Gravitational wave induced baryon acoustic oscillations

arXiv: 2107.10283

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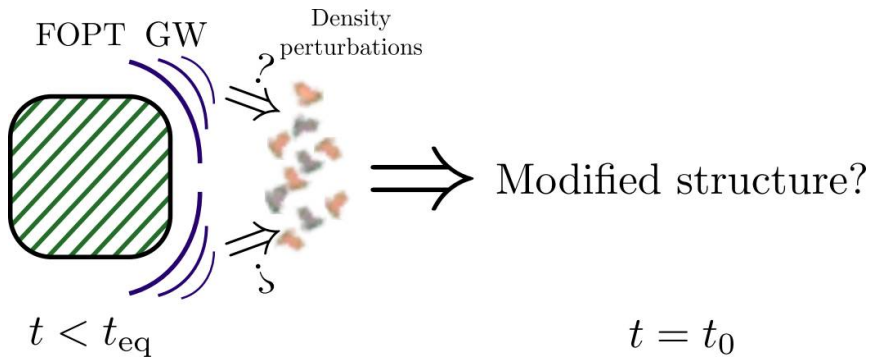
Matthias Bartelmann

Question: Can GWs from FOPTs impact structure formation?
If so, can we infer bounds on the FOPT parameters from SF?



Motivation

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- Many models beyond the standard model predict first order phase transitions (FOPT)
- Thinking about: What else than gravitational waves can we do?
- How do these events speak/interact with other event/processes in the early universe?

Overview

- Short review of cosmological first order phase transitions (FOPT) and gravitational waves (GWs)
- Short review on structure formation (SF)
- Physical idea
- Methods
- Results
- Summary

Short intro to FOPTs

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Particle (scalar) model

$$\mathcal{L} \supset \partial_\mu \phi \partial^\mu \phi^* - V^{(0)}(\phi)$$

$$V_{\text{eff}}(\phi, T) = V^{(0)}(\phi) + V^{(1)}(\phi) + V_T^{(1)}(\phi)$$

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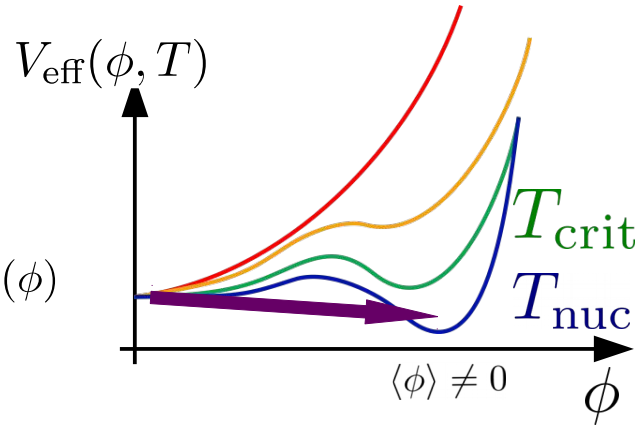

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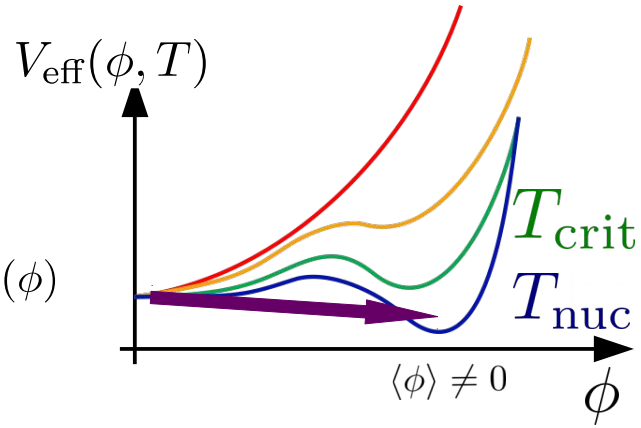

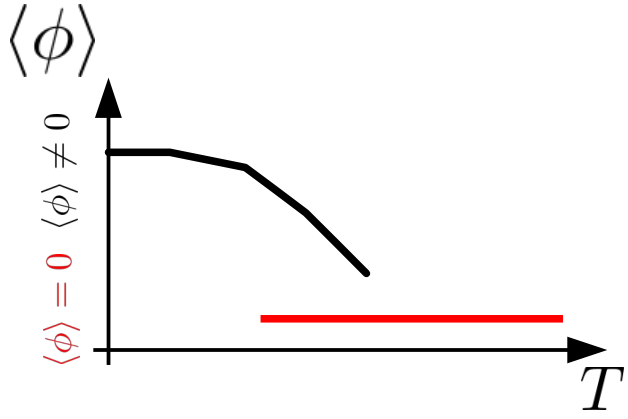

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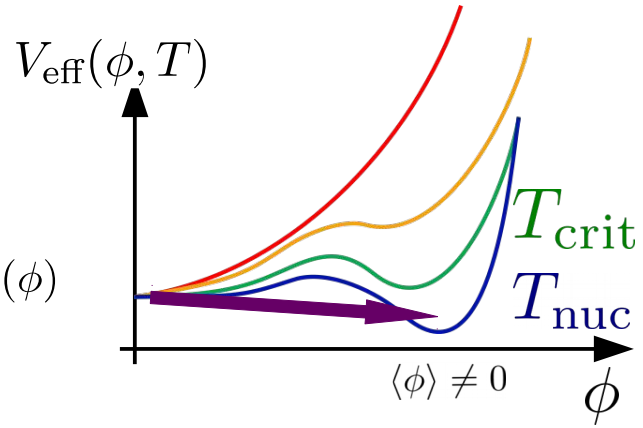

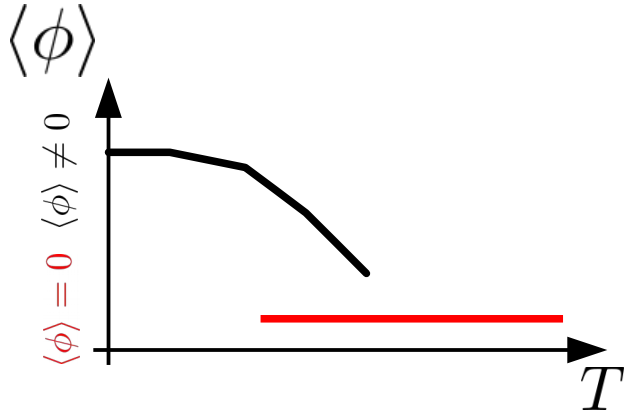

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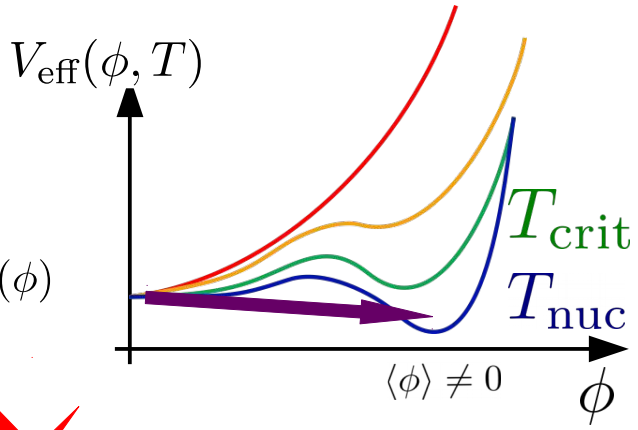

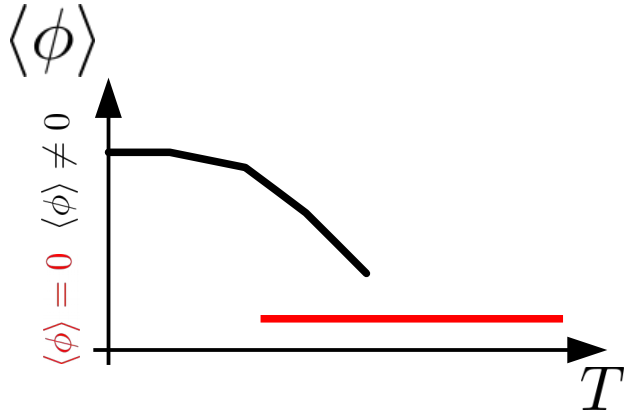

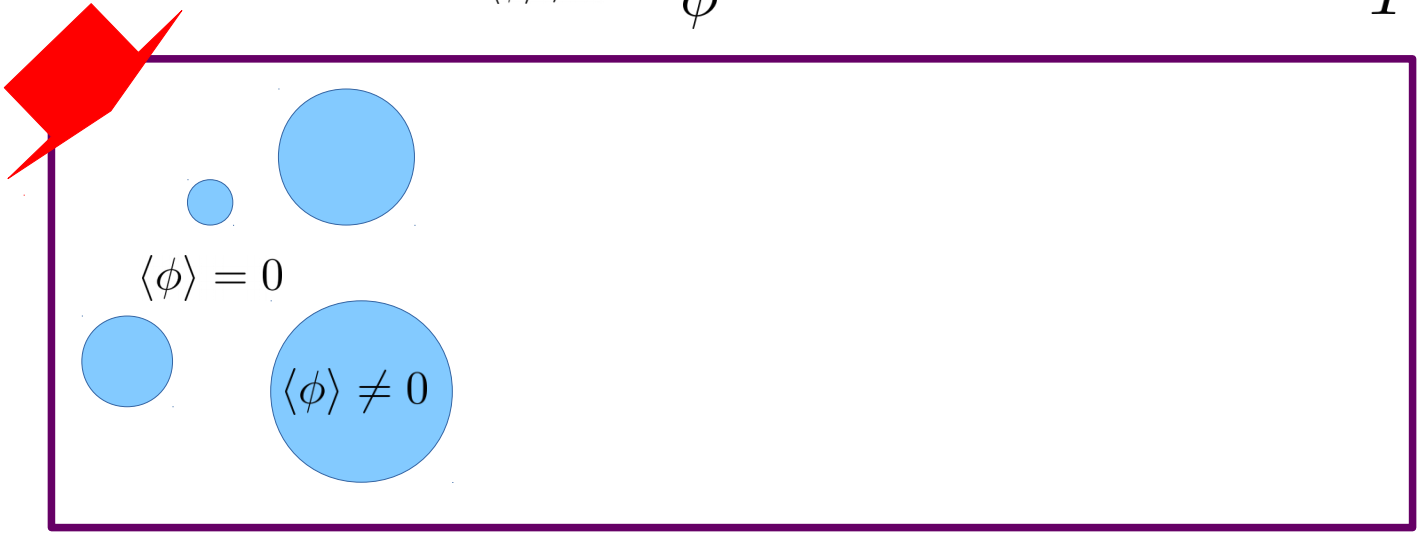

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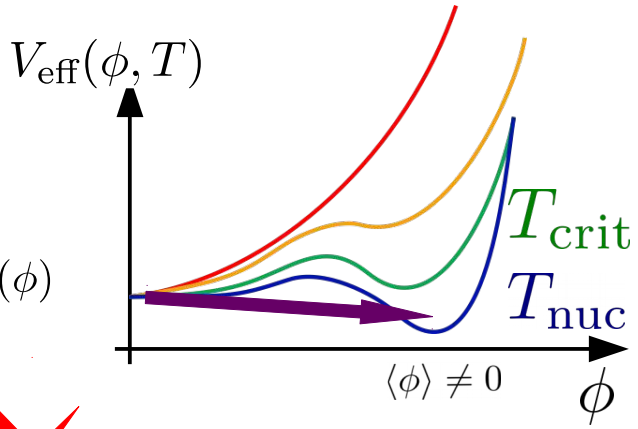

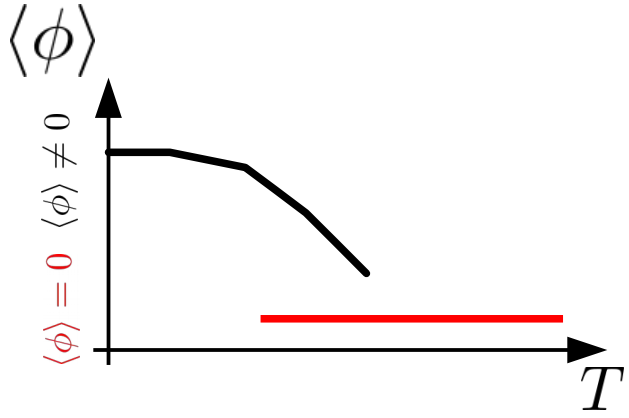

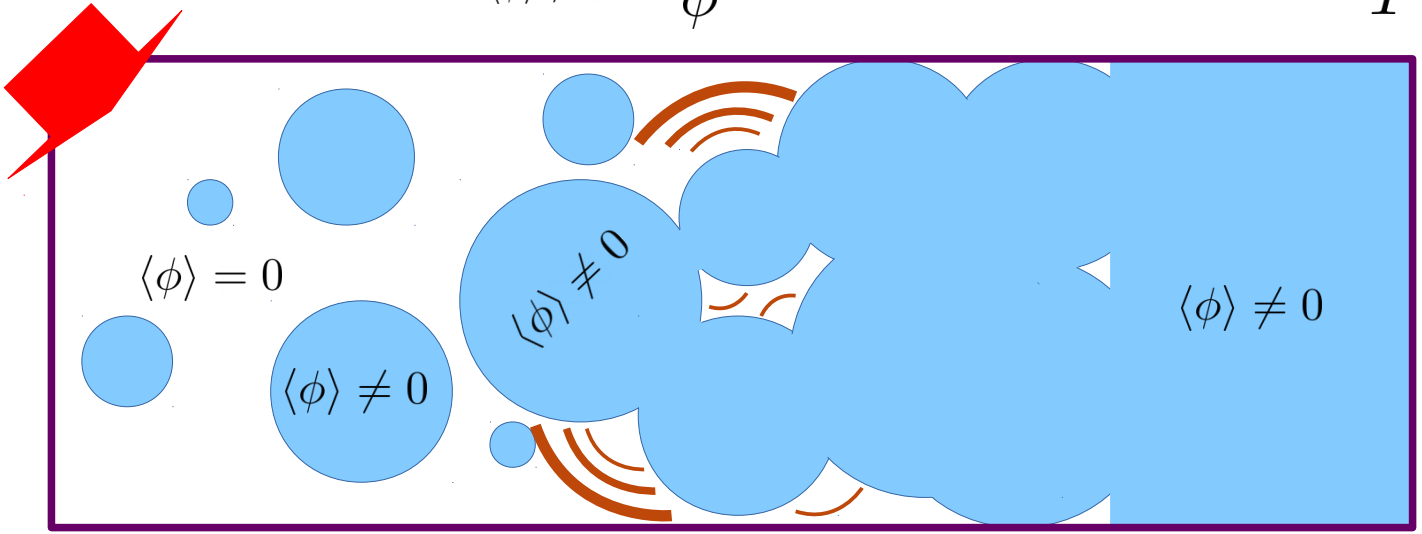

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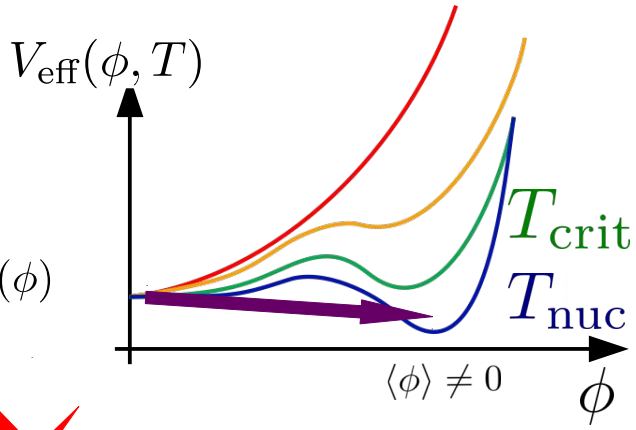
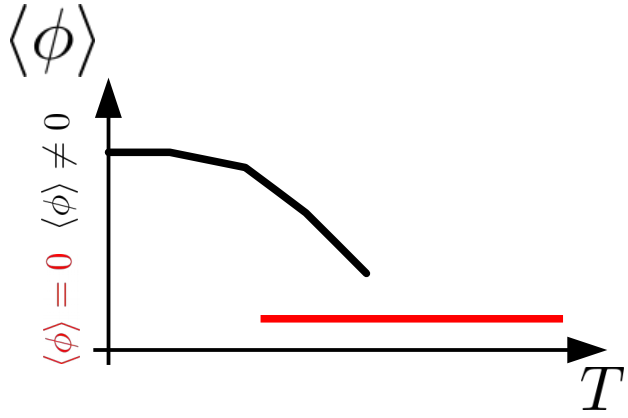
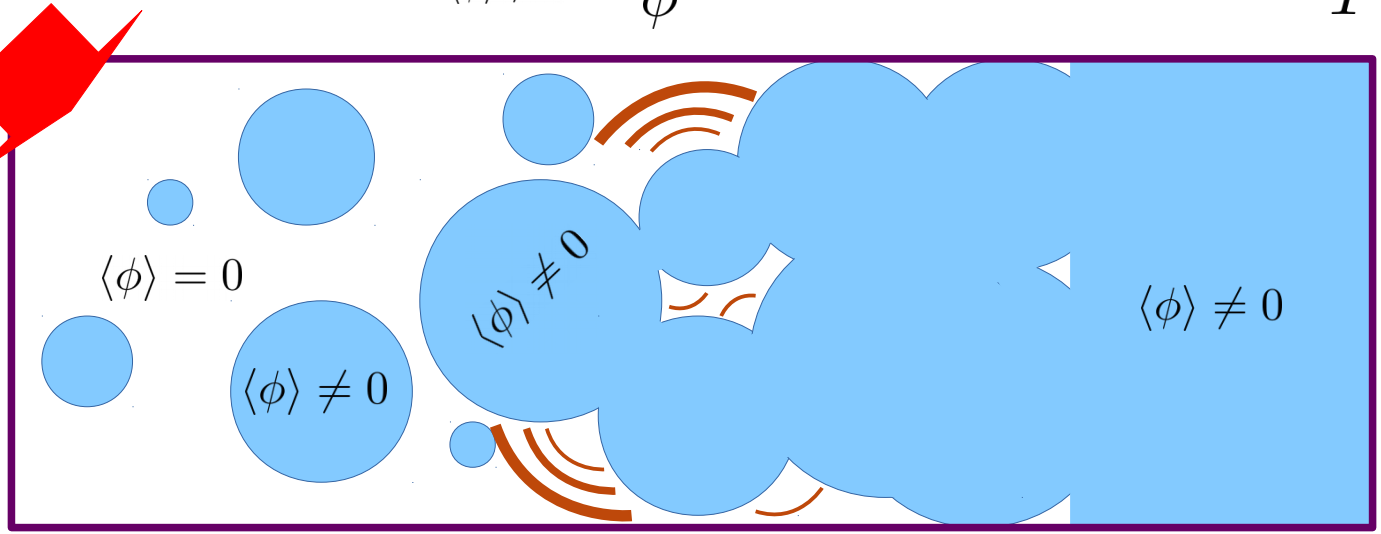




- Strength $\alpha := \dfrac{\rho_{\rm vac}}{\rho_{\rm rad}}$
- Duration β^{-1}
- Scale/Temperature $T_{\rm nuc}/T_*$



[Review: C. Caprini, D. Figueroa, Class.Quant.Grav. 35 (2018) 16, 163001, arXiv:1801.04268]

Bubble nucleation in supercooled water

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Source:
https://www.youtube.com/watch?v=_9N-Y2CyYhM

Bubble nucleation in supercooled water

Nucleation in old phase

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Linear structure formation

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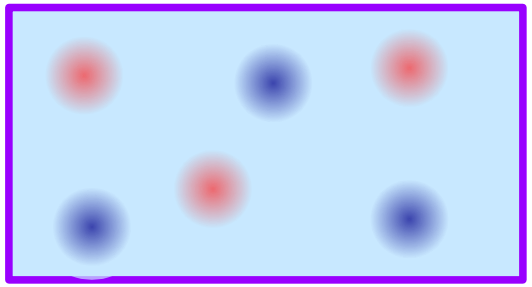
Linear structure formation



$$\rho \approx \rho^{(0)} + \rho^{(1)}$$

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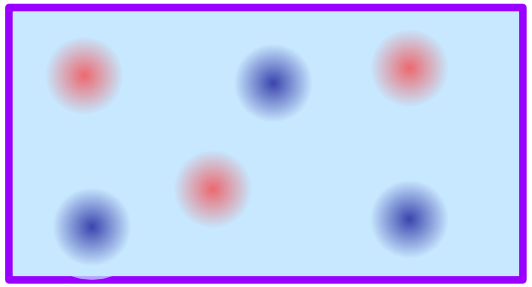

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Perturbed equations:

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$$
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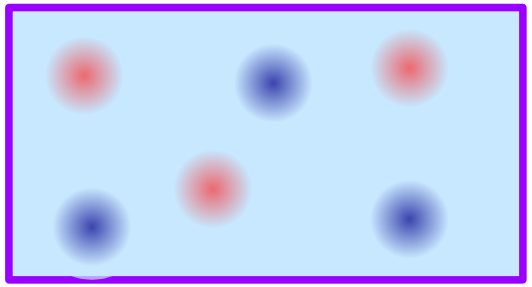

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evolution equations for density contrast

$$\delta(k,t) := \frac{\rho^{(1)}}{\rho^{(0)}}(k,t)$$

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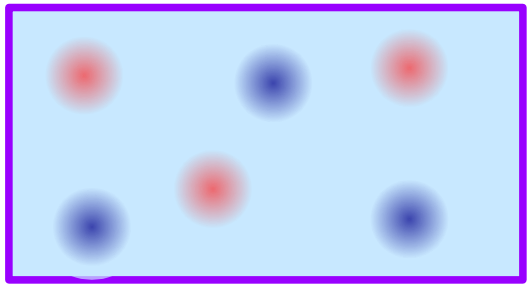

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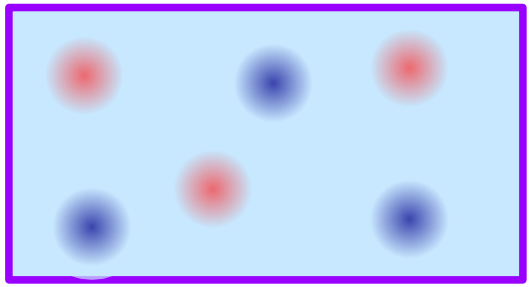

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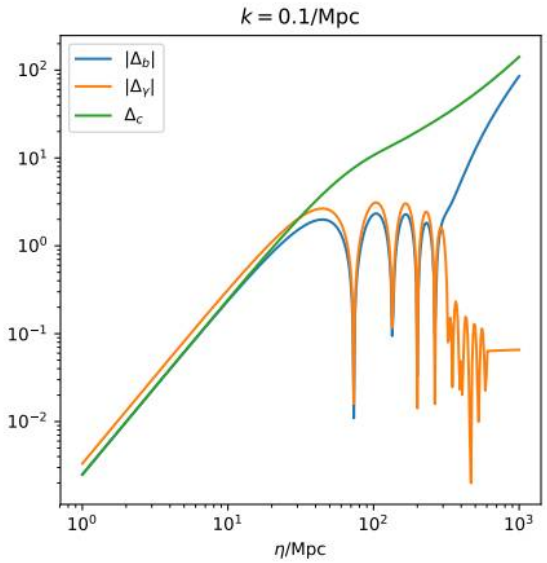

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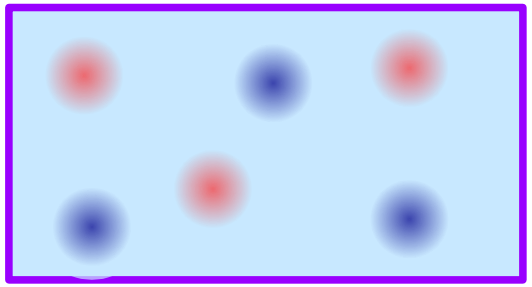

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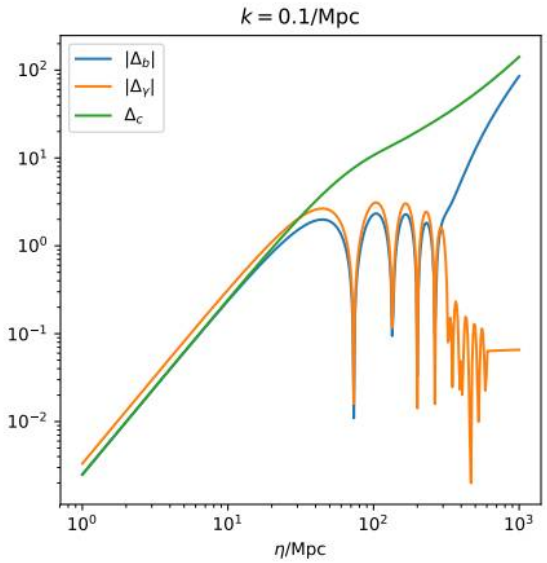

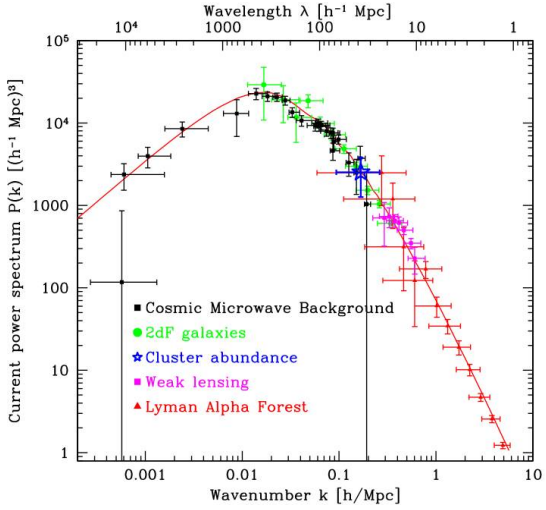

[M. Tegmark, M. Zaldarriaga, 2002, Phys. Rev. D, 66, 103508]

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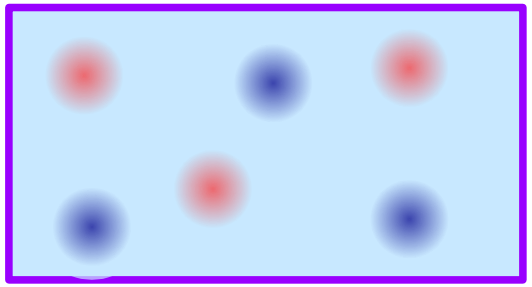

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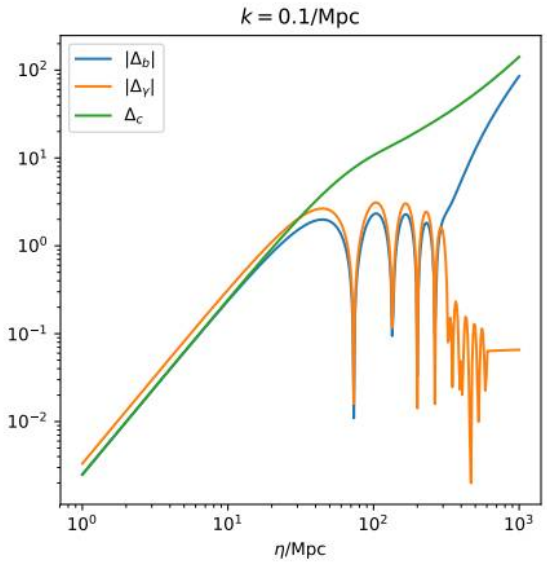

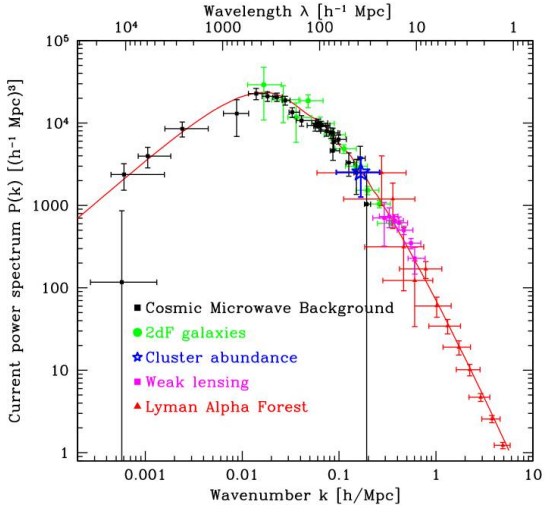

E.g. : affected by neutrino free streaming

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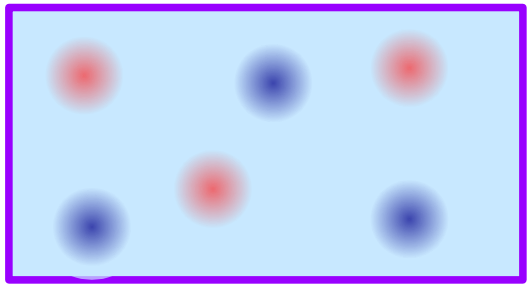

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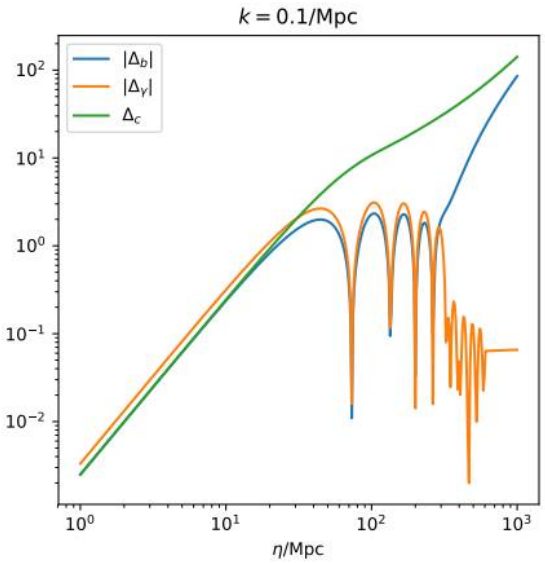

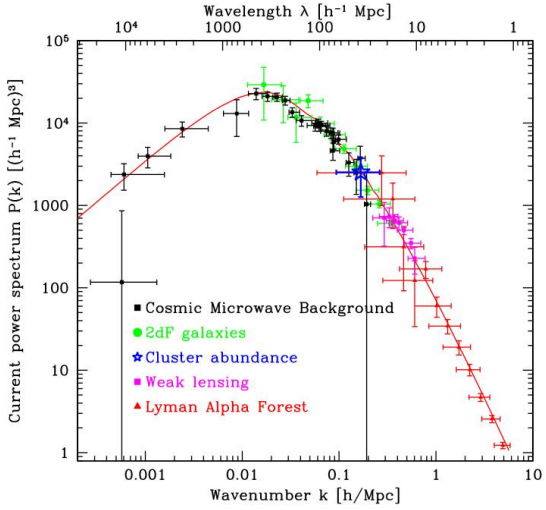

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=> supression of small scales

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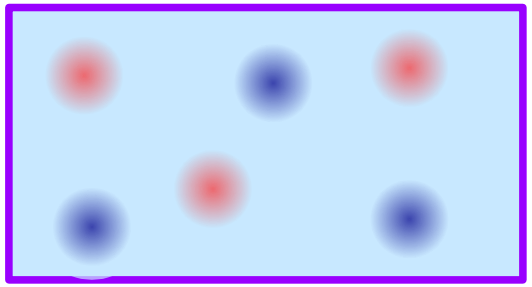

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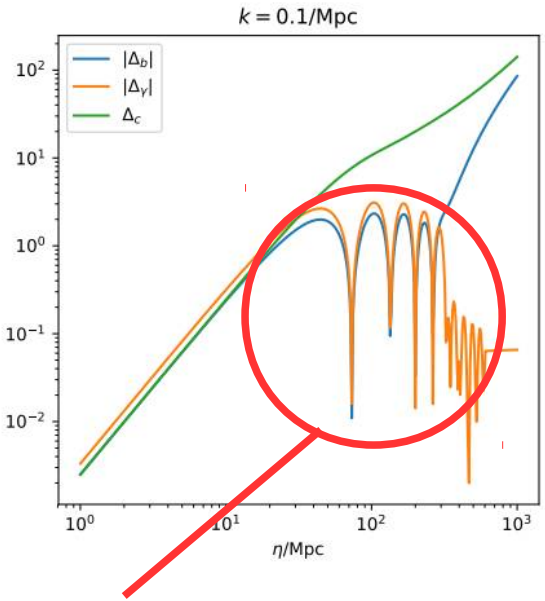

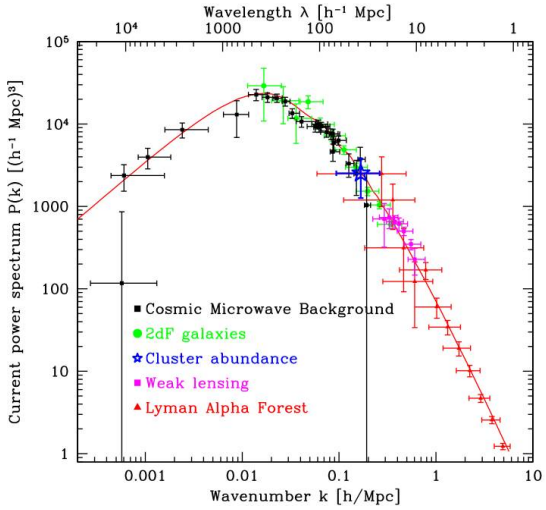

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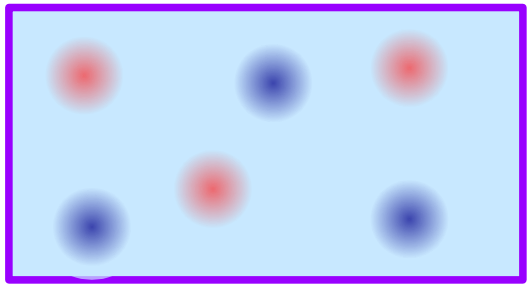

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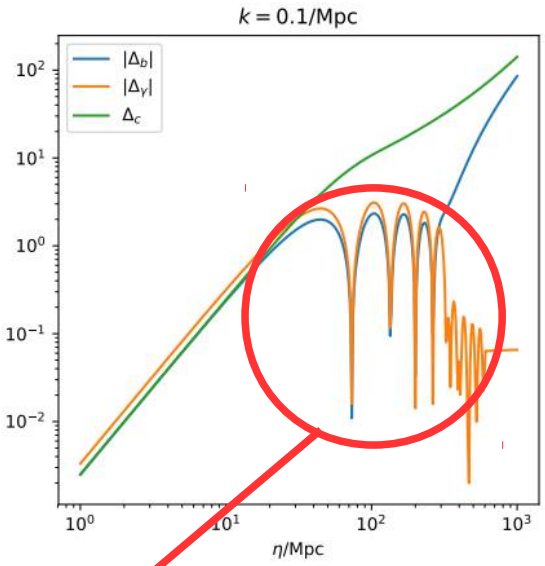
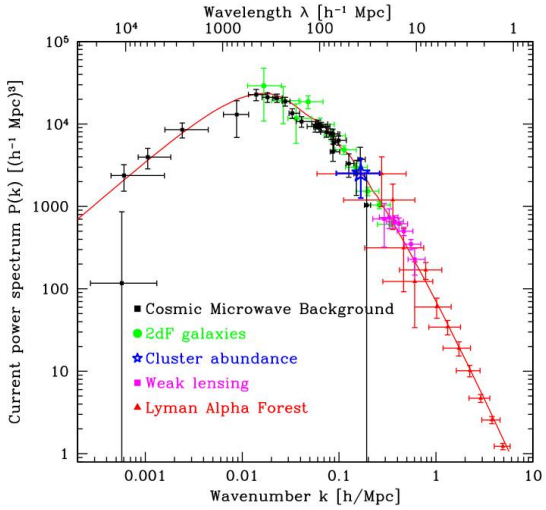


Baryon acoustic oscillations (BAOs)



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$$\ddot{\delta}_\gamma + c_s^2 \frac{k^2}{a^2}\delta = \frac{4}{3} 4\pi G \left(\rho_d^{(0)}\delta_d + \rho_b^{(0)}\delta_b\right)$$

wiggles on MP spectrum

Density perturbations in the environment of a FOPT

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FOPT properties:

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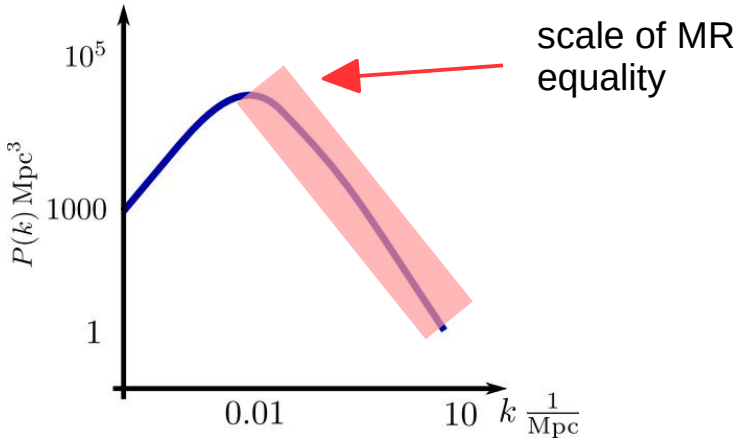

Density perturbations in the environment of a FOPT

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FOPT properties:

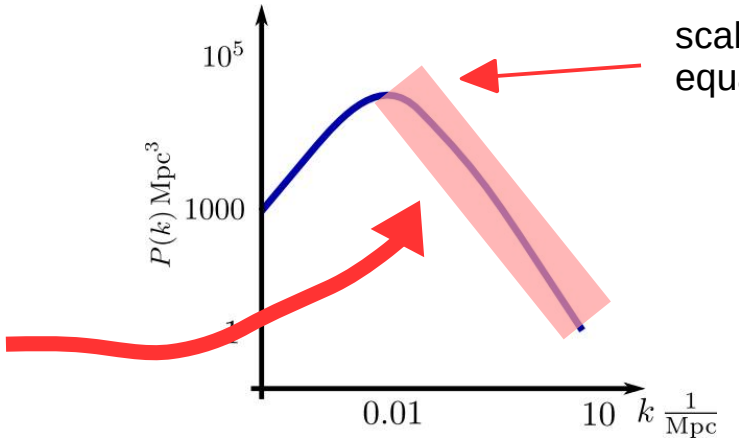
- Takes place on sub-horizon scales

Horizon $k \gtrsim a_* H_*$

- May complete within a Hubble time

$\beta^{-1} \lesssim H_*^{-1}$

➡ Can only impact scales (and smaller) at which the transition occurs

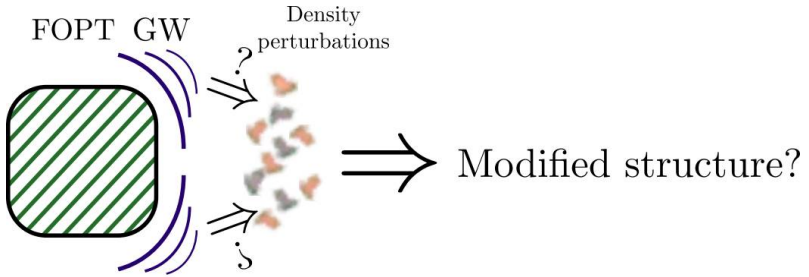


FOPT needs to occur at *late times:*

$t_* \;:\; 10^6\,\mathrm{s} - 10^{12}\,\mathrm{s}$

$T \sim (100 - 1)\,\mathrm{eV}$

GW induced density perturbations Technicalities



GW induced density perturbations Technicalities

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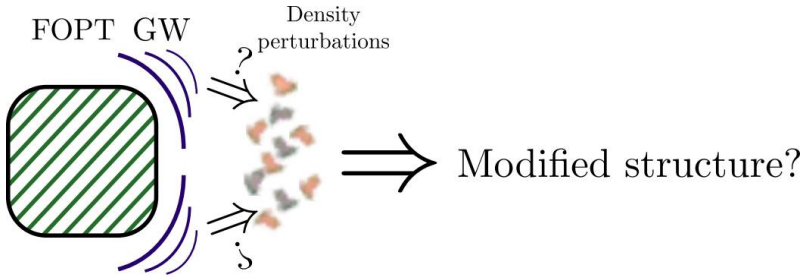
Rad. Dom era



GW induced density perturbations Technicalities

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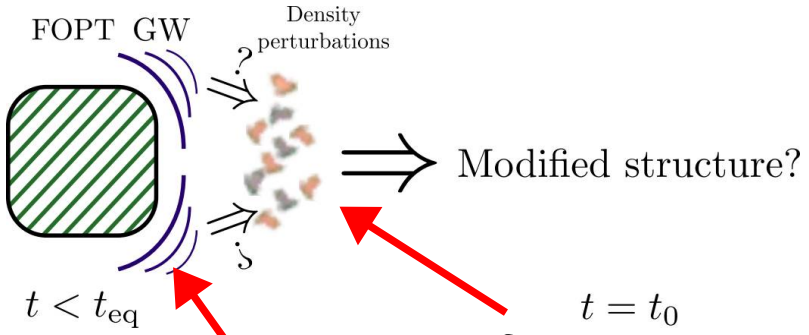

GW induced density perturbations Technicalities

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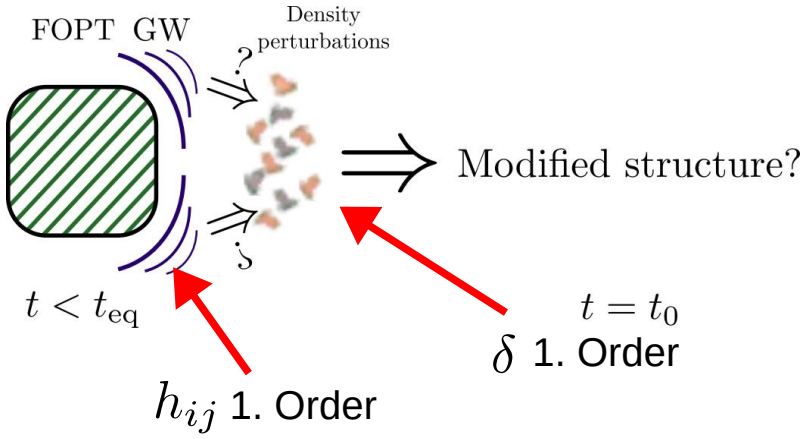

Interaction 2nd order

GW induced density perturbations Technicalities

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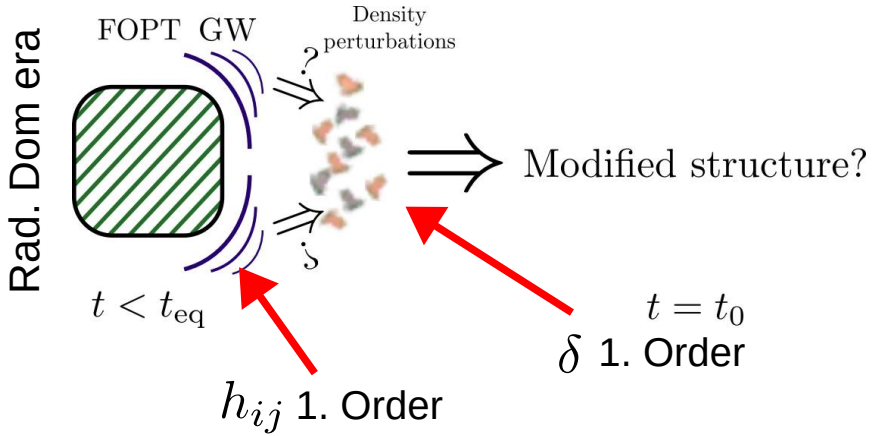
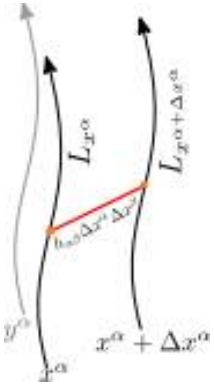


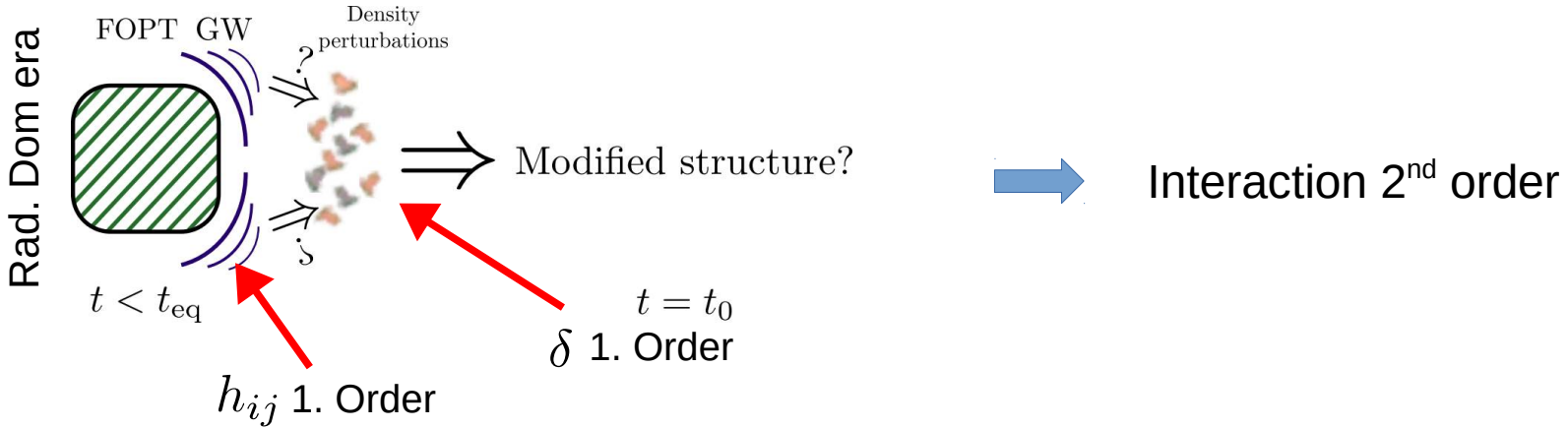
Interaction 2nd order

1+3 covariant approach



GW induced density perturbations Technicalities

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK HEIDELBERG



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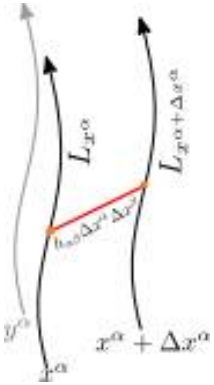
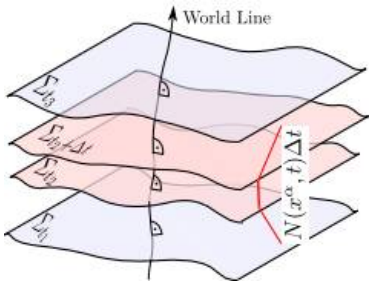


vs

3+1 approach



GW induced density perturbations Technicalities

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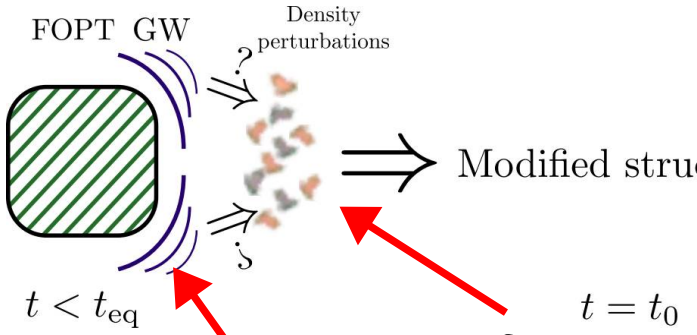

Interaction 2nd order

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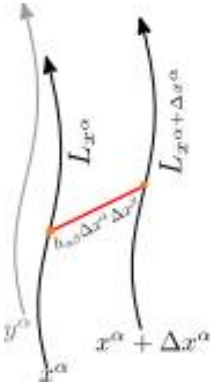

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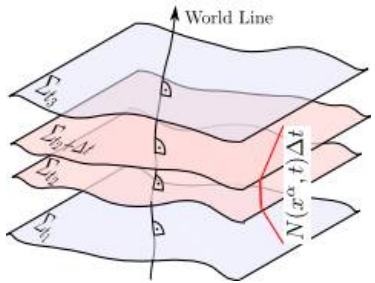

[C. Tsagas, A. Challinor, R. Maartens, arXiv:0705.4397v3]
[G.F.R. Ellis, H. van Elst, Cargèse lectures 1998, arXiv:gr-qc/9812046v5]

[G.F.R. Ellis, M. Bruni, Phys. Rev. D 40 (Sep. 1989) 1804-1818]

GW induced density perturbations Technicalities

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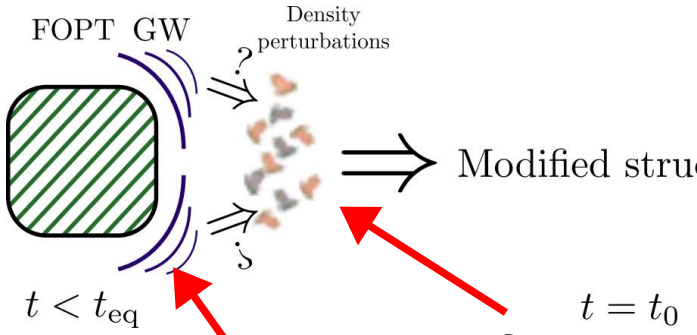

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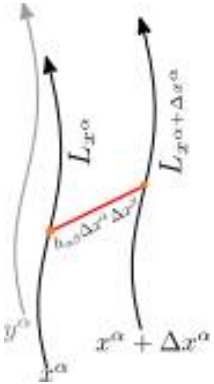

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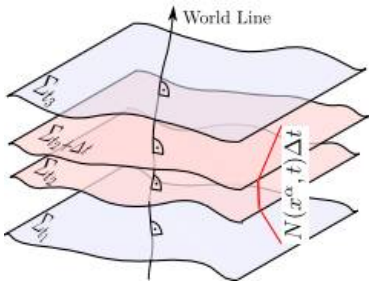

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[G.F.R. Ellis, M. Bruni, Phys. Rev. D 40 (Sep. 1989) 1804-1818]

Similar calculation
Matter dom. & superhorizon

[D. Pazouli, C.G. Tsagas, Phys. Rev. D93 no. 6 (2016) 063529, arXiv:1512.02932]

GW induced density perturbations Technicalities

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1+3 framework

$$\dot{\Delta}_a = f(\Delta_a, Z_a, \sigma_{ab}, \dots)$$
$$\dot{Z}_a = g(Z_a, \Delta_a, \sigma_{ab}, \dots)$$
$$\Delta_a := \frac{a}{\rho} \mathrm{D}_a \rho \,, \; Z_a := a \mathrm{D}_a \Theta$$
$$\sigma_{ab} = a^2 \dot{h}_{\alpha\beta}$$

GW induced density perturbations Technicalities

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Step 1:

perturbe to 2nd order

$$\mathrm{WE}\left(\ddot{\Delta}_a^{(2)}\right) = \tilde{f}(\dot{\Delta}_a^{(1)}, \sigma_{ab}^{(1)}, \dots)$$

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$$\delta^{(2)\prime\prime}(\kappa,\tau) + \frac{1}{3}\kappa^2\delta^{(2)}(\kappa,\tau) = 8\cdot\Omega_{\mathrm{GW}}(\kappa,\tau)$$

Analytic estimate: [C. Caprini, R. Durrer, T Konstandin, G. Servant: Phys. Rev. D, 79:083519, 2009]

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Interpretation: Second order baryon acoustic oscillations driven by GW from FOPT

GW induced density perturbations Technicalities

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$$T^2(k) = 1 + \left(\frac{\delta^{(2)}}{\delta^{(1)}}(k)\right)^2$$

GW induced density perturbations Technicalities

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK HEIDELBERG

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Step 4:

Matter-Power Spectrum

$$\widetilde{\mathcal{P}}(k) \sim T^2(k)\mathcal{P}(k)$$

Impact on linear MP spectrum

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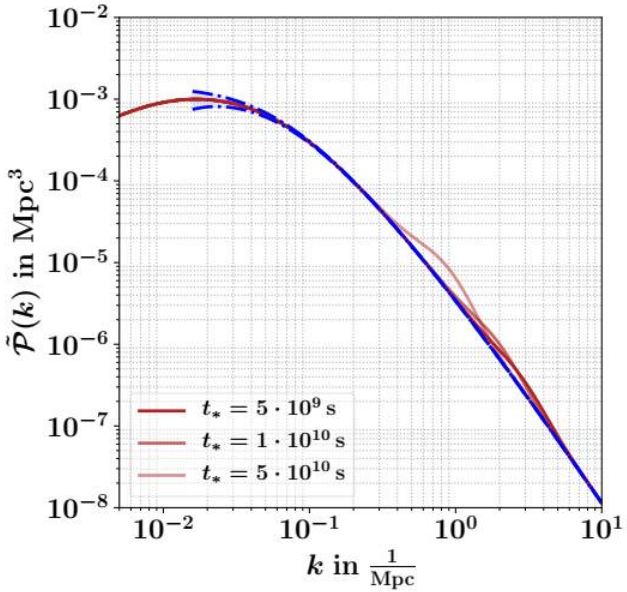
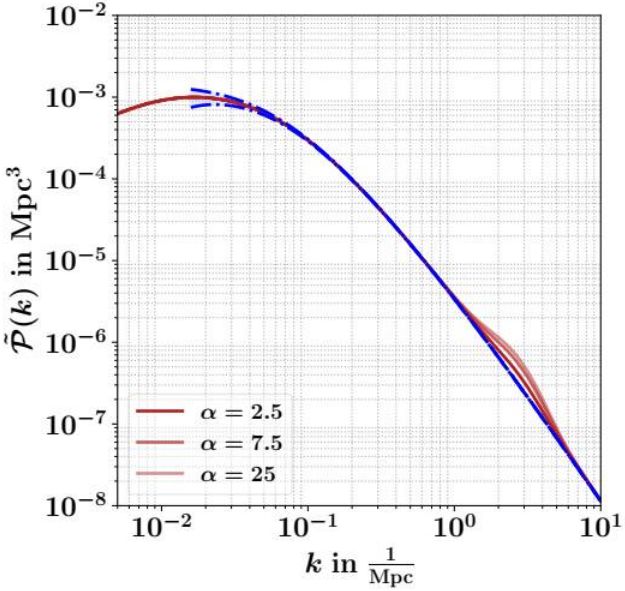
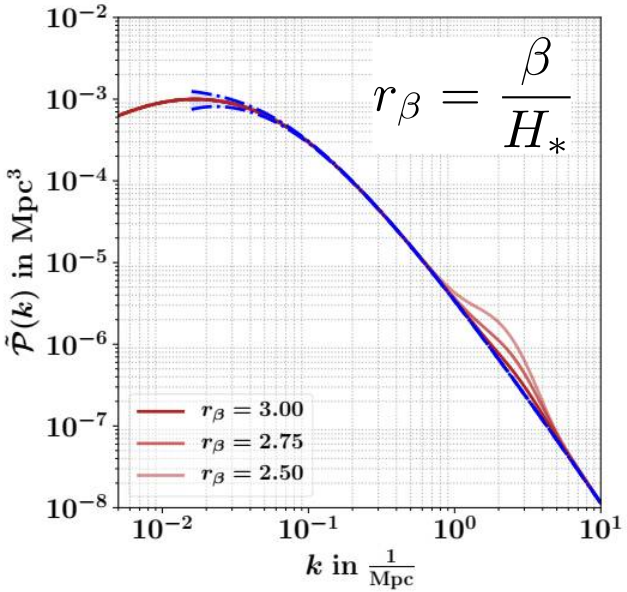


Changing scale



Changing strength



Changing duration

Blue line: Cosmic Variance Bound

Impact on linear MP spectrum

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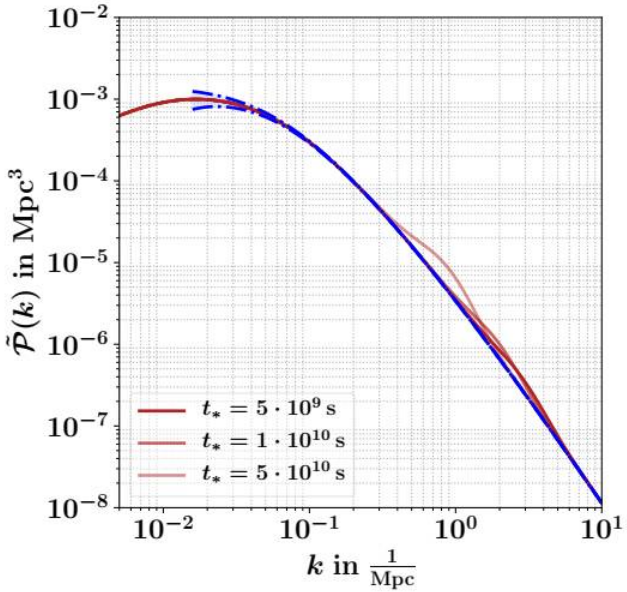

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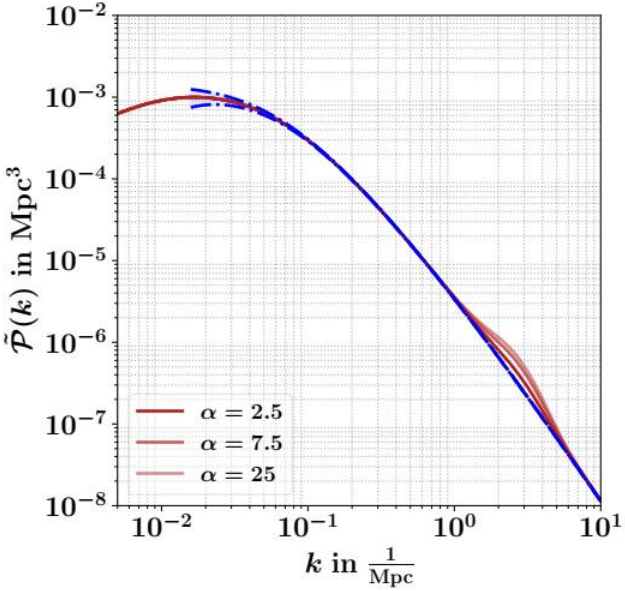
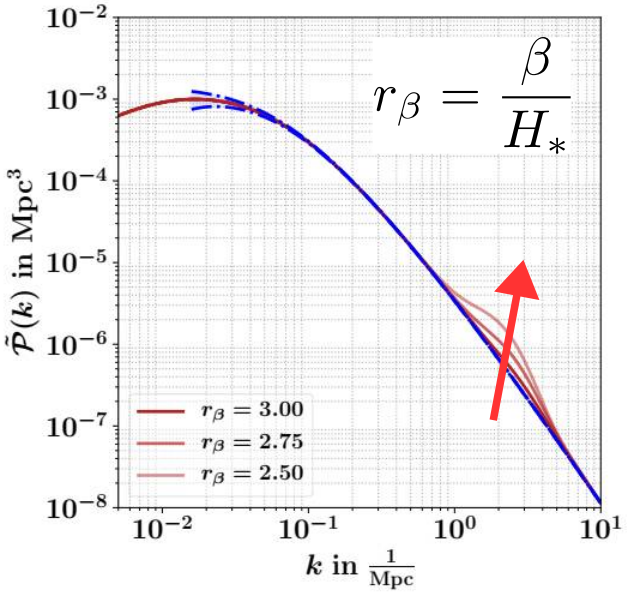


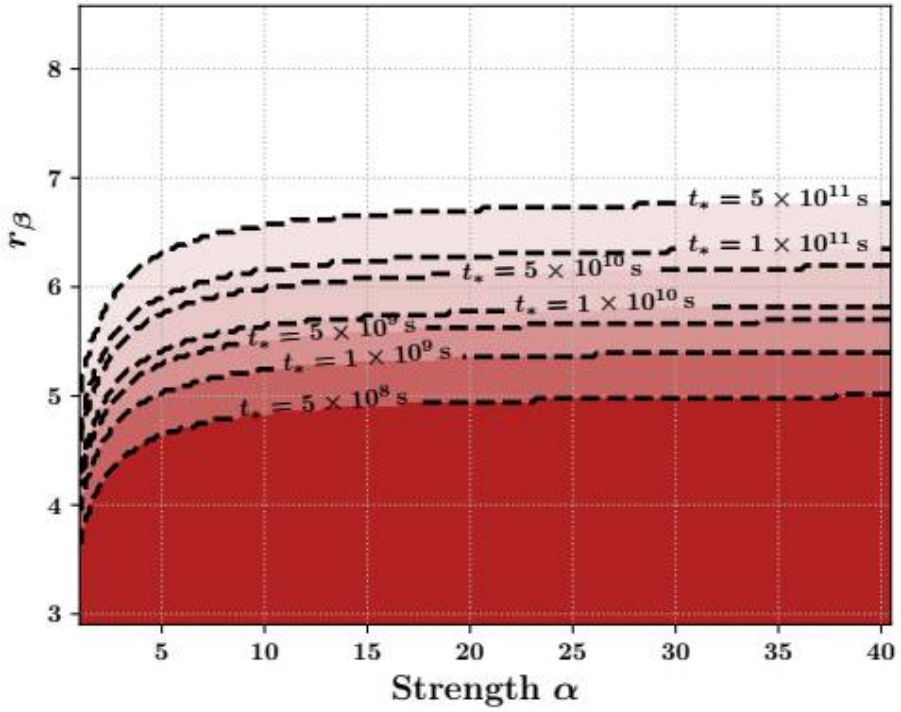
Changing strength

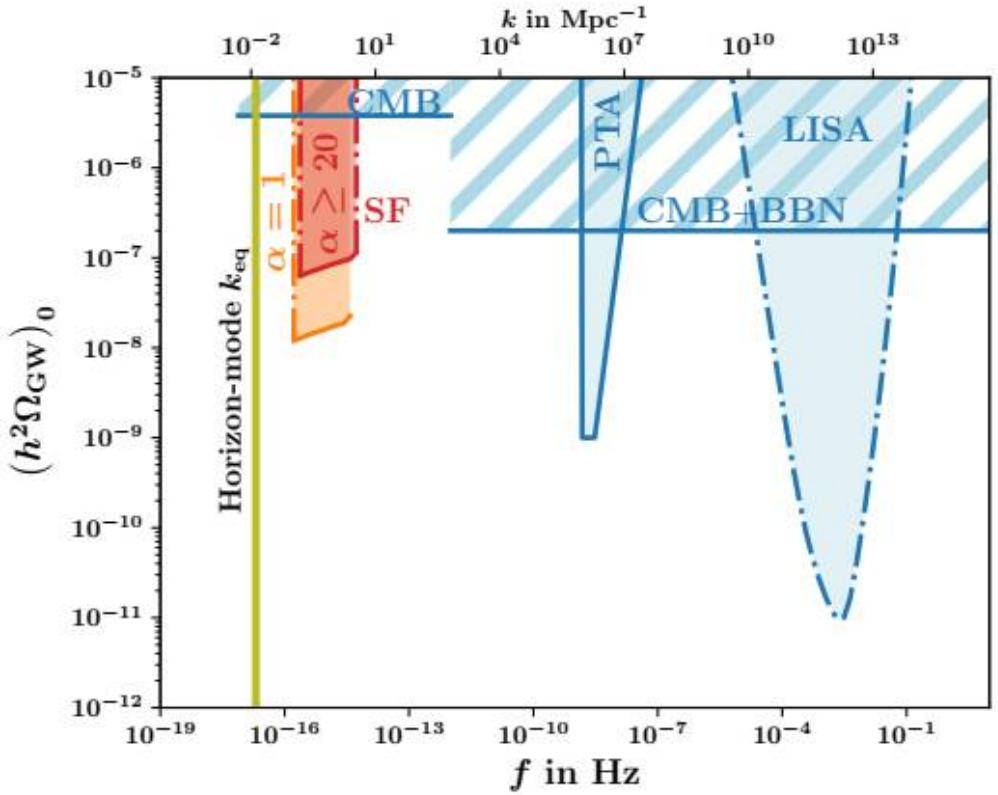


Changing duration

Blue line: Cosmic Variance Bound

Limits from cosmic variance





Particle models that can achieve this: e.g. conformal models

Summary

- GWs from FOPTs can seed density perturbations at second order
- Effect is bound to the scale at which the FOPT occurs → late FOPTs
- Only very strong and long FOPTs can have significant impact
- Cosmic variance bound leads to new limit on very small GW frequencies

Backup slides

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1+3 Decomposition

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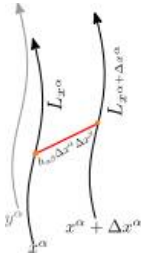
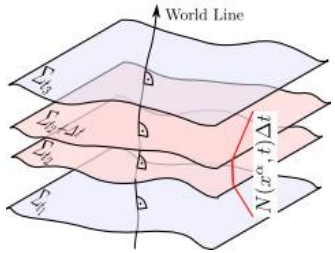
Spacetime decomposition:

$$u^a = \frac{\mathrm{d}x^a}{\mathrm{d}\tau} \qquad h_{ab} := g_{ab} + u_a u_b$$



1+3 approach



3+1 approach

Motion of test particle

$$\nabla_b u_a = \sigma_{ab} + \omega_{ab} + \frac{1}{3}\Theta h_{ab} - A_a u_b$$

volume expansion

shear vorticity acceleration

density perturbation

$$\Delta_a : \frac{a}{\rho}\mathrm{D}_a\rho$$

volume gradient

$$Z_a := a\mathrm{D}_a\Theta$$

$$a\mathrm{D}_b\Delta_a = \frac{1}{3}\Delta h_{ab} + \Delta_{\langle ab\rangle} + \Delta_{[ab]}$$

Stewart & Walker Lemma:

$$S^{(1)} \to S^{(1)} + \epsilon\mathfrak{L}_\xi S^{(0)}$$

Gauge invariant if zero

J. M. Stewart, M. Walker, Proc. R. Soc. Lond. A 341 no. 49, (1974)

P. K. S. Dunsby, M. Bruni, G.F.R. Ellis, Class. Quant. Grav. 14 (1997) 1215-1222

Evolution equations

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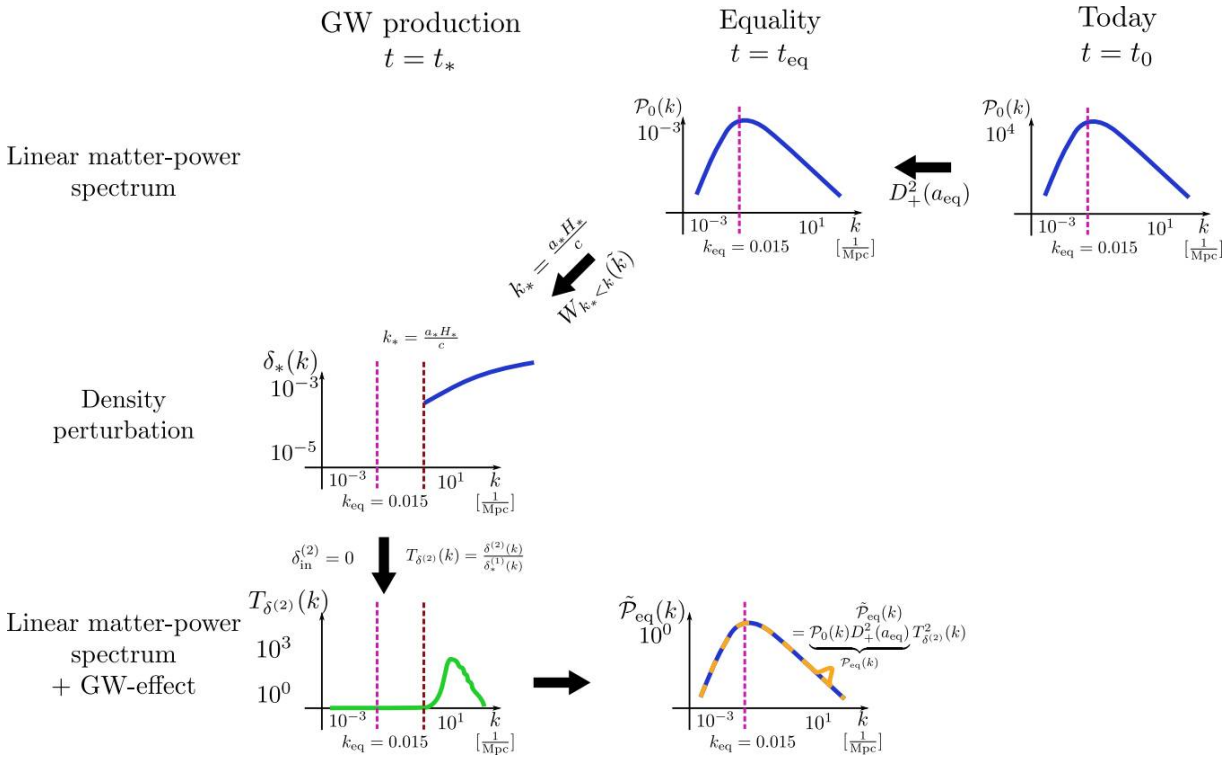
$$\dot{\Delta}_{\langle a\rangle} = \frac{p}{\rho}\,\Theta\Delta_a - \left(1+\frac{p}{\rho}\right) Z_a + a\frac{\Theta}{\rho}\left(\dot{q}_{\langle a\rangle} + \frac{4}{3}\,\Theta q_a\right) - \frac{a}{\rho}\,{}^{b}_{a} q_b + a\frac{\Theta}{\rho}\,{}^{b}\pi_{ab}$$
$$- \left(\sigma^{b}{}_{a} + \omega^{b}{}_{a}\right)\Delta_b - \frac{a}{\rho}\,{}_{a}\left(2A^b q_b + \sigma^{bc}\pi_{bc}\right) + a\frac{\Theta}{\rho}\left(\sigma_{ab}+\omega_{ab}\right) q^b + a\frac{\Theta}{\rho}\,\pi_{ab}A^b$$
$$+ \frac{1}{\rho}\left({}^{b}q_b + 2A^b q_b + \sigma^{bc}\pi_{bc}\right)\left(\Delta_a - aA_a\right)$$

$$\dot{Z}_{\langle a\rangle} = -\frac{2}{3}\,\Theta Z_a - \frac{1}{2}\,\kappa\rho\Delta_a - \frac{3}{2}\,\kappa a_a p - a\left[\frac{1}{3}\,\Theta^2 + \frac{1}{2}\,\kappa(\rho+3p) - \Lambda\right] A_a + a^{b}_{a} A_b$$
$$- \left(\sigma^{b}{}_{a} + \omega^{b}{}_{a}\right) Z_b - 2a_a\left(\sigma^2 - \omega^2\right) + 2aA^{b}_{a} A_b$$
$$- a\left[2\left(\sigma^2 - \omega^2\right) - {}^{b}\,A_b - A^b A_b\right] A_a$$

Transferfunction

Estimating the linear density perturbation from the linear MP spectrum:



Examples for late PTs

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- J. Frieman,C. Hill, R Watkins: Phys. Rev. D, 46:1226-1238, 1992
- I. Wasserman: Phys. Rev. Lett, 57:2234-2236, 1986
- A. Patwardhan, G. Fuller: Phys. Rev. D, 90(6):063009, 2014
- Xiao-chun Luo, D. Schramm: Astrophys. J., 421:393-399, 1994