One-loop Matching for Perturbative Unitary Theories

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Based on 1911.06822, 2104.10930
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Content

- Perturbative Unitarity and Gauge Theories
- Slavnov Taylor Identities
  - to eliminate unphysical d.o.f.
  - constrain physical coupling constants
- Sketch renormalisation of FCNCs

Perturbative Unitary Theories

- We have N^nLO results for the SM
- and LO / NLO results for some BSM models.
- Can we derive results valid for any BSM model?
- RGEs for arbitrary gauge groups are known. [Jack et.al.'07]
- What about broken gauge theories?
  - Start with loop induced FCNCs

Renormalisable Theories?

- Perturbative Unitary $\leftrightarrow$ massive vectors from SSB [Llewellyn Smith '73; Cornwall et.al. 73/74]
- Need correct high-energy behaviour in loops:
  - Gauge-structure from Slavnov-Taylor Identities (STs)
  - Traditionally used in high-energy scattering ("Goldstone-boson Equivalence Theorem")
- UV behaviour controls renormalization properties
  - This talk: renormalise loop induced FCNCs

Remnants of gauge symmetry

- Massive vector bosons originate from a spontaneously broken gauge symmetry

- Fix the gauge for massive vector ($\sigma_{V^\pm} = \pm i$, $\sigma_V = 1$)

$$\mathcal{L}_{\text{fix}} = -\sum_{v} (2\xi_v)^{-1} F_{\bar{v}} F_v \,, \qquad F_v = \partial_\mu V_v^\mu - \sigma_v \xi_v M_v \phi_v \,,$$

- BRST invariant field combination $s(\ldots)_{\text{ph}} = 0$

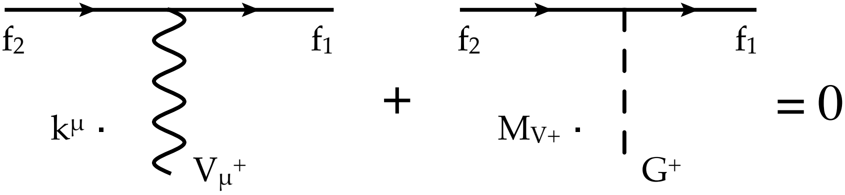
- STs from $s\langle T \bar{u}_v (\ldots)_{\text{ph}} \rangle = 0$ at required order:

$$\langle T \Big(k^\mu \underline{V_v^\mu} - i\sigma_{\bar{v}} M_v \underline{\phi_v} \Big) (\ldots)_{\text{ph}} \rangle = 0$$

Eliminate Goldstone Boson Couplings

- E.g. for $(\ldots)_{\text{ph}} = \bar{f}_1 f_2$ we have

$$\langle T\Big(k^\mu \underline{V}^\mu_{v^+} + M_{v^+}\underline{\phi}_{v^+}\Big)(\bar{f}_1 f_2)_{\text{ph}}\rangle = 0$$



$$y^{L/R}_{\phi_1 \bar{f}_1 f_2} = \frac{1}{M_{v_1}}\Big(m_{f_1} g^{L/R}_{v_1 \bar{f}_1 f_2} - g^{R/L}_{v_1 \bar{f}_1 f_2} m_{f_2}\Big)$$

Elimination of All Goldstone boson couplings

- From $(VV)_{\text{ph}}, (Vh)_{\text{ph}}, (hh)_{\text{ph}}$ we obtain 3-point STs:

$$g_{v_1\phi_2\phi_3} = \sigma_{v_2}\sigma_{v_3}\,\frac{M_{v_2}^2+M_{v_3}^2-M_{v_1}^2}{2\,M_{v_2}M_{v_3}}\,g_{v_1v_2v_3}\,, \quad g_{v_1\phi_2 s_1} = -i\sigma_{v_2}\,\frac{1}{2\,M_{v_2}}\,g_{v_1v_2s_1}\,,$$

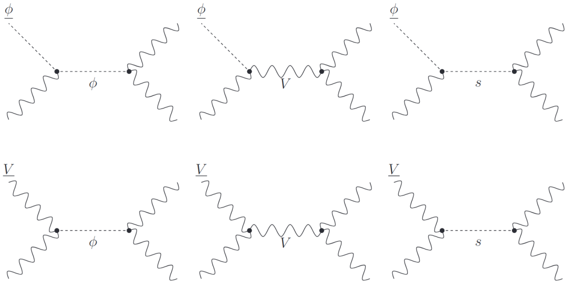
$$g_{v_1v_2\phi_3} = -i\sigma_{v_3}\,\frac{M_{v_1}^2-M_{v_2}^2}{M_{v_3}}\,g_{v_1v_2v_3}\,, \quad g_{\phi_1s_1s_2} = i\sigma_{v_1}\,\frac{M_{s_1}^2-M_{s_2}^2}{M_{v_1}}\,g_{v_1s_1s_2}\,,$$

$$g_{\phi_1\phi_2 s_1} = -\sigma_{v_1}\sigma_{v_2}\,\frac{M_{s_1}^2}{2\,M_{v_1}M_{v_2}}\,g_{v_1v_2s_1}\,, \quad g_{\phi_1\phi_2\phi_3} = 0\,.$$

- Plus higher vertices allows us to eliminate all Goldstone couplings [1903.05116]

Constraints on Physical Couplings

- $s\langle T\bar{u}_v(VVV)_{\mathrm{ph}}\rangle = 0$ relates quartic coupling and

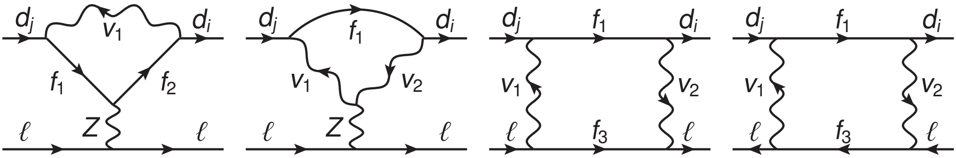


- Results in quadratic (Jacobi) constraints on gauge couplings

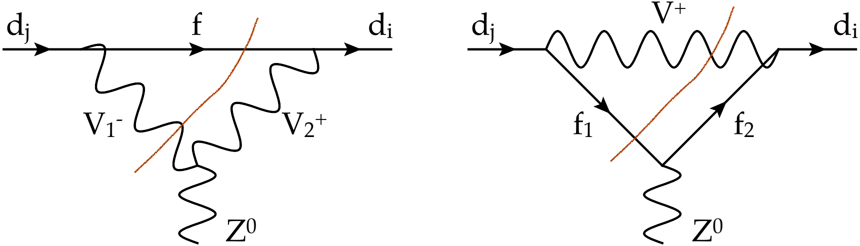
$$\sum_{v_5} g_{v_1 v_2 v_5} g_{v_3 v_4 \bar{v}_5} + g_{v_2 v_3 v_5} g_{v_1 v_4 \bar{v}_5} + g_{v_3 v_1 v_5} g_{v_2 v_4 \bar{v}_5} = 0$$

$$g_{v_1 v_2 v_3 v_4} = \sum_{v_5} g_{v_1 v_4 v_5} g_{v_2 v_3 \bar{v}_5} + g_{v_1 v_3 v_5} g_{v_2 v_4 \bar{v}_5}$$

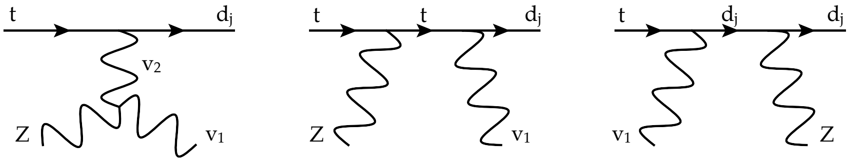
Consider loop FCNC from massive vectors



- Idea: Renormalisation via high energy tree-level properties derived in 1903.05116.



Identities for $d > 4$ Green's functions



- Setting $v_3 = Z$, $f_2 = d_j$ there are two additional STs:

$$g^L_{Z\bar{t}t} g^L_{v_1^+\bar{t}d_j} = g^L_{v_1^+\bar{t}d_j} g^L_{Z\bar{d}_j d_j} + \sum_{v_2} g_{Zv_1^+v_2^-} g^L_{v_2^+\bar{t}d_j}$$

$$g^R_{Z\bar{t}t} g^L_{v_1^+\bar{t}d_j} = \frac{1}{2} g^L_{v_1^+\bar{t}d_j}\left(g^L_{Z\bar{t}t} + g^L_{Z\bar{d}_j d_j}\right) + \sum_{v_2} \frac{M^2_{v_1} - M^2_Z}{2M^2_{v_2}} g_{Zv_1^+v_2^-} g^L_{v_2^+\bar{t}d_j}$$

- Which can be used to eliminate $g^{L/R}_{Z\bar{t}t}$ from the expression
- For $v_1^+ v_2^- d_i d_j$ external states we obtain generalised GIM

Generic Vector Interactions in (B)SM

$$\mathcal{L}_3^V = g^{L/R}_{v_1\bar{f}_1 f_2} V_{v_1,\mu}\bar{\psi}_{f_1}\gamma^\mu P_{L/R}\psi_{f_2} + \frac{i}{6} g^{abc}_{v_1 v_2 v_3}\Big(V_{v_1,\mu} V_{v_2,\nu}\partial^{[\mu} V^{\nu]}_{v_3} + \dots\Big).$$

- In SM, for $K \to \pi\nu\bar{\nu}$ we would need the following:
  - $g^L_{W^+\bar{u}_j d_k} = \dfrac{e}{s_w\sqrt{2}} V_{jk}, \quad y^L_{G^+\bar{u}_j d_k} = \dfrac{m_{uj}}{M_W}\dfrac{e}{s_w\sqrt{2}} V_{jk}$
  - $g^L_{Z\bar{f}_j f_k} = \dfrac{2e}{s_{2w}}\left(T_3^f - Q_f s_w^2\right)\delta_{jk}, \quad g^R_{Z\bar{f}_j f_k} = -\dfrac{2e}{s_{2w}} Q_f s_w^2 \delta_{jk}$
  - $g_{ZW^+W^-} = \dfrac{e}{t_w}, \; g_{ZW^+G^-} = -t_w^2\dfrac{e}{t_w}, g_{ZG^+G^-} = \left(1 - \dfrac{1}{2c_W^2}\right)\dfrac{e}{t_w}$
- We can also combine Z/γ-Penguin and Boxes using:

$$\sum_Z g^\sigma_{Z\bar{\ell}\ell} g_{Z v_2 \bar{v}_1} = -\delta_{\bar{v}_1 v_2} g^\sigma_{\gamma\bar{\ell}\ell} g_{\gamma v_2 \bar{v}_1} - \sum_{f_3}\left(g^\sigma_{\bar{v}_1\bar{\ell} f_3} g^\sigma_{v_2 \bar{f}_3 \ell} - g^\sigma_{v_2 \bar{\ell} f_3} g^\sigma_{\bar{v}_1 \bar{f}_3 \ell}\right)$$

Gauge independent result for $s \to d\nu\bar{\nu}$

$$C_{L\sigma}^{sd\nu} = \sum_{v_1 v_2 f_1 f_3} \frac{g^L_{\bar{v}_2 \bar{s} f_1} g^L_{v_1 \bar{f}_1 d}}{M_{v_1}^2} g^\sigma_{v_2 \bar{\nu} f_3} g^\sigma_{\bar{v}_1 \bar{f}_3 \nu} F_V^{\sigma, B' Z}(x_{v_1}^{f_0}, x_{v_1}^{f_1}, x_{v_2}^{v_1}, x_{v_1}^{f_3})$$
$$+ \sum_{Z v_1 v_2 f_1 f_2} \frac{g^\sigma_{Z\bar{\nu}\nu} g^L_{v_1 \bar{f}_1 d} g^L_{\bar{v}_2 \bar{s} f_2}}{M_Z^2} \Big\{ \delta_{f_1 f_2} g_{Z \bar{v}_1 v_2} F^Z_{V''}(x_{v_1}^{f_0}, x_{v_1}^{f_1}, x_{v_2}^{v_1})$$
$$+ \delta_{v_1 v_2} \left[g^L_{Z \bar{f}_2 f_1} F_V^Z(x_{v_1}^{f_1}, x_{v_1}^{f_2}) + g^R_{Z \bar{f}_2 f_1} F^Z_{V'}(x_{v_1}^{f_1}, x_{v_1}^{f_2}) \right] \Big\},$$

- Extends the Penguin Box Coefficients to generic theories ($X_t \leftrightarrow F_V^{\sigma, B' Z}(0, x_W^t, 1, 0)$ & $F^Z_{V^{(\prime)}}(x, x) = F^Z_{V''}(x, y, 1) = 0$)
- Results for various external states includes also extra scalars [2104.10930 and Github].

Z' model with flavour off-diagonal couplings

- [1704.06005] for $b \to s\ell\ell$: vector-like T quark charged under spontaneously broken $U(1)'$

- Express $b \to s\ell\ell$ in terms of generalized penguin-box functions $F^{Z}_{V^{(\prime)}}(x^t_W, x^T_W)$

$$C^{\mu,\mathrm{NP}}_{9/10} = \frac{s_R^2}{2} q' q'_{\mu,V/A} \frac{m_t^2}{M_{Z'}^2} \frac{\tilde{g}^2}{e^2} \left\{ \frac{1}{2} \log\left(x^T_W\right) + \frac{1}{c_R^2} + \frac{3}{2(x_t - 1)} \right.$$
$$\left. -1 - \frac{1}{2}\left(\frac{3}{(x_t-1)^2} + 1\right) \log(x^t_W) \right\}$$

Outlook

- STs can be used to derive gauge independent matching conditions for generic theories
- Resulting sumrules constrain generic theory
  - have been rederived using on-shell amplitudes [2204.13119]
- STs work for renormalisable models. Integrating out leads to polynomial corrections that reproduce SMEFT effects [1909.10551]
- Todo:
  - Extend to Higgs potential and proof that the system closes
  - Derive phenomenological formulae e.g. for direct detection and Higgs physics