

# *Primordial Black Holes*

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*LMU Munich*

*PASCOS 2022*

*Heidelberg, 29th of July 2022*

# *A Diplomatic Remark\*:*

There is a distinction of primordial black holes being *the* dark matter (ie. all of it) or a part of it; the latter could well be both *microscopic* and *macroscopic*.

\*since most conference participants work on particle dark matter

# What are Primordial Black Holes (PBHs)?

- ★ Black holes formed in the early Universe (in particular: *non-stellar*).
- ★ First proposed by Novikov and Zel'dovič in the late 1960th, but their conclusion was negative for the existence of PBHs.

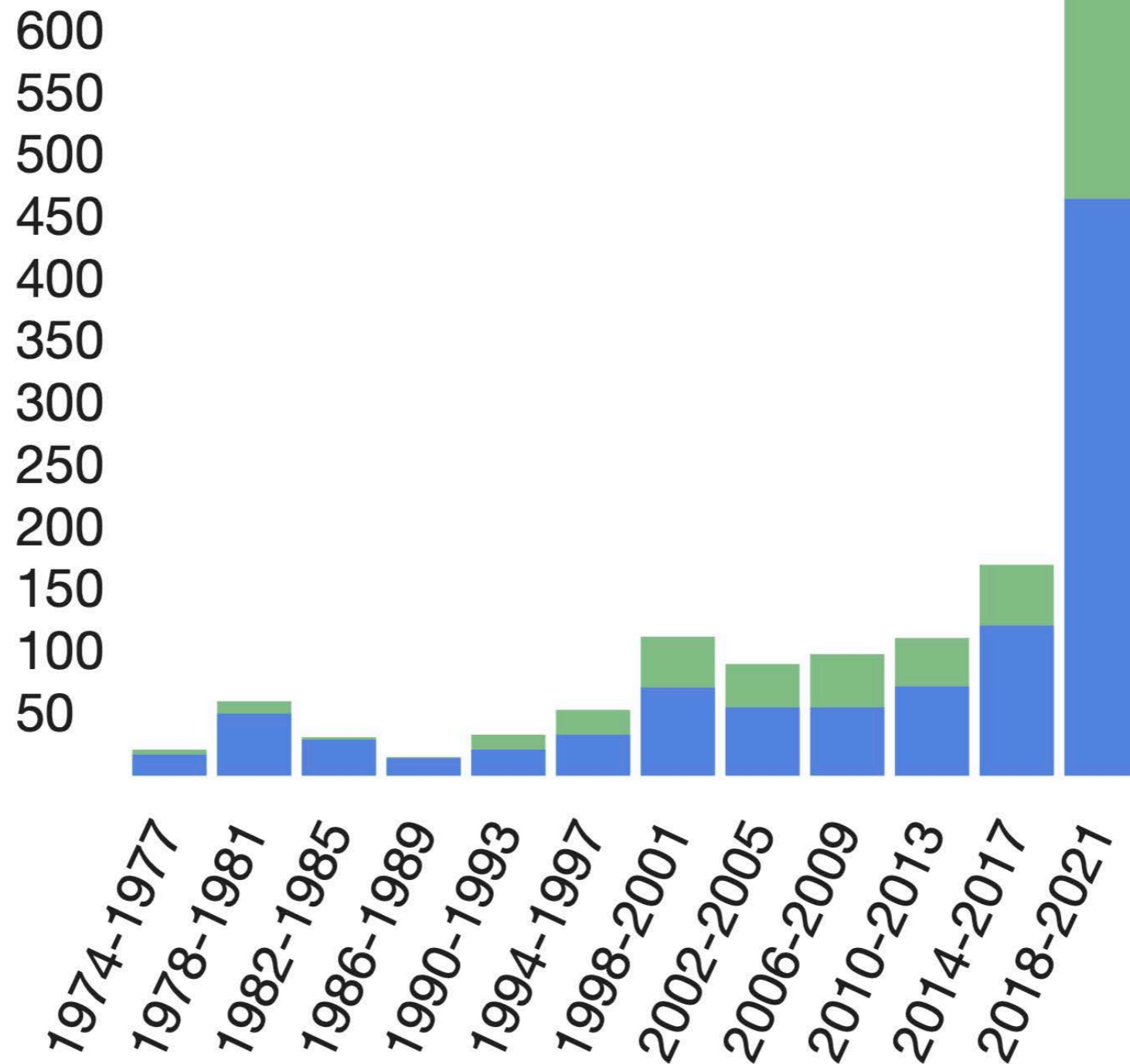


- ★ Conclusion disproved by Carr & Hawking (1974), reinvigorated PBH research (nearly 2000 papers to date).



# *Primordial Black Holes are Popular!*

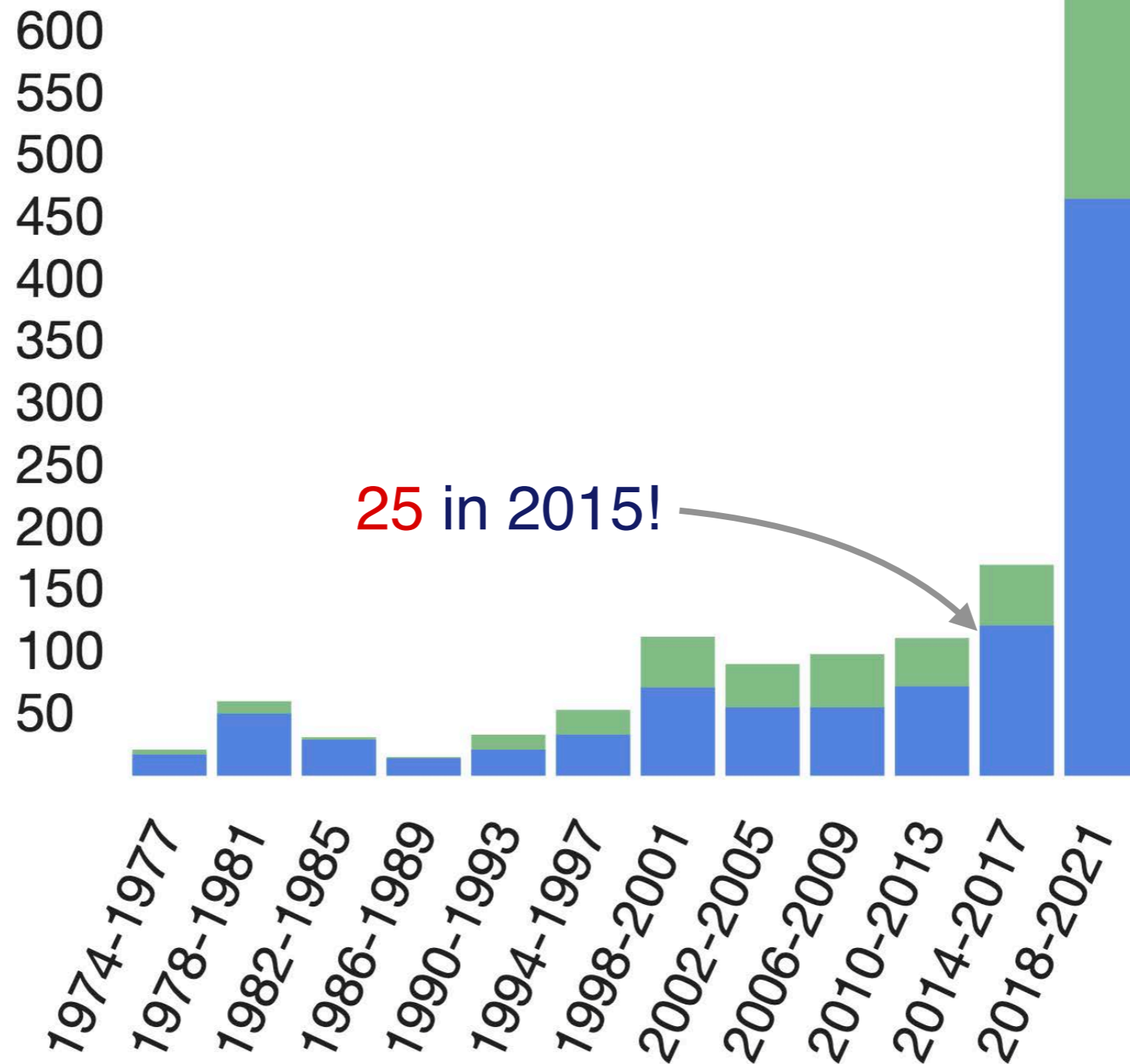
■ refereed   ■ non refereed



[SAO/NASA  
Astrophysics  
Data System]

# *Primordial Black Holes are Popular!*

■ refereed ■ non refereed



[SAO/NASA  
Astrophysics  
Data System]

*Primordial Black Hole  
Formation*

# PBH Formation Mechanisms

★ Large density perturbations (inflation)

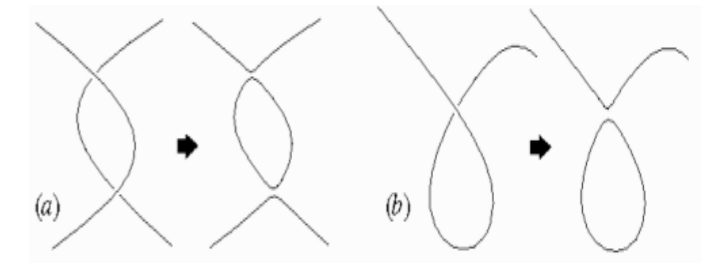
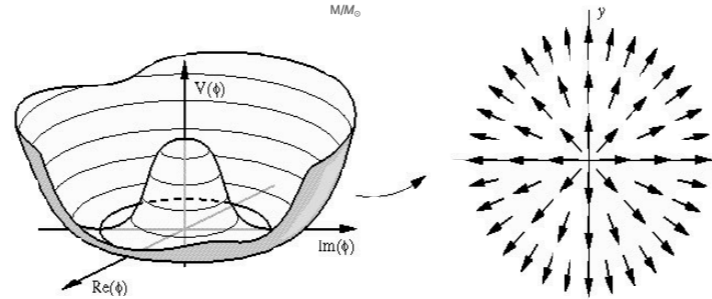
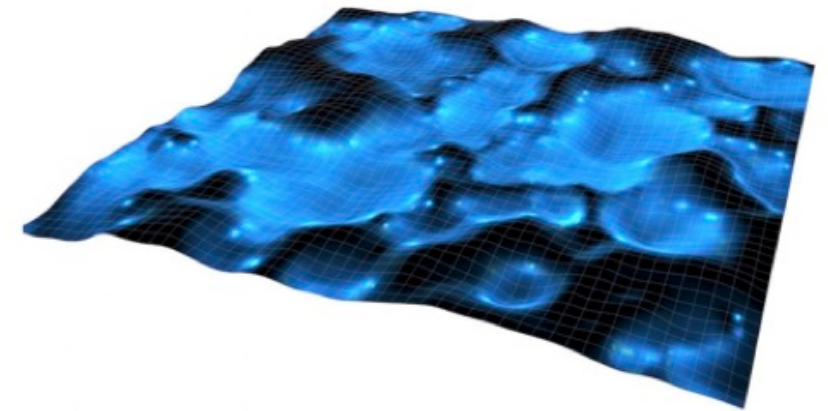
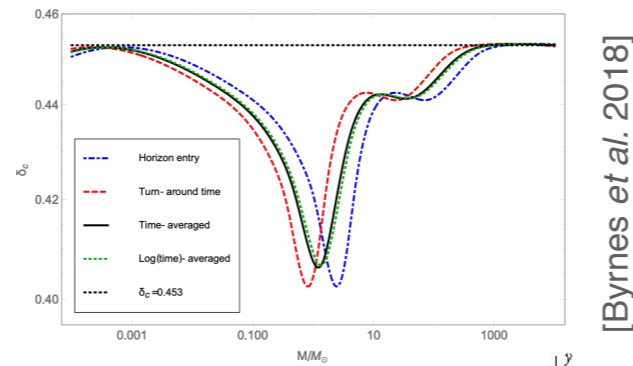
★ Pressure reduction

★ Cosmic string loops

★ Bubble collisions

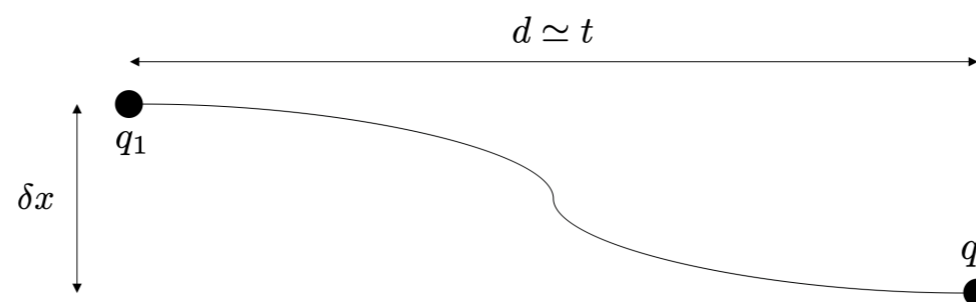
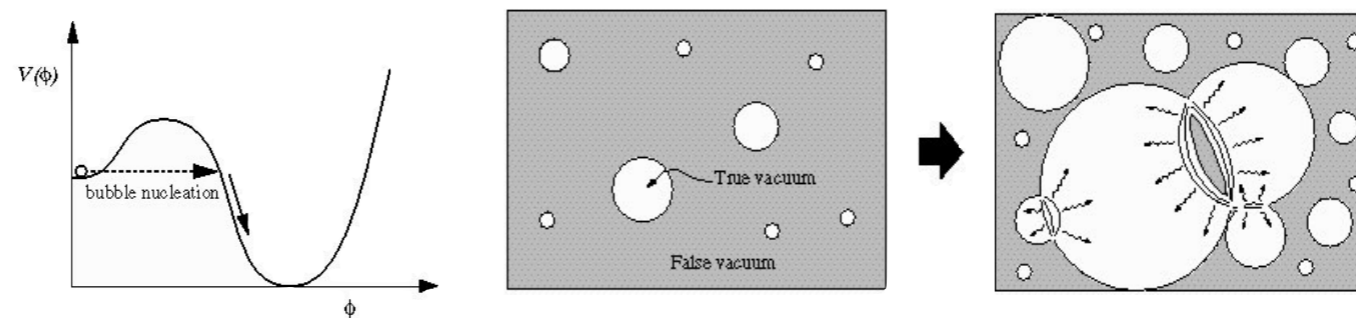
★ Quark confinement

★ Q-balls, Multiverse...

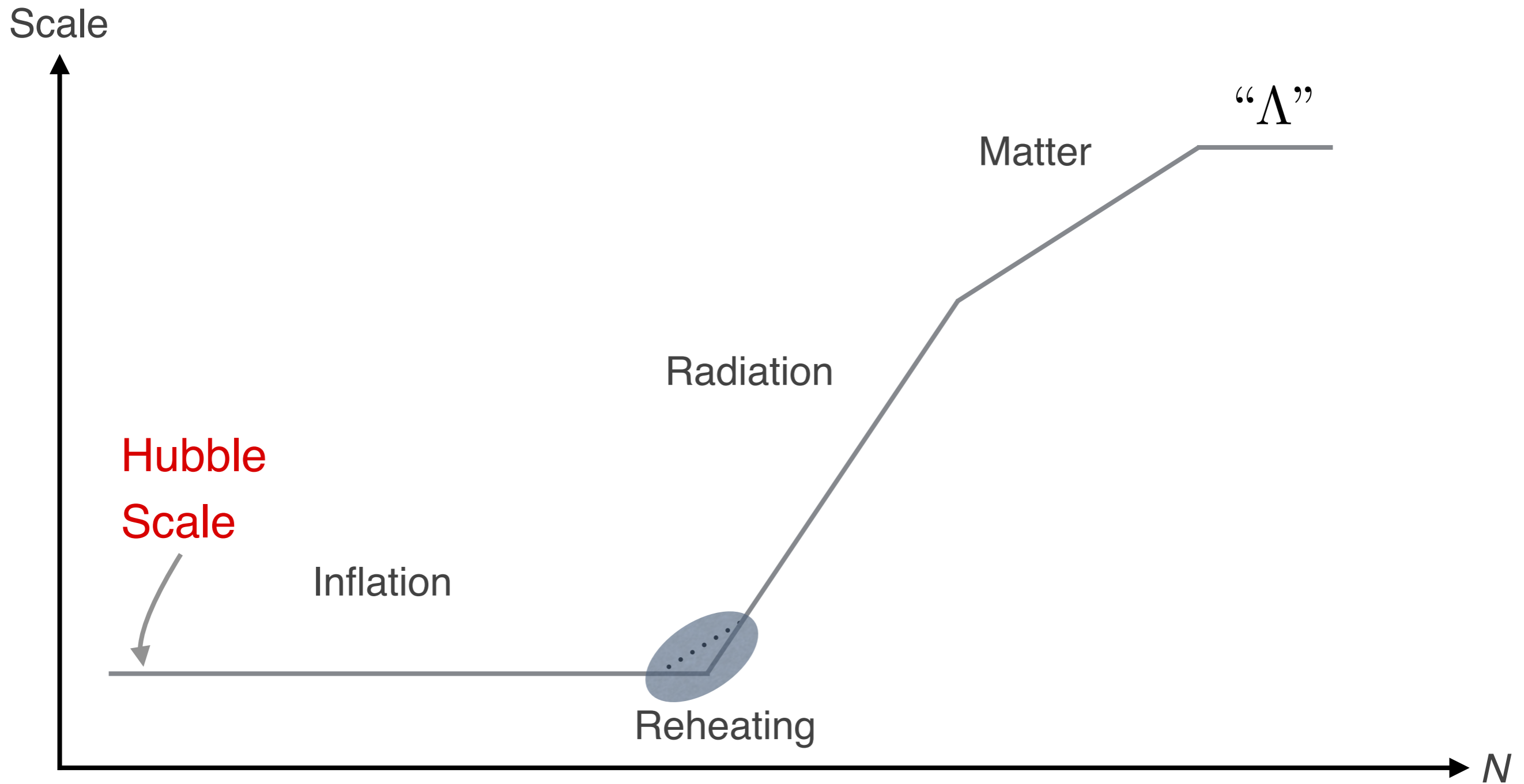


[http://www.damtp.cam.ac.uk/research/gr/public/cs\\_phase.html](http://www.damtp.cam.ac.uk/research/gr/public/cs_phase.html)

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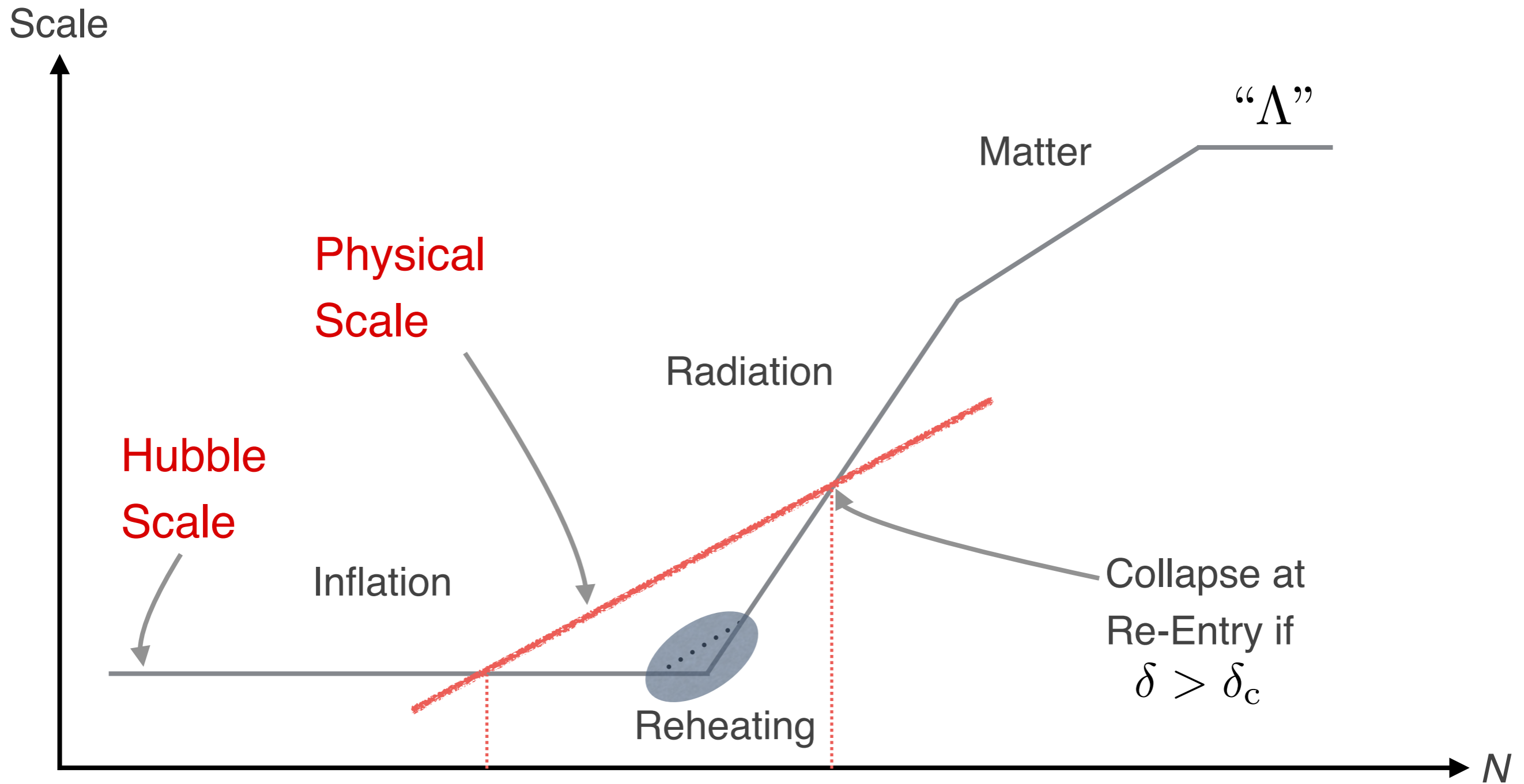


# *PBH Formation from Inflationary Overdensities*



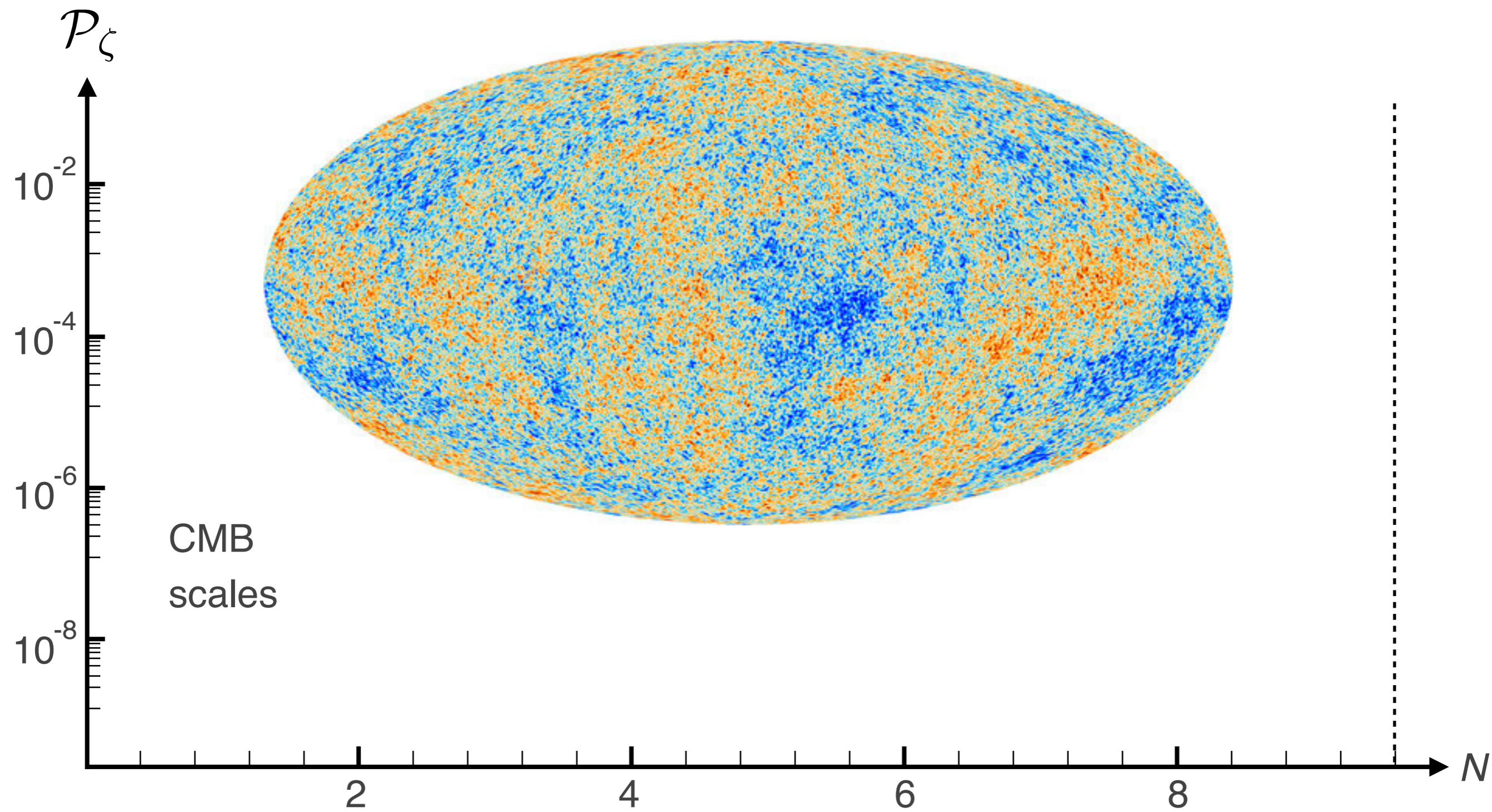


# *PBH Formation from Inflationary Overdensities*

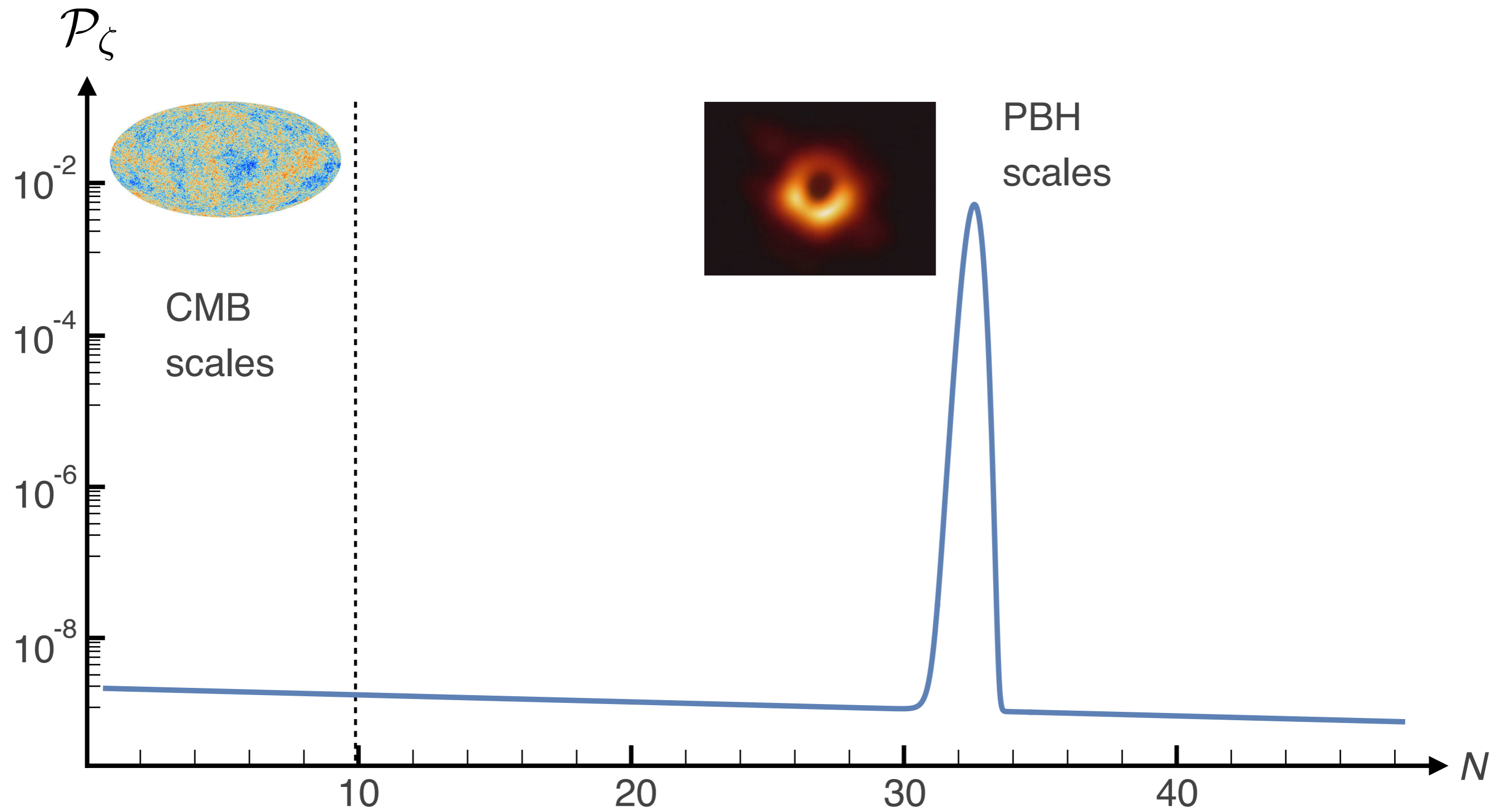




# *PBH Formation — Scales*



# *PBH Formation — Scales*

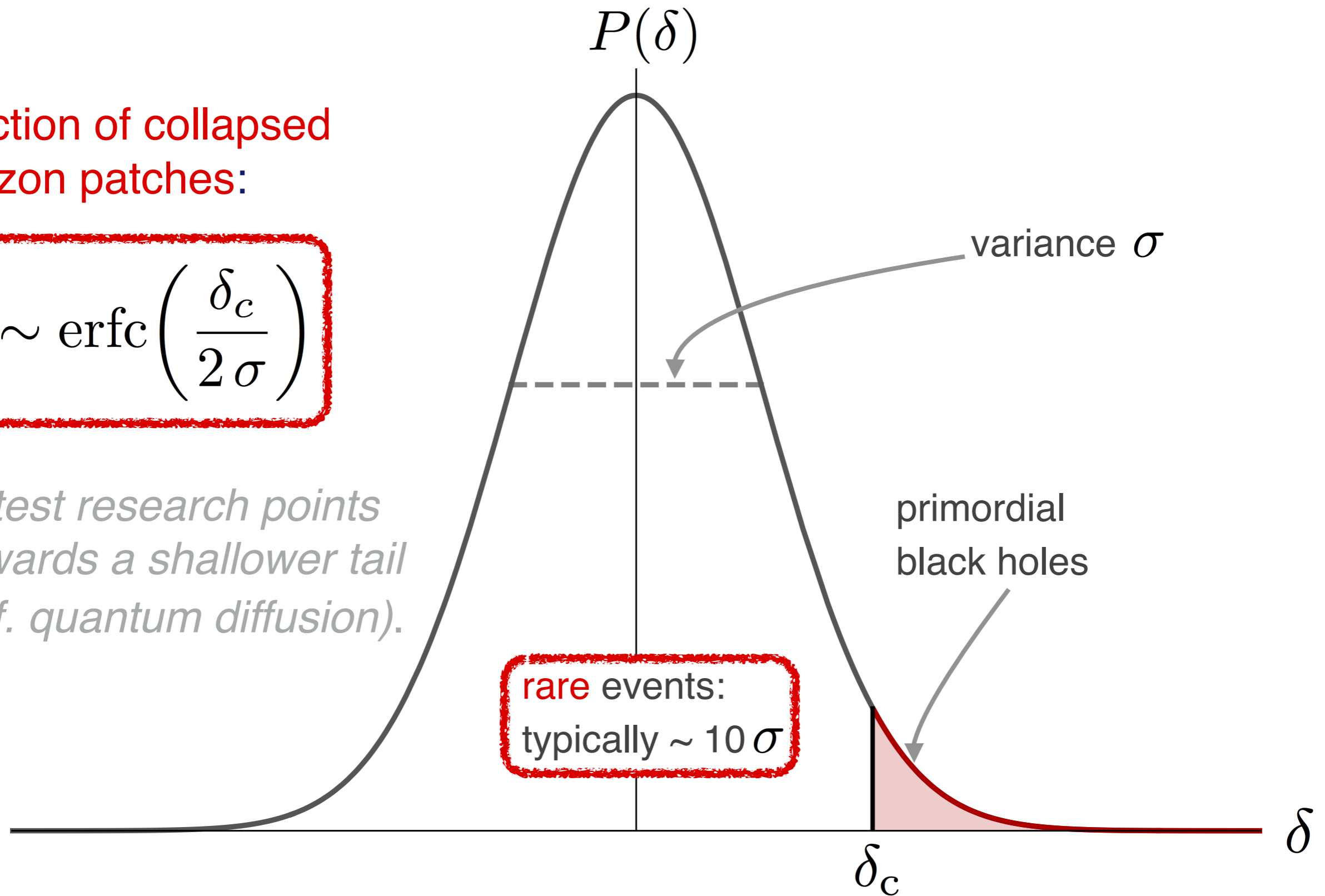


# *PBH Formation — Rare Events*

Fraction of collapsed horizon patches:

$$\beta \sim \text{erfc} \left( \frac{\delta_c}{2\sigma} \right)$$

*Latest research points towards a shallower tail (c.f. quantum diffusion).*



# *PBH — Some Numbers*

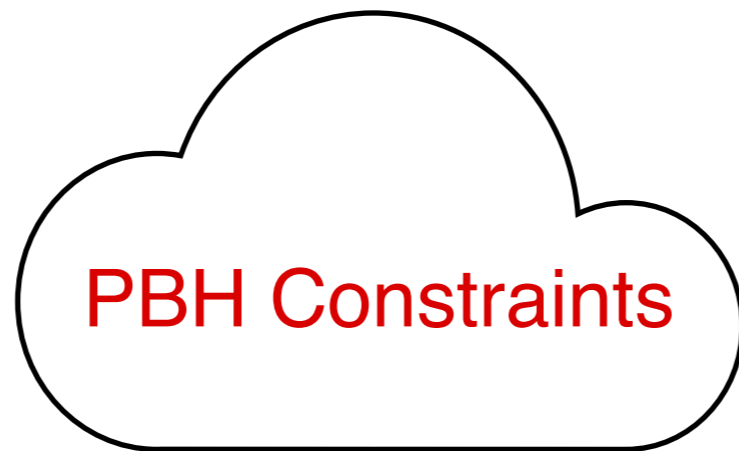
★ If **primordial black holes** constituted **all** of the **dark matter**:

★ Assume that all PBH have mass:  $10^{20}$  g

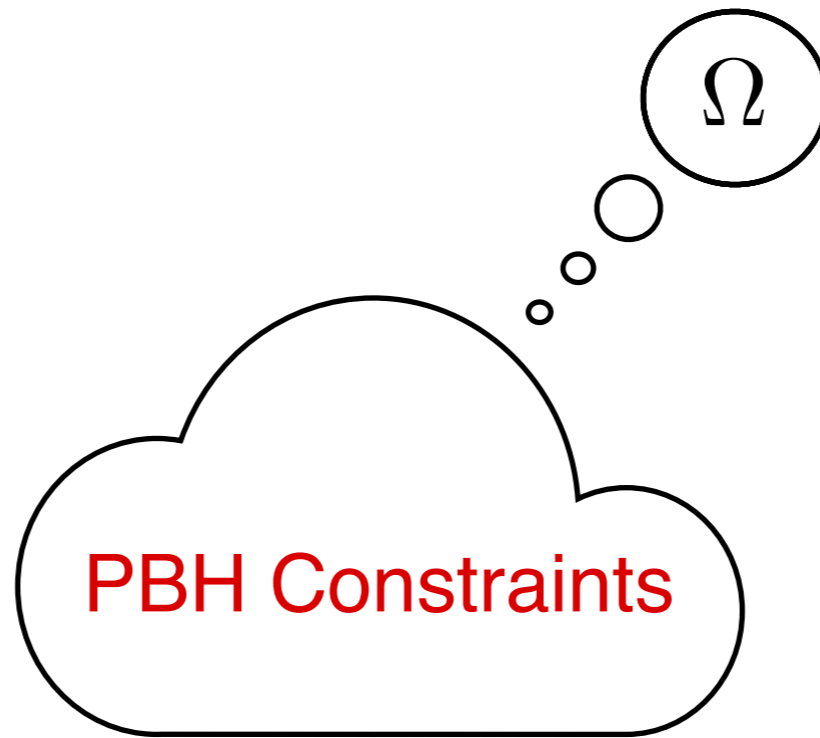
★ Size:  $10^{-8}$  cm

★ Number in our Galaxy:  $10^{25}$

★ Distance: 10 AU



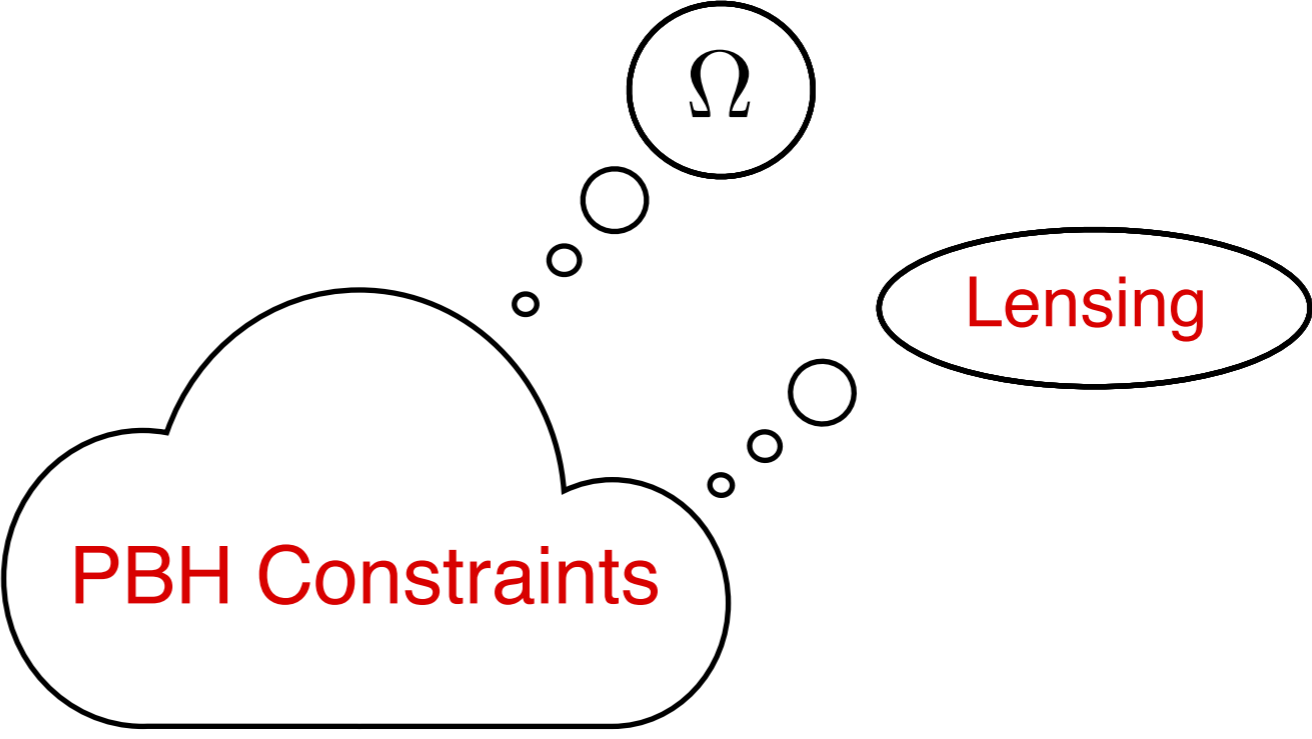
**PBH Constraints**

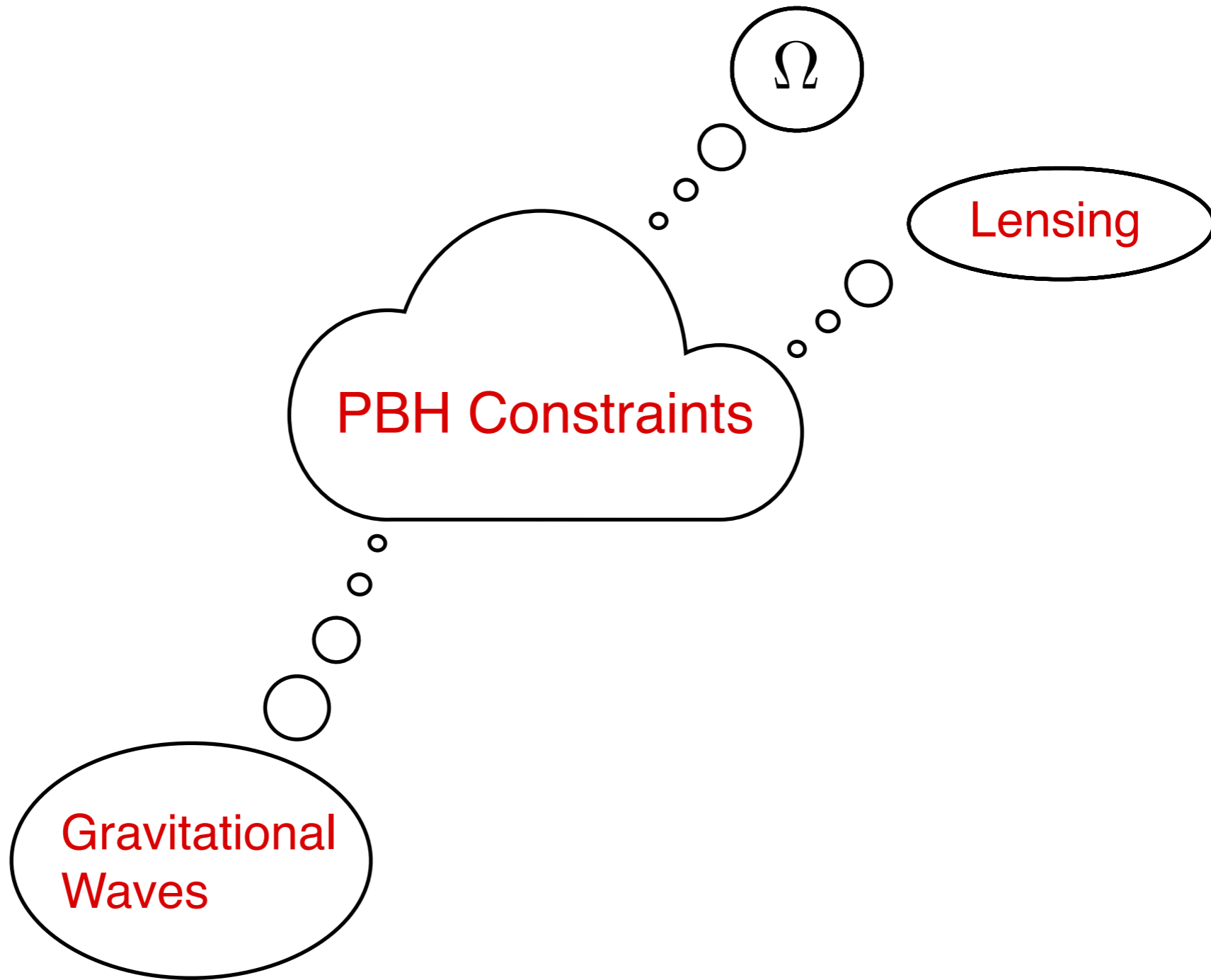


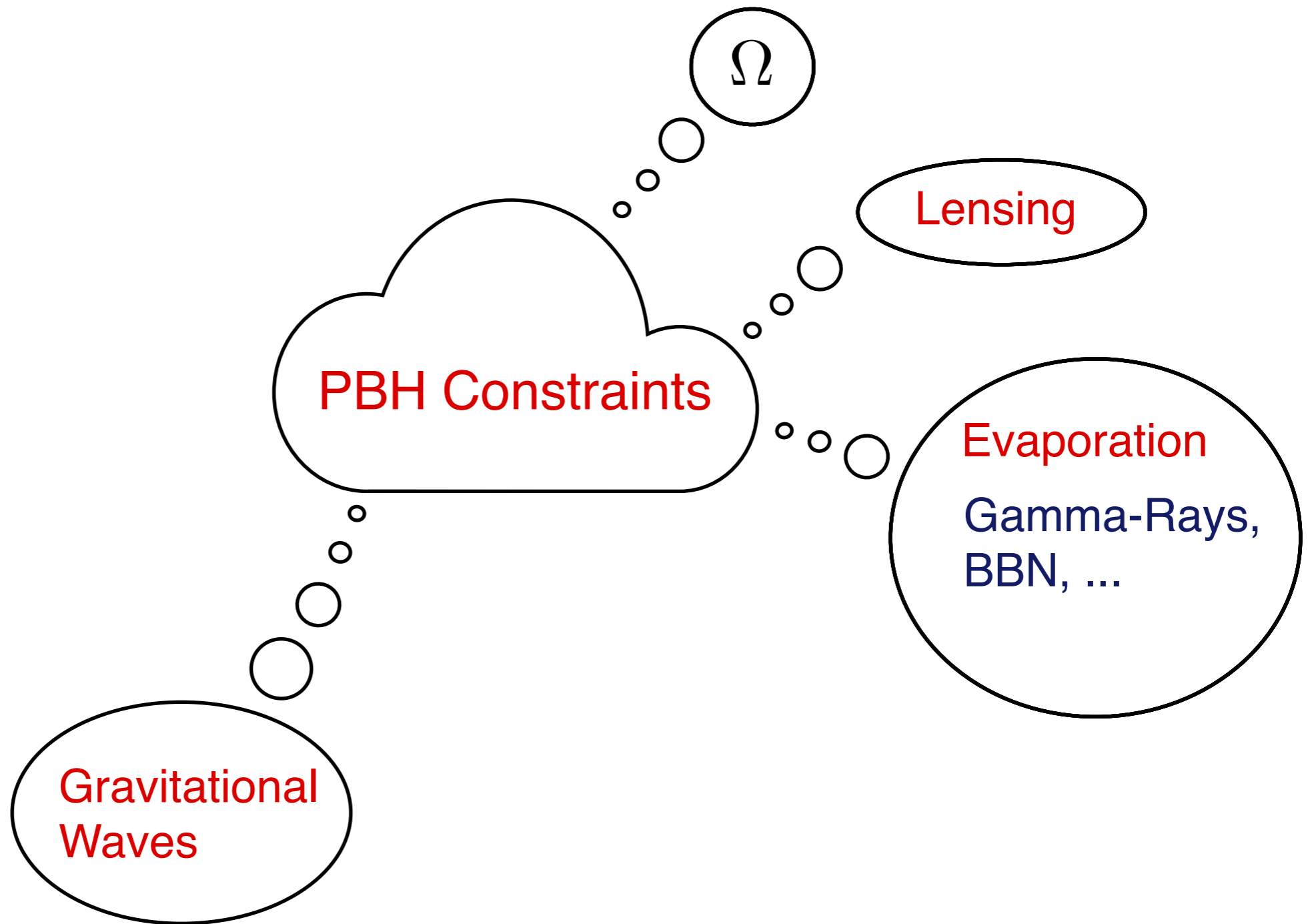
**PBH Constraints**

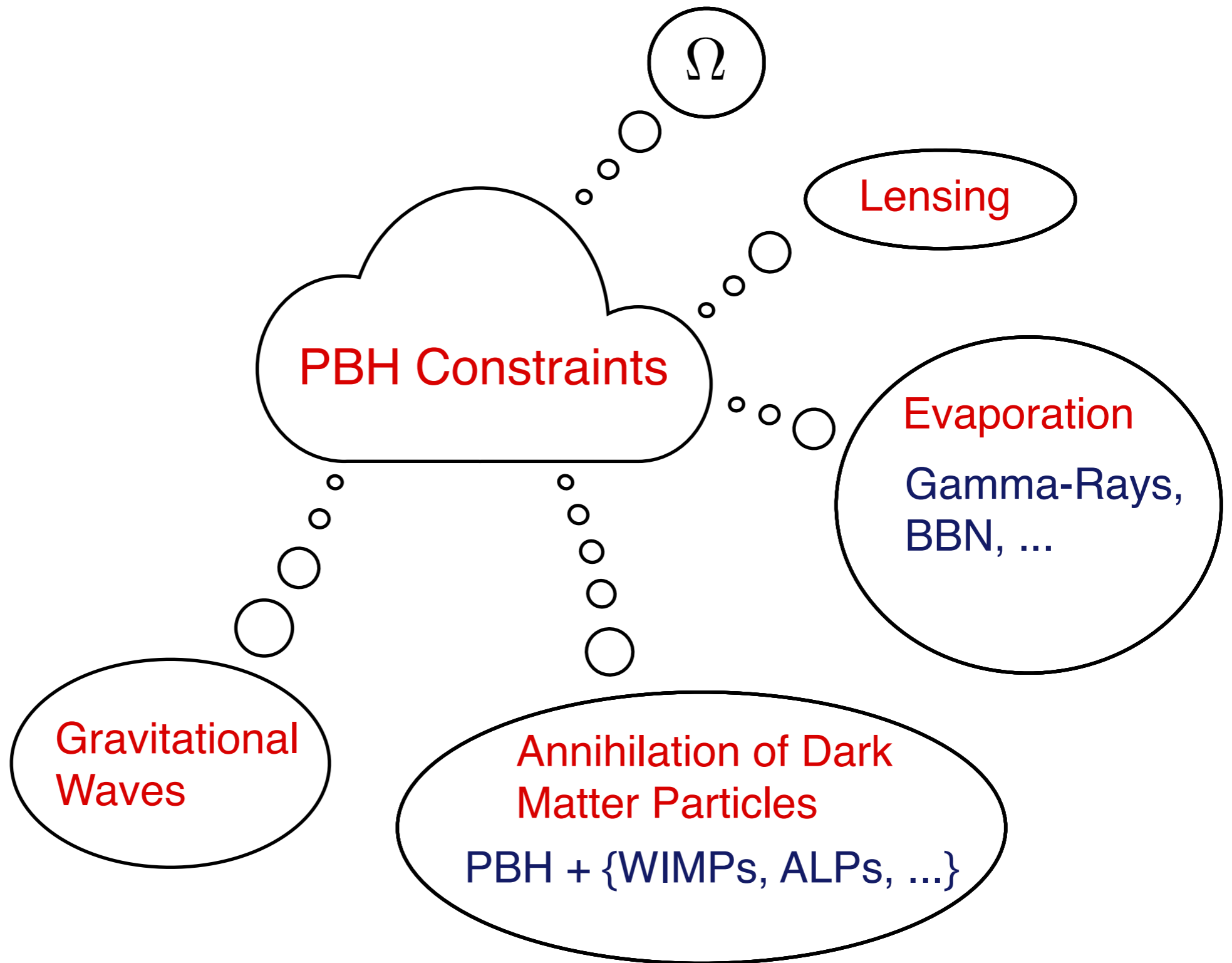
$\Omega$

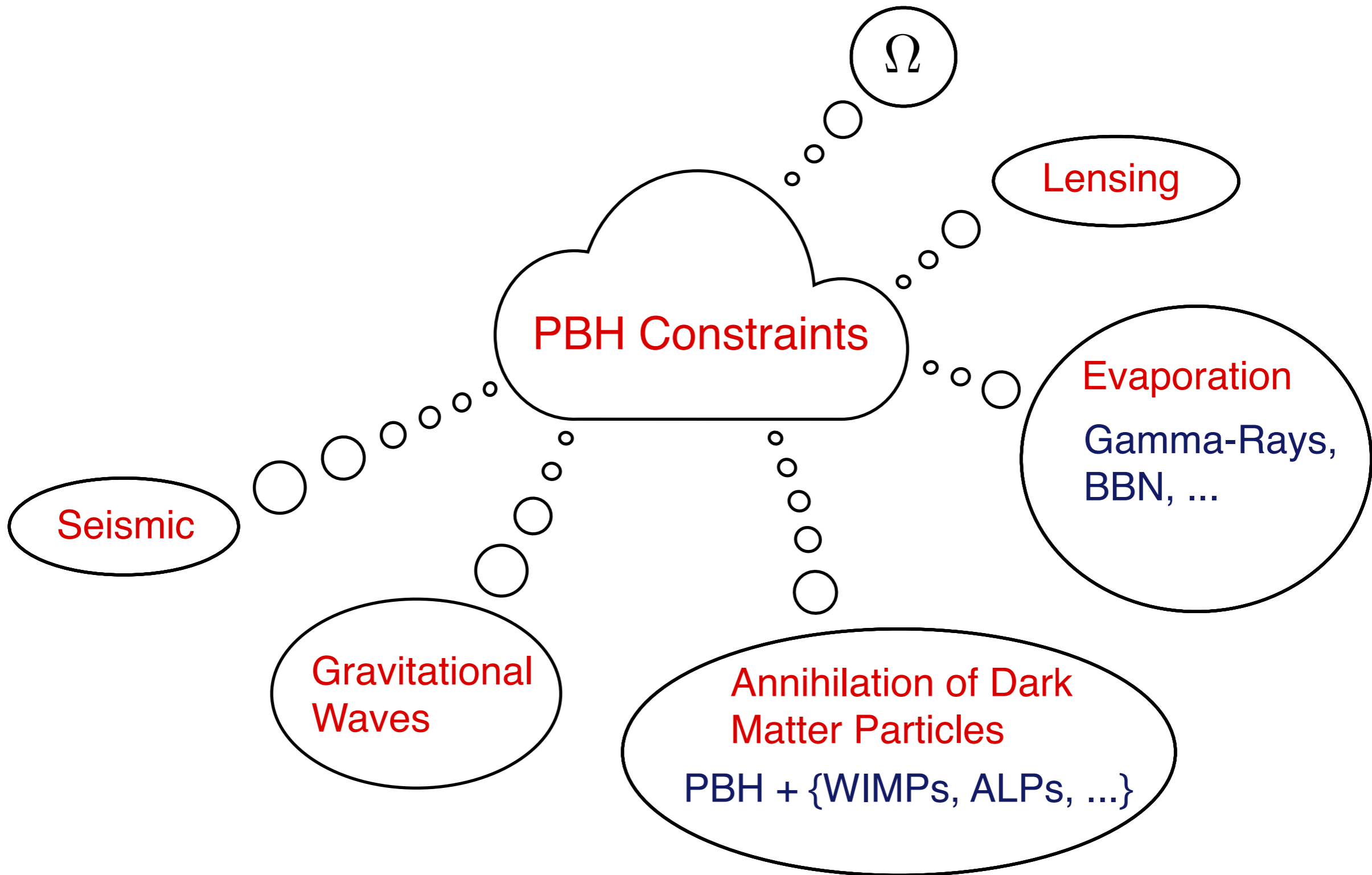












**PBH Constraints**

**Seismic**

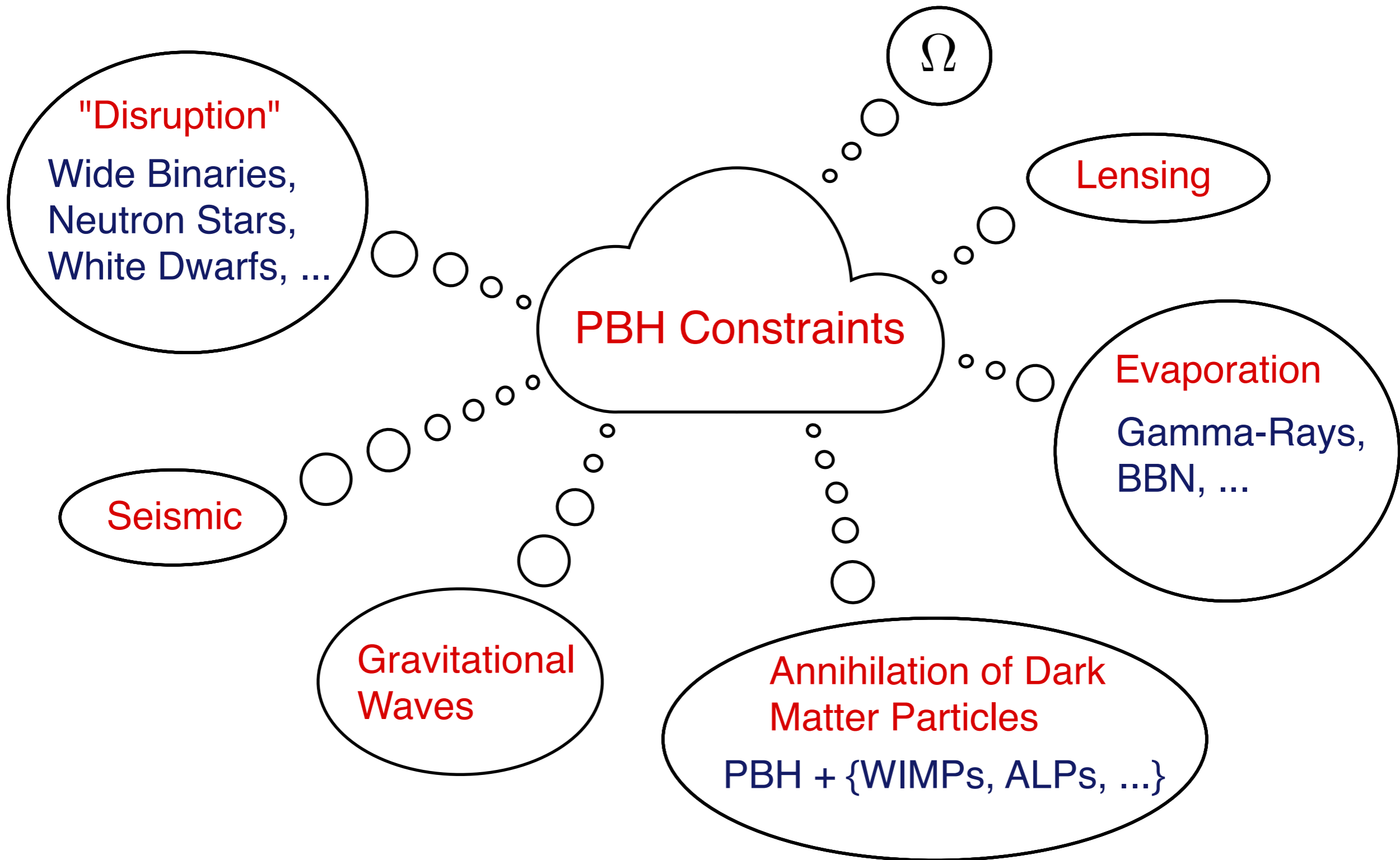
**Gravitational  
Waves**

**Annihilation of Dark  
Matter Particles**  
PBH + {WIMPs, ALPs, ...}

**Evaporation**  
Gamma-Rays,  
BBN, ...

**Lensing**

$\Omega$



**"Disruption"**

Wide Binaries,  
Neutron Stars,  
White Dwarfs, ...

$\Omega$

**Lensing**

**PBH Constraints**

**Evaporation**

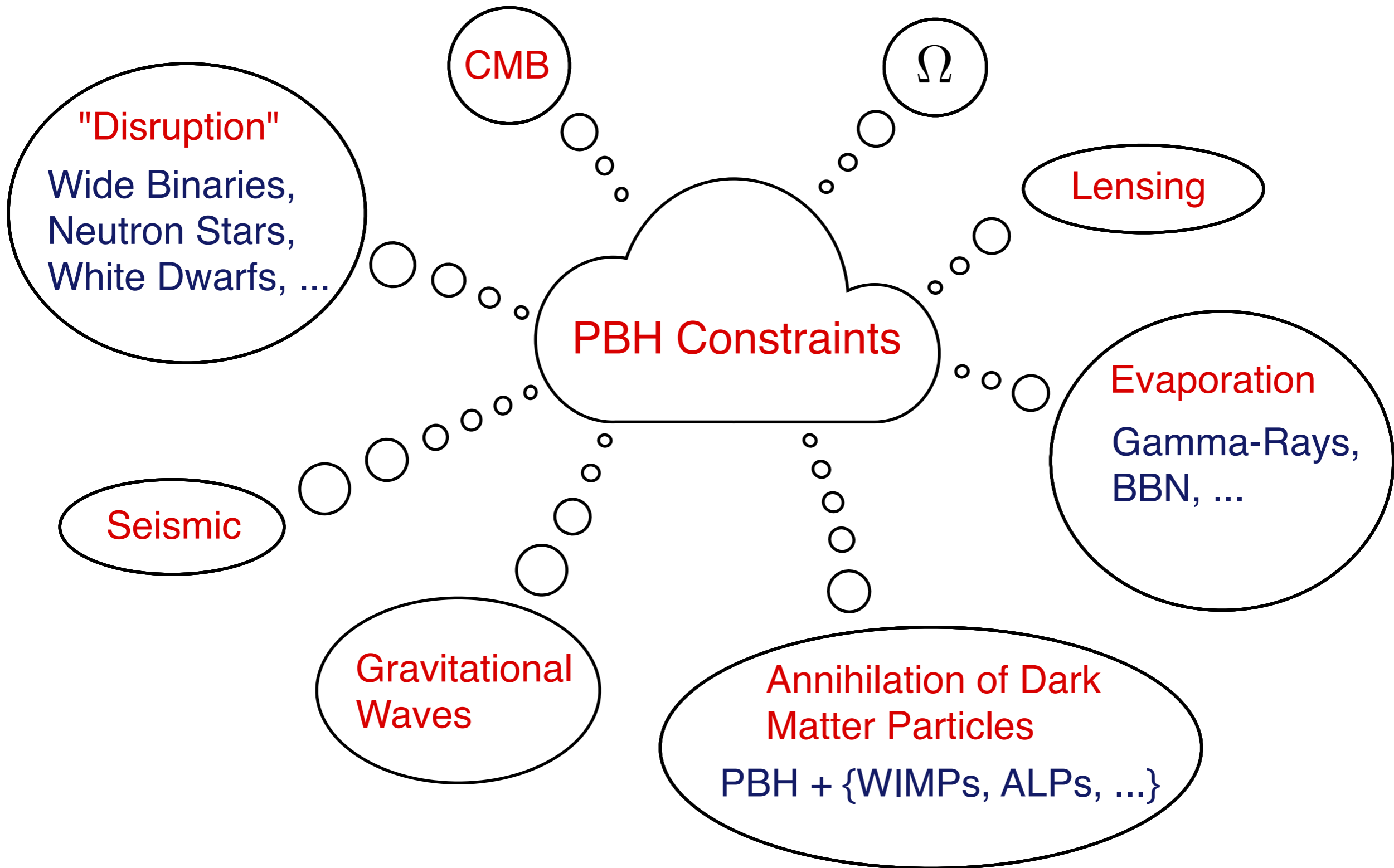
Gamma-Rays,  
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**Seismic**

**Gravitational  
Waves**

**Annihilation of Dark  
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PBH + {WIMPs, ALPs, ...}



CMB

$\Omega$

"Disruption"

Wide Binaries,  
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White Dwarfs, ...

Lensing

PBH Constraints

Evaporation

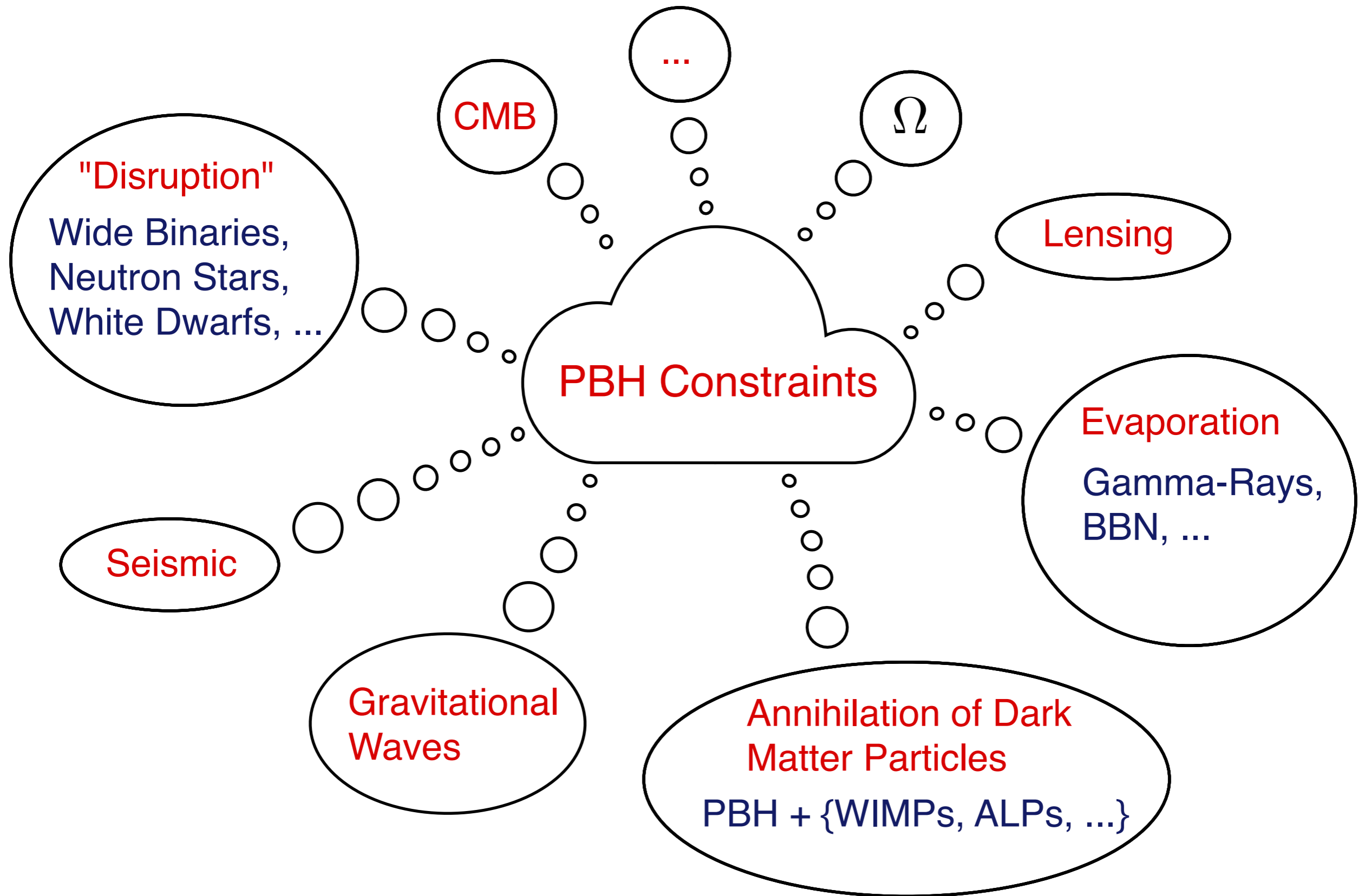
Gamma-Rays,  
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Seismic

Gravitational  
Waves

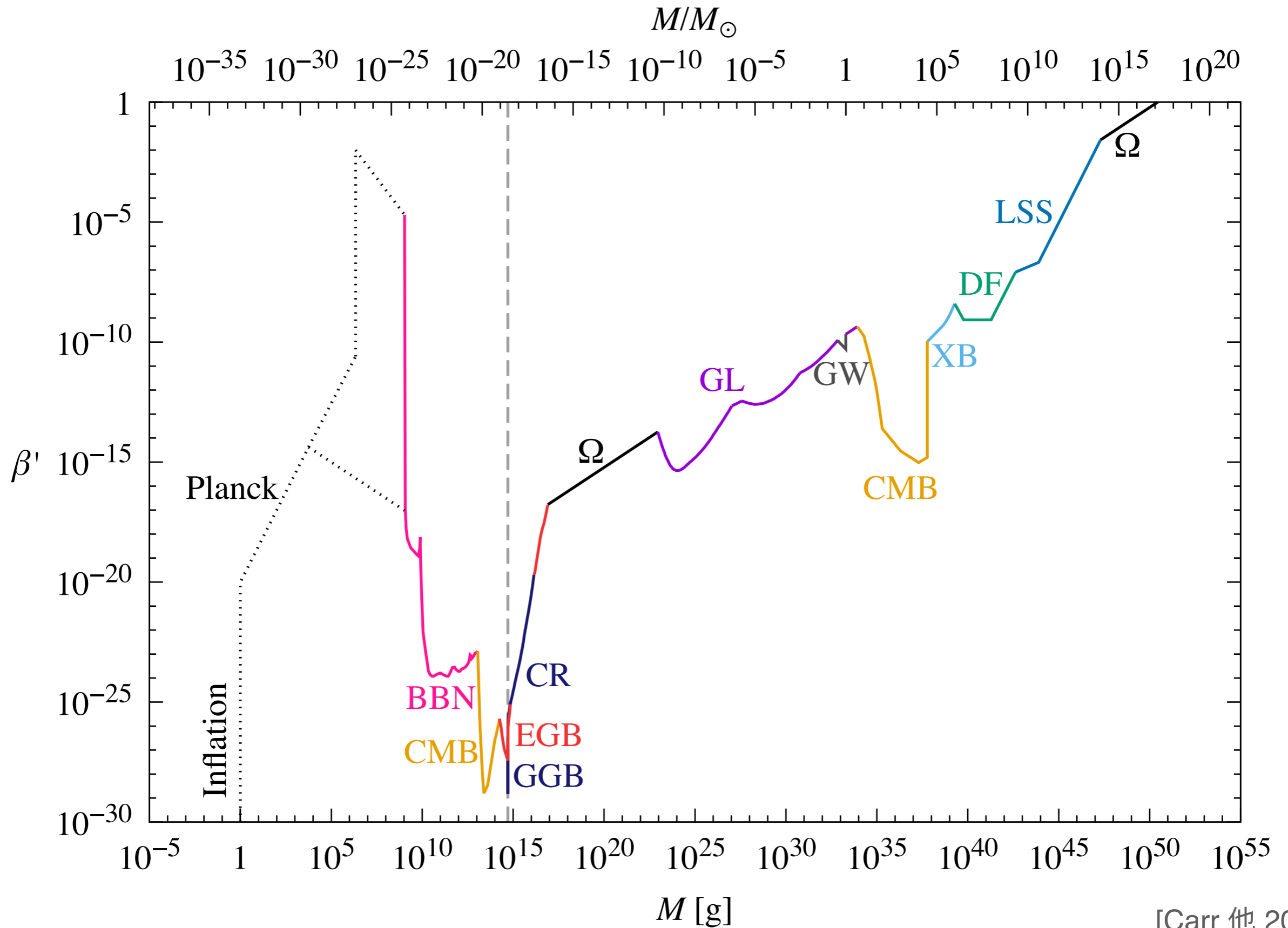
Annihilation of Dark  
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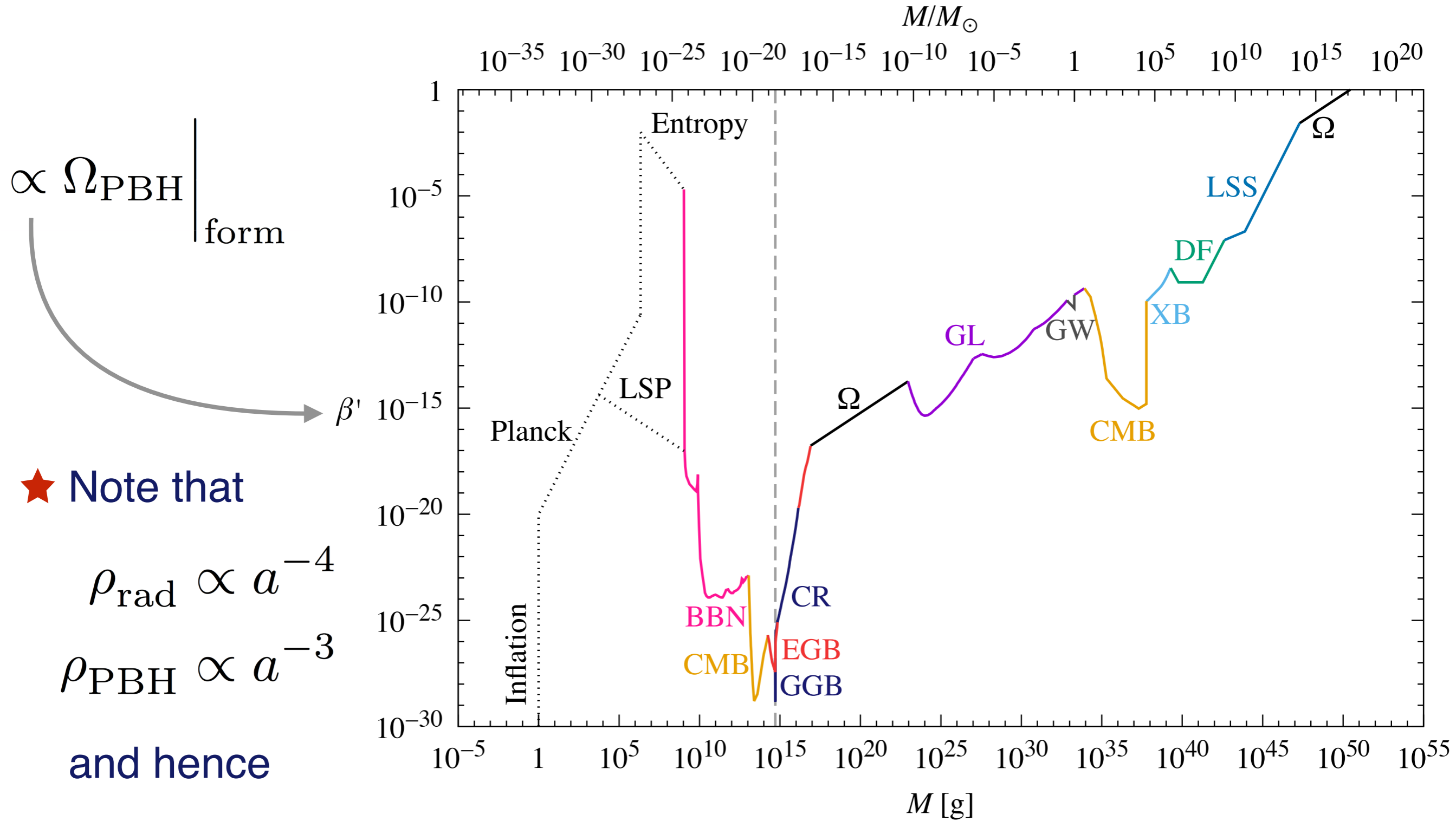




# *PBH Constraints at Formation*

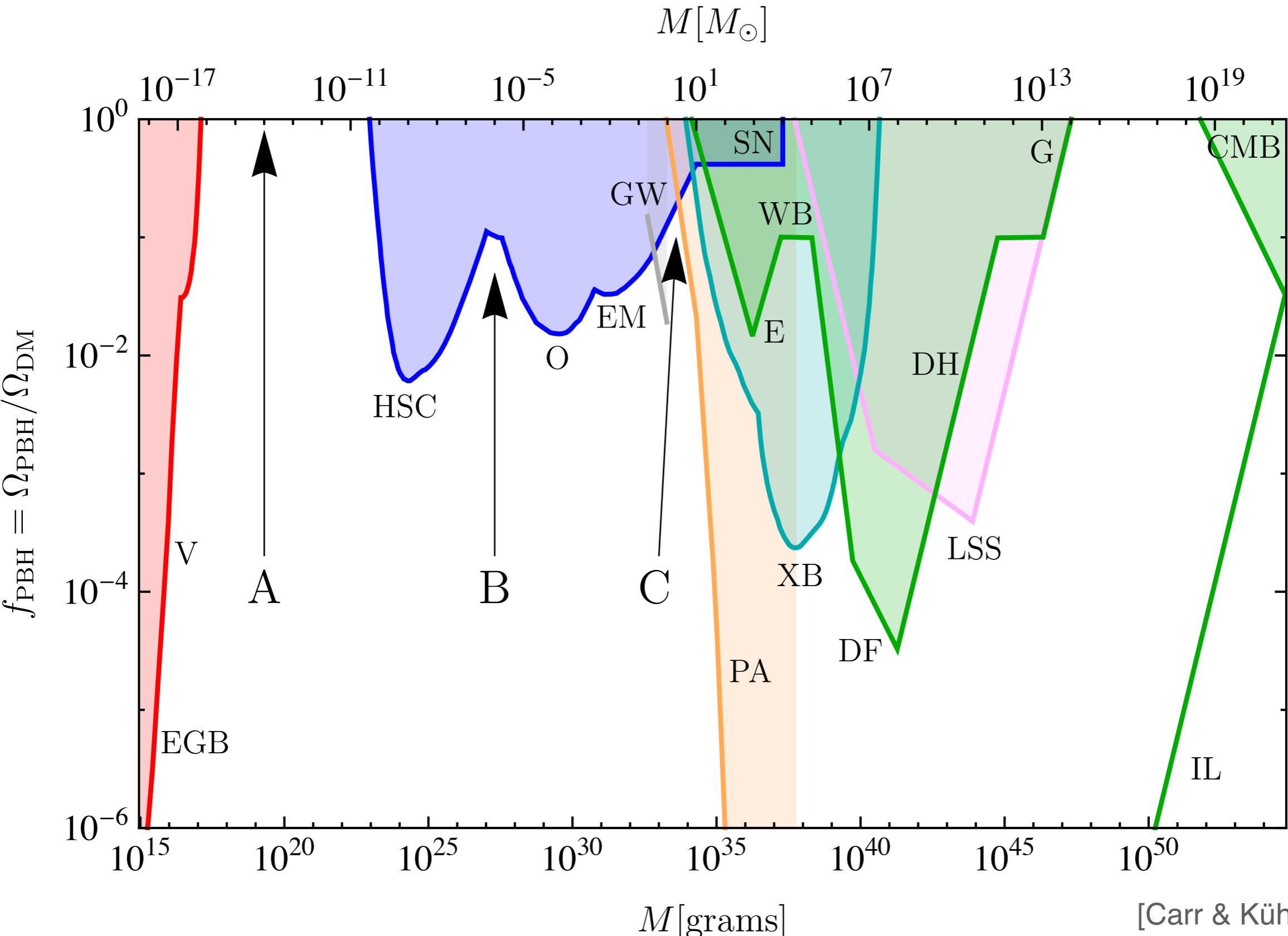


# PBH Constraints at Formation



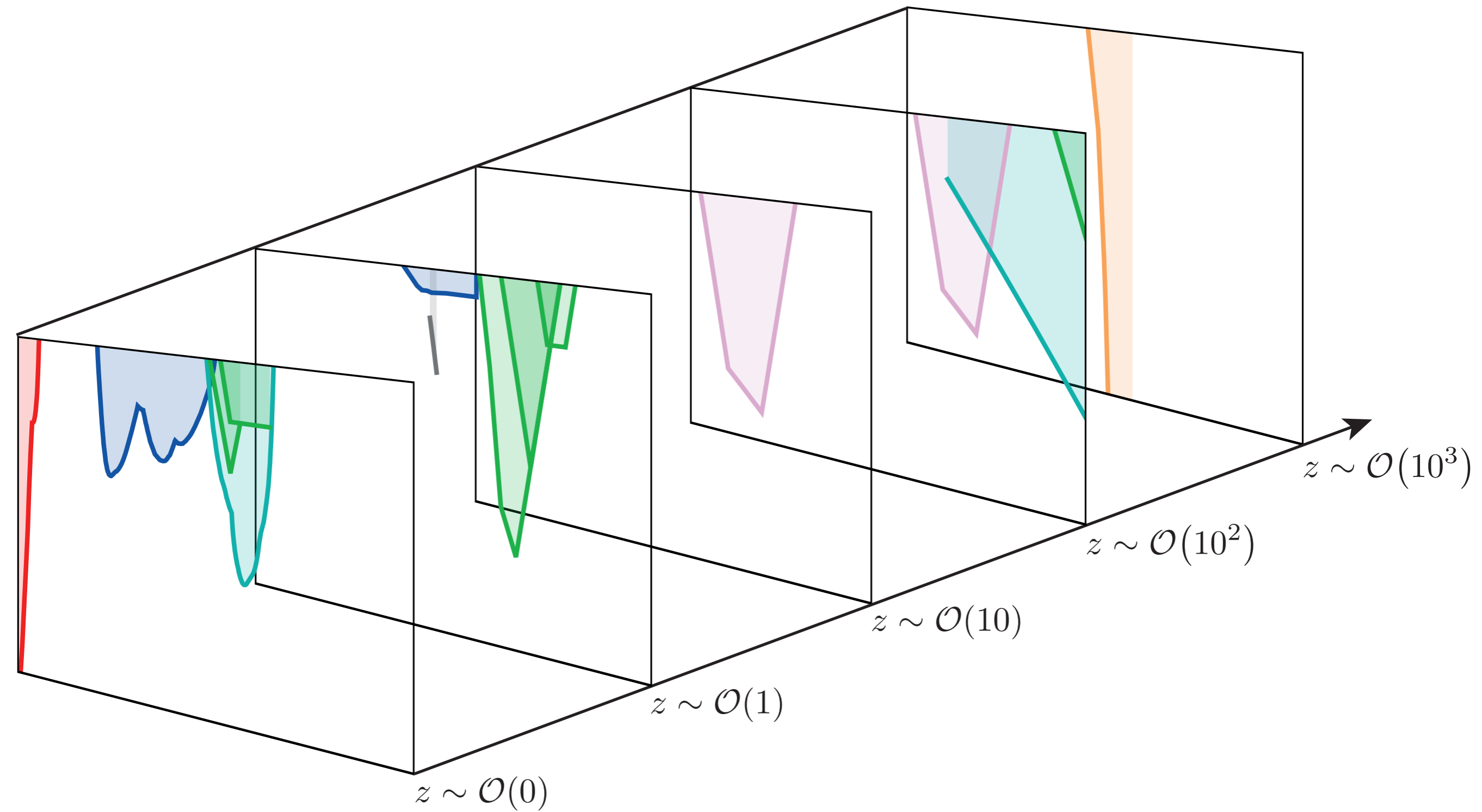
[Carr 他 2021]

# Current PBH Constraints



[Carr & Kühnel 2020]

# *PBH Constraints — Redshift Dependence*



*Observational Hints  
for Primordial Black Holes*

*Evidence?*

*Observational ~~Hints~~*

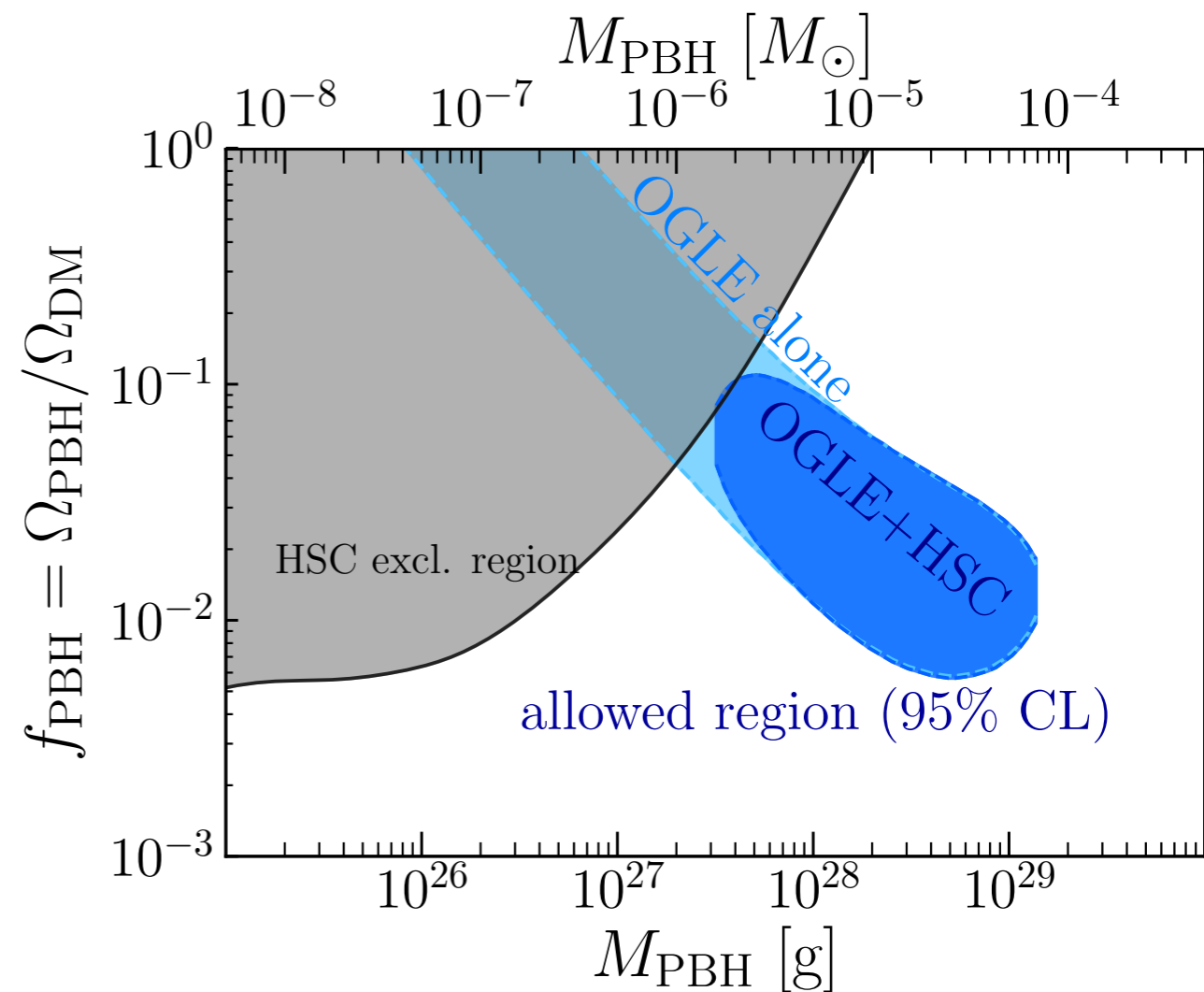
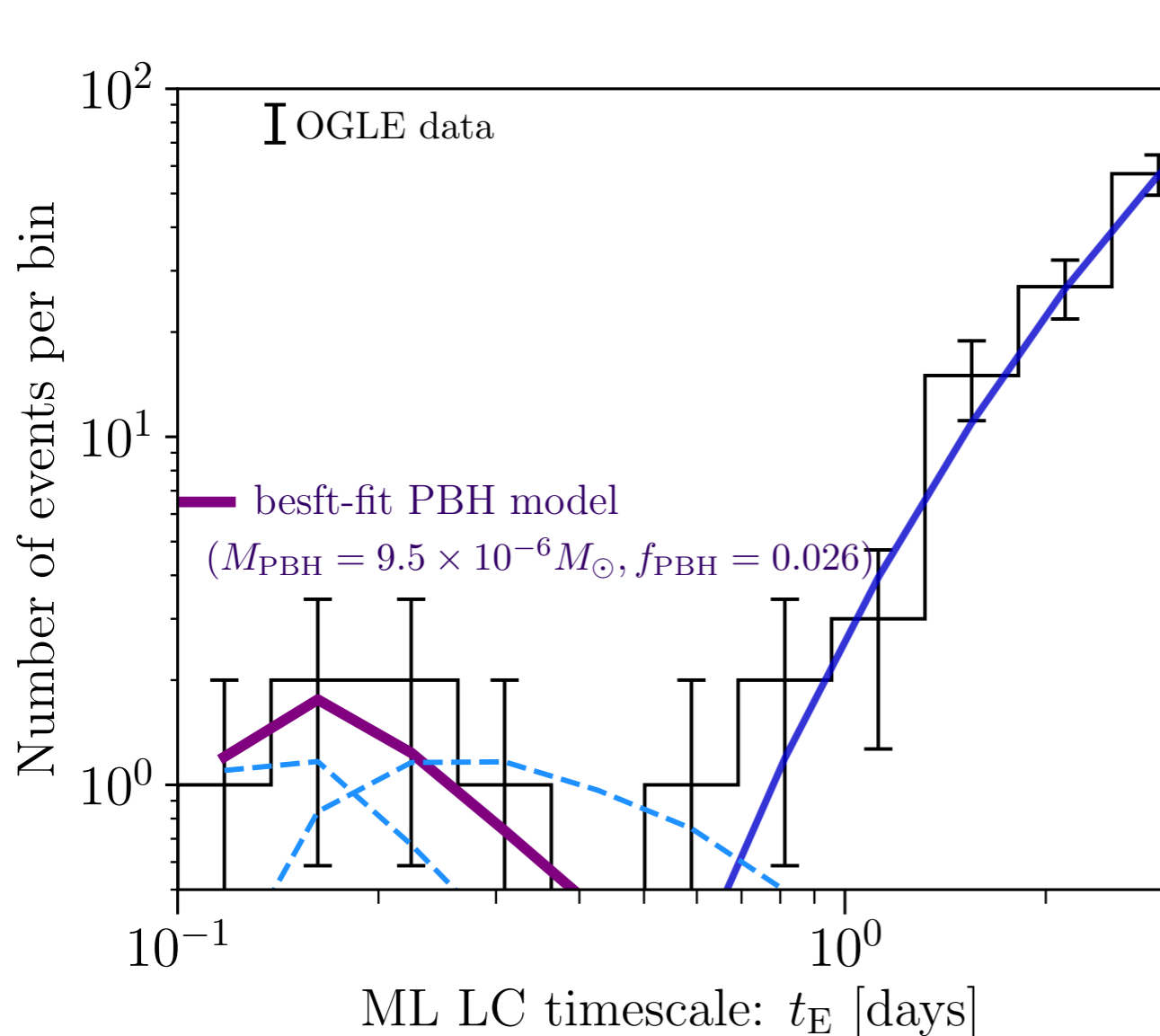
*for Primordial Black Holes*

# Planetary-Mass Microlensing

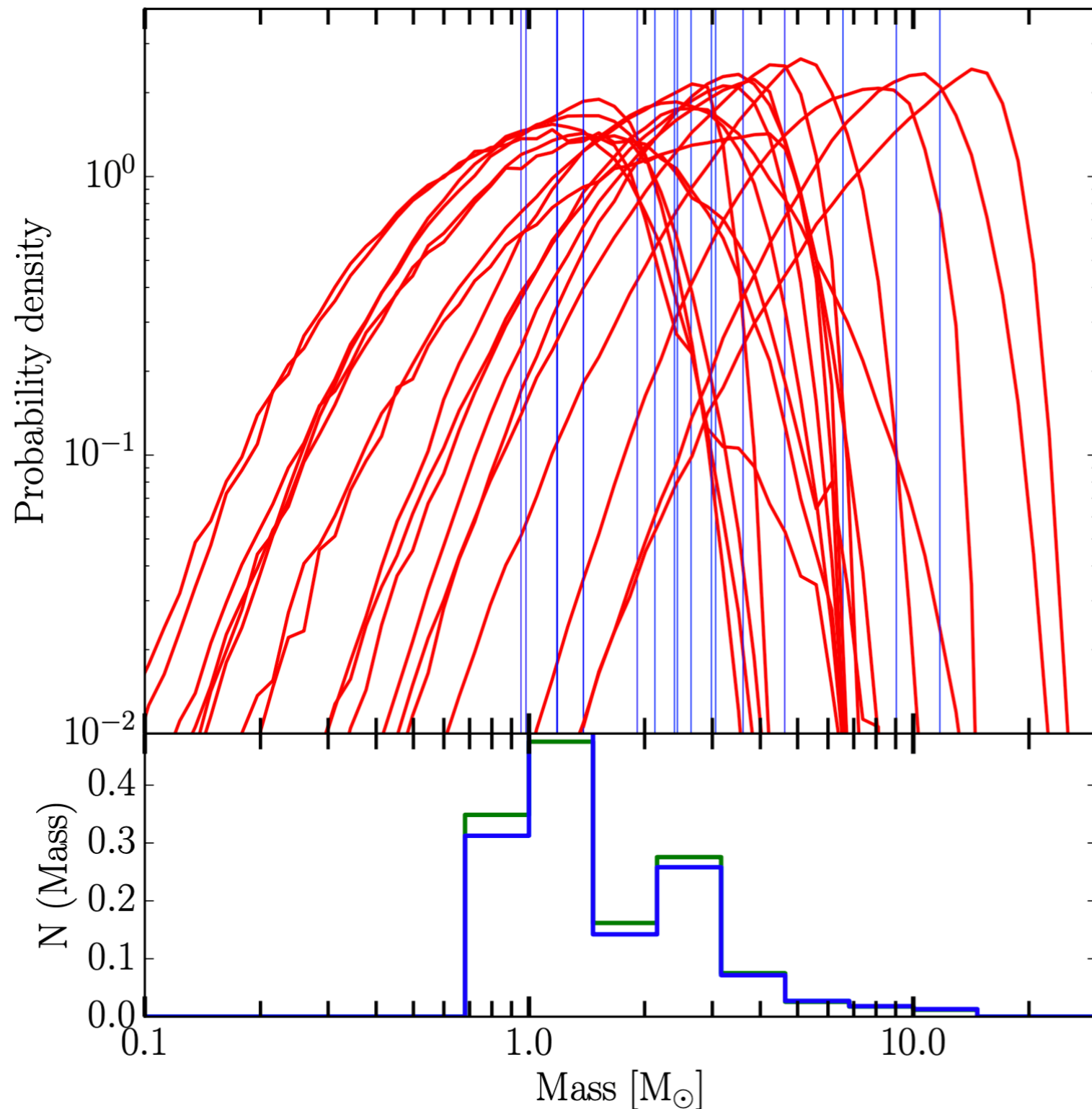
★ OGLE detected a particular **population** of microlensing events:

★ **0.1 - 0.3 days** light-curve timescale - origin **unknown!**

Could be free-floating planets... or **PBHs!**



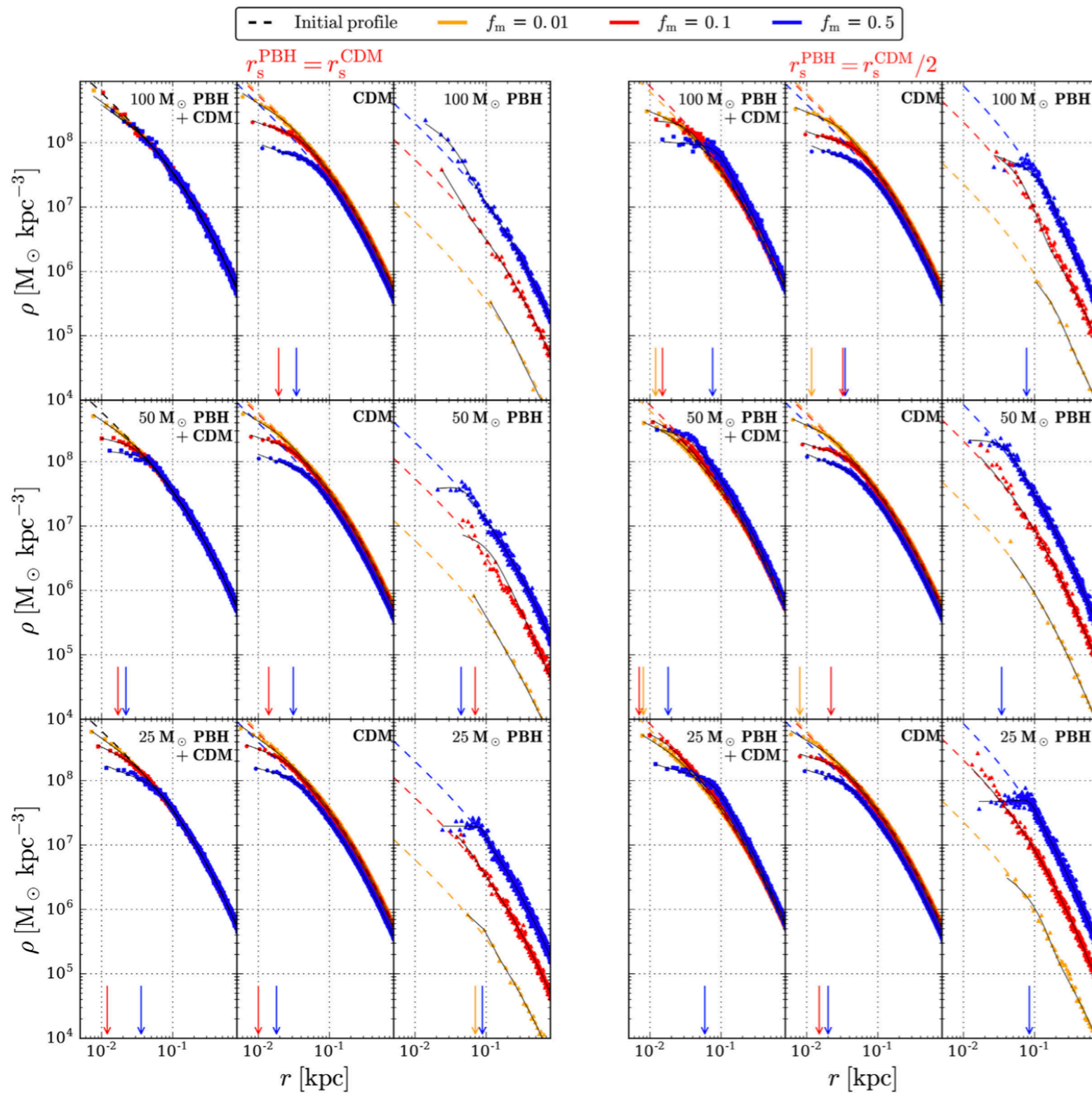
# Excess of Lenses in Galactic Bulge



- ★ OGLE has detected 58 long-duration microlensing events in the Galactic bulge.
- ★ 18 of these cannot be main-sequence stars and are very likely black holes.
- ★ Their mass function overlaps the low mass gap from 2 to 5  $M_{\odot}$ .
- ★ These are not expected to form as the endpoint of stellar evolution.

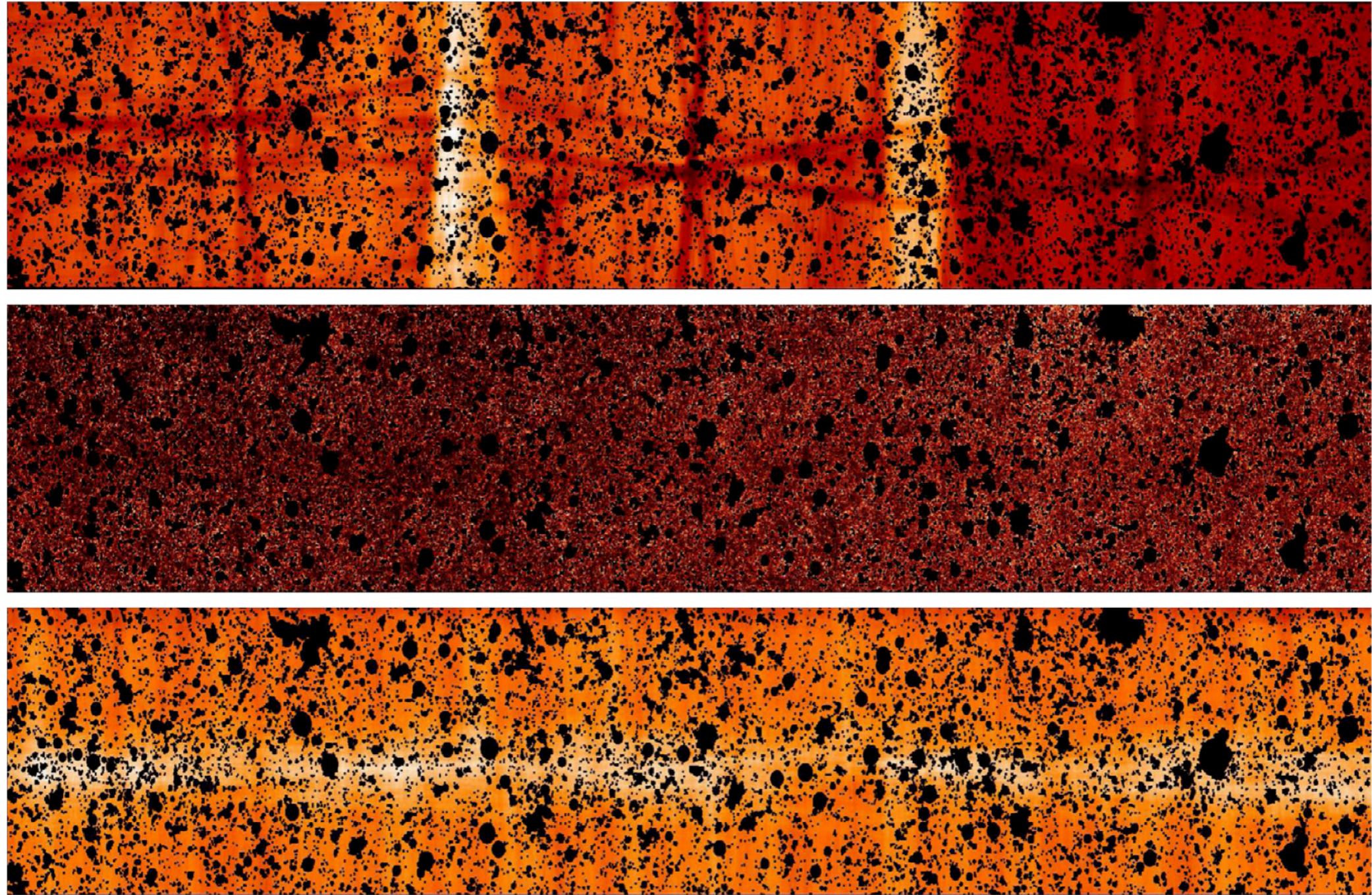


# Ultra-faint Dwarf Galaxies



- ★ **Non-detection** of dwarf galaxies smaller than  $\sim 10 - 20$  pc
- ★ Ultra-faint dwarf galaxies are **dynamically unstable** below some critical radius in the presence of PBH CDM!
- ★ This works with **a few percent of PBH DM** of  $25 - 100 M_\odot$ .

# Correlations of Cosmic Infrared/X-Ray Backgrounds



[Capelluti *et al.* 2013]

★ PBHs generate early structure and respective backgrounds

# GRAVITATIONAL WAVE MERGER DETECTIONS

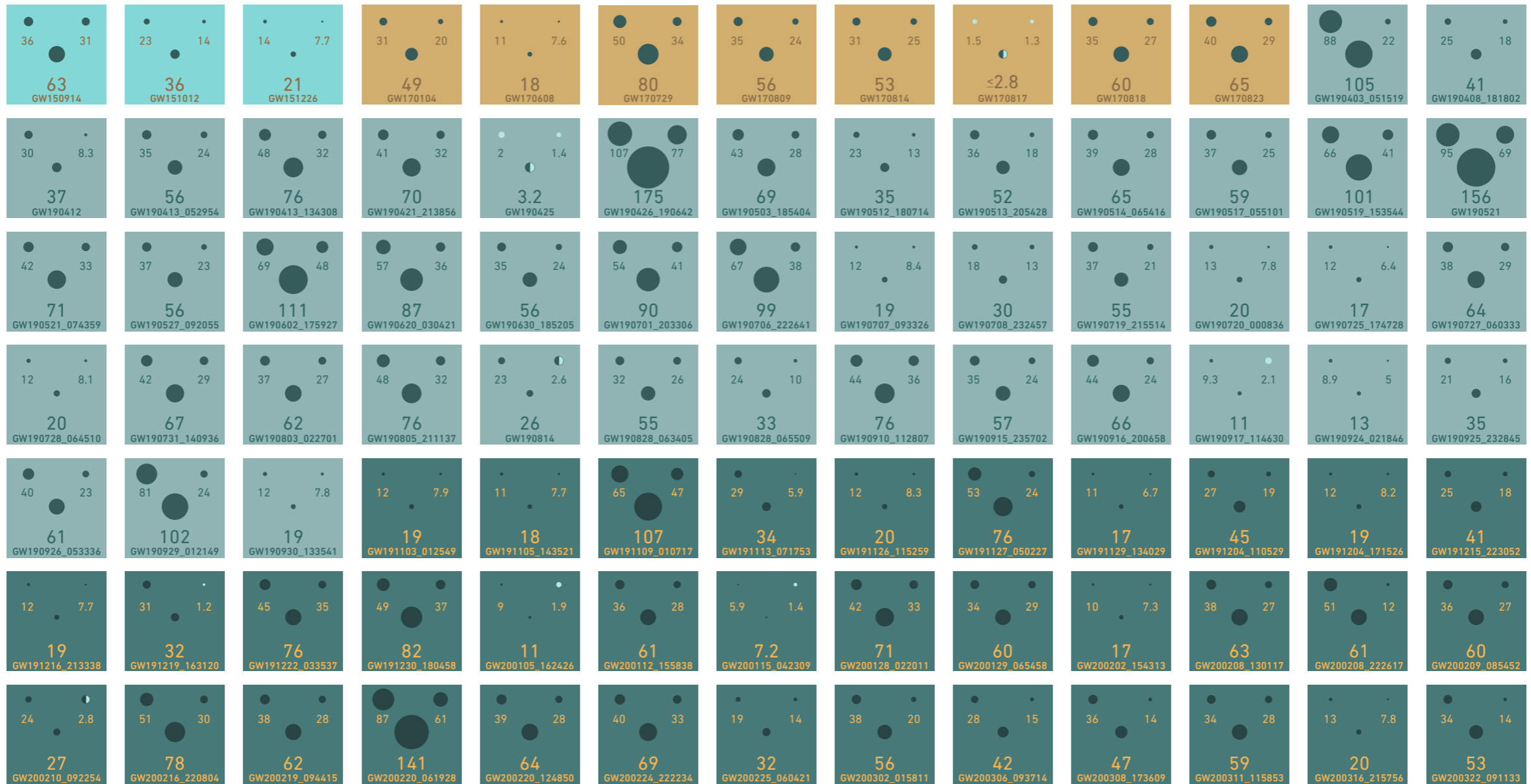
→ SINCE 2015

OBSERVING RUN

01 2015-2016

02 2016-2017

03a+b 2019-2020



# GRAVITATIONAL WAVE MERGER DETECTIONS

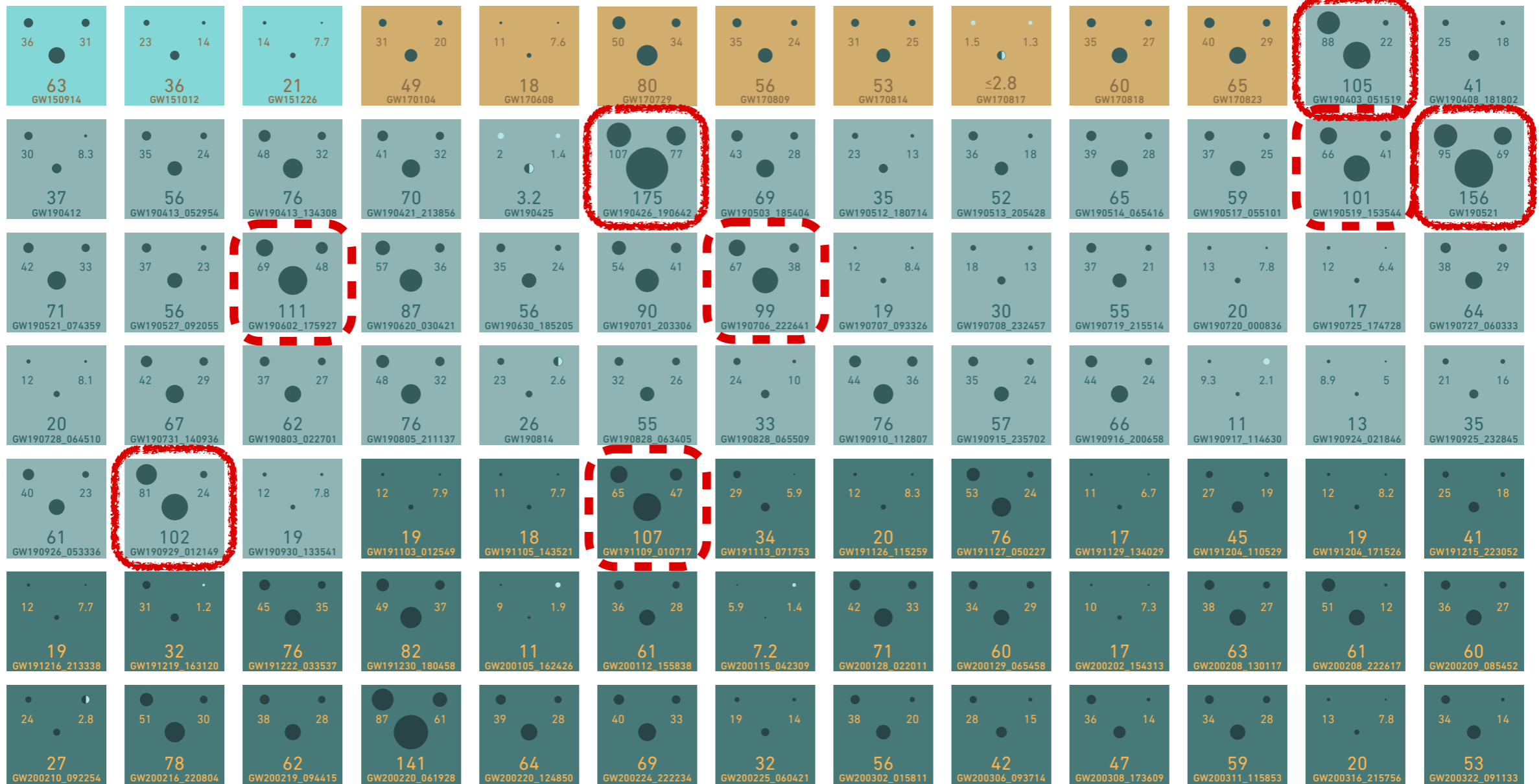
→ SINCE 2015

OBSERVING RUN

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★ Black hole progenitors in the **pair-instability mass gap** (i.e. above  $\sim 60 M_{\odot}$ )



# GRAVITATIONAL WAVE MERGER DETECTIONS

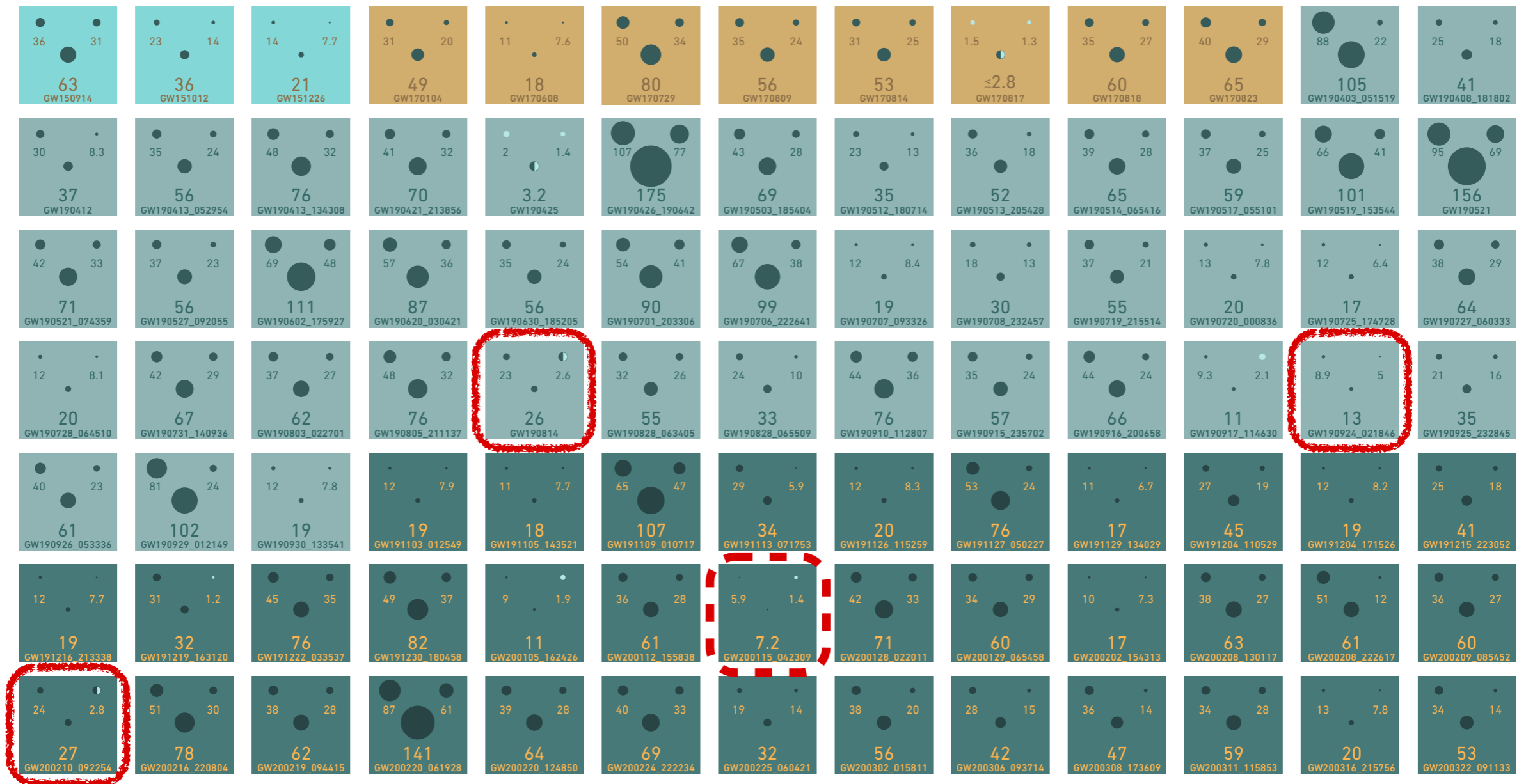
→ SINCE 2015

OBSERVING RUN

01 2015-2016

02 2016-2017

03a+b 2019-2020



★ Black hole progenitors in the **lower mass gap** (i.e. between 2 and 5  $M_{\odot}$ )



# GRAVITATIONAL WAVE MERGER DETECTIONS

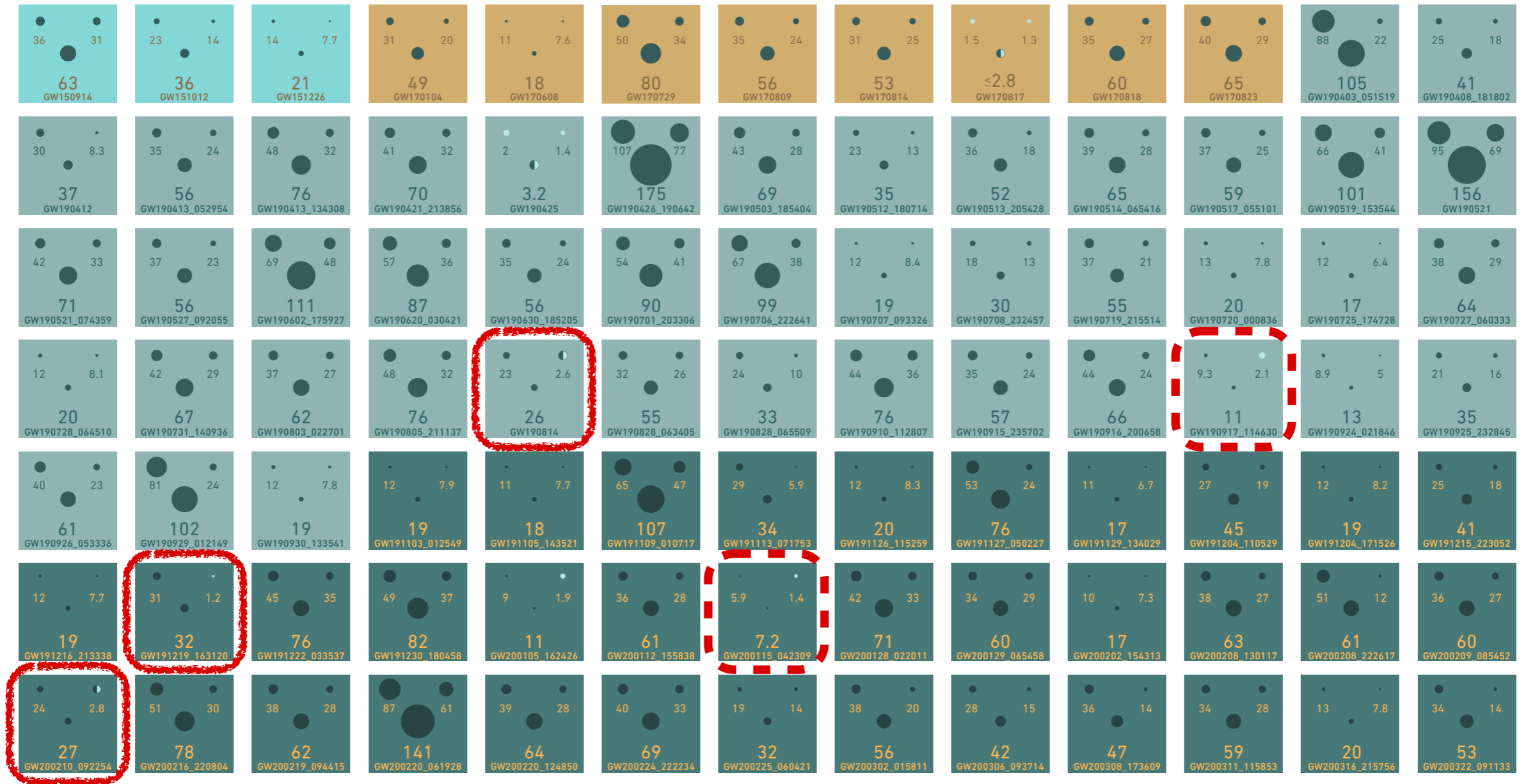
→ SINCE 2015

OBSERVING RUN

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03a+b 2019-2020



★ **Asymmetric** black hole progenitors (mass ratio  $q < 0.25$ )





## **GW190814: Gravitational Waves from the Coalescence of a 23 Solar Mass Black Hole with a 2.6 Solar Mass Compact Object**

R. Abbott<sup>1</sup>, [...]

### **Abstract**

We report the observation of a compact binary coalescence involving a  $22.2\text{--}24.3 M_{\odot}$  black hole and a compact object with a mass of  $2.50\text{--}2.67 M_{\odot}$  [...] **the combination of mass ratio, component masses, and the inferred merger rate for this event challenges all current models of the formation and mass distribution of compact-object binaries.**

★ **Asymmetric** black hole progenitors (mass ratio  $q < 0.25$ )



# Subsolar Black Holes - The Smoking Gun!

- ★ Recent reanalysis of LIGO data by *Phukon et al.* '21 with updated merger rates and low mass ratios:

| FAR [ $\text{yr}^{-1}$ ] | $\ln \mathcal{L}$ | UTC time            | mass 1 [ $M_{\odot}$ ] | mass 2 [ $M_{\odot}$ ] |
|--------------------------|-------------------|---------------------|------------------------|------------------------|
| 0.1674                   | 8.457             | 2017-03-15 15:51:30 | 3.062                  | 0.9281                 |
| 0.2193                   | 8.2               | 2017-07-10 17:52:43 | 2.106                  | 0.2759                 |
| 0.4134                   | 7.585             | 2017-04-01 01:43:34 | 4.897                  | 0.7795                 |
| 1.2148                   | 6.589             | 2017-03-08 07:07:18 | 2.257                  | 0.6997                 |

- ★ **Four subsolar candidates** with  $\text{SNR} > 8$  and a  $\text{FAR} < 2 \text{ yr}^{-1}$
- ★ Note that an **order-one dark matter fraction** of subsolar PBHs is still **possible!**



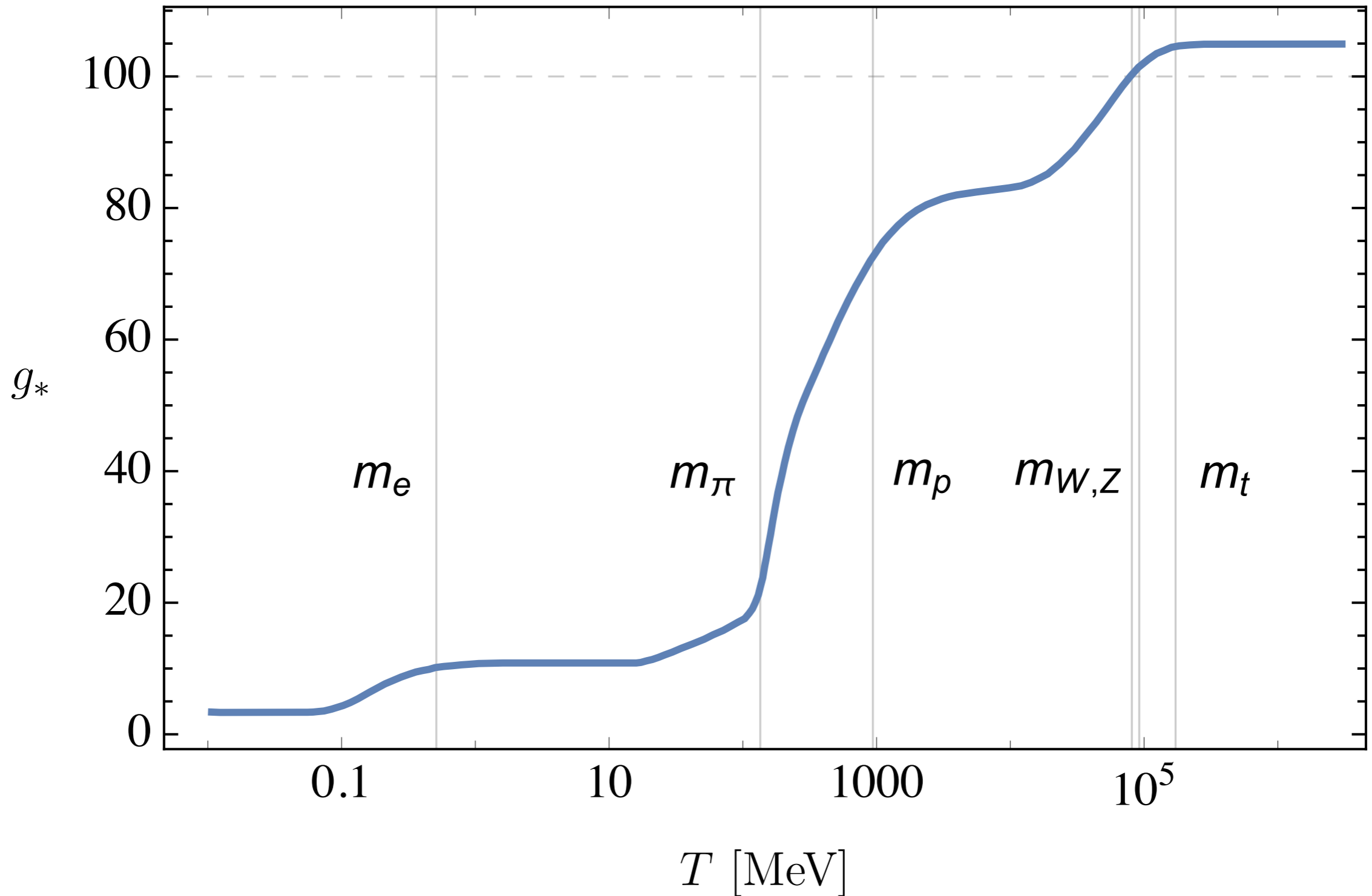
# *Further Support / Evidence for PBHs*

- ★ High-redshift quasars (up to  $10^8 M_{\odot}$  at  $z = 13$ )
- ★ Fast radio bursts
- ★ Missing-pulsar problem
- ★ Excess of lenses in Galactic bulge
- ★ Clumping of dark matter
- ★ ...

# *Thermal History*

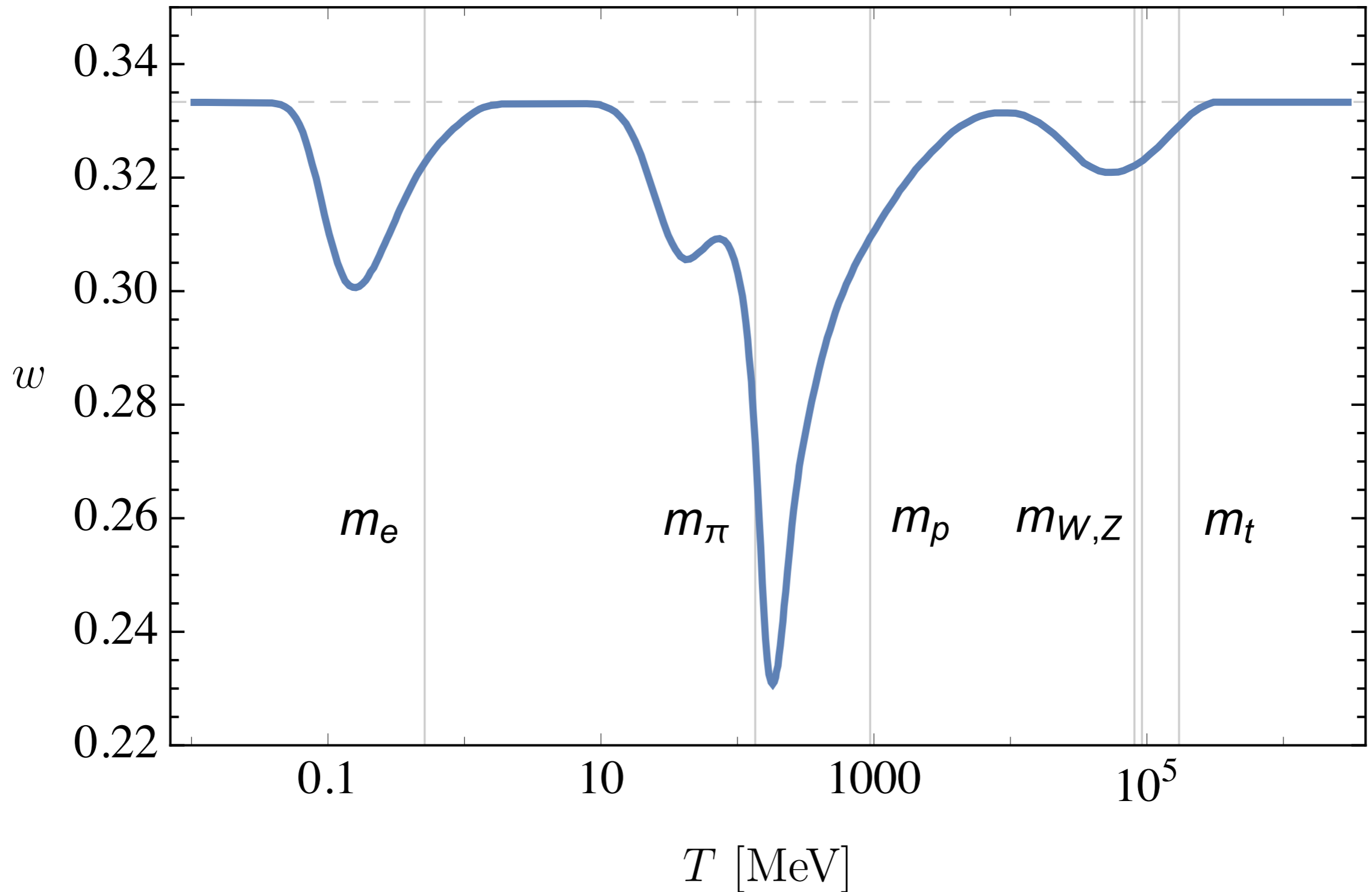
# Thermal History of the Universe

★ Changes in the relativistic degrees of freedom:



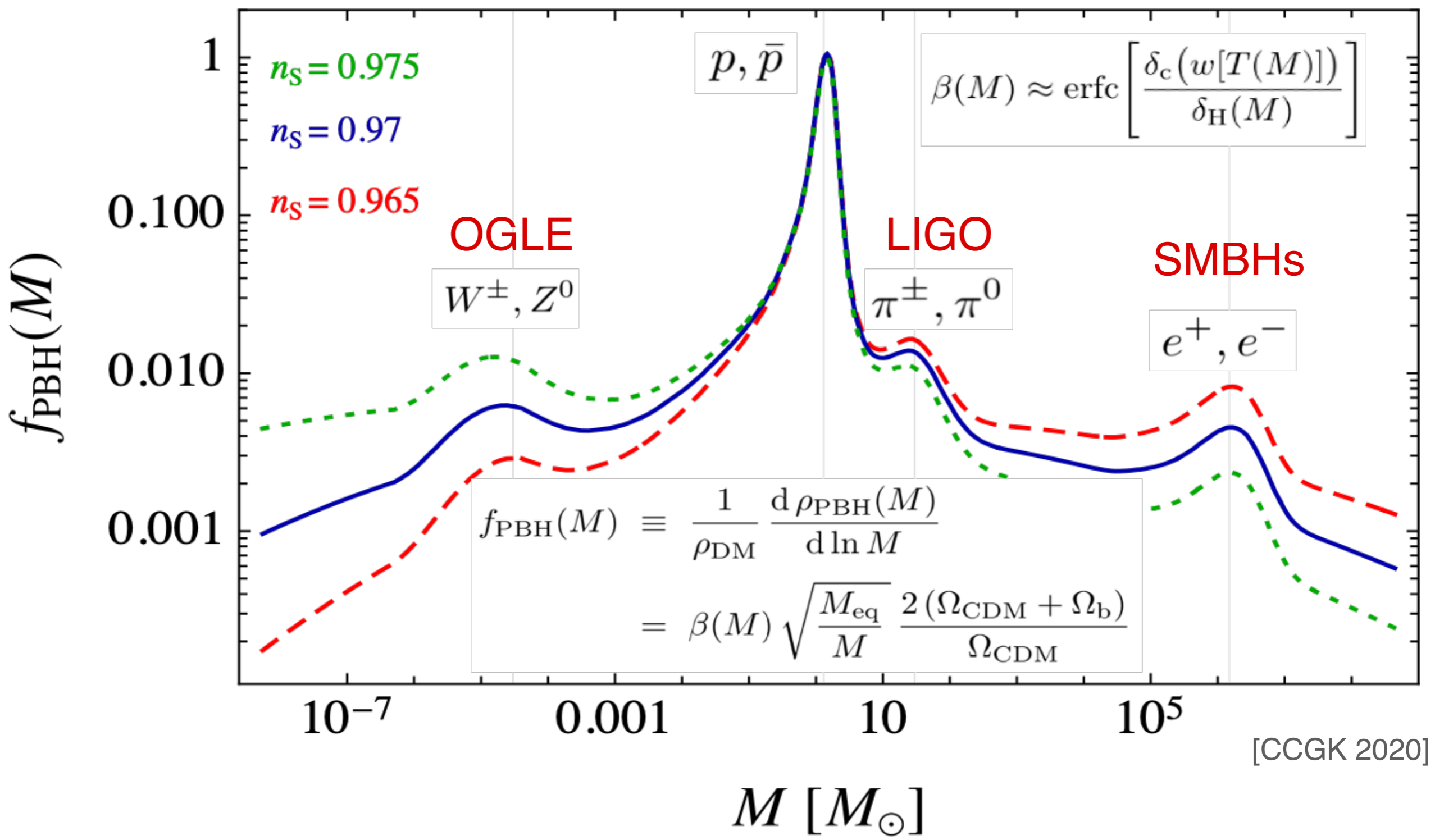
# *Thermal History of the Universe*

★ Changes in the **equation-of-state parameter**  $w = p/\rho$ :



# Thermal History of the Universe

★ An essentially **featureless power spectrum** leads to:



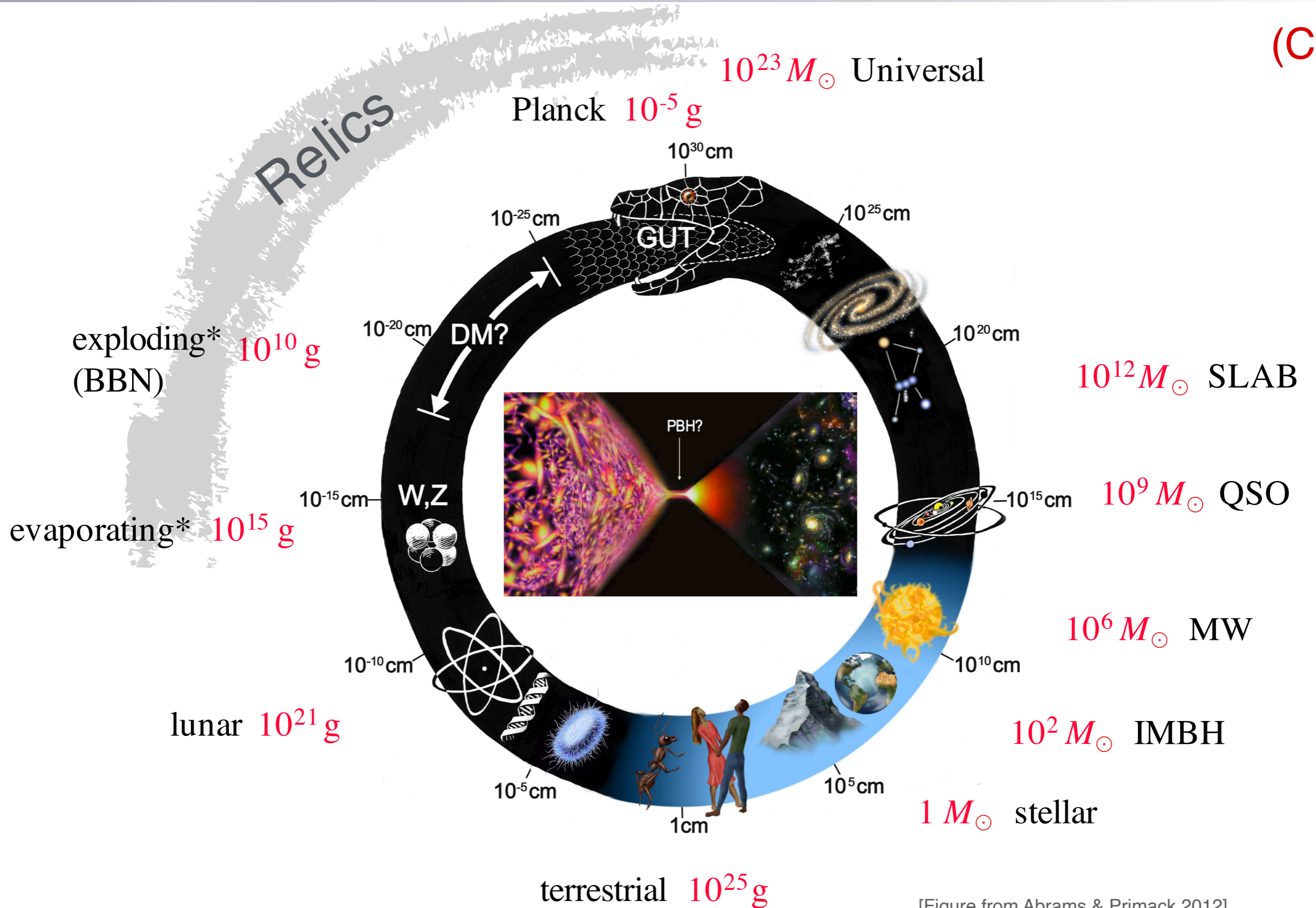
*Conclusion*

# Conclusion

- ★ Primordial black holes influence physics on many different scales, and manifest themselves via a plethora of different signatures.
- ★ At present, they are *not* tightly constraint in general and can easily constitute 100% of the dark matter, even in several mass ranges.
- ★ There are many hints for their existence from OGLE and other microlensing surveys, LIGO/Virgo gravitational-wave events etc.
- ★ The thermal history of the Universe naturally provides peaks in the PBH mass function at several relevant scales.

# Black Holes as a Link between Micro and Macro Physics

(Carr)



[Figure from Abrams & Primack 2012]





*Addendum*



*Primordial Black Holes  
from Confinement*

*work with Dvali & Zantedeschi*

# *Important Issues*

- ★ The standard approach of PBH formation has **two main issues**:
  - ★ In order to have a given percentage of PBH dark matter requires **exponential fine-tuning**.
  - ★ PBH formation happens in the **strong-coupling regime**.

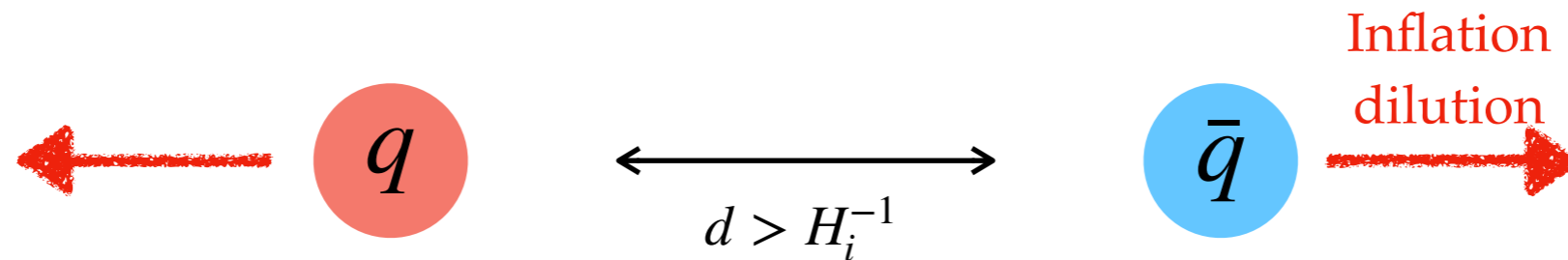
# *A New Approach*

★ We propose a novel PBH formation mechanism which is

- ★ assumption-minimal,
- ★ free of exponential fine-tuning,
- ★ avoids strong coupling,
- ★ works with standard QCD\*,
- ★ compatible with observations.

# Confinement Formation Mechanism

★ **1. Ingredient:** de Sitter fluctuations produce quarks during inflation.



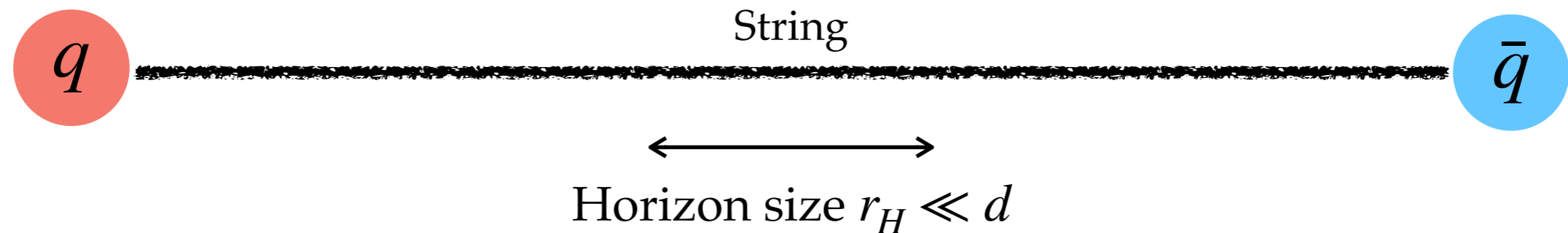
★ Focus on a simple pair case.

★ Distance grows as  $d \propto e^{N_e}$ .

★ Quarks quickly move out of causal contact.

# Confinement Formation Mechanism

★ 2. Ingredient: **Confinement** at energy scale  $\Lambda_c$ ,  $M_q/\Lambda_c \gg 1$



★ Flux tubes form connecting quark/anti-quark pairs.

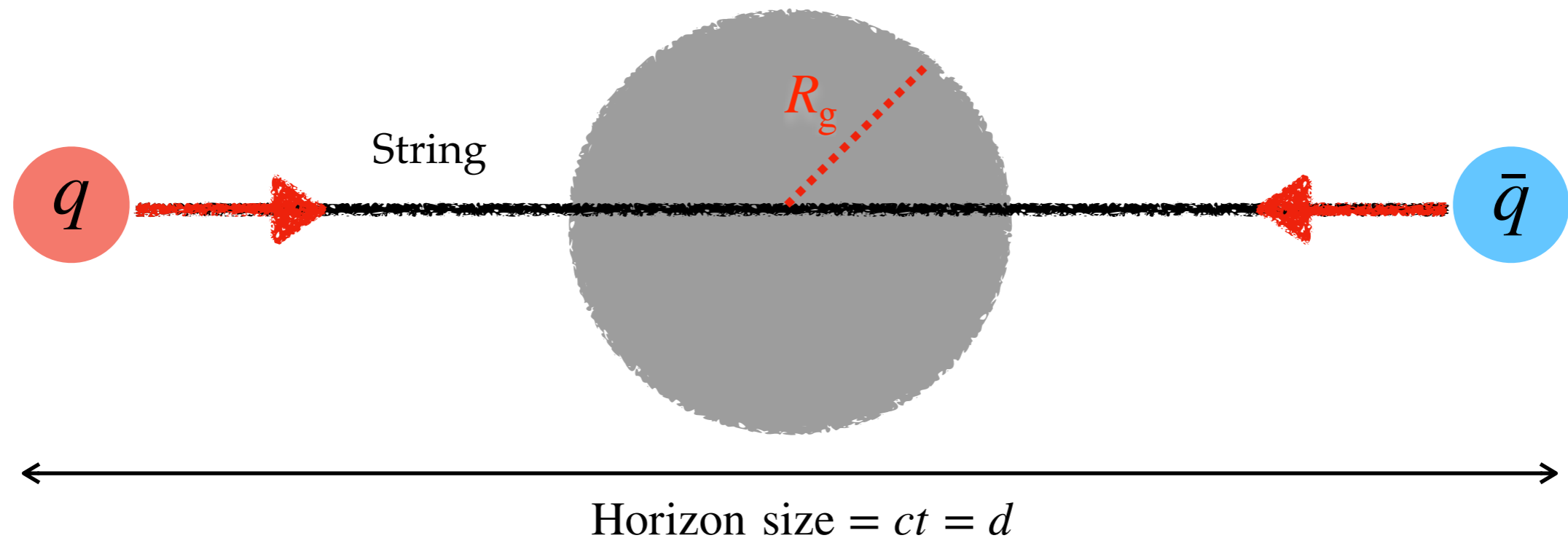
★ The system cannot collapse as long as  $d > r_H$ .

★ String breaking into quarks pair,  $P_{\text{tunnel}} \propto e^{-\pi \left( M_q/\Lambda_c \right)^2}$ ,  
suppressed as long as  $M_q/\Lambda_c \gg 1$ .



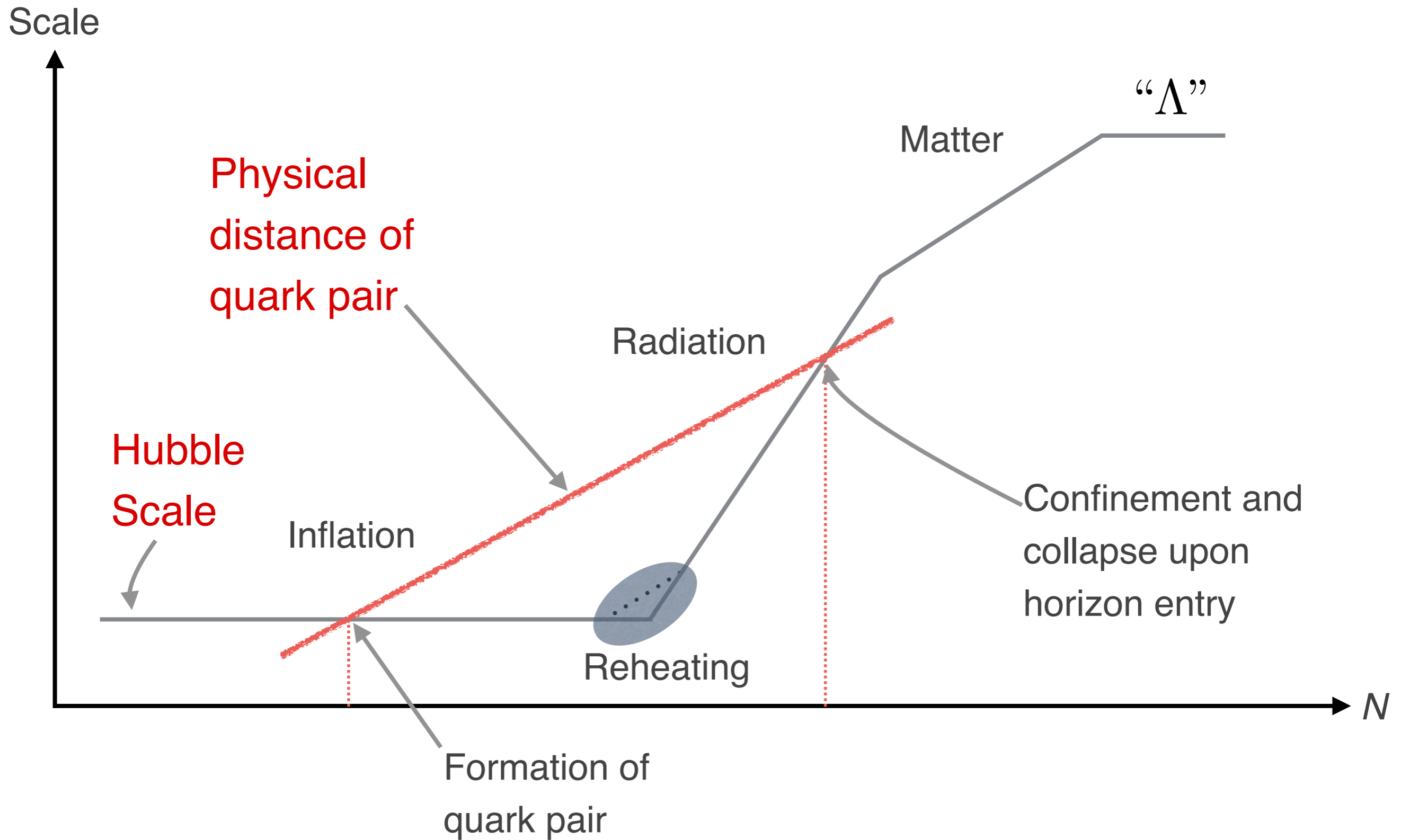
# Confinement Formation Mechanism

★ **3. Ingredient:** **Black hole formation** upon horizon entry



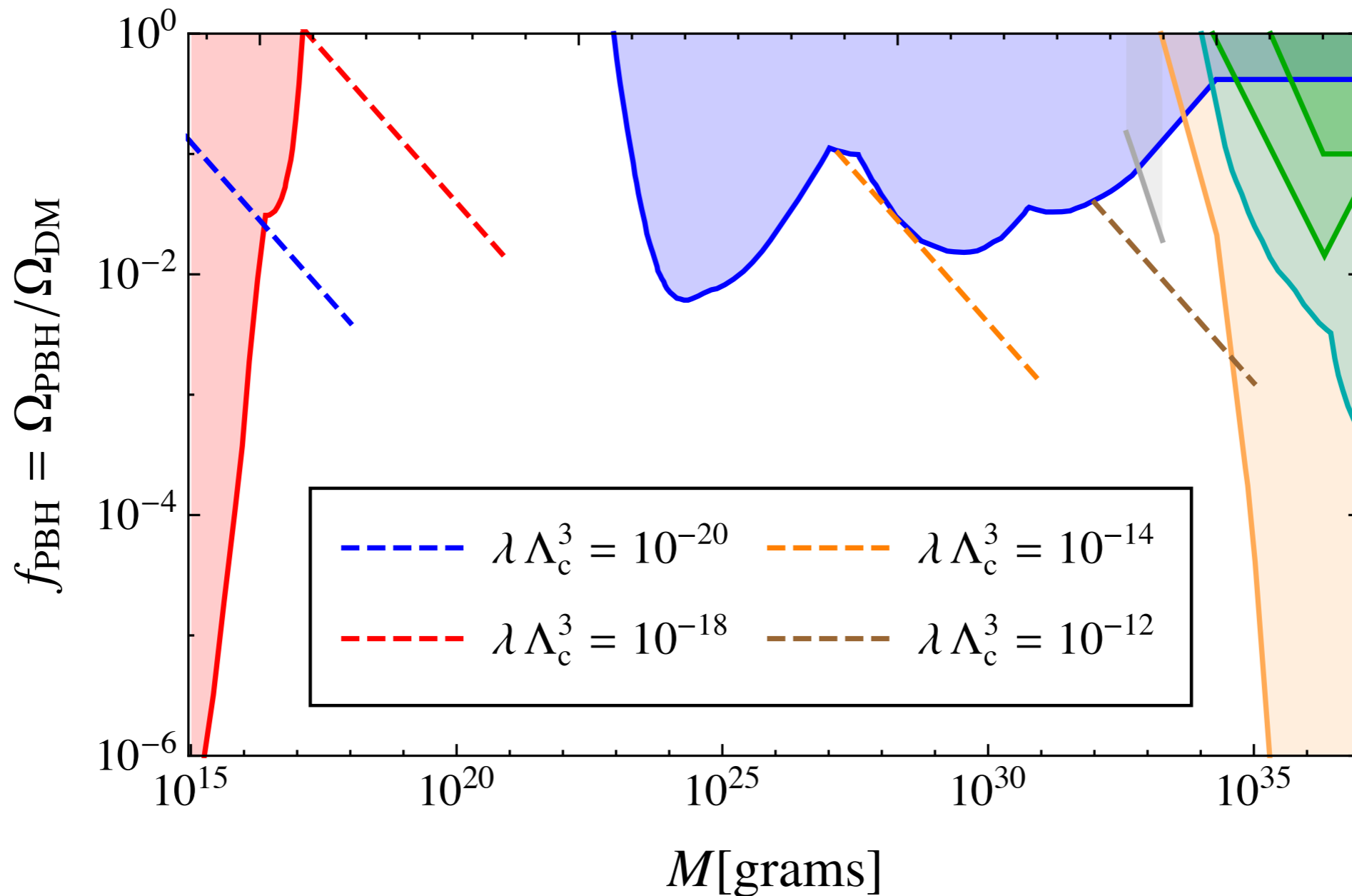
- ★ Acceleration of the quarks  $a = \Lambda_c^2/m_q$  quickly leads to their ultra-relativistic motion.
- ★ The energy stored in the string is  $E \simeq \Lambda_c^2 t \simeq M_g$ ,  $R_g \gg \Lambda_c^{-1}$ .
- ★ PBHs from inflationary overdensities are heavier by a factor  $\sim \Lambda_c^2$ .

# Formation Scales



# Dark Matter from Confinement

★ Present-day **dark matter distribution** vs *monochromatic* constraints:

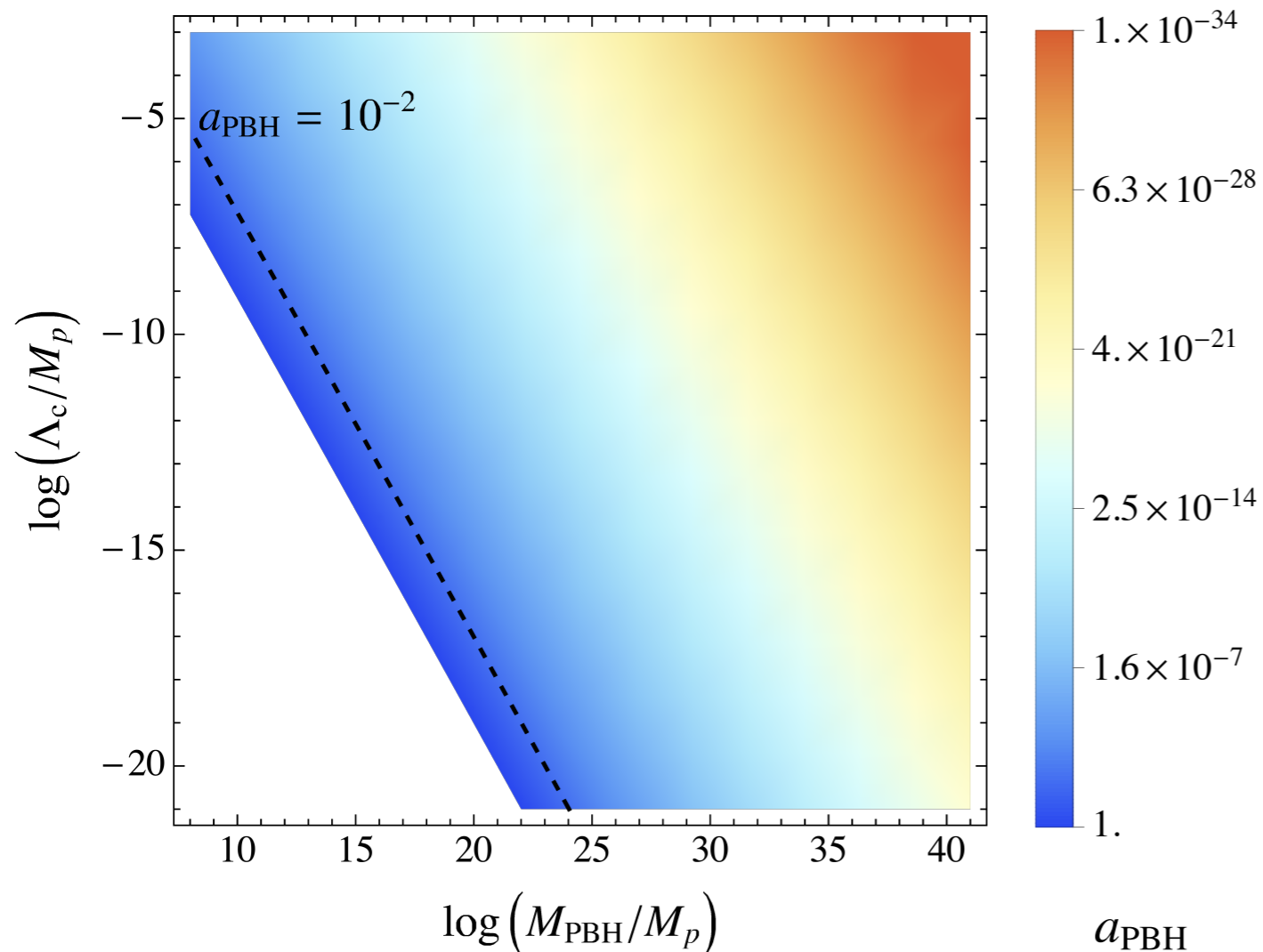
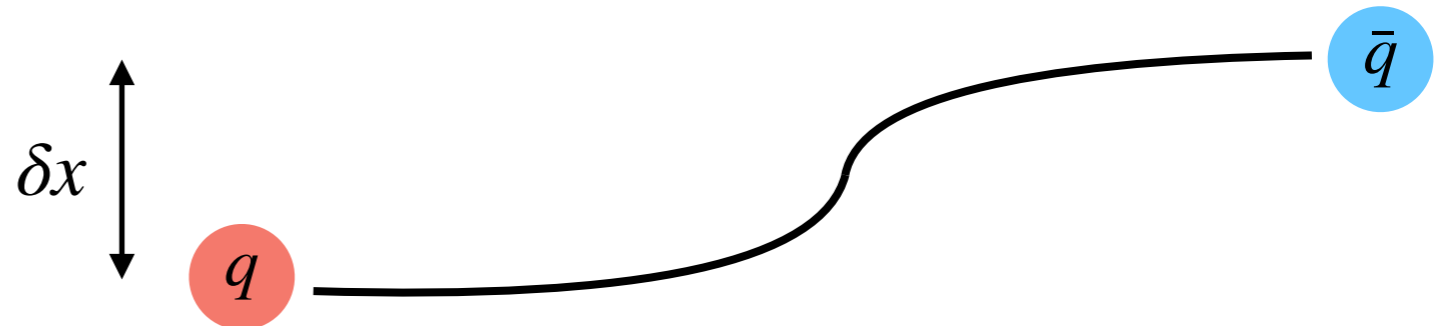


★ Find: 
$$f_{\text{PBH}} \equiv \frac{\rho_{\text{PBH}}(t)}{\rho_{\text{CDM}}(t)} = \frac{32\pi}{3} \lambda \Lambda_c^3 \left( \frac{M_{\text{PBH}}}{M_{\text{eq}}} \right)^{-1/2}$$

# High-Spin Subsolar PBHs

- ★ During inflation, the string undergoes a **Brownian motion**, induced by de Sitter quantum fluctuations, leading to **deviation from straightness**:

$$\delta x \simeq \sqrt{N_e} H_i^{-1}$$

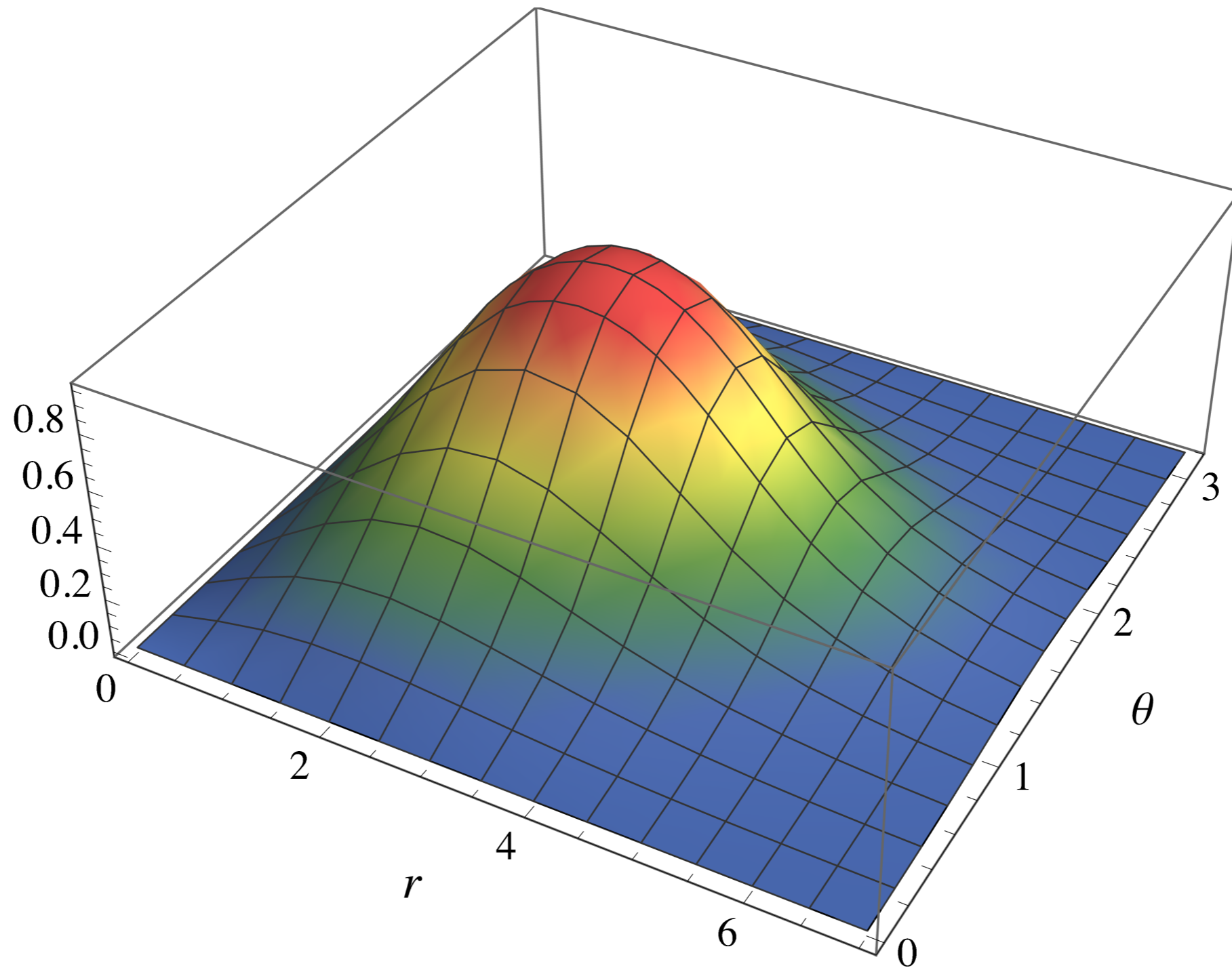


- ★ This leads to potentially **significant spin**:

$$a_{\text{PBH}} \simeq \frac{\delta x}{R_g}$$

$$\simeq \frac{1}{H M_{\text{PBH}}} \log \left( \frac{H M_{\text{PBH}}}{\Lambda_c^2} \right)^{1/2}$$

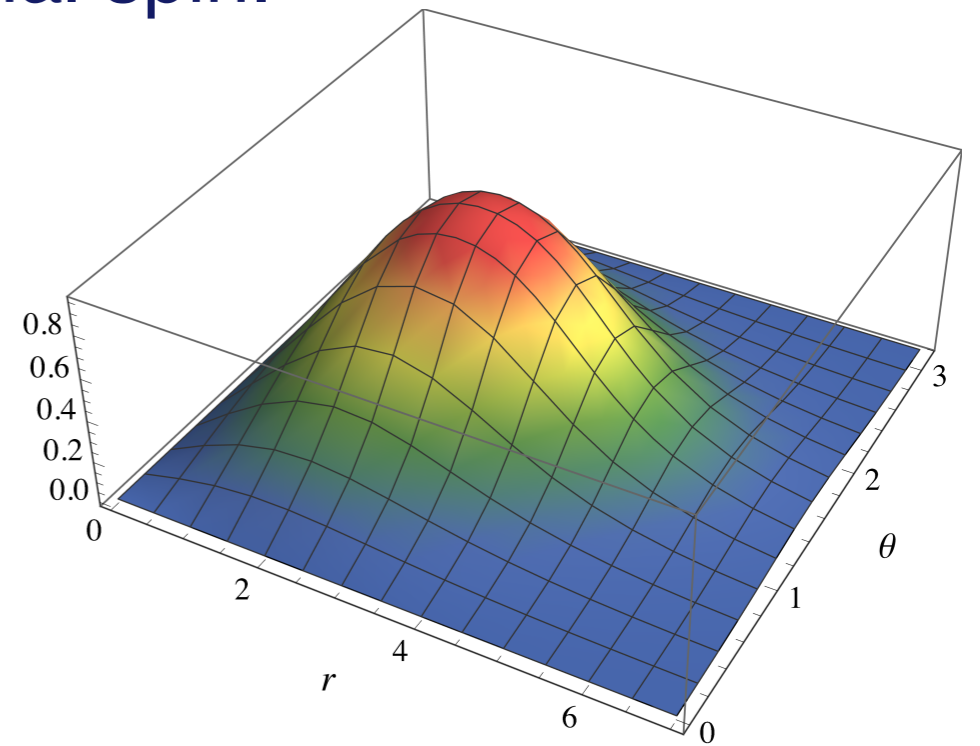
# *Formation of Vortices*



[Dvali, Kühnel, Zantedeschi 2021]

# Formation of Vortices

- ★ Black Holes can be understood as **saturons** (see talk by Dvali)
- ★ We showed that these admit **vortex structure** (see talk by Zantedeschi), in the case of near-extremal spin.
- ★ PBHs from confinement could provide **ideal prerequisites for vortex formation** due to highly spinning light PBHs.
- ★ If these PBHs provide the dark matter, their vorticity might explain **primordial magnetic fields**.
- ★ Besides, vorticity provides a **topological meaning to the stability of extremal black holes**.



[Dvali, Kühnel, Zantedeschi 2021]

*More on Constraints*

# Constraints — A Worthwhile Remark

★ These constraints are not just nails in a coffin!

(Carr)



★ All constraints have caveats and might change.

★ Each constraint is a potential signature.

★ PBHs are important even if  $f_{\text{PBH}} \ll 1$ .



# *Constraints — Uncertainties*

★ May constraints rely on rather on **uncertain, restrictive, simplistic** or even **incorrect assumptions!**

➔ We have to understand better:

- ★ Galactic dark-matter profile
- ★ Clustering
- ★ Accretion
- ★ Characteristics of the lensed sources (size, variability, ...)
- ★ Composition of "probes" in general
- ★ Velocity distribution
- ★ Hawking radiation
- ★ ...

# Constraints — Uncertainties on Hawking Radiation

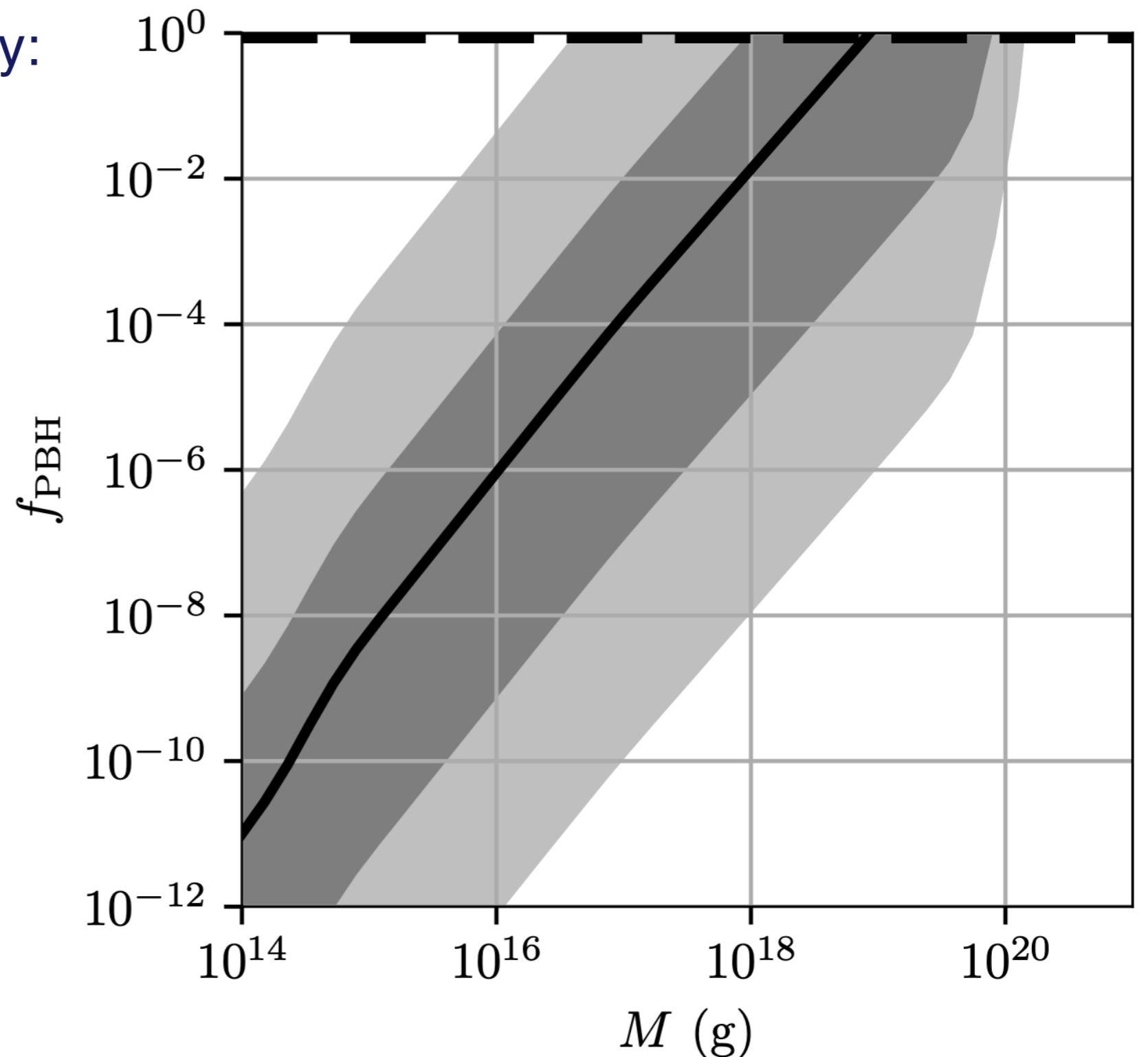
★ Uncertainties induced by:

★ instrument characteristics

★ computation of the (extra)galactic photon fluxes

★ statistical treatment

★ computation of the Hawking radiation

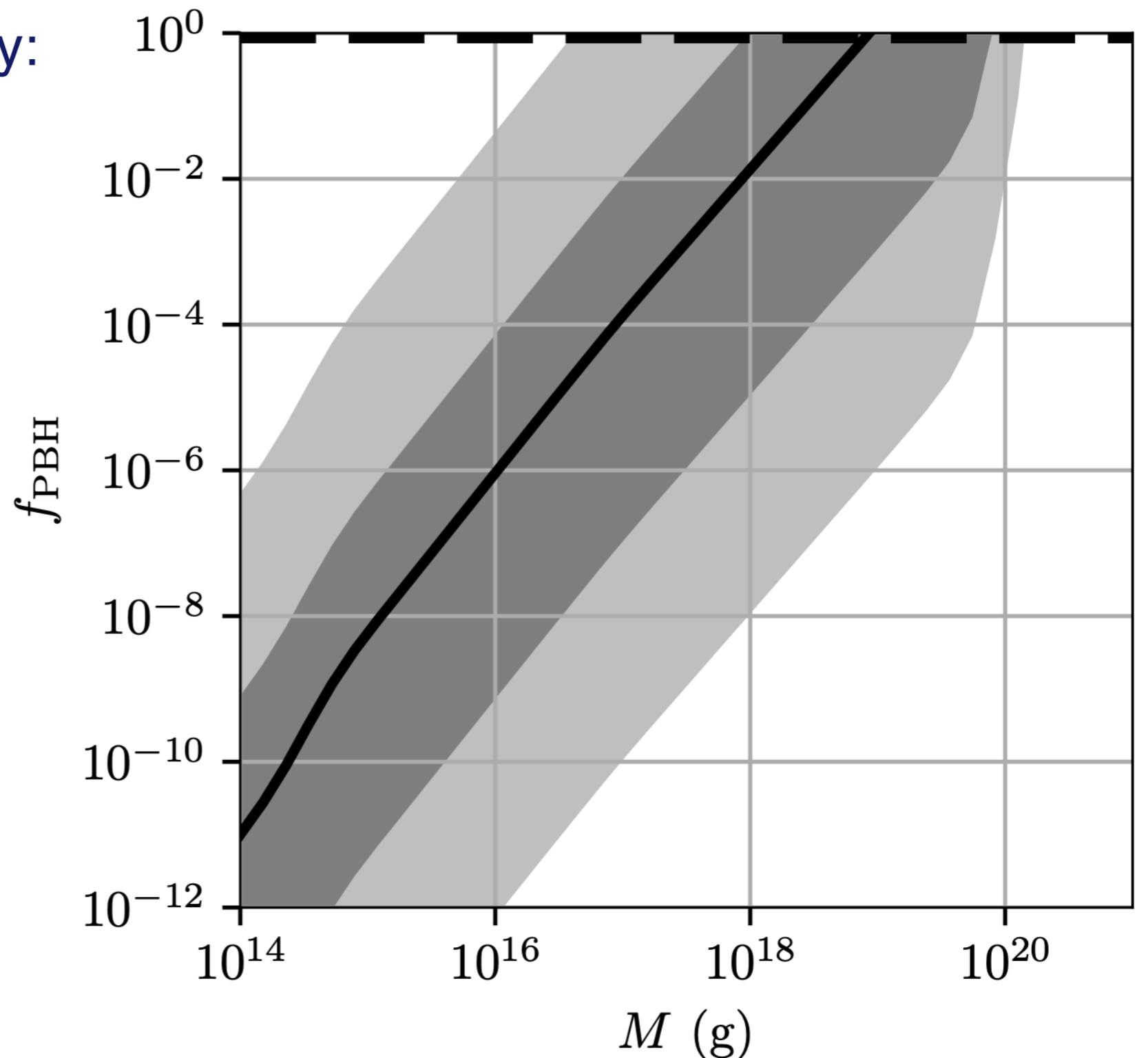


# Constraints — Uncertainties on Hawking Radiation

★ Uncertainties induced by:

- ★ instrument characteristics
- ★ computation of the (extra)galactic photon fluxes
- ★ statistical treatment
- ★ computation of the Hawking radiation

These constraints might even *entirely disappear*, due to quantum back-reaction!



[Auffinger 2022]

(see work by Dvali *et al.*)

*Micro & Macro*

# *PBH @ Particle Dark Matter*

★ Always when  $f_{\text{PBH}} < 1$  there **must** be another DM component!

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# PBH @ Particle Dark Matter

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  - ★ Study **WIMP annihilations** in PBH halos:
    - ★ The annihilation rate  $\Gamma \propto n^2$ .
    - ★ Halo profile does matter; **enhancement** of  $\Gamma$  in density spikes.
      - 1) Derive the **density profile** of the captured WIMPs;
      - 2) calculate the **annihilation rate**;
      - 3) and **compare to data**.

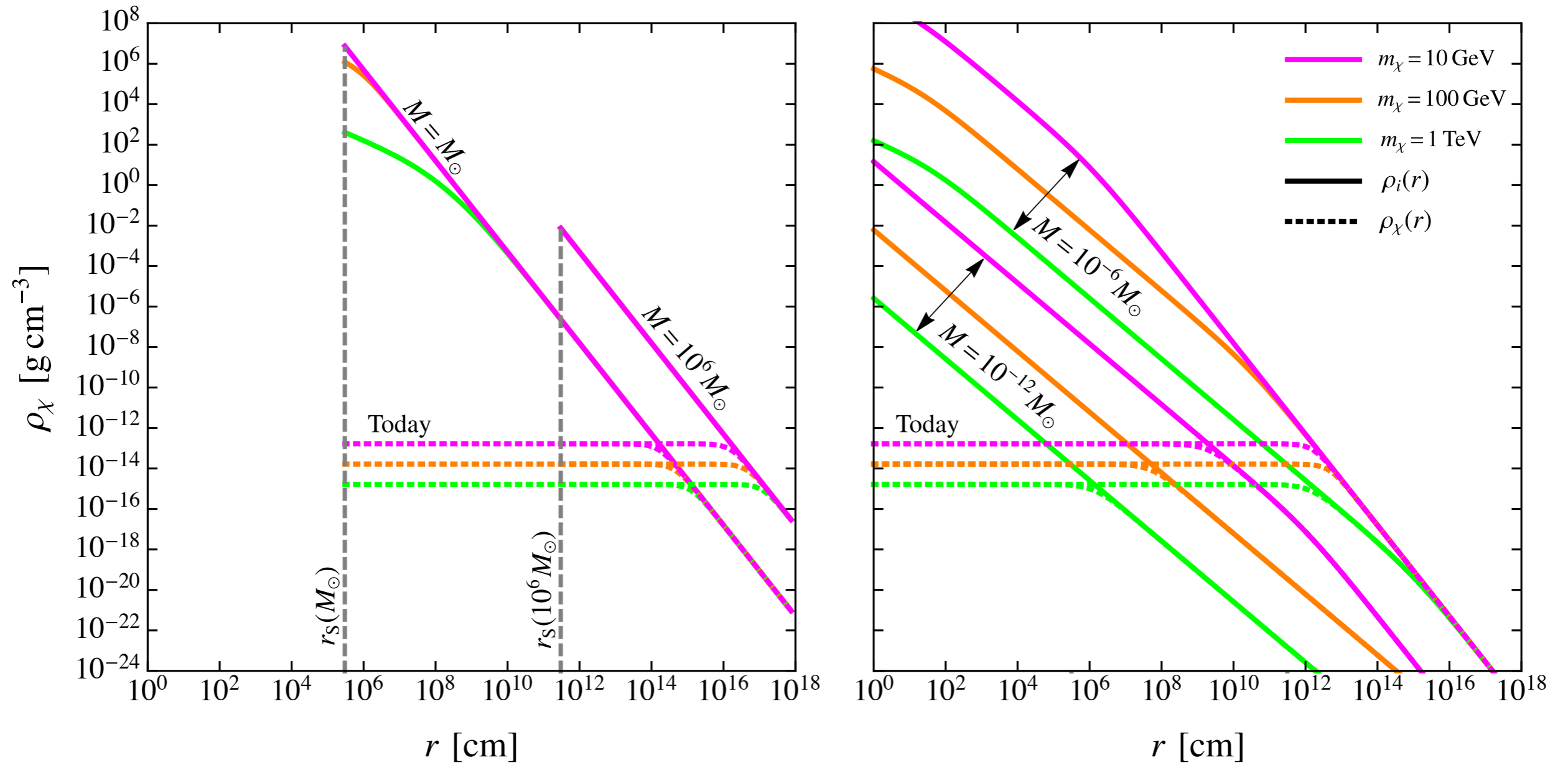
[Eroshenko 2016,

Boucenna *et al.* 2017,

Adamek *et al.* 2019,

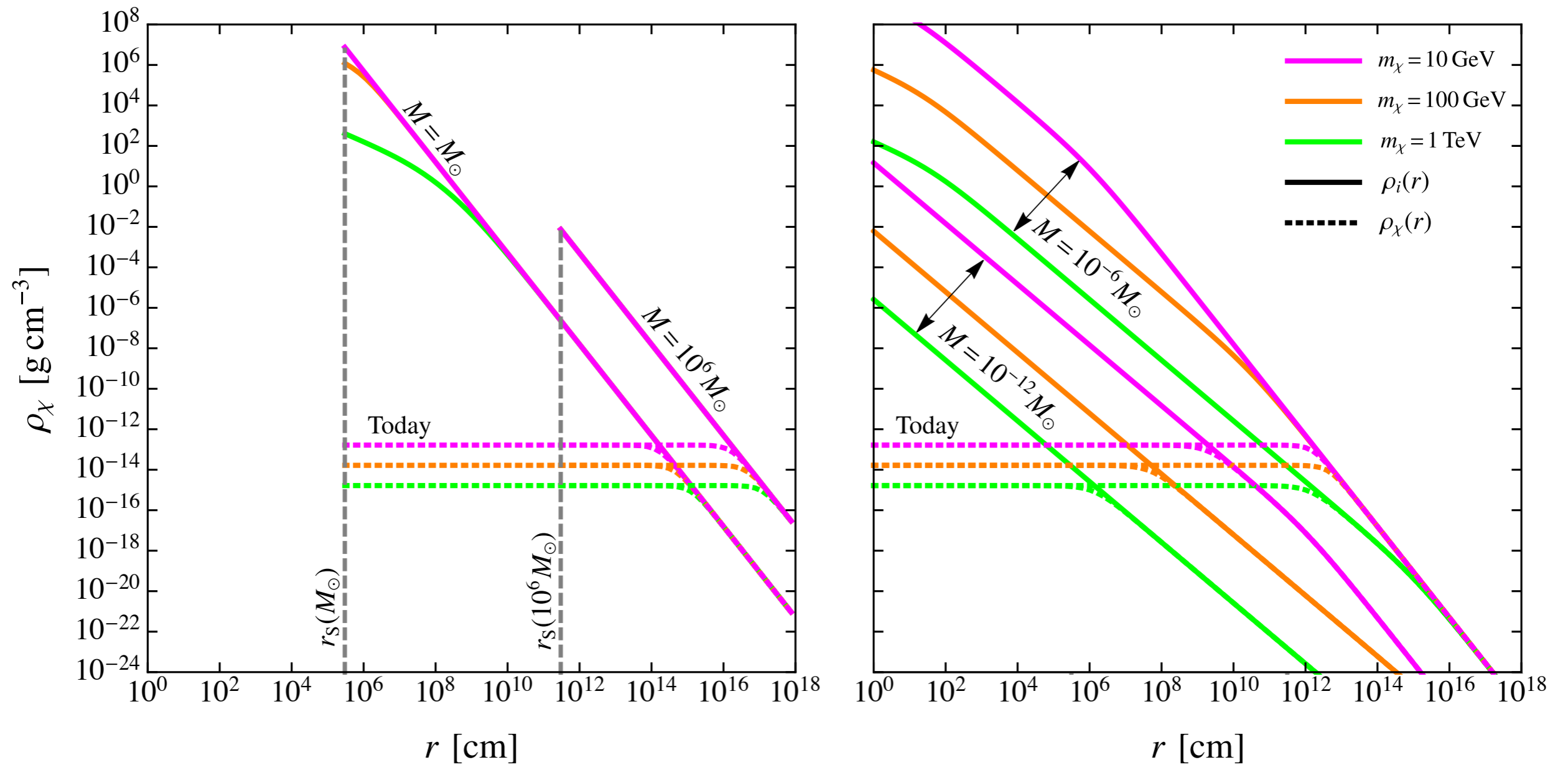
Carr, Kühnel, Visinelli 2020 & 2021]

# PBHs @ WIMPs



[Carr, Kühnel, Visinelli 2021]

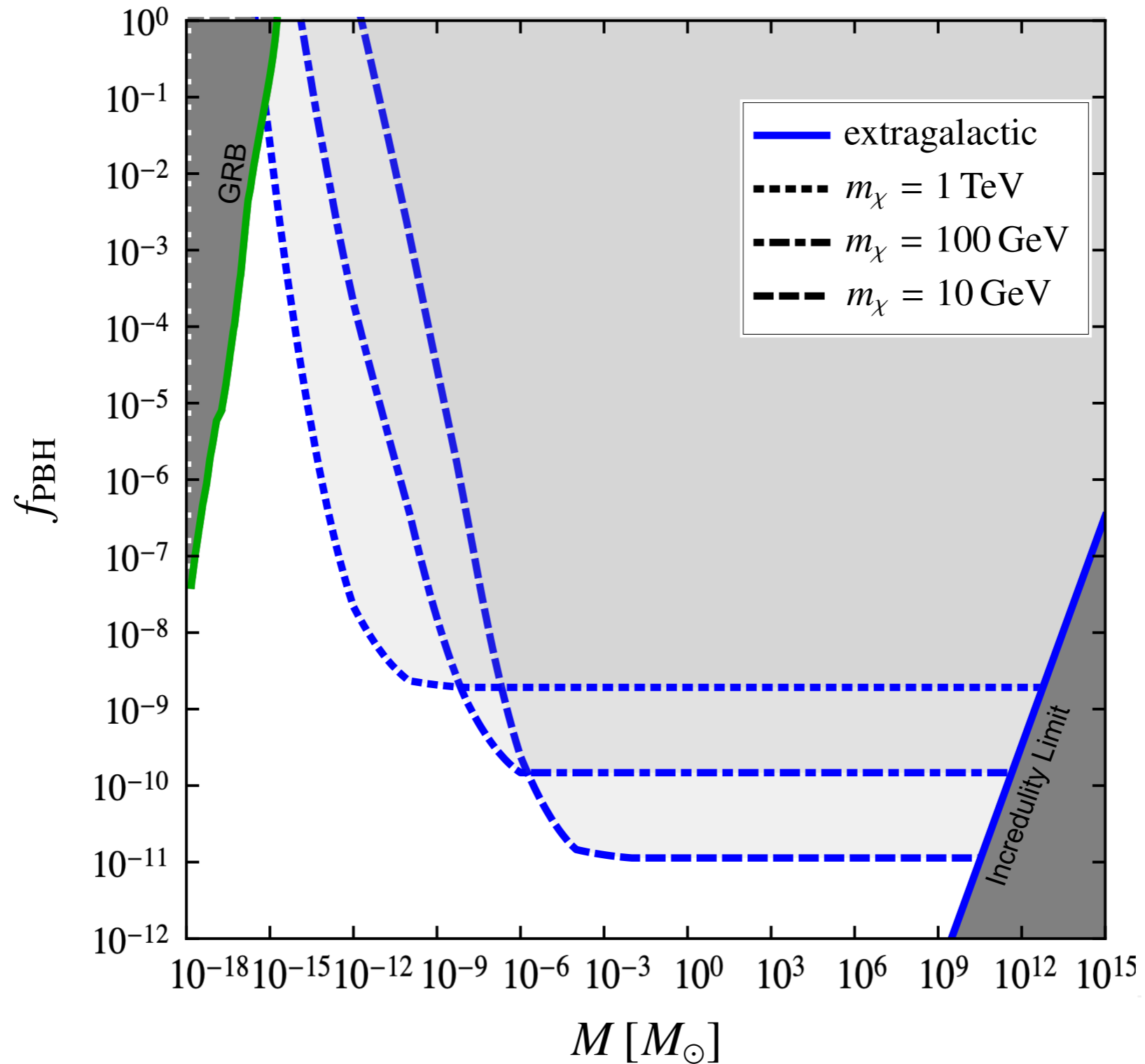
# PBHs @ WIMPs



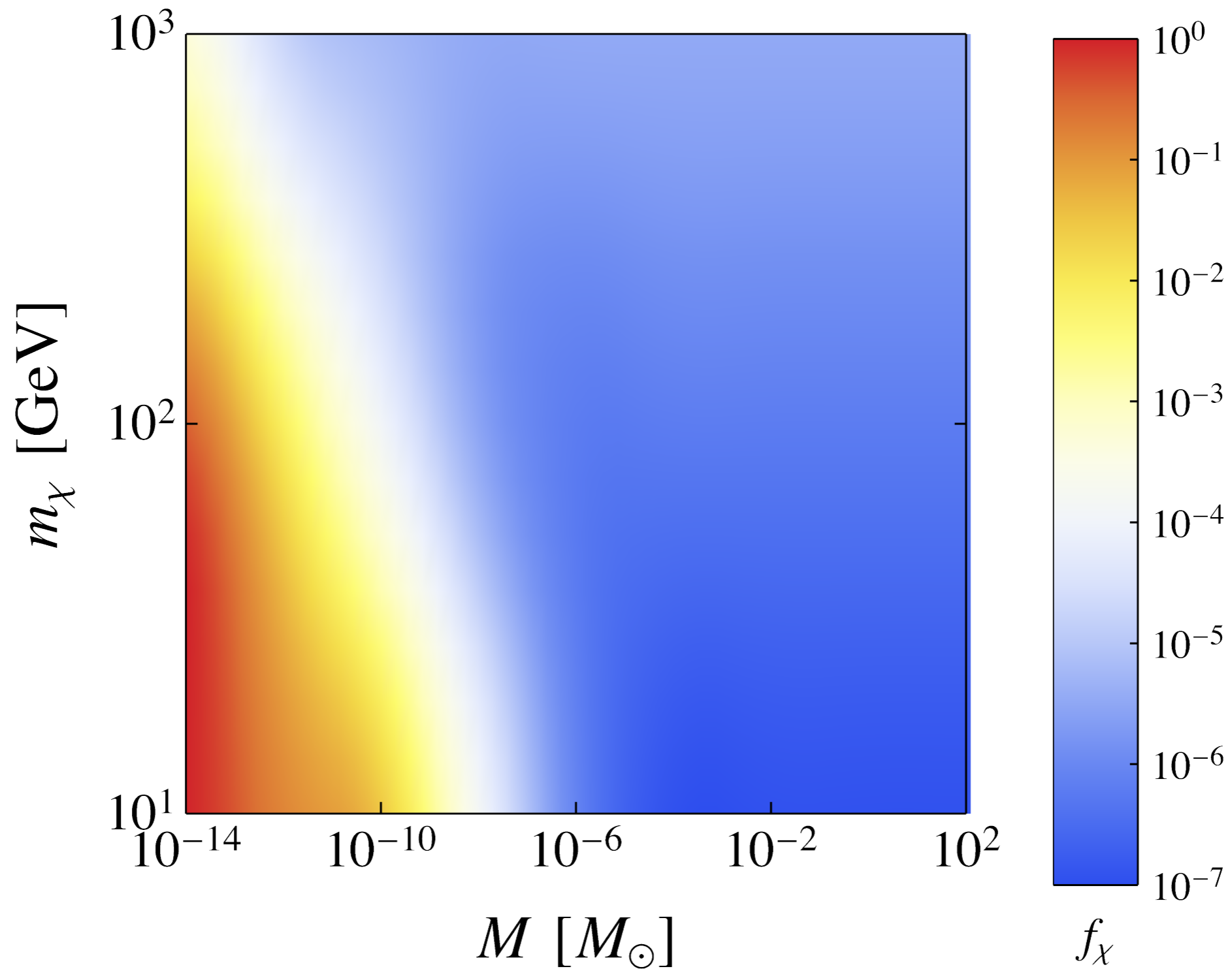
[Carr, Kühnel, Visinelli 2021]

★ **Annihilations** lead to **plateaux** in the present-day halos.

# PBHs @ WIMPs



# *PBHs @ WIMPs*



*Primordial Black Holes  
from Confinement*

# *Important Issues*

- ★ The standard approach of PBH formation has **two main issues**:
  - ★ In order to have a given percentage of PBH dark matter requires **exponential fine-tuning**.
  - ★ PBH formation happens in the **strong-coupling regime**.

# *A New Approach*

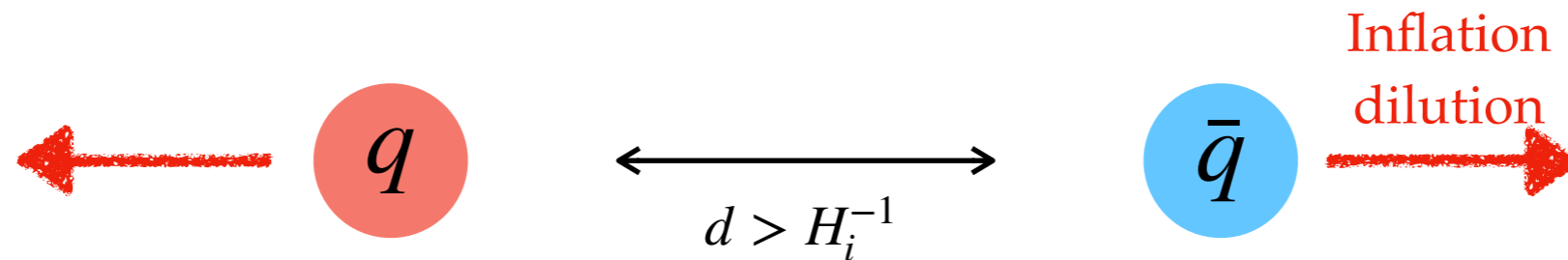
★ We propose a novel PBH formation mechanism which is

- ★ assumption-minimal,
- ★ free of exponential fine-tuning,
- ★ avoids strong coupling,
- ★ works with standard QCD\*,
- ★ compatible with observations.



# Confinement Formation Mechanism

★ **1. Ingredient:** de Sitter fluctuations produce quarks during inflation.



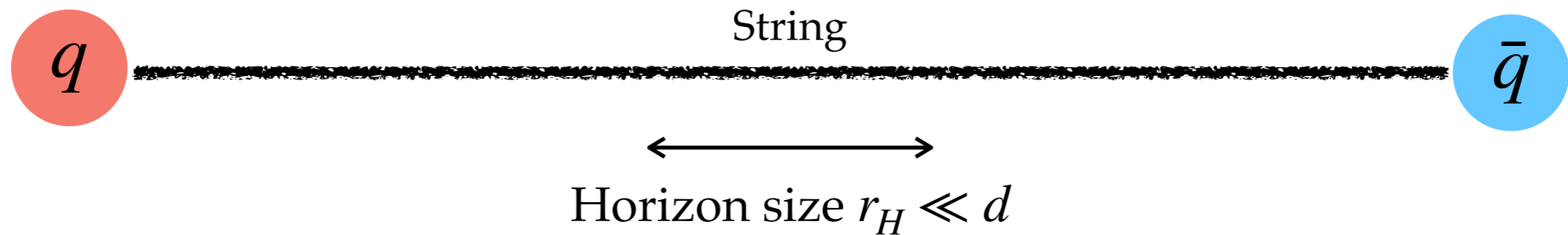
★ Focus on a simple pair case.

★ Distance grows as  $d \propto e^{N_e}$ .

★ Quarks quickly move out of causal contact.

# Confinement Formation Mechanism

★ 2. Ingredient: **Confinement** at energy scale  $\Lambda_c$ ,  $M_q/\Lambda_c \gg 1$



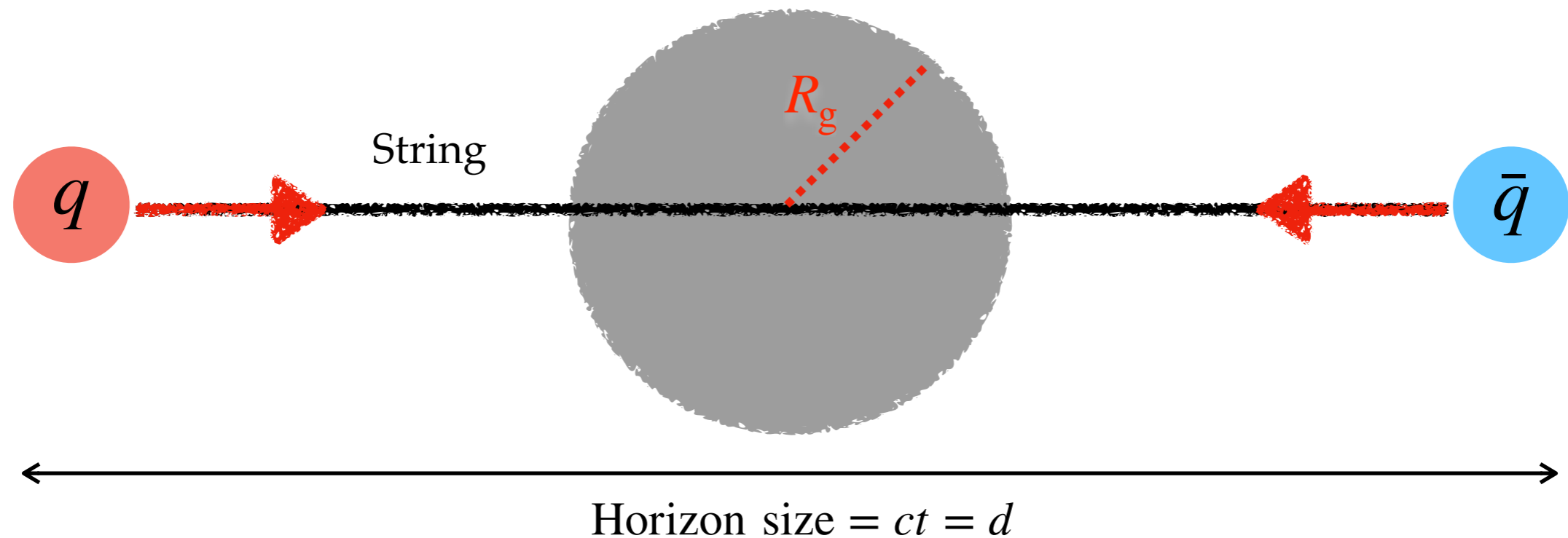
★ Flux tubes form connecting quark/anti-quark pairs.

★ The system cannot collapse as long as  $d > r_H$ .

★ String breaking into quarks pair,  $P_{\text{tunnel}} \propto e^{-\pi \left( M_q/\Lambda_c \right)^2}$ ,  
suppressed as long as  $M_q/\Lambda_c \gg 1$ .

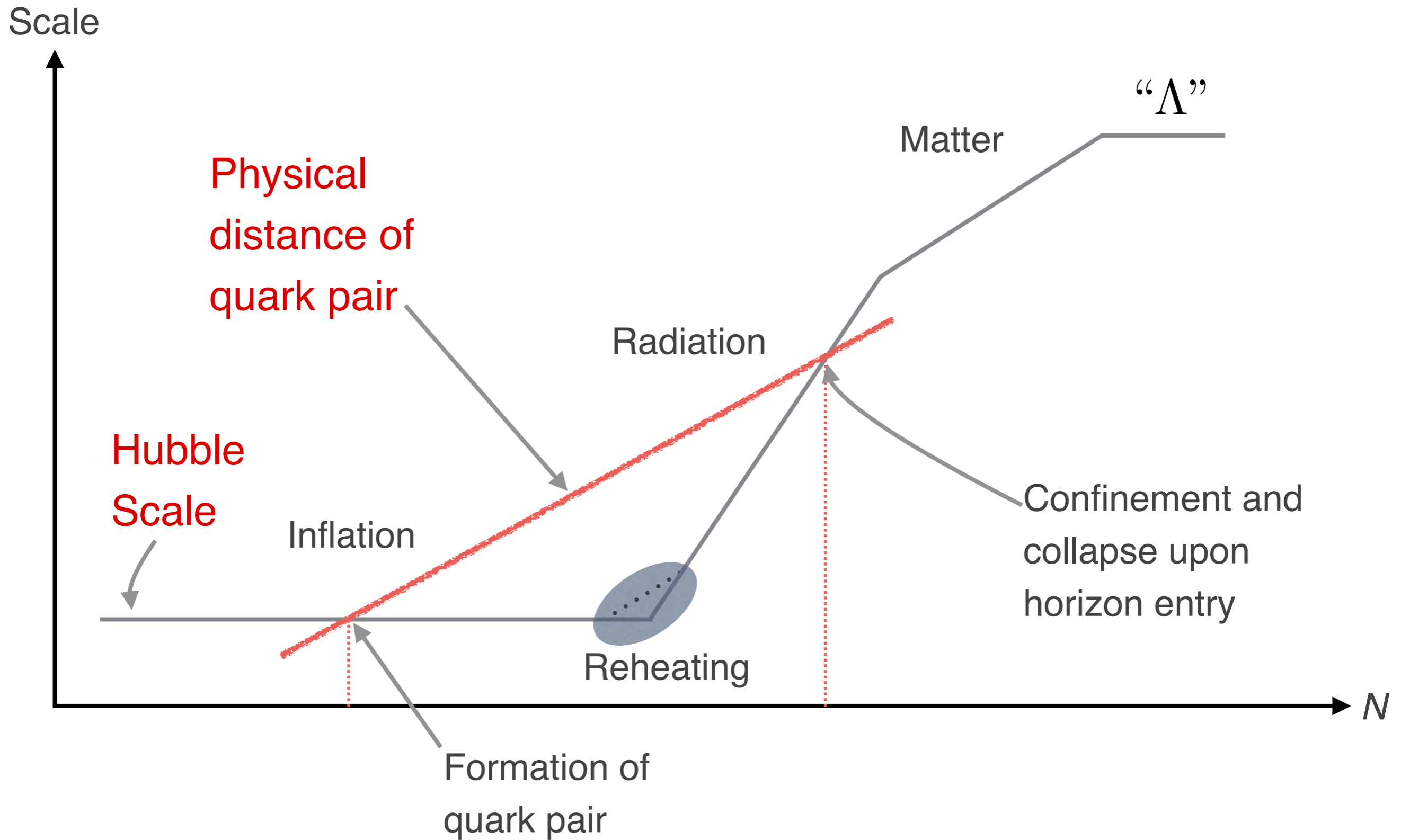
# Confinement Formation Mechanism

★ **3. Ingredient:** **Black hole formation** upon horizon entry



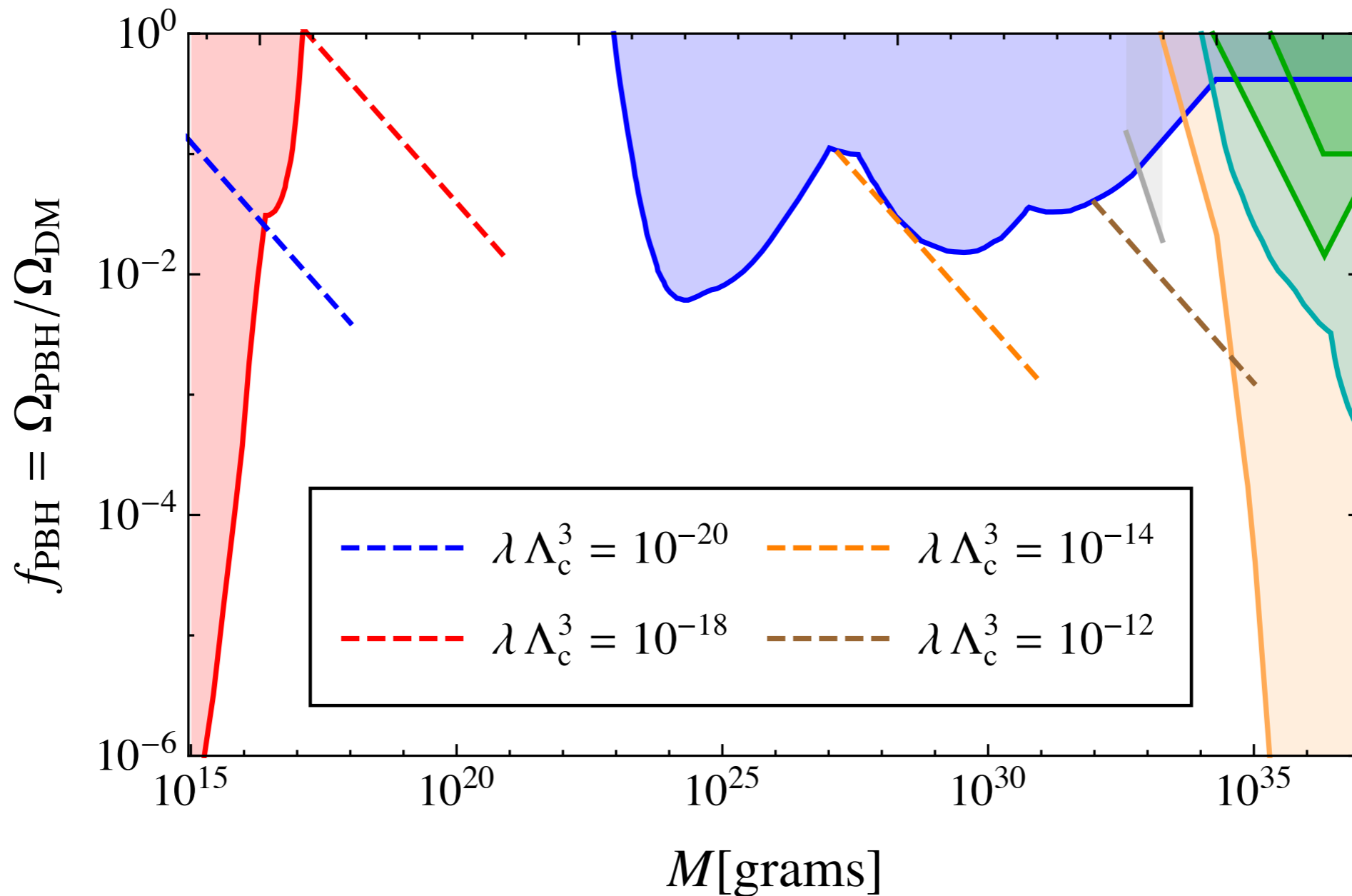
- ★ Acceleration of the quarks  $a = \Lambda_c^2/m_q$  quickly leads to their ultra-relativistic motion.
- ★ The energy stored in the string is  $E \simeq \Lambda_c^2 t \simeq M_g$ ,  $R_g \gg \Lambda_c^{-1}$ .
- ★ PBHs from inflationary overdensities are heavier by a factor  $\sim \Lambda_c^2$ .

# Formation Scales



# Dark Matter from Confinement

★ Present-day **dark matter distribution** vs *monochromatic* constraints:

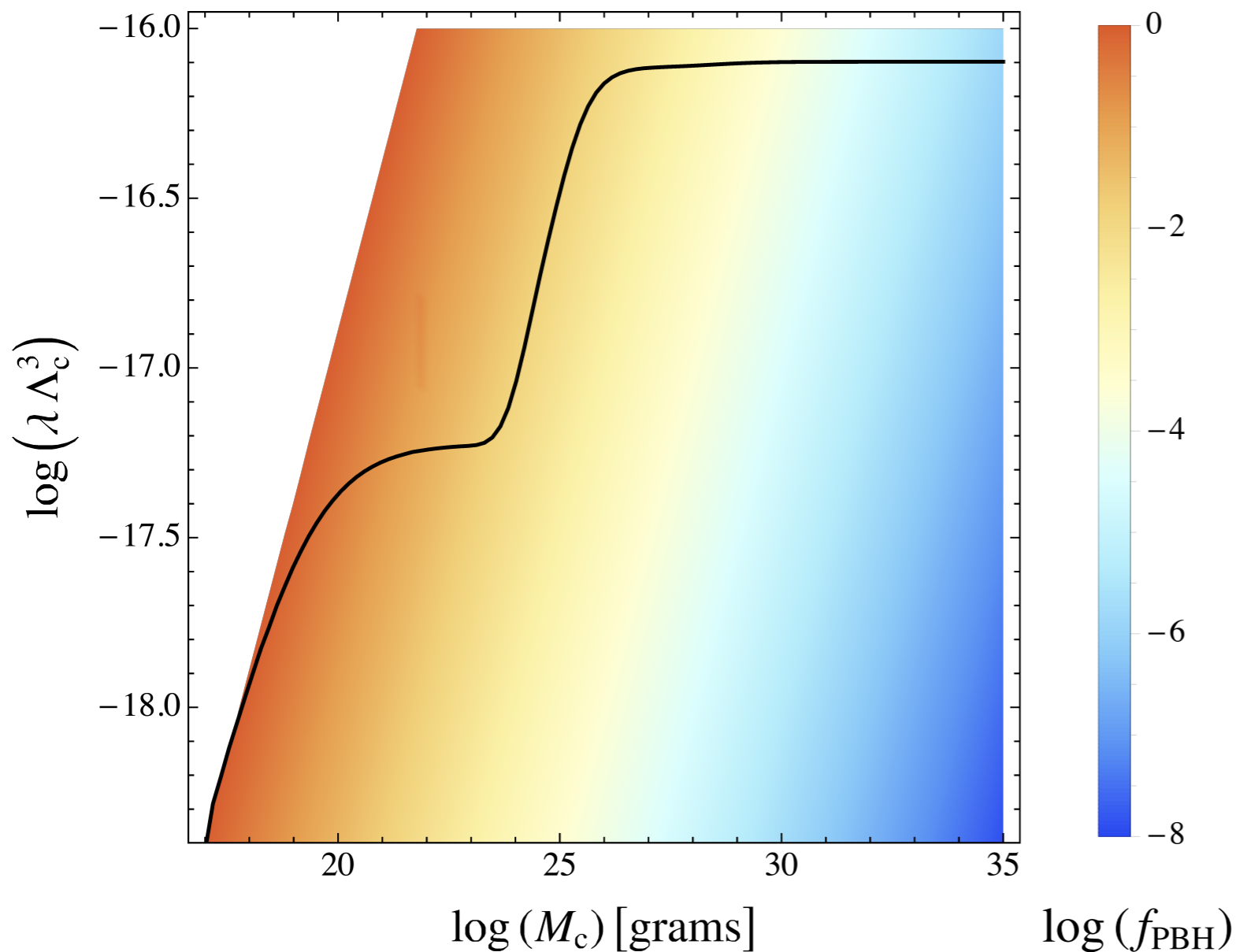


★ Find: 
$$f_{\text{PBH}} \equiv \frac{\rho_{\text{PBH}}(t)}{\rho_{\text{CDM}}(t)} = \frac{32\pi}{3} \lambda \Lambda_c^3 \left( \frac{M_{\text{PBH}}}{M_{\text{eq}}} \right)^{-1/2}$$

# Extended-Constraint Analysis

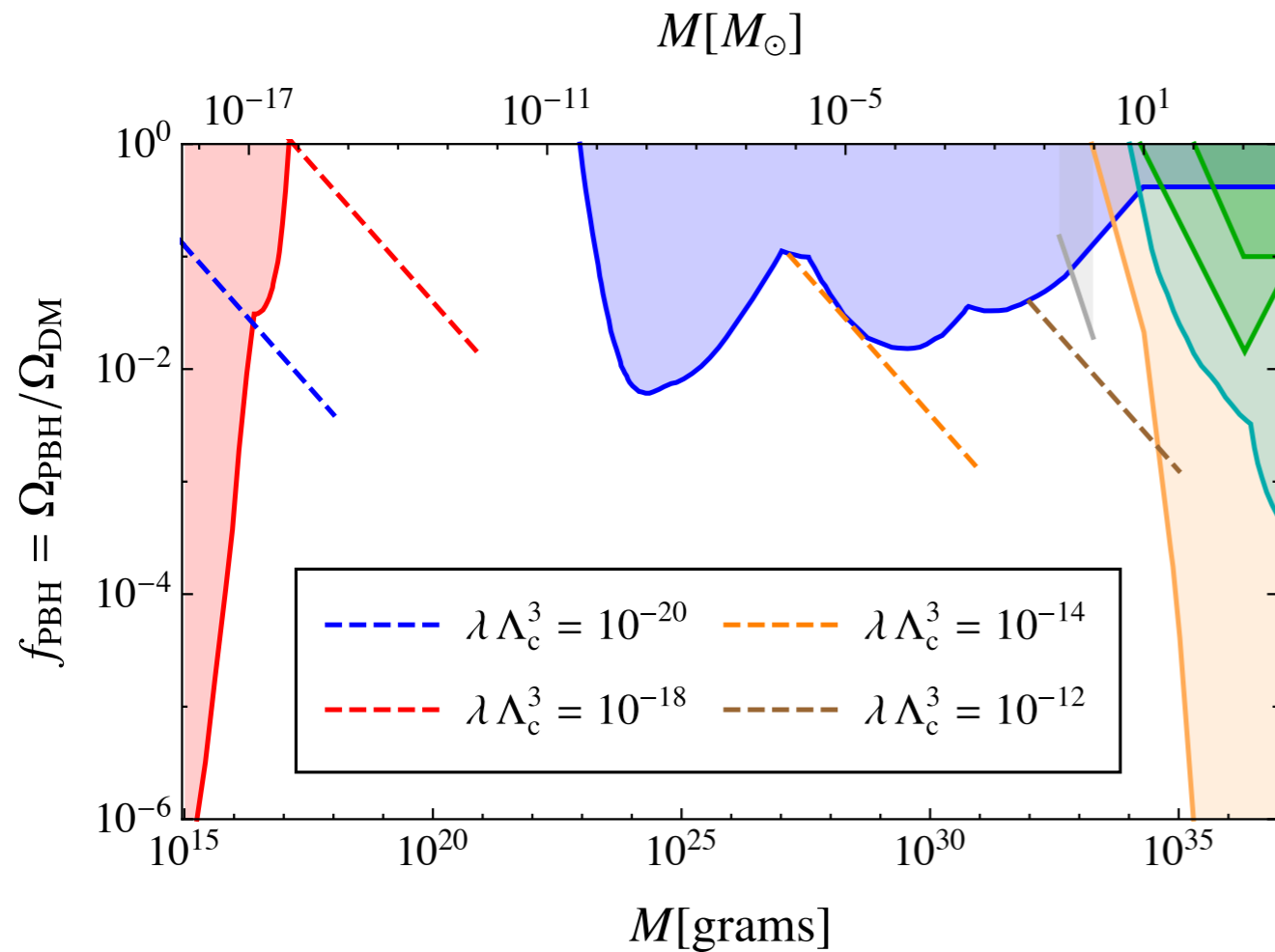
★ **Require:** 
$$\int_{M_1}^{M_2} d \ln M_{\text{PBH}} \frac{d f_{\text{PBH}}(M_{\text{PBH}})}{d \ln M_{\text{PBH}}} \frac{1}{f_{\text{max}}(M_{\text{PBH}})} \stackrel{!}{\leq} 1$$

[Carr *et al.* 2017]



- ★ **Full compatibility with observations below the black line, here, exemplary for  $M_{\text{peak}} \sim 10^{17}$  g.**
- ★ **Results: Possible to accommodate 100% of PBH dark matter...**
- ★ **... at the same time provide seeds for supermassive black holes in galactic centres.**

# Dark Matter



## Monochromatic spectrum

$$f_{\text{PBH}} \equiv \frac{\rho_{\text{PBH}}(t)}{\rho_{\text{CDM}}(t)} = \frac{32\pi}{3} \lambda \Lambda_c^3 \left( \frac{M_{\text{PBH}}}{M_{\text{eq}}} \right)^{-1/2}$$

## Extended spectrum

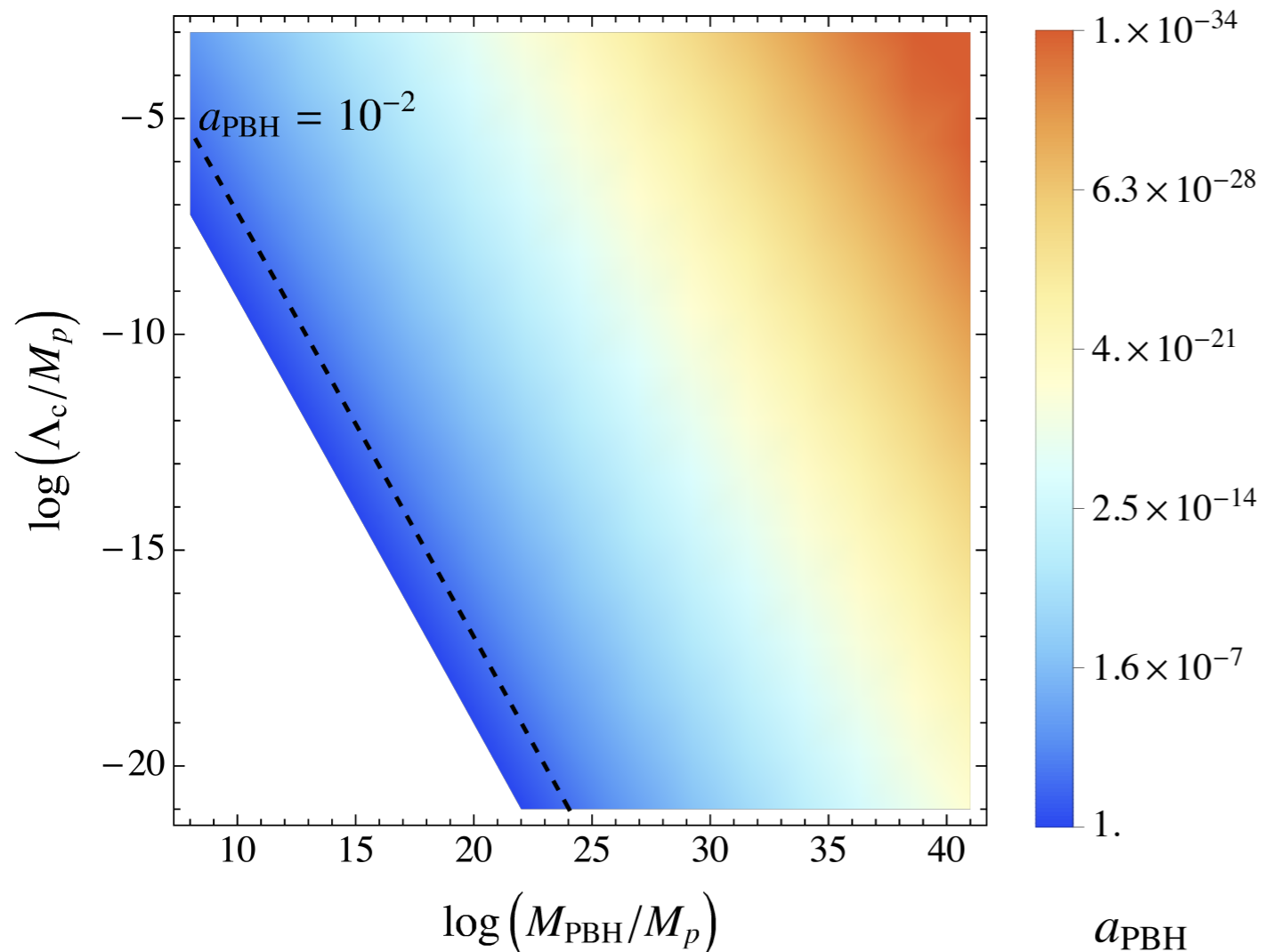
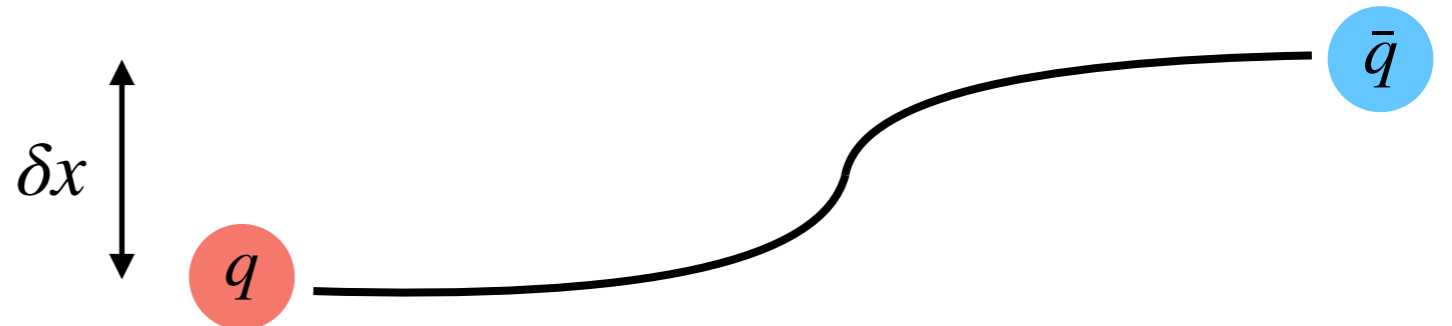
$$\int_{M_1}^{M_2} d \ln M_{\text{PBH}} \frac{d f_{\text{PBH}}(M_{\text{PBH}})}{d \ln M_{\text{PBH}}} \frac{1}{f_{\text{max}}(M_{\text{PBH}})} \stackrel{!}{\leq} 1$$

100% of dark matter!

# High-Spin Subsolar PBHs

- ★ During inflation, the string undergoes a **Brownian motion**, induced by de Sitter quantum fluctuations, leading to **deviation from straightness**:

$$\delta x \simeq \sqrt{N_e} H_i^{-1}$$



- ★ This leads to potentially **significant spin**:

$$a_{\text{PBH}} \simeq \frac{\delta x}{R_g}$$

$$\simeq \frac{1}{H M_{\text{PBH}}} \log \left( \frac{H M_{\text{PBH}}}{\Lambda_c^2} \right)^{1/2}$$



# *Embedding within Standard QCD\**

- ★ Remember, our **required assumption**, for the string not to break:

$$\Lambda_c < M_q$$

- ★ However, standard QCD values indicate the **opposite**:  $\Lambda_c > M_q$ .

- ★ It looks like, our mechanism cannot work with QCD...

# Embedding within Standard QCD\*

- ★ It is natural for the confinement scale and mass **to change in the early Universe!**

$$g_y \bar{\psi}_L \psi_R \phi \quad \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu}$$

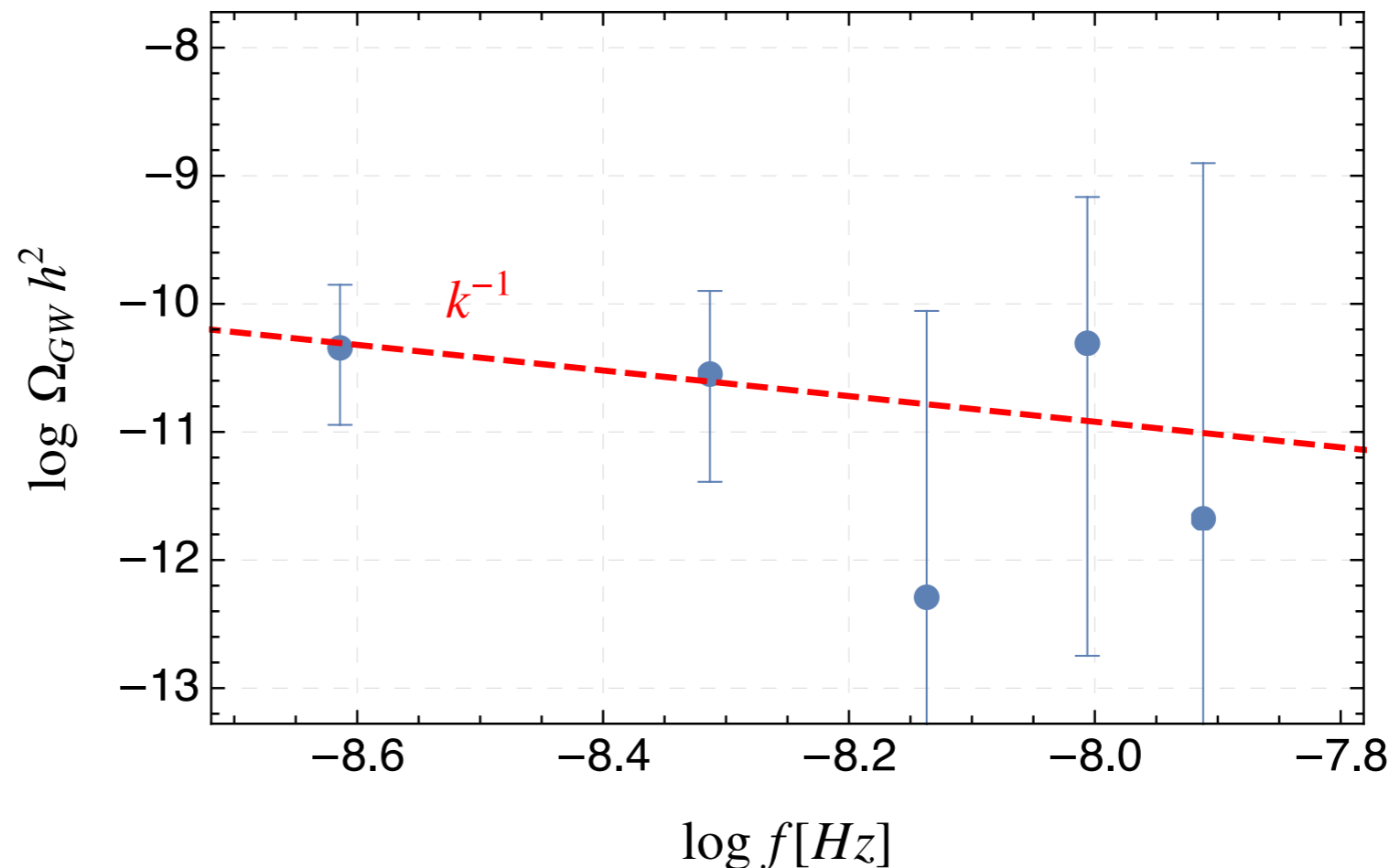
- ★ Couplings are expectation values of fields and can be very different in the early Universe.

- ★ Requirement: Low-temperature expectation value should **set the right coupling values.**

This should happen before BBN,  
leaving **large room for PBH production**  
via the confinement mechanism.

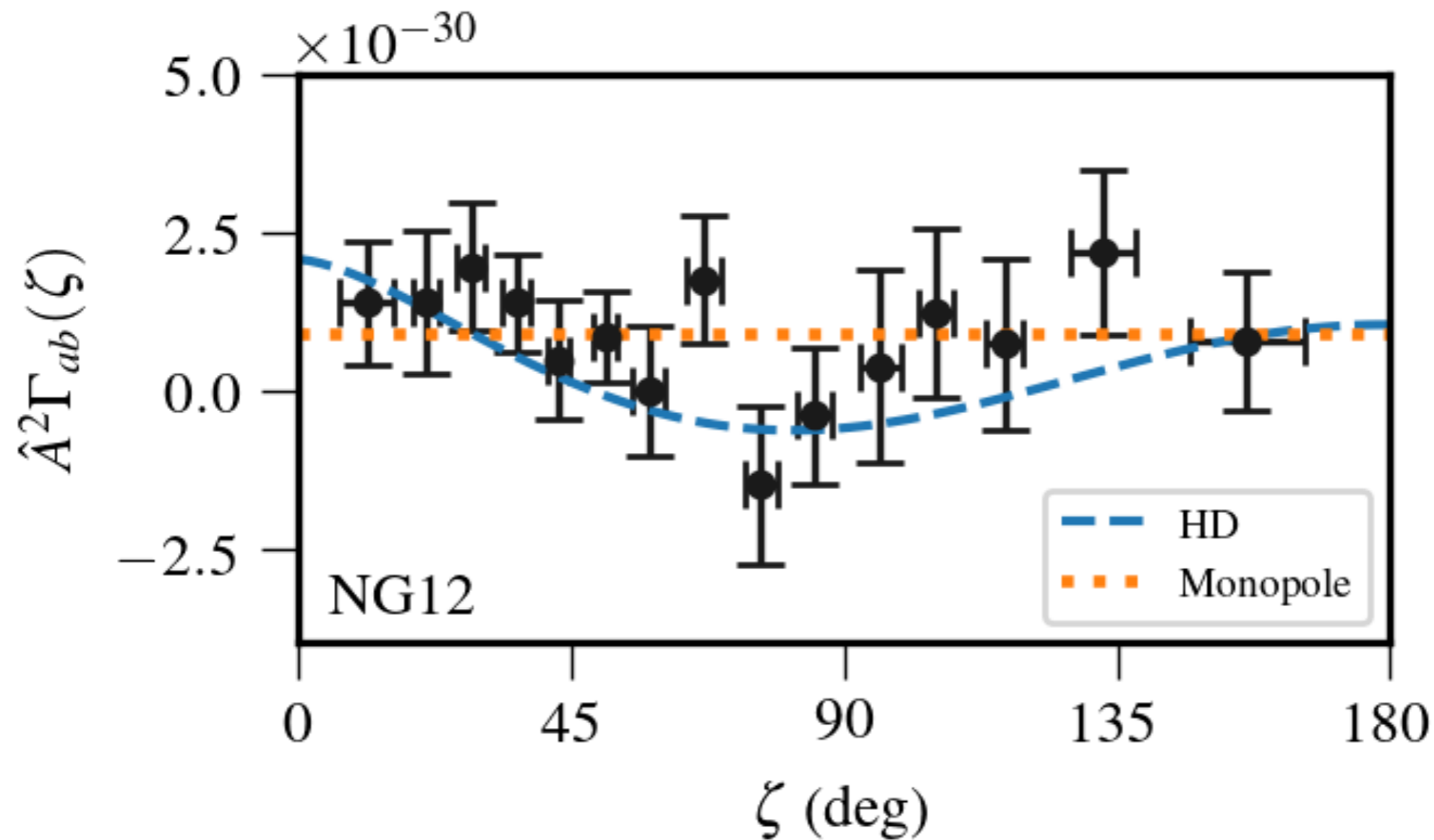
# Gravitational Waves

- ★ After horizon entry, the quarks quickly move towards each other, **emitting gravitational waves**.
- ★ This is similar to dual to systems of dual monopole/anti-monopole pairs connected by a string. [cf. Martin & Vilenkin 1997; Leblond, Shlaer, Siemens 2009]
- ★ **NANOGrav** data from pulsar-timing observations indicate the presence of a **stochastic gravitational-wave background**.



# NANOGrav

- ★ There might be a lack of **Hellings-Downs correlation**. - *still unclear*



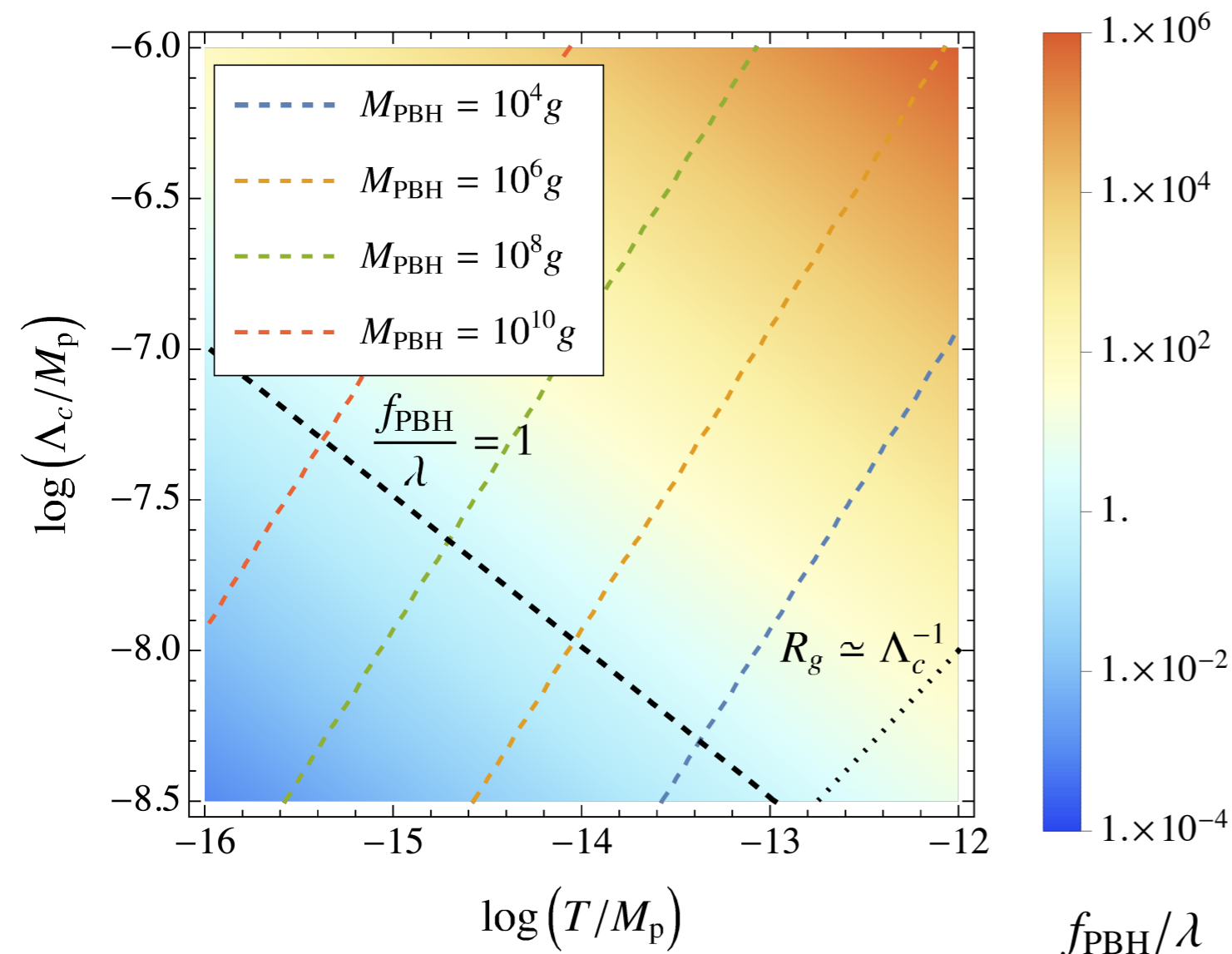
[Arzoumanian *et al.* 2021]

- ★ We can easily generate a **monopolar** signal upon adding e.g.  $\sim \phi \bar{q}q$  with coupling strength relatively weaker by  $\sim 10^{-3}$ .

# Light PBH Dark Matter?

- ★ The exclusion of light ( $M_{\text{PBH}} \lesssim 10^{15}$  g) PBHs is based on the **validity of semiclassical** Hawking radiation throughout **most of the evaporation**.
- ★ This is **unjustified** (and likely to be entirely false), as suggested by recent studies of black holes on the **full quantum level**.

[Dvali *et al.* 2020]



- ★ Results suggest that due to the holes' enormous memory capacity, their **lifetime  $\tau$**  might be **significantly prolonged**.

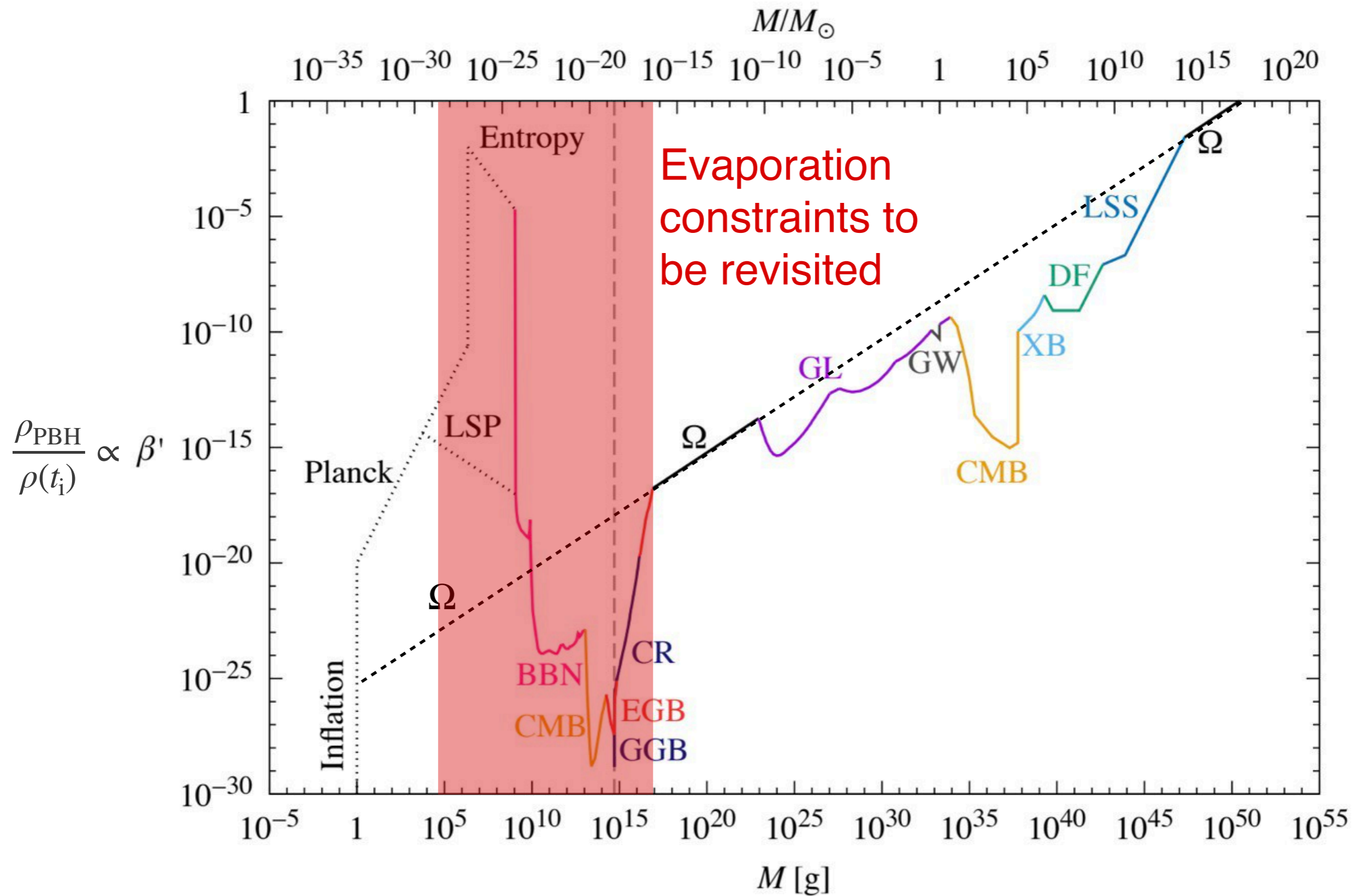
- ★ A conservative estimate is:

$$\tau \rightarrow \tilde{\tau} \geq \tau S^2$$

Entropy of the black hole

- ★ This opens up a large window for light PBH dark matter.

# Light PBH Dark Matter?



*Formation II:  
Critical Collapse*

# Critical Collapse

★ Usually: Assume

$$M_{BH} \propto M_H$$

↑  
horizon mass

★ Critical scaling:

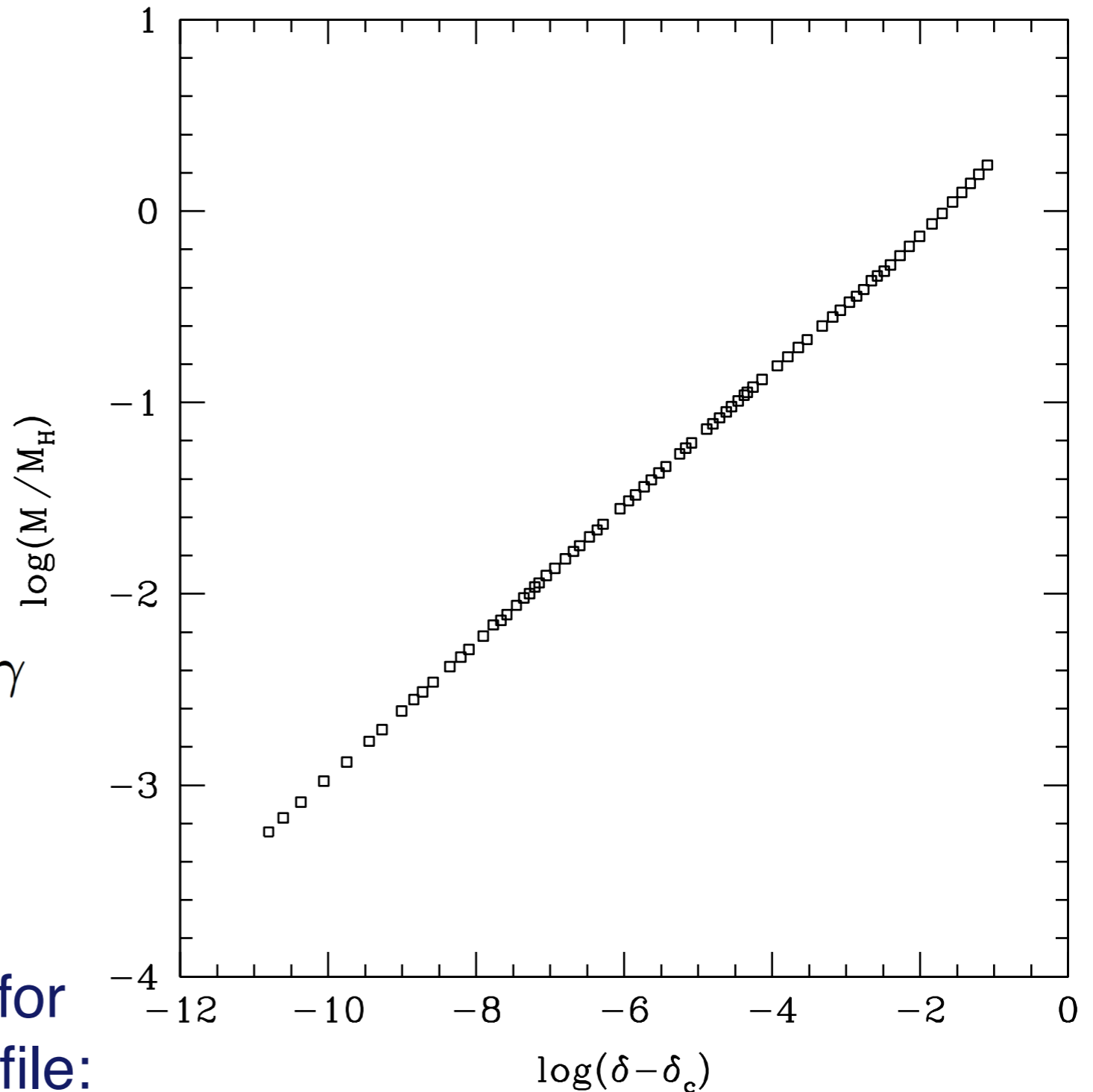
[Choptuik '93]

$$M_{BH} = k M_H (\delta - \delta_c)^\gamma$$

↑  
density contrast

★ Radiation domination and for spherical Mexican-hat profile:

$$k \approx 3.3, \quad \delta_c \approx 0.45, \quad \gamma \approx 0.36$$

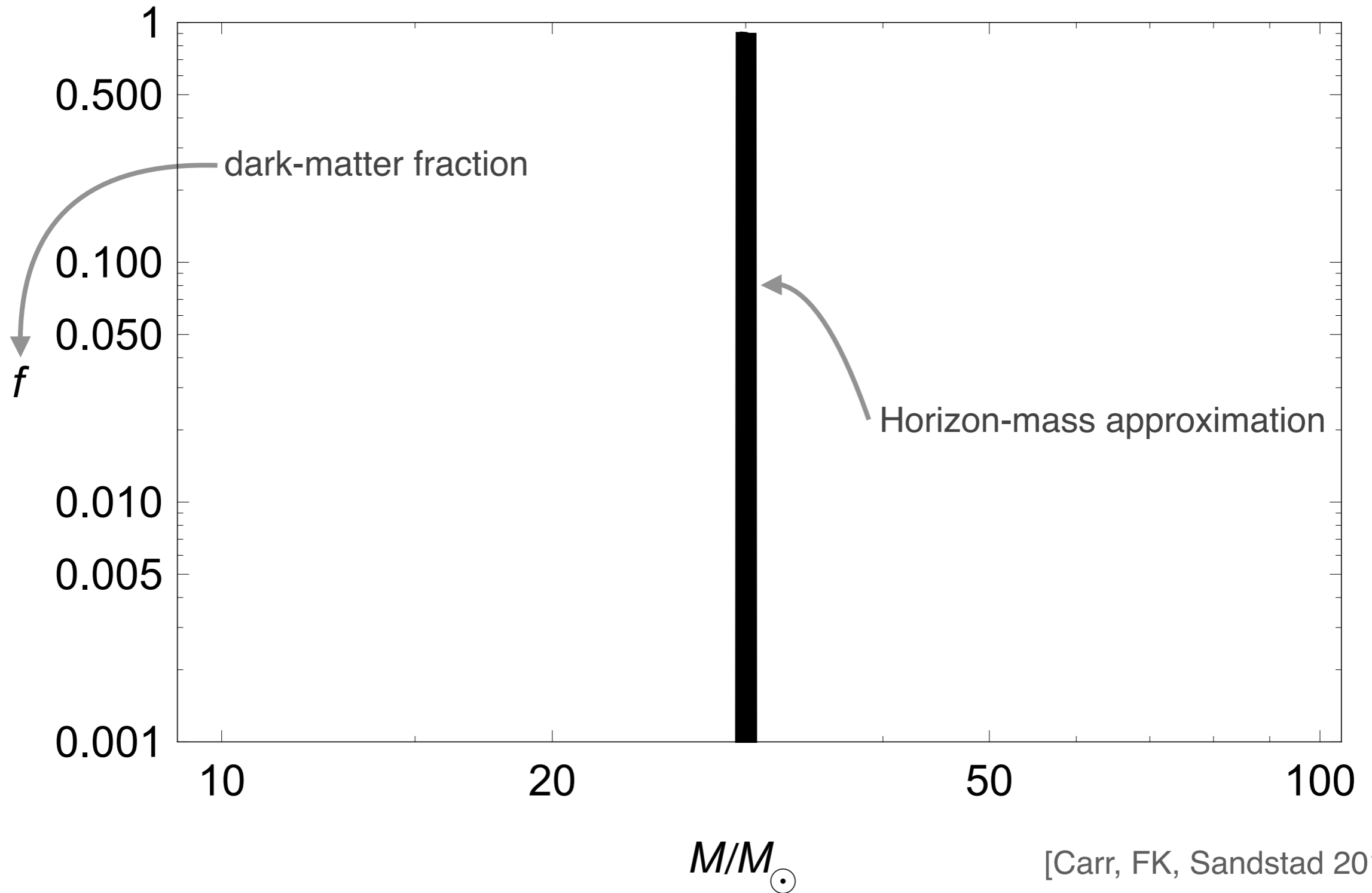


[Musco, Miller, Polnarev 2008]



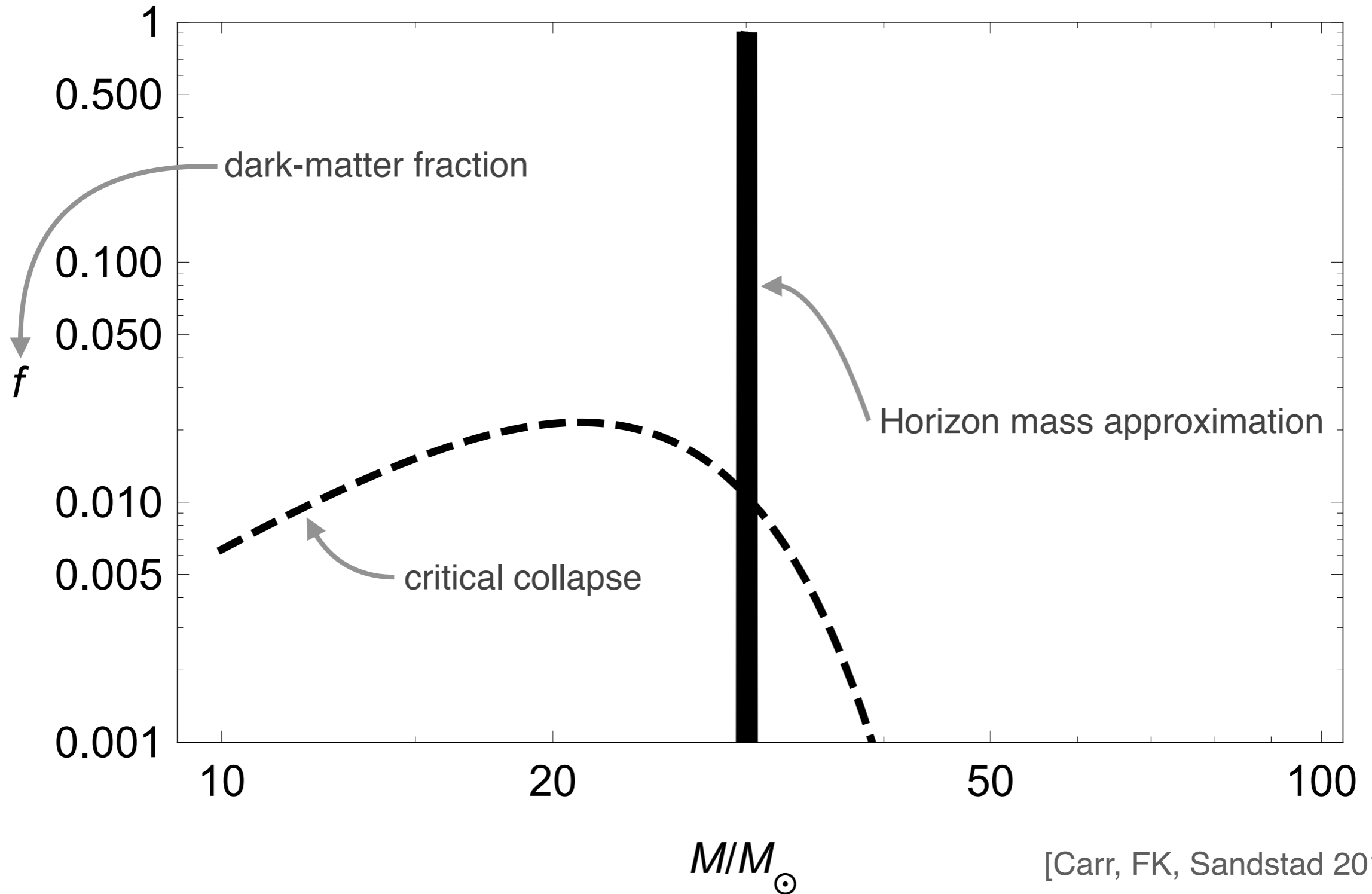
# Critical Collapse

★ How would this look for **monochromatic** mass function?



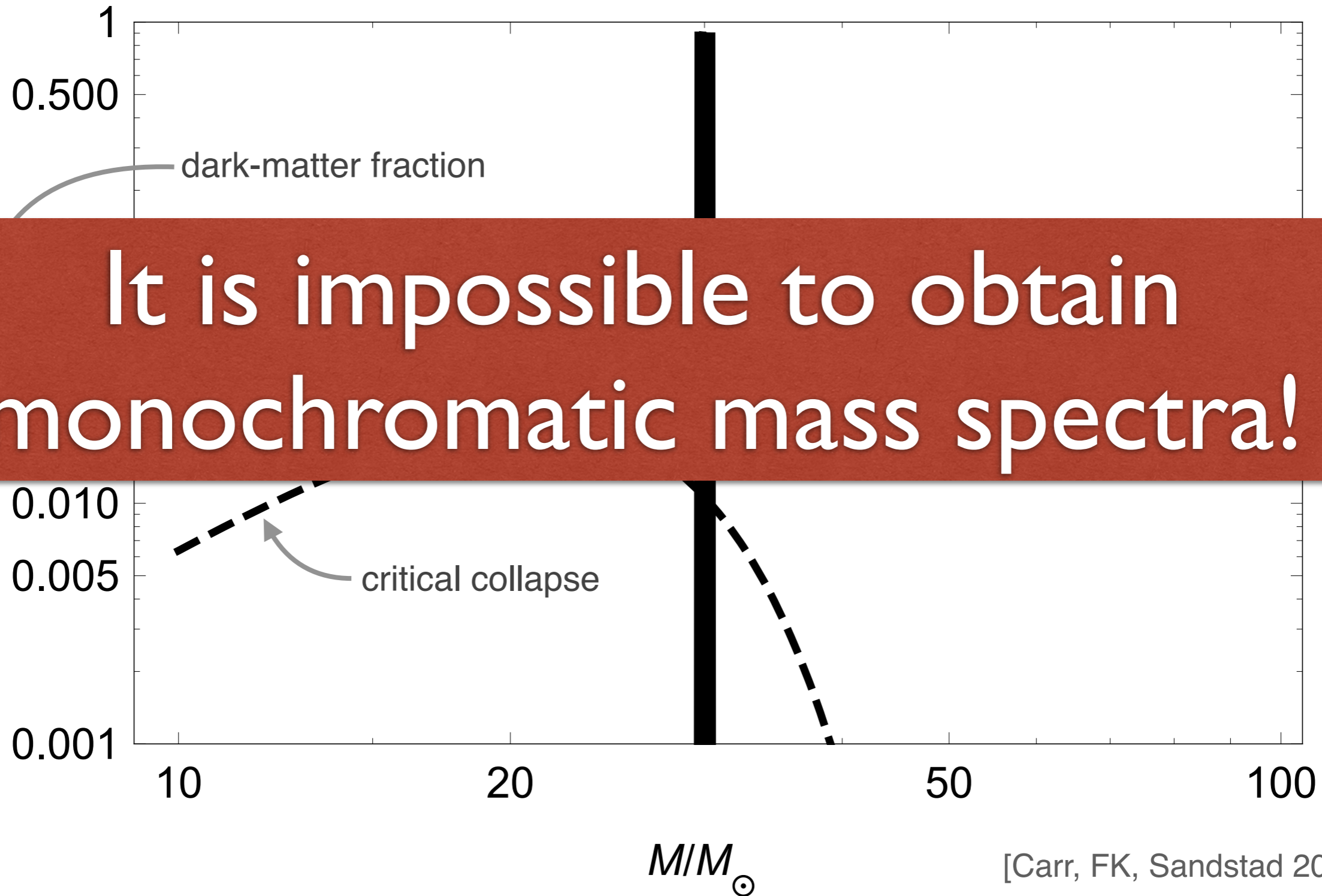
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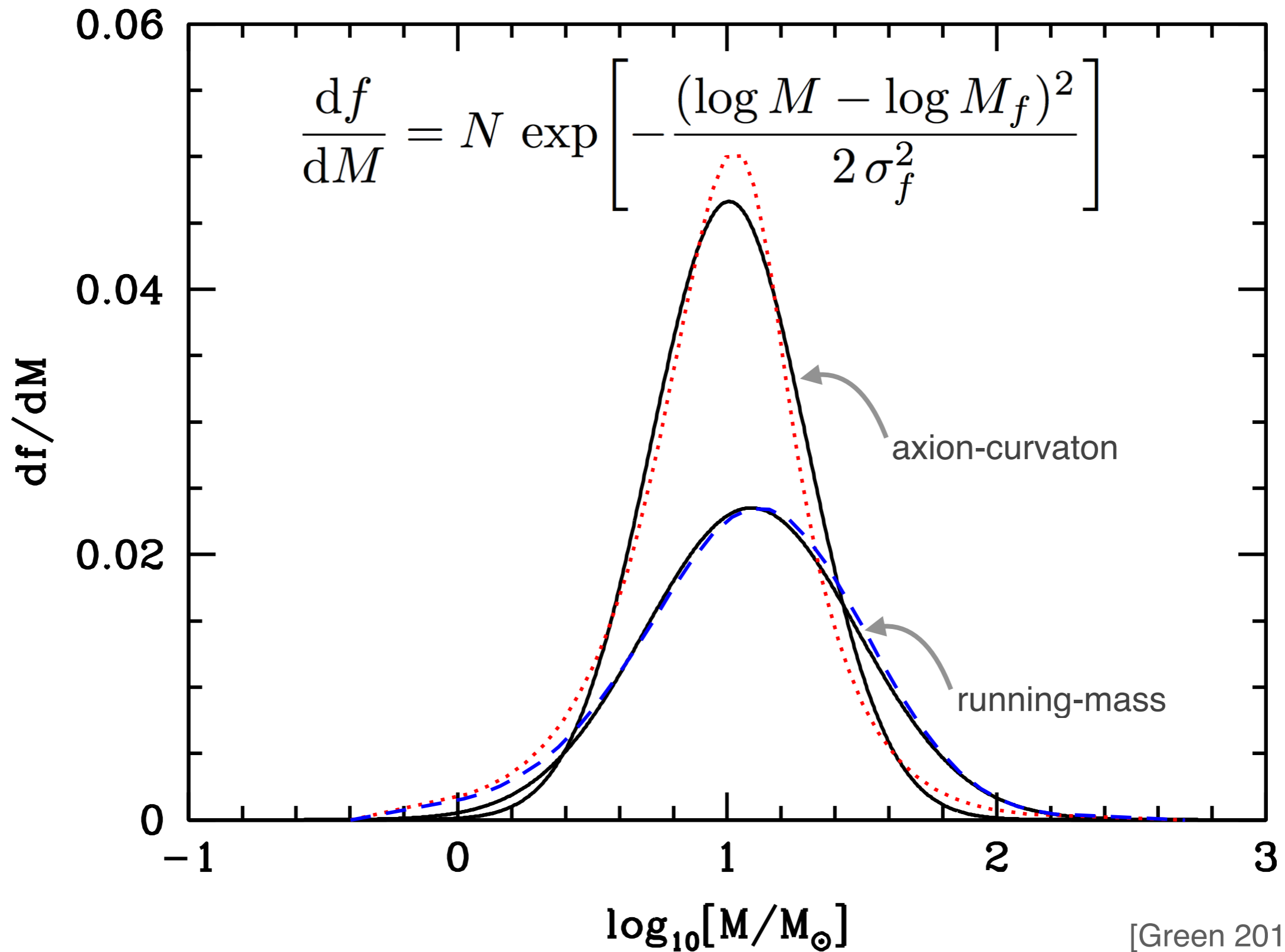


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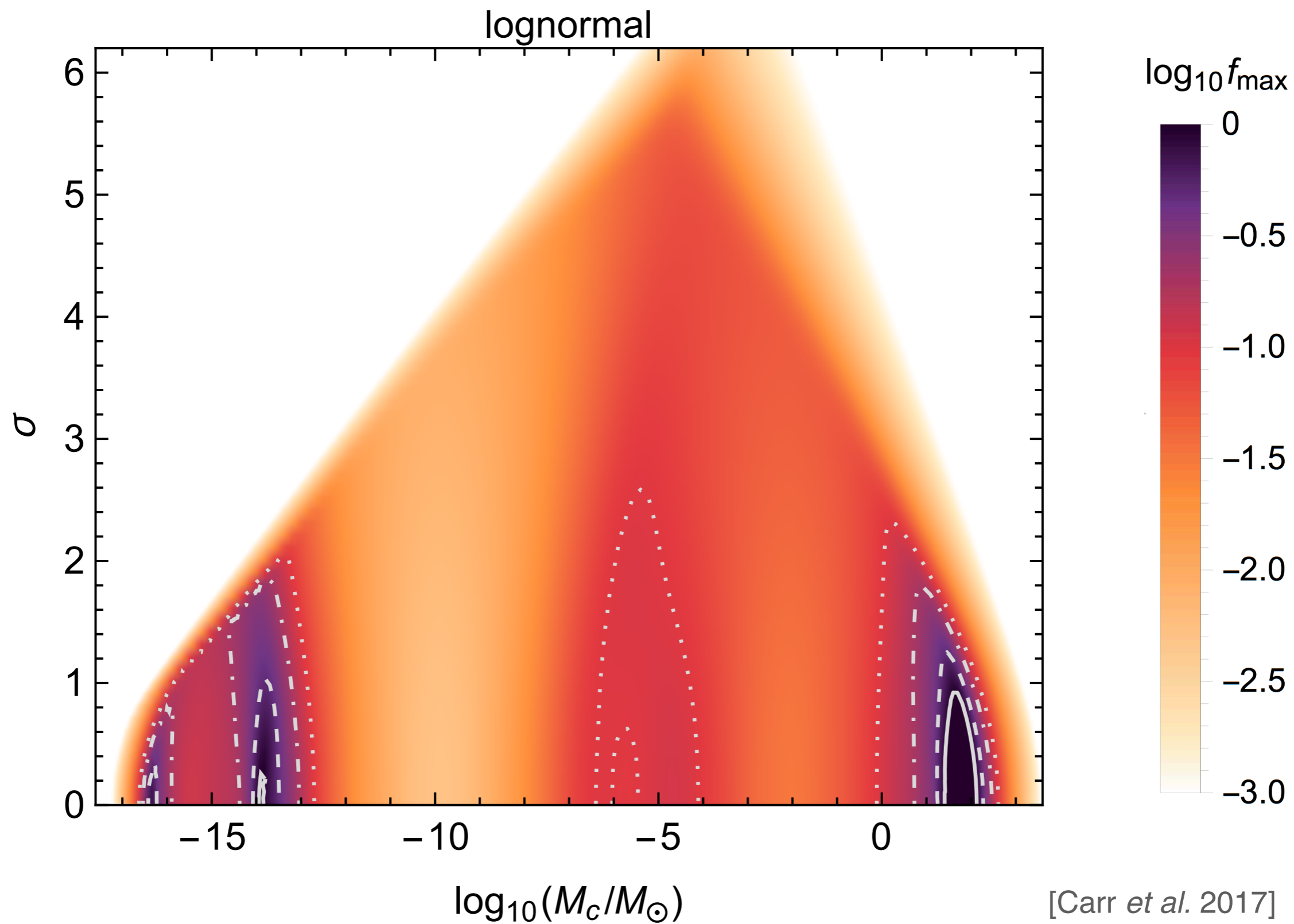
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# More Systematic Study



# More Systematic Study



# *Lepton Flavour Asymmetry*

# Lepton Flavour Asymmetry

★ **Lepton flavour asymmetries** are defined as

$$l_\alpha \equiv \frac{n_\alpha - n_{\bar{\alpha}} + n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{s}, \quad \alpha \in \{e, \mu, \tau\}$$

$n_\alpha, n_{\bar{\alpha}}, n_{\nu_\alpha}, n_{\bar{\nu}_\alpha}$  number densities of (anti)leptons and corresponding (anti)neutrinos

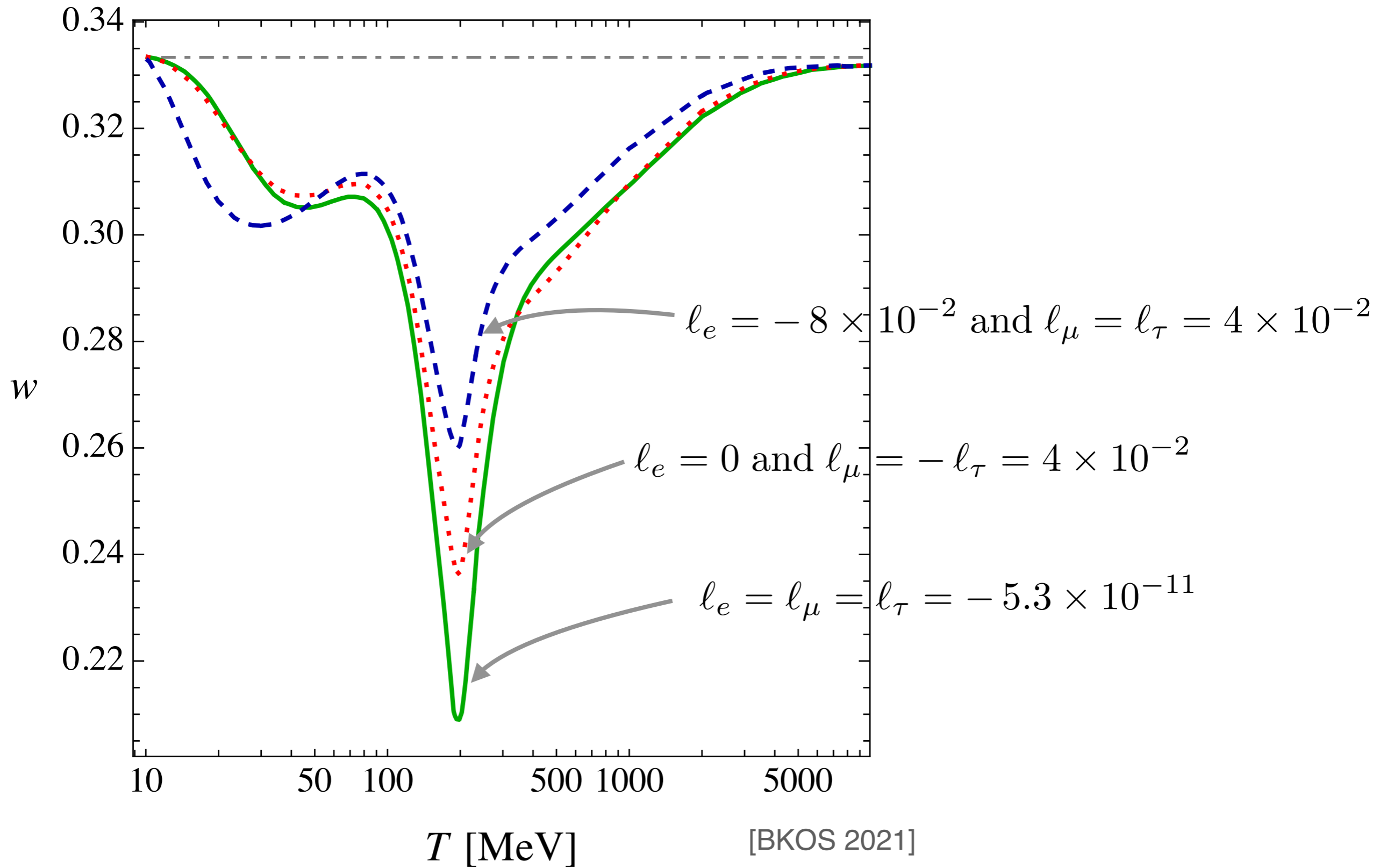
$s$  entropy densities

★ **CMB constraints are quite weak:**

$$|l_e + l_\mu + l_\tau| < 1.2 \times 10^{-2}$$

(unlike baryon asymmetry:  $b = 8.7 \times 10^{-11}$ )

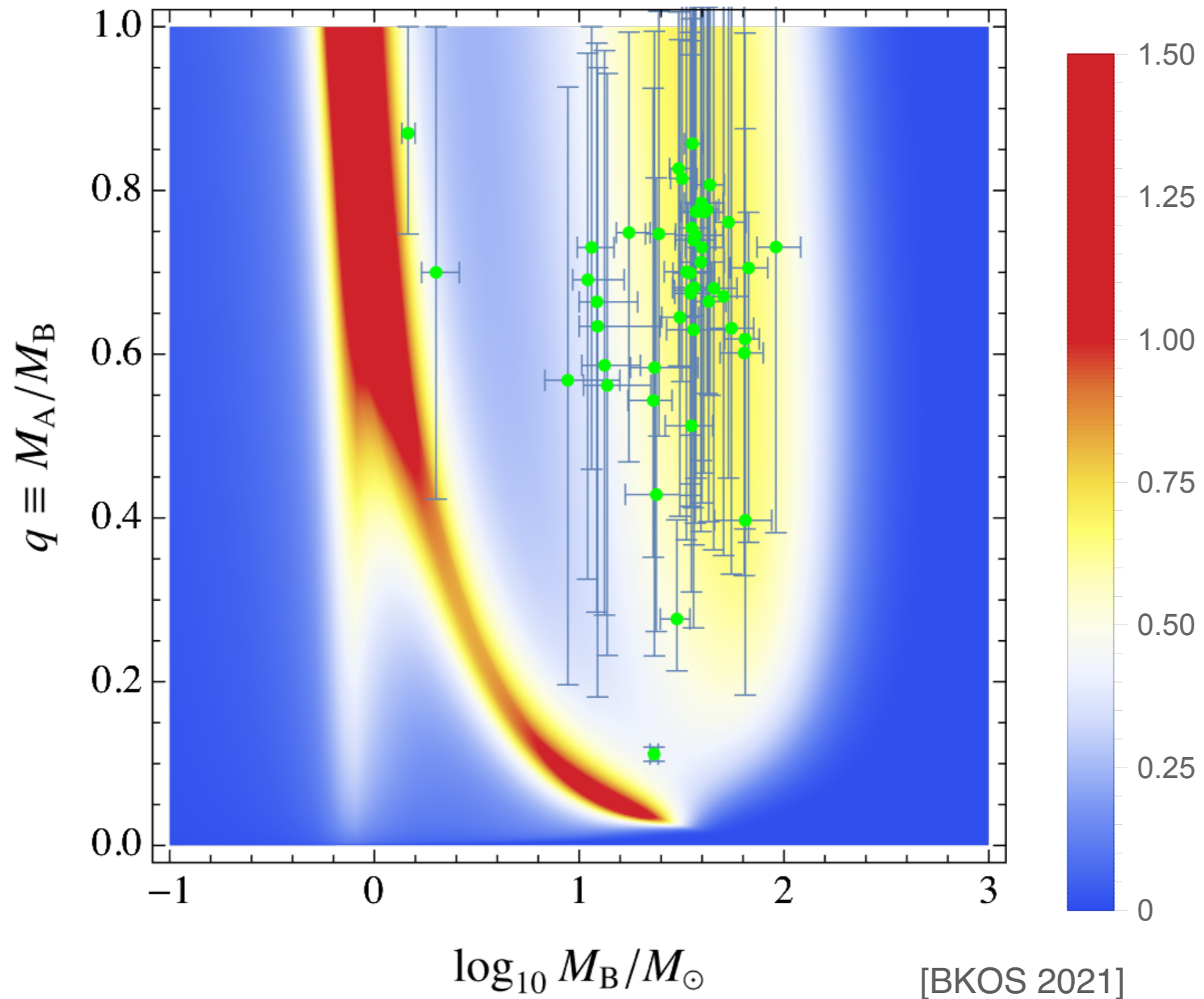
# Lepton Flavour Asymmetry





# *Lepton Flavour Asymmetry*

$$l_e = l_\mu = l_\tau = -5.3 \times 10^{-11}$$



# *Lepton Flavour Asymmetry*

$$\ell_e = -8 \times 10^{-2} \text{ and } \ell_\mu = \ell_\tau = 4 \times 10^{-2}$$

