$ and $s > 4m^2$

$$\Sigma_{IR} = \frac{1}{2\pi i}\int_\Gamma ds \frac{\mathcal{A}(s)}{(s-\mu^2)^3} = \int_{4m^2}^\infty \frac{ds}{\pi}\left(\frac{Im\mathcal{A}(s)}{(s-\mu^2)^3} + \frac{Im\mathcal{A}^+(s)}{(s-4m^2+\mu^2)^3}\right)$$

$$\Sigma_{IR} = \frac{1}{2}\mathcal{A}''(s) > 0$$ – rules out exact galileon symmetry

Positivity bounds: massive vs massless

Dispersive relations



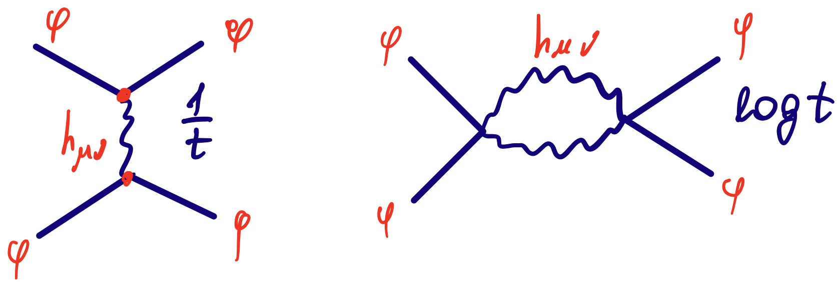
$$\Sigma_m = \frac{1}{2}A_{ss}(\mu^2) = \frac{1}{2\pi i}\int_\Gamma \frac{A(s)ds}{(s-\mu^2)^3} = \frac{1}{\pi}\int_{4m^2}^{\infty}\left(\frac{ImA(s)ds}{(s-\mu^2)^3} + \frac{ImA_*(s)ds}{(s+\mu^2-4m^2)^3}\right)$$

$$\Sigma_0 = \frac{\mathcal{A}_{ss}(\mu^2)}{16} - \frac{3i\mathcal{A}_s(\mu^2)}{16\mu^2} = \frac{1}{2\pi i}\int_\Gamma \frac{s^3A(s)ds}{(s^2+\mu^4)^3} = \int_0^\infty \frac{ImA(s)s^3ds}{(s^2+\mu^4)^3} + \text{crossed}$$

Herrero-Valea, Santos-Garcia, AT'20

Dispersive relations with graviton exchange

Divergences at $t \to -0$



$$A(s, t \to -0) = A_0 \frac{s^2}{M_P^2 t} + A_1 \frac{s^2}{M_P^4} \log\left(\frac{-t}{\mu_0^2}\right) + \text{higher loops} + O(t)$$

$$\Sigma_0 = \frac{1}{2}\left(\frac{A_0}{t} + A_1 \log t\right) + (\text{loops}) + O(t)$$

Where are the same divergences in the right hand side?

How to cancel $1/t$ and $\log t$?

The only source of the divergences is an infinite tail of the integral.

$$\int_{M^2}^{\infty} \mathrm{Im}\mathcal{A}(s,t)ds\left(\frac{1}{s^3}+\frac{1}{(s+t)^3}\right) = f(t) + (\text{finite at } t\to 0)$$

Assume that after some scale M

$$\mathrm{Im}\mathcal{A} = s^{2+jt}\left(1+\frac{\xi}{\log s}\right)$$

This form allows to get $1/t$ and $\log t$ (Herrero-Valea, Santos-Garcia, AT'20). Generalisation:

$$\mathrm{Im}\mathcal{A} = s^{2+jt}\phi(s,t)$$

$$\phi(s,t) = \phi(s,0) + \phi_t(s,0)t + \frac{1}{2}\phi_{tt}(s,0)t^2 + \dots$$

$$\int_0^{\infty} 2\,\phi(s,0)s^{jt}\left(\frac{ds}{s}\right) = \int_0^{\infty} 2M^{2jt}\,\phi(\sigma,0)e^{j\sigma t}d\sigma = f(t) + (\text{finite at } t\to 0)$$

$s = M^2 e^{\sigma}$

UV and IR are connected by the Laplace transformation

$$\phi(\sigma, 0) = \mathcal{L}^{-1}[f(t)] + O(t)$$

Next orders in t? Up to subleading terms in $t \to 0$ limit:

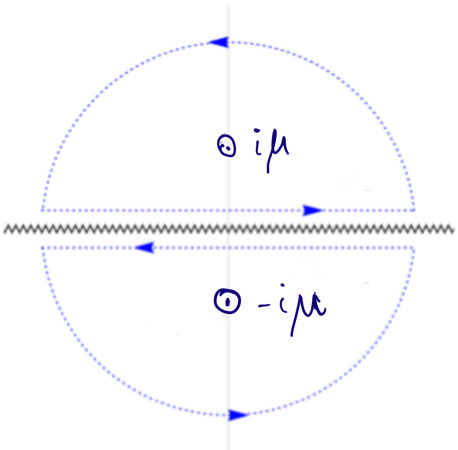
$$\phi(\sigma, 0) = a_0 L^{-1}[f(t)],$$
$$\phi_t(\sigma, 0) = a_1 L^{-1}\left[\frac{f(t)}{t}\right],$$
$$\phi_{tt}(\sigma, 0) = a_2 L^{-1}\left[\frac{f(t)}{t^2}\right], \ \ldots$$

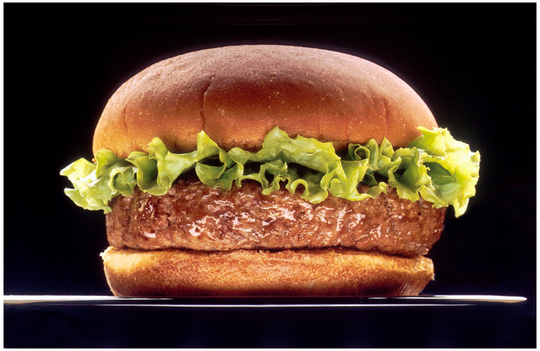
$$f(t) = \frac{A}{t}, \quad \phi(\sigma, t) = \sum a_n \sigma^n t^n = \phi(\sigma t) = \phi(t \log s), \quad \phi(0) \neq 0$$

Recall that $A(s) < s^2$ at any $t \neq 0$. The dispersion relation allows to get the UV amplitude in the limit $t \log s \to 0$ while $t \to 0$ and $s \to \infty$.

Herrero-Valea, Koshelev, AT'22

Infinite arc integral





We learned that in the UV at the leading order

$$A = \mathrm{Re}\ A + i\gamma s^{2+jt}$$

Let's check the infinite arc contribution

$$I_R = \frac{1}{2\pi i}\oint ds \frac{A(s)}{s^3} = \frac{1}{2\pi i}\int_0^{2\pi} id\theta \frac{A(Re^{i\theta})e^{-2i\theta}}{R^2}$$

$$I_R = \frac{i\gamma}{2\pi}\int_0^{2\pi} d\theta R^{jt} e^{ijt\theta} + \frac{1}{2\pi}\int_0^{2\pi}\frac{d\theta}{R^2} ReA(Re^{i\theta})e^{-2i\theta} = i\gamma + (\text{real part})$$

Herrero-Valea, Koshelev, AT'22

Reconstructing the amplitude from the imaginary part

$$\mathrm{Im}A(s,t) = i\gamma s^{2+jt}$$

$$\mathcal{A}(s,t) = \frac{s^2}{2\pi i}\oint_{\gamma_s}\frac{\mathcal{A}(z,t)dz}{z^2(z-s)} = F(s,t) + F(-s-t,t)$$

$$F(s,t) = \frac{s^2}{\pi}\int_0^\infty \frac{\mathrm{Im}\mathcal{A}(z,t)dz}{z^2(z-s)} = \frac{s^2}{\pi}\int_0^{M_*^2}\frac{\mathrm{Im}\mathcal{A}(z,t)dz}{z^2(z-s)} + \\ + \frac{s^2}{\pi}\int_{M_*^2}^\infty \frac{a_0 z^{jt}dz}{z-s} + \mathcal{O}\left(t\log(s)\right)$$

$$A(s,t) = -\frac{\gamma e^{-i\pi jt}}{\sin\left(\pi\alpha' t\right)}\left(s^{2+\alpha' t} + (-s-t)^{2+jt}\right) + \mathcal{O}\left(t\log(s)\right)$$

Real and imaginary parts are connected by analiticity!

Herrero-Valea, Koshelev, AT'22

Infinite arc contribution

Contribution from the arc R

$$\Sigma_\infty = -\frac{2\gamma e^{-i\pi jt}}{\sin(\pi jt)}\frac{R^{jt}}{2\pi}\frac{e^{2\pi ijt}-1}{ijt} = -\frac{2\gamma}{\pi jt}R^{jt} = -\frac{2\gamma}{\pi jt}$$

Contribution from ImA

$$\Sigma_{UV} = \frac{2}{\pi}\int_{M_*^2}^{\infty}\frac{ds\,\mathrm{Im}\mathcal{A}(s,t)}{s^3} = \frac{2}{\pi}\int_{M_*^2}^{R}\frac{ds}{s}a_0 s^{jt} = \frac{2\gamma}{\pi jt}\left(R^{jt} - (M_*^2)^{jt}\right)$$

Two limits can be considered: $R \to \infty$ with finite $t < 0$ and $t \log R \to 0$. The result is the same.

Conclusions

- UV and IR amplitudes are connected under the assumptions of unitarity, locality and analyticity of the fundamental theory
- Assumptions about UV lead to positivity bounds for IR theory
- IR singularities in the forward limit open the possibilities to find the form of UV amplitude in the limit $t \log s \to 0$, $s \to \infty$
- Gravity invalidates positivity bounds for $A''(s)$ but they still can be obtained from $A^{(4)}(s)$ and higher...

Thank you for your attention!