Microscopic and Macroscopic Effects in the Decoherence of Neutrino Oscillations

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IMPRS

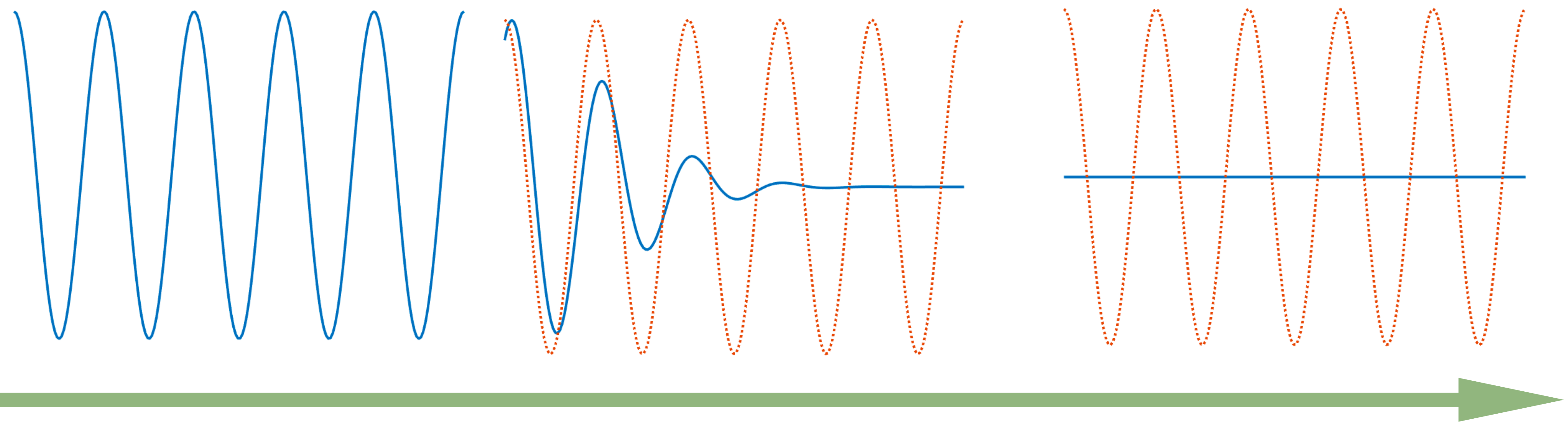
for Precision Tests of Fundamental Symmetries

INTERNATIONAL MAX PLANCK RESEARCH SCHOOL

Decoherence

FTP: $P_{3,\alpha\to\beta} = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i\frac{\Delta m^2_{jk} L_0}{2E_0}} \phi_{jk}(\overrightarrow{\sigma}, L, E)$ $\quad 0 \leq |\phi_{jk}| \leq 1$

Coherent (Quantum) Decoherent Classical



Damping + Phase shift (if ϕ_{jk} has an imaginary part)

- Open quantum system: lose information
- Wavepacket seperation: lose mixing

Decoherence - what has been done?

- QM Wavepackets $\phi_{jk}(L,E) = \exp\left[-\left(\frac{\Delta m_{jk}^2 L \sigma}{2\sqrt{2}E^2}\right)^2\right]$ (WP seperation)
  [Gitunti 1991]

- S-Matrix formalism (QFT) & Consider Wavepackets

$$\phi_{jk}(L,E) = \exp\left[-\left(\frac{\Delta m_{jk}^2 L \sigma_{eff}}{2\sqrt{2}E_{eff}^2}\right)^2 - \xi\right]$$

(WP seperation)
[Beuthe 2001], [Akhmedov, Kopp 2010]

- Density Matrix formalism (QM) — Open Quantum System

Lindblad Equation: $\frac{\partial \rho}{\partial t} = -i[H,\rho] - \mathscr{D}[\rho]$ 9×9 entries

$$\phi_{jk}(L,E) = \exp\left[-\left(\frac{\alpha m_{jk}^2 L}{E^n}\right)^2\right],$$

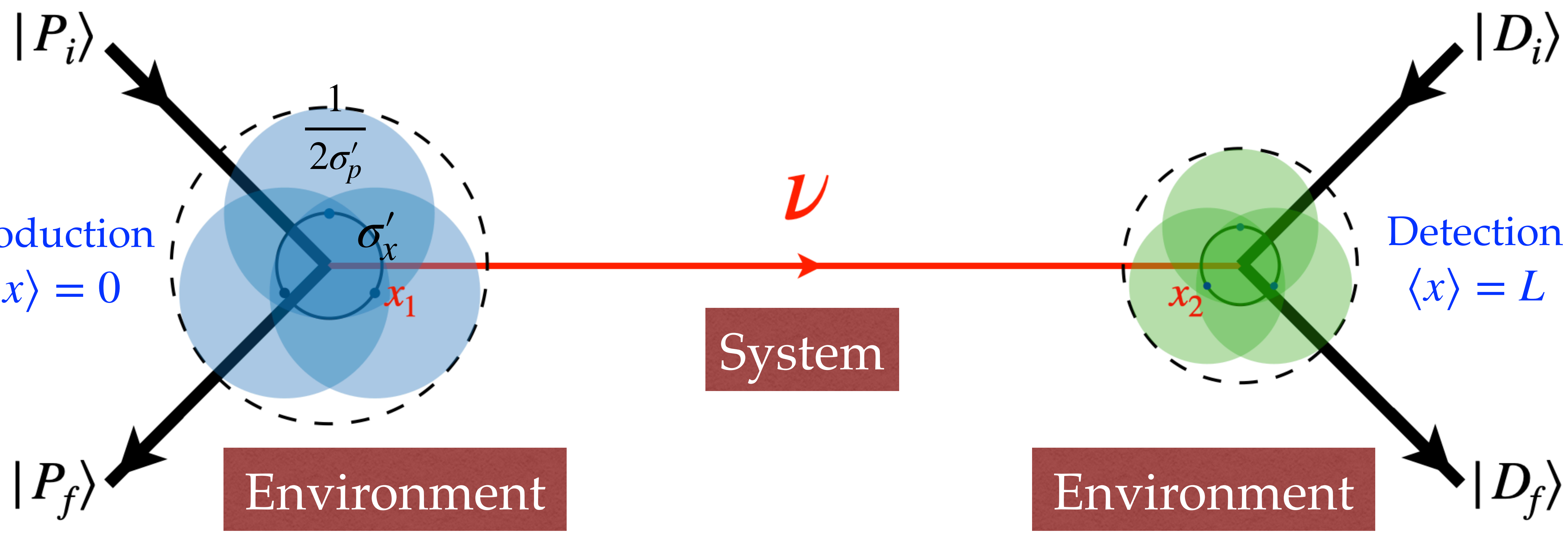
e.g. $n = -2$: matter effect fluctuation
$n = 0,2$: quantum gravity
$n = 1$: neutrino absorption

[2112.14450 as a review]

QFT + Open Quantum System

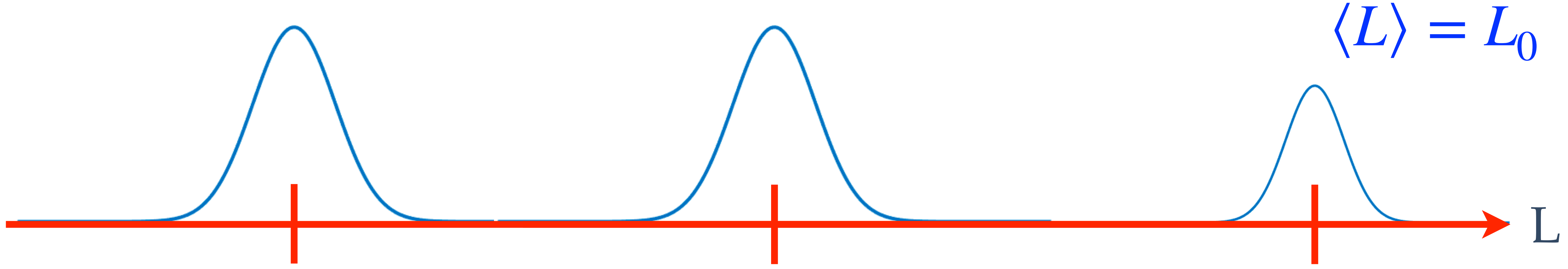
- Intrinsic uncertainties: unavoidable, can only be squeezed
- σ_x : (off-shell) neutrino interactions (related to the form factor)
- σ_p : (on-shell) external particle's WP size (mean free path, life time, …)

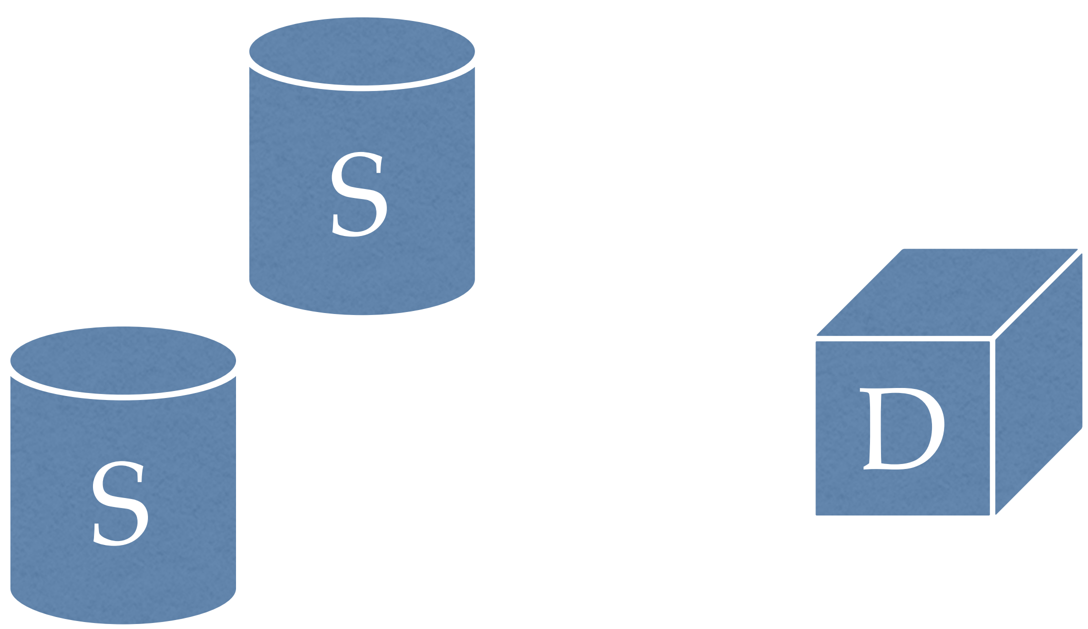
(Just a parameterisation not the total position/momentum uncertainty !)



Classical Uncertainties

- Statistical averaging — sum over the probability (amplitude^2)
- Reflects our ignorance — avoidable
- What do we get in the end? Expectation value of the FTP





- σ_L : production profile, space time (gravitation) fluctuation …
- σ_E : energy reconstruction model, energy resolution …

The (Phase Space) Layer Structure

Measurement layer (layer 3)

- Kinematic variables : $\{ T_0 , \mathbf{L}_0 , \mathbf{P}_0 \}$ or $\{T_0 , L , E_0 , \Omega \}$ $\xrightarrow{\text{Consistent emission}}$ $\{L_0 , E_0 , \Omega\}_{T_0=0}$
- Uncertainties : Those from lower layers

$\sigma_L \; \sigma_E$ $\uparrow$ $\mathscr{LMO}^2$

Physical layer (layer 2)

- Kinematic variables : $\{T , \mathbf{L} , \mathbf{P}\}$
- Uncertainties : Macroscopic + Those from layer 1

$\sigma_p \; \sigma_x$ $\uparrow$ $\mathscr{LMO}^1$ $\qquad$ $\uparrow$ $\mathscr{LMO}^{\bar{1}}$ $\sigma_p \; \sigma_x$

Microscopic layer (layer 1) — Fock space

- Kinematic variables : $\{x , p\} \xrightarrow{\text{on shell}} \{x , \mathbf{p}\}$
- Uncertainties : Microscopic

Microscopic layer (layer 1) — Wigner space

- Kinematic variables : $\{\bar{t} , \bar{\mathbf{x}} , \bar{\mathbf{p}}\}$ (on shell)
- Uncertainties : Microscopic

The layer moving operators ($\mathscr{LMO}$) contain the uncertainty parameters

The Layer Moving Operator

- The layer moving operator:

$$B_{i+1}(x_{i+1}, p_{i+1}) = \mathscr{L}\mathscr{M}\mathscr{O}^i B_i(x_i, p_i) = \int d^4x_i \int d^3p_i\, W_i(x_i, p_i; x_{i+1}, p_{i+1}) B_i(x_i, p_i)\,,$$

Layer $i+1$ Layer i Weighting function

- width as uncertainties
- Spread through (x_i, p_i)
- Centered at (x_{i+1}, p_{i+1})

- Flavor transition probability:

$$P_3(T_0, \mathbf{L}_0, \mathbf{P}_0) = \mathscr{L}\mathscr{M}\mathscr{O}^2 P_2(T, \mathbf{L}, \mathbf{P})$$ Layer $2 \rightarrow 3$

$$P_2(T, \mathbf{L}, \mathbf{P}) = A_2^*(T, \mathbf{L}, \mathbf{P}) A_2(T, \mathbf{L}, \mathbf{P}), \quad A_2(T, \mathbf{L}, \mathbf{P}) = \mathscr{L}\mathscr{M}\mathscr{O}^1 A_1(\mathrm{x}, \mathrm{p})$$ Layer $1 \rightarrow 2$

$$P_2(T, \mathbf{L}, \mathbf{P}) = \mathscr{L}\mathscr{M}\mathscr{O}^{\bar{1}} P_{\bar{1}}(\bar{\mathbf{x}}, \bar{\mathbf{p}})\,.$$ Layer $\bar{1} \rightarrow 2$

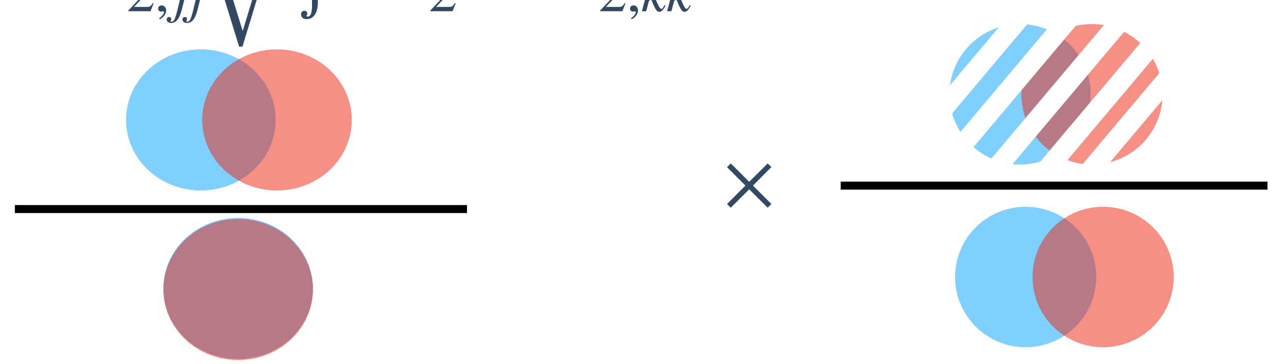
General Neutrino Decoherence

- Operation definition of FTP:

Normalised automatically

$$P_{3,jk}(X_3) = \frac{\overbrace{\mathscr{L}\mathscr{M}\mathscr{O}_2}\;\boxed{\int dX_2\, H(X_2;X_3)}\,\boxed{\Gamma_{2,jk}(X_2;X_3)}\ \text{unnormalised } P_{2,jk}}{\sqrt{\int dX_2\, H(X_2;X_3)\,\Gamma_{2,jj}(X_2;X_3)}\sqrt{\int dX_2\, H(X_2;X_3)\,\Gamma_{2,kk}(X_2;X_3)}},$$

$$= \frac{\int dX_2 H\,|\,\Gamma_{2,jk}\,|}{\sqrt{\int dX_2\, H\,\Gamma_{2,jj}}\sqrt{\int dX_2\, H\,\Gamma_{2,kk}}} \times \frac{\int dX_2\, H\,\Gamma_{2,jk}}{\int dX_2 H\,|\,\Gamma_{2,jk}\,|}$$



- PWO effect: $\dfrac{\int dx\,\Gamma(x;L)}{\int dx\,|\Gamma(x;L)|} = e^{i(\eta(x)|_{x=L}-\beta)}\Phi(L)$, for $\Gamma(x;L) \equiv |\Gamma(x;L)|e^{i\eta(x)}$

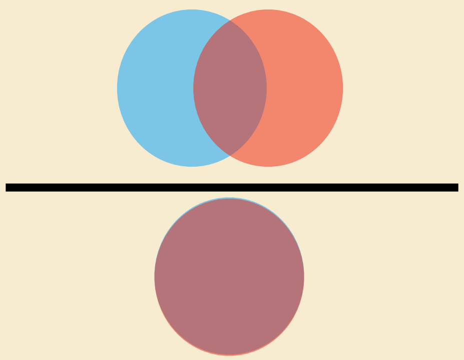
Classification of Decoherence

$\phi_{jk} =$ State Decoherence $\times$ Phase Decoherence

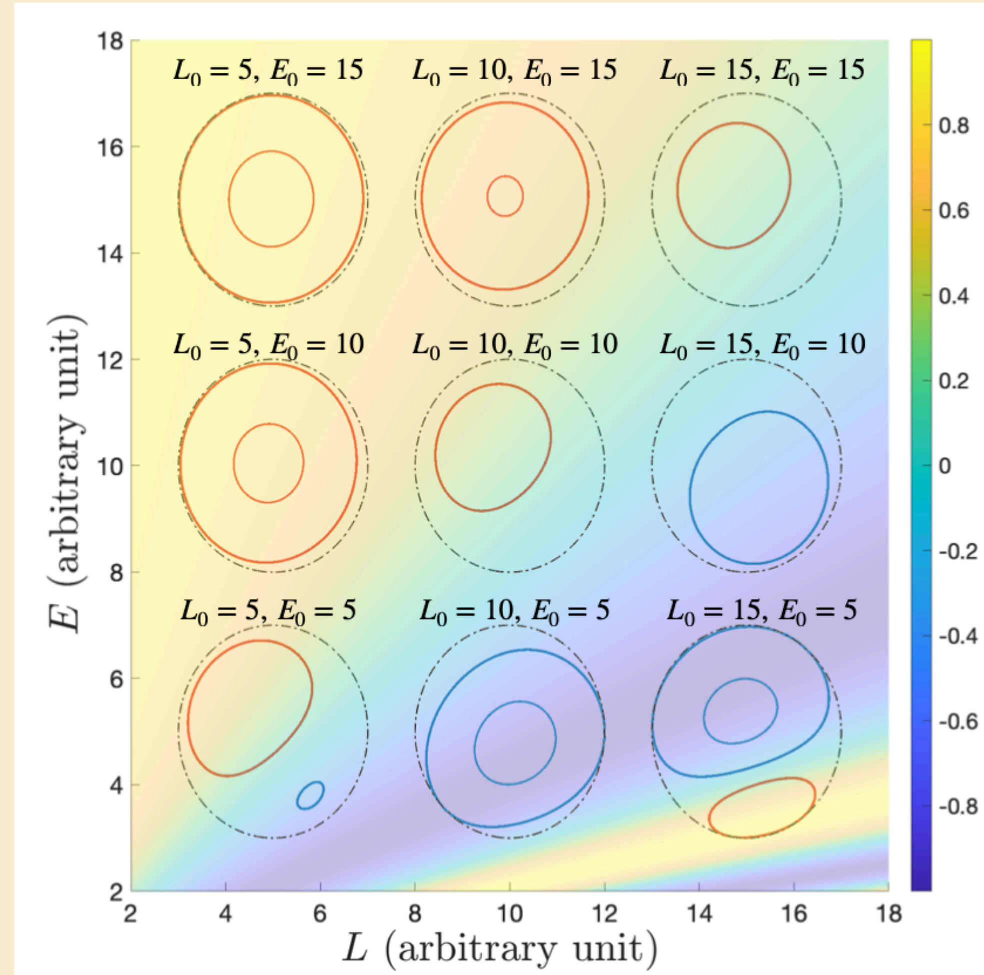
Quantum effects σ_x, σ_p

Dominated by classical effects σ_L, σ_E

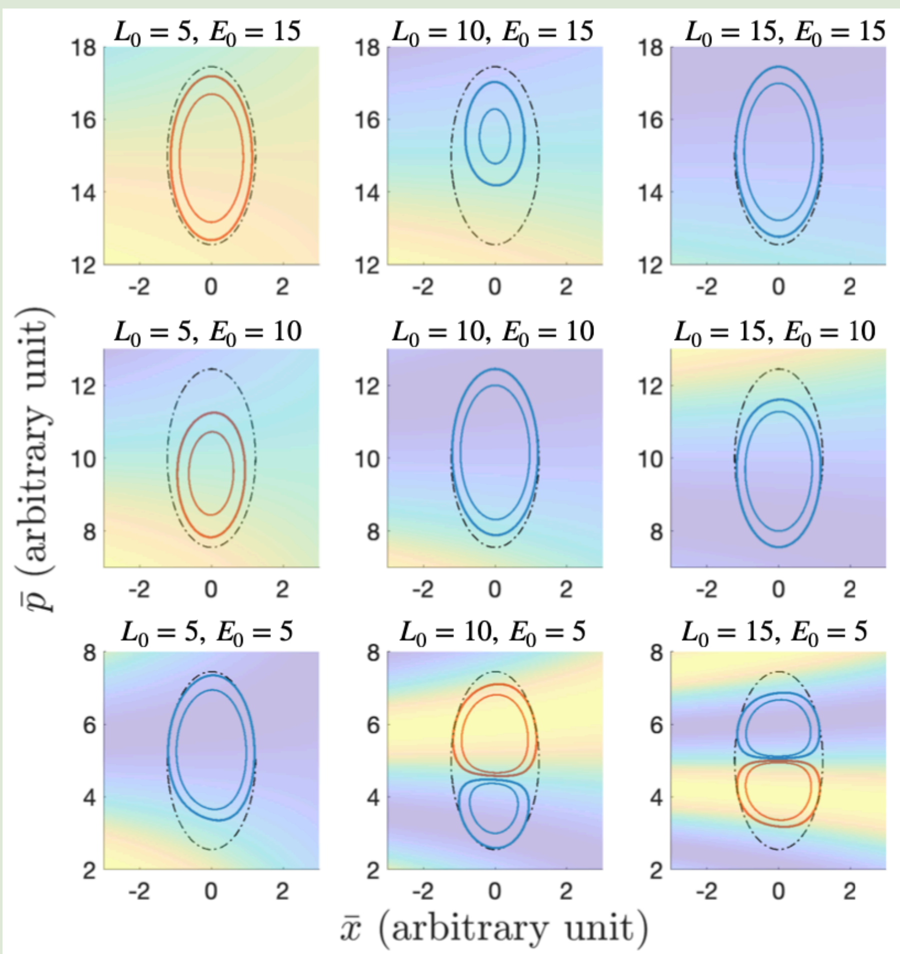
Physical Layer



$\times$



Micro. Layer, Wigner PS

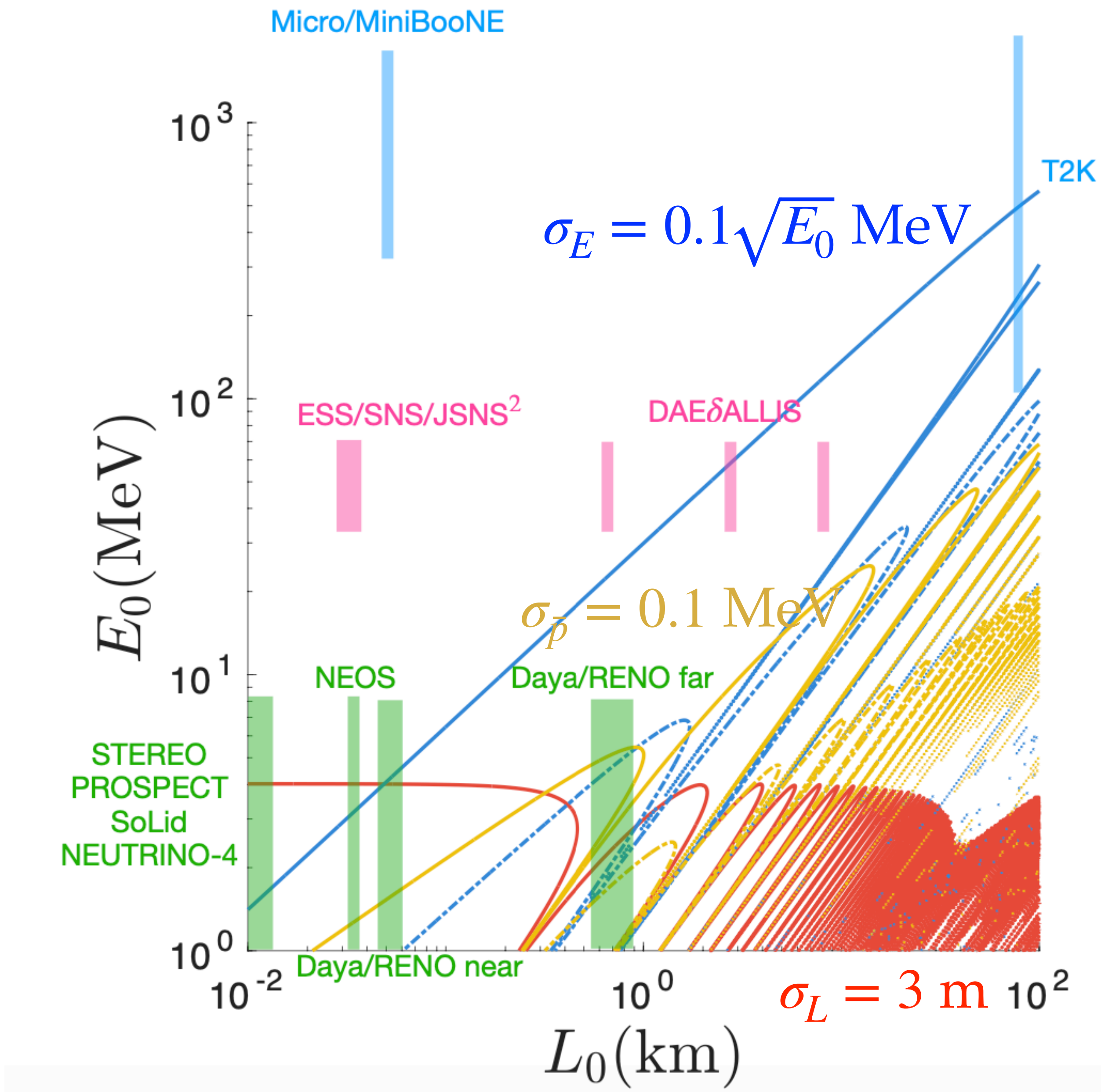


- Phase wash-out effect
- Non-symmetric uncertainty distribution: Imaginary part in ϕ_{jk} (phase shift signatures)

Sensitivity

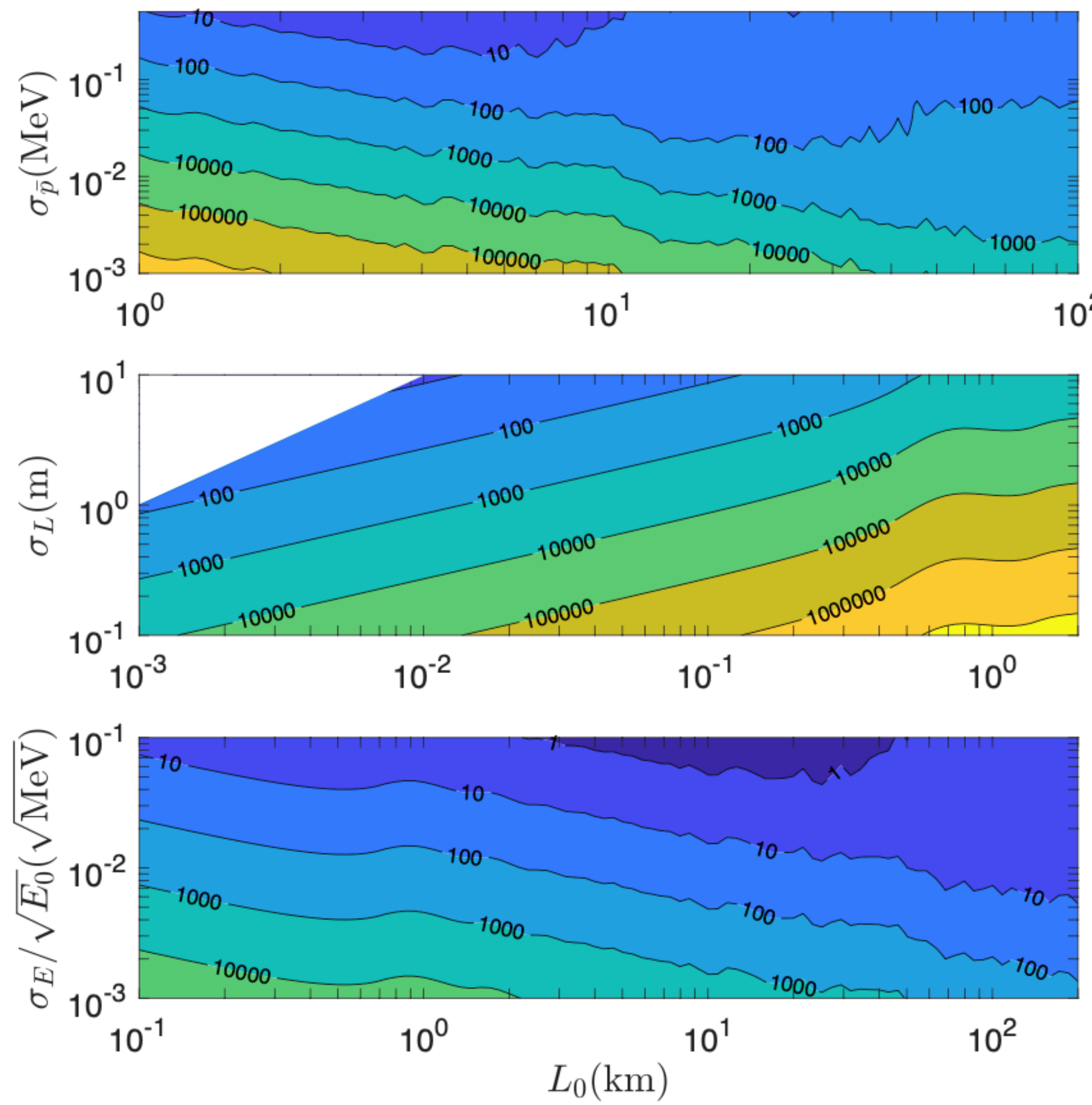
Where to look?

$$|P_{\alpha\beta}(\sigma_n) - P_{\alpha\beta}(\sigma_n = 0)| = 10^{-6}$$



How far are we?

Benchmark: how much more statistic than RENO?



Summary

- Neutrino oscillation experiments are going into a precision era — able to see decoherence by damping and/or phase shift signatures
- Formulate a structure to parameterise neutrino decoherence by its quantum or classical origins → theory input
- Find that decoherence can be describe by a phase averaging effects (PWO effects) of the oscillation phase or the phase in the Wigner phase space → allows simple numerical calculation
- Observe decoherence by measuring the oscillation phase, spectrum and traveling distance → experimental & analysis design

Outlook

Neutrino oscillation going to the precision era

This work

Fundamental

Generic

Useful

Nov. 2002

The dream

Experimental design, e.g. phase measuring method, squeezed states, interferometry

Ground based Neutrinos @ Low energy

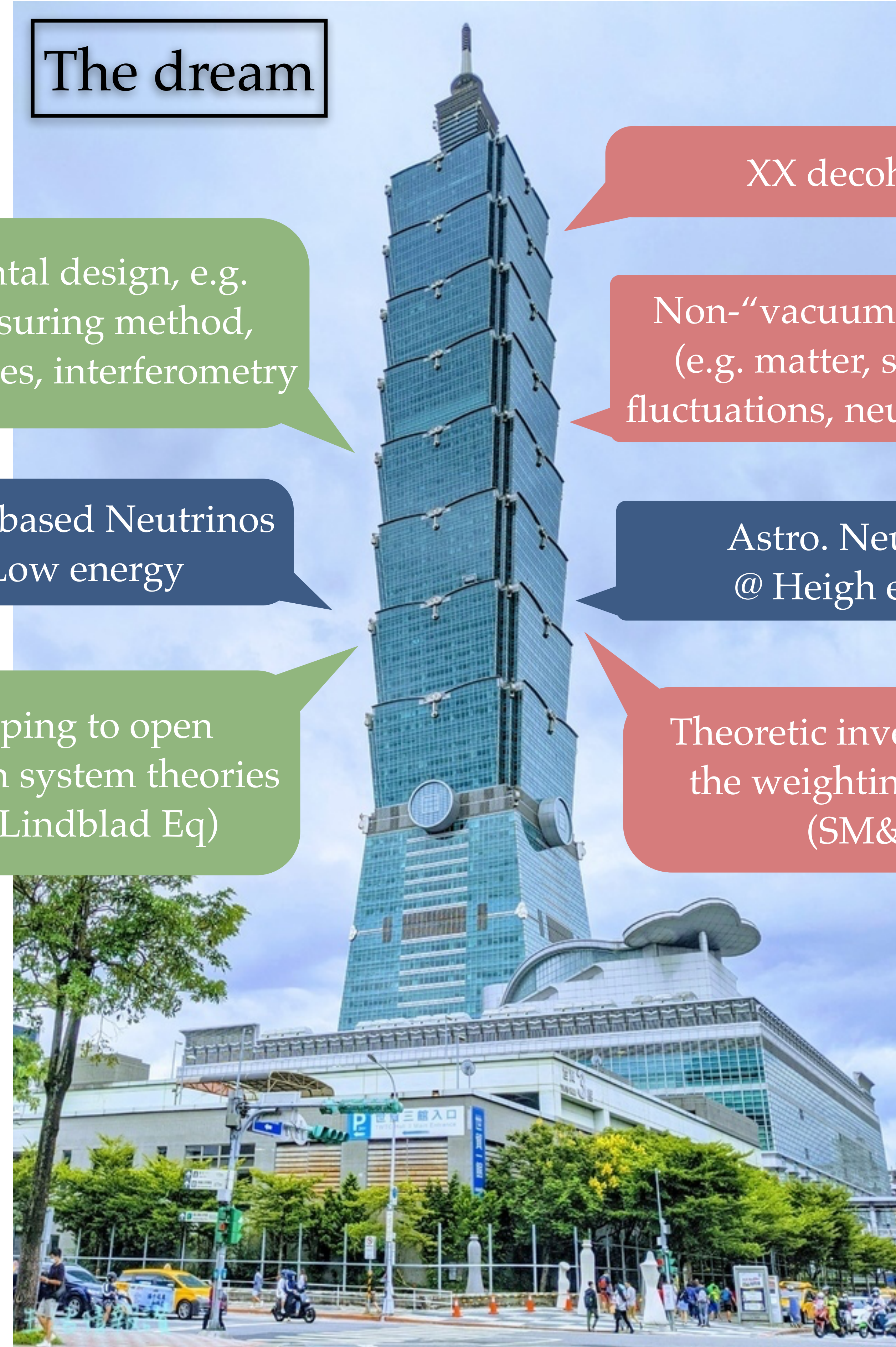
Mapping to open quantum system theories (e.g. Lindblad Eq)

XX decoherence

Non-“vacuum” scenario , (e.g. matter, space-time fluctuations, neutrino decay)

Astro. Neutrinos @ Heigh energy

Theoretic investigation on the weighting functions (SM&NP)



Dec. 2004

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Backup Sildes

The Microscopic Layer

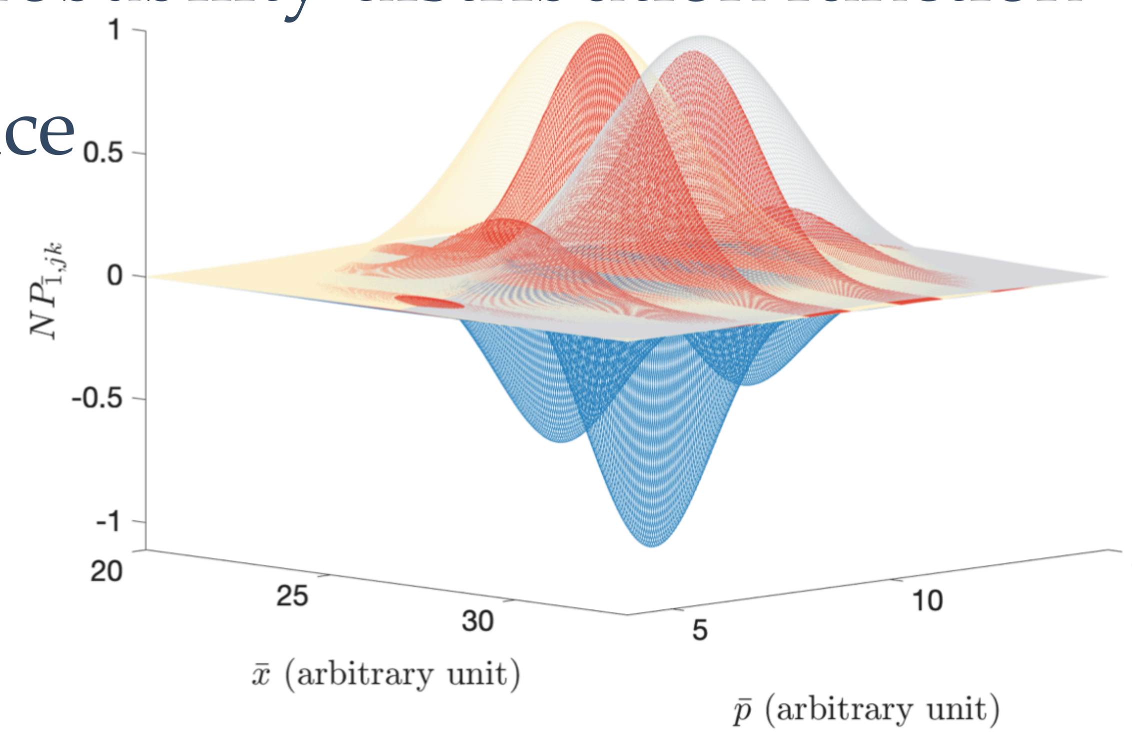
- Representation of a layer: how particle states are described

Layer 1: QFT Transition Amplitude, $A_1(\mathrm{x}, \mathrm{p})$

- Fock phase space — Fock states (second quantisation)

Layer $\bar{1}$: Wigner Transition Probability, $P_{\bar{1}}(\bar{\mathbf{x}}, \bar{\mathbf{p}})$

- Wigner phase space — Wigner quasi probability distribution function
- A bridge of QM to statistical phase space



$$P_2(T, \mathbf{L}, \mathbf{P}) = \mathscr{L}\mathscr{M}\mathscr{O}^{\bar{1}} P_{\bar{1}}(\bar{\mathbf{x}}, \bar{\mathbf{p}}) \quad \sigma_{\bar{p}} \quad \sigma_{\bar{x}}$$

$$= \underset{\sigma_p\ \sigma_x}{\mathscr{L}\mathscr{M}\mathscr{O}^1} A_1^*(x', p') \underset{\sigma_p\ \sigma_x}{\mathscr{L}\mathscr{M}\mathscr{O}^1} A_1(\mathrm{x}, \mathrm{p}) \, .$$

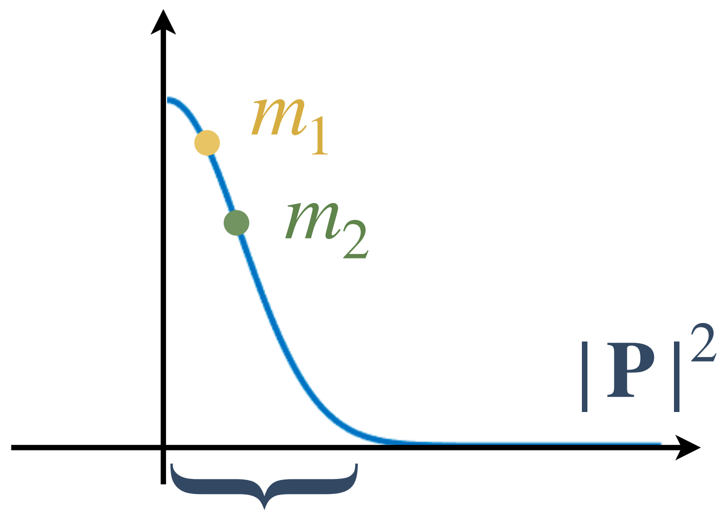
The Physical Layer

- Relativistic representation — phase space for massless neutrinos
- Different neutrino masses are "allowed" by the uncertainties

$$|\mathbf{P}_j| \equiv E - \delta E_j$$

$$|\mathbf{P}| = E$$

Energy budget



$$\mathbf{P}_j = \mathbf{P} - \vec{\xi}_j \frac{m_j^2}{2E}$$

$$\mathbf{L}_j = \mathbf{L} - \vec{v}_j T$$

Relativistic rep.

- Additional classical uncertainty — In energy (σ_E) and distance (σ_L)

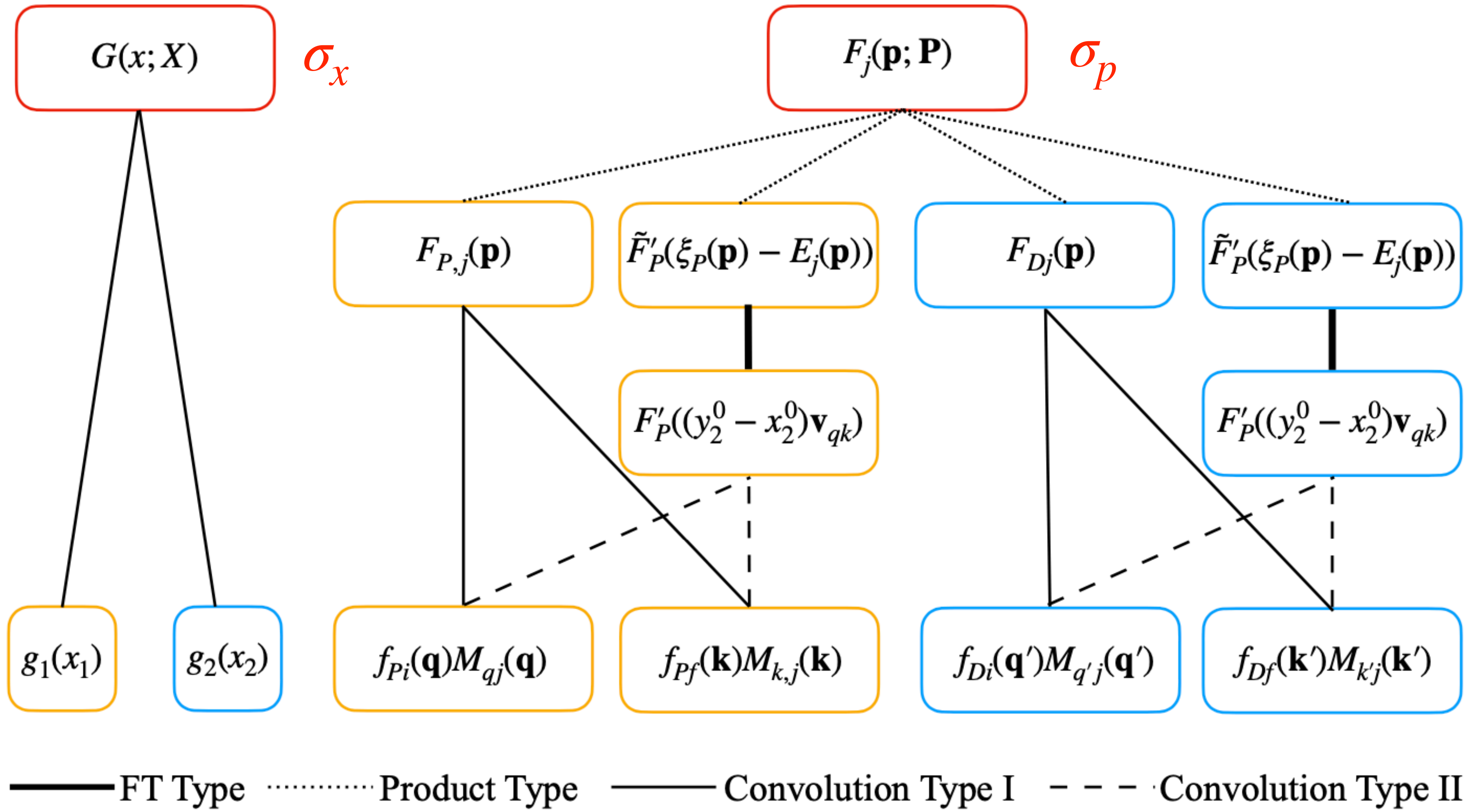
FTP on its ith layer: $P_{i,\nu_\alpha \to \nu_\beta} = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} P_{i,jk}$

$$P_{3,jk}(X_3) = \int dX_2\, P_{2,jk}(X_2)\, H(X_2 - X_3) \quad \Rightarrow \text{Convolution}$$

($P_{2,jk}$: σ_p & σ_x; H: σ_E & σ_L)

$X_3 = \{L_0, E_0\}$,
$X_2 = \{L, E\}$

Quantum Uncertainty Decomposition



Type (notation for σ_h)	Function relation	Width relation	Gaussian case
FT type ($\tilde{\sigma}_f$)	$h = \mathcal{FT}(f)$	NC	$\sigma_h = \frac{1}{2\sigma_f}$
Product type (σ_{fg})	$h = f \times g$	PC, $\sigma_h < \{\sigma_f, \sigma_g\}$	$\frac{1}{\sigma_h^2} = \frac{1}{\sigma_f^2} + \frac{1}{\sigma_g^2}$
Convolution type I (σ_{f*g})	$h = f * g$	PC, $\sigma_h > \{\sigma_f, \sigma_g\}$	$\sigma_h^2 = \sigma_f^2 + \sigma_g^2$

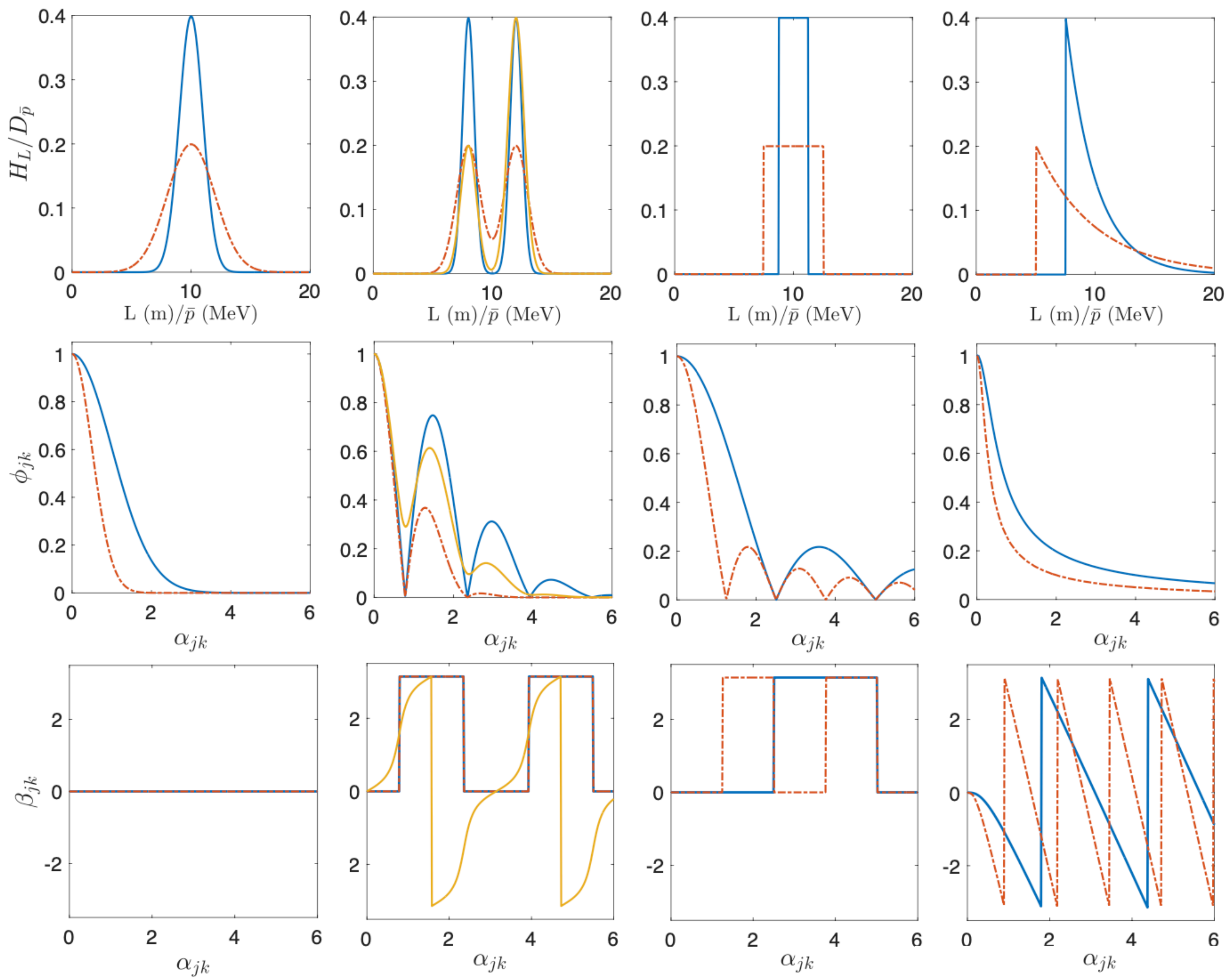
Function (width notation), Type	Width relation	Gaussian case
$H(y)$ (σ_H), Convolution type I	PC , $\sigma_H > \{\sigma_f, \sigma_g\}$	$\sigma_H = \sigma_{f*g}$
$I(p)$ (σ_I), Convolution type II	NC	$\sigma_I = 1/\sigma_{fg}$

Damping and Phase Shift — σ_L & $\sigma_{\bar{p}}$

Weighting function:

Damping term:

Phase shift term:



For H_L: $\alpha_{jk} = \dfrac{\Delta m_{jk}^2}{2E_0}$

For $D_{\bar{p}}$: $\alpha_{jk} = \dfrac{\Delta m_{jk}^2 L_0}{2E_0^2}$

Damping and Phase Shift — σ_E

Weighting function:

Damping term:

Phase shift term: