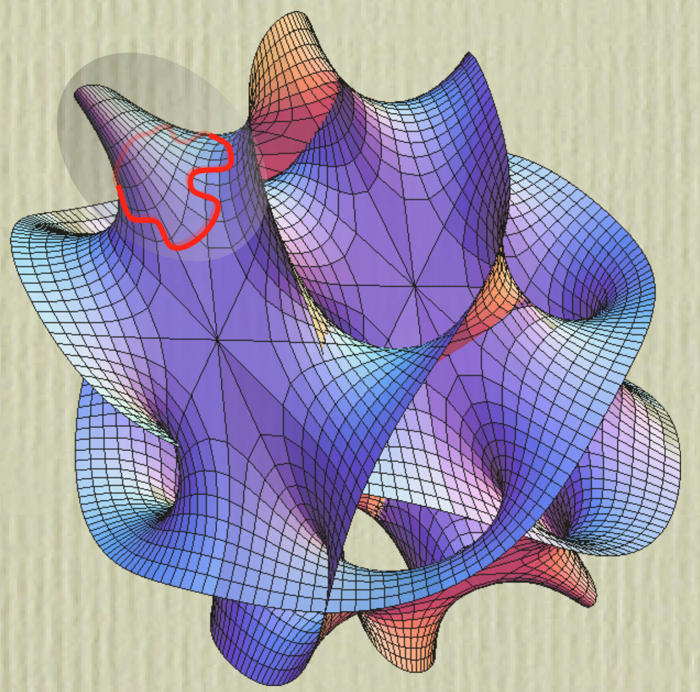
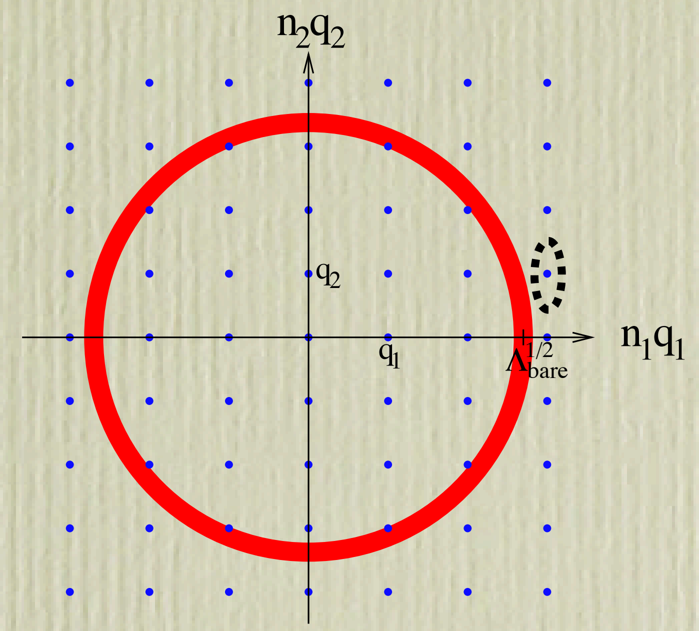
A Quantum-Mechanical Mechanism for Reducing the Cosmological Constant





picture from [Bousso & Polchinski '00]

N. Kaloper & AW '22 — arXiv:2204.13124
and [N. Kaloper — arXiv:2202.06977 , arXiv:2202.08860]

Alexander Westphal
(DESY)

- had the difficult choice from 3 subjects to talk about

Festina Lente for axions - w/ V. Guidetti, N. Righi & G. Venken ‘22

Fuzzy Dark Matter from Strings - w/ M. Cicoli, V. Guidetti & N. Righi ‘21

-> for PQ/QCD axions in KKLT - see talk by Jakob Moritz

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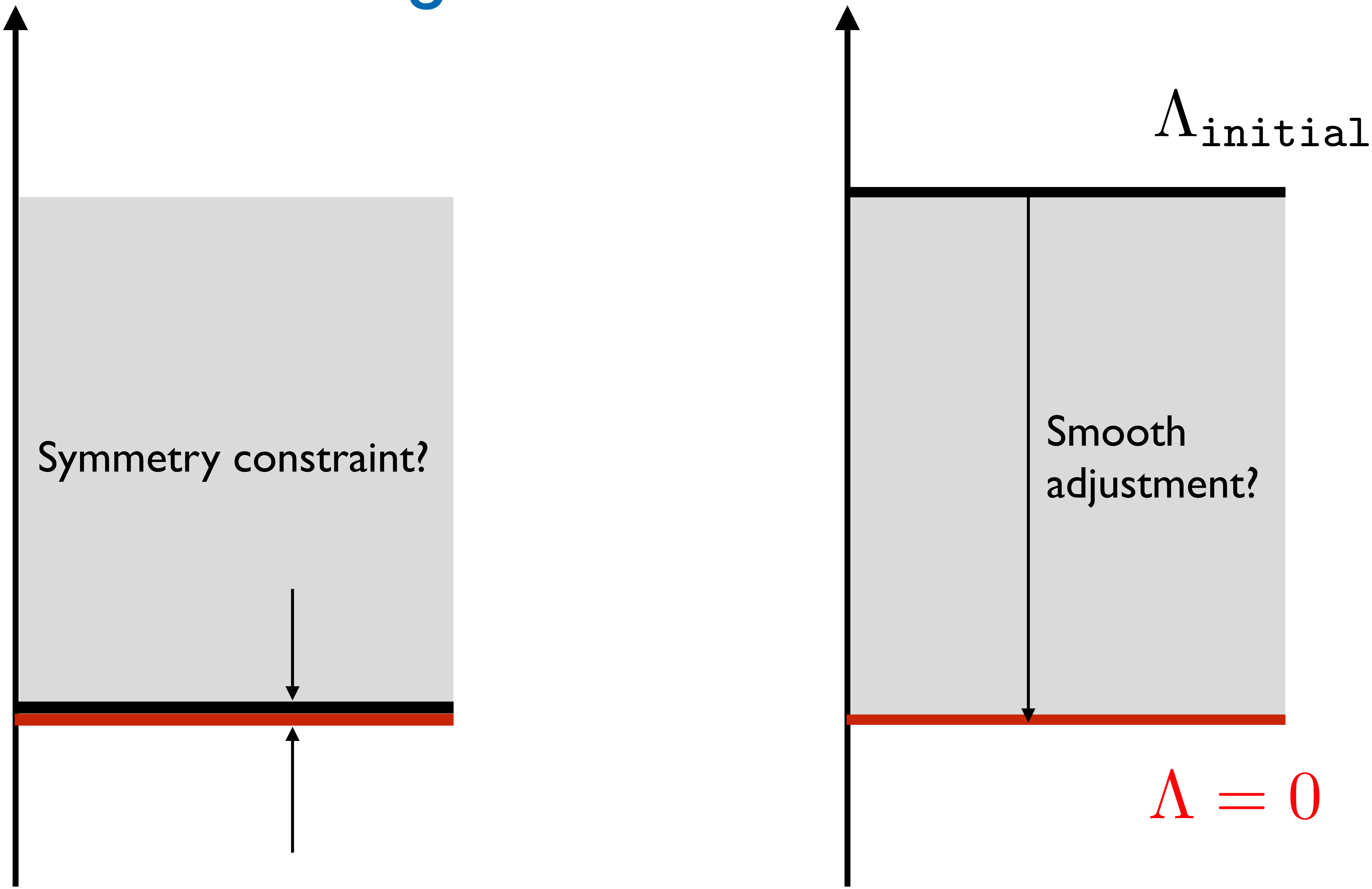
-> need to explain 3 features - tasks for string theory:

- existence of small CC vacua
  … despite natural scale M_S^4
- why we observe small CC
- dS vacua should be meta-stable

Andriot, Horer & Marconnet ’22
Basile ’21/’22
Brinkmann, Cicoli, Dibitetto & Pedro ’22
Friedrich, Hebecker, Salmhofer, Strauss & Walcher ’22
Demirtas, Gendler, Long, McAllister & Moritz ’21
Bardzell, Gonzalo, Rajaguru & Wrase ’22
…

Cosmological Constant Problem



[Weinberg '89]

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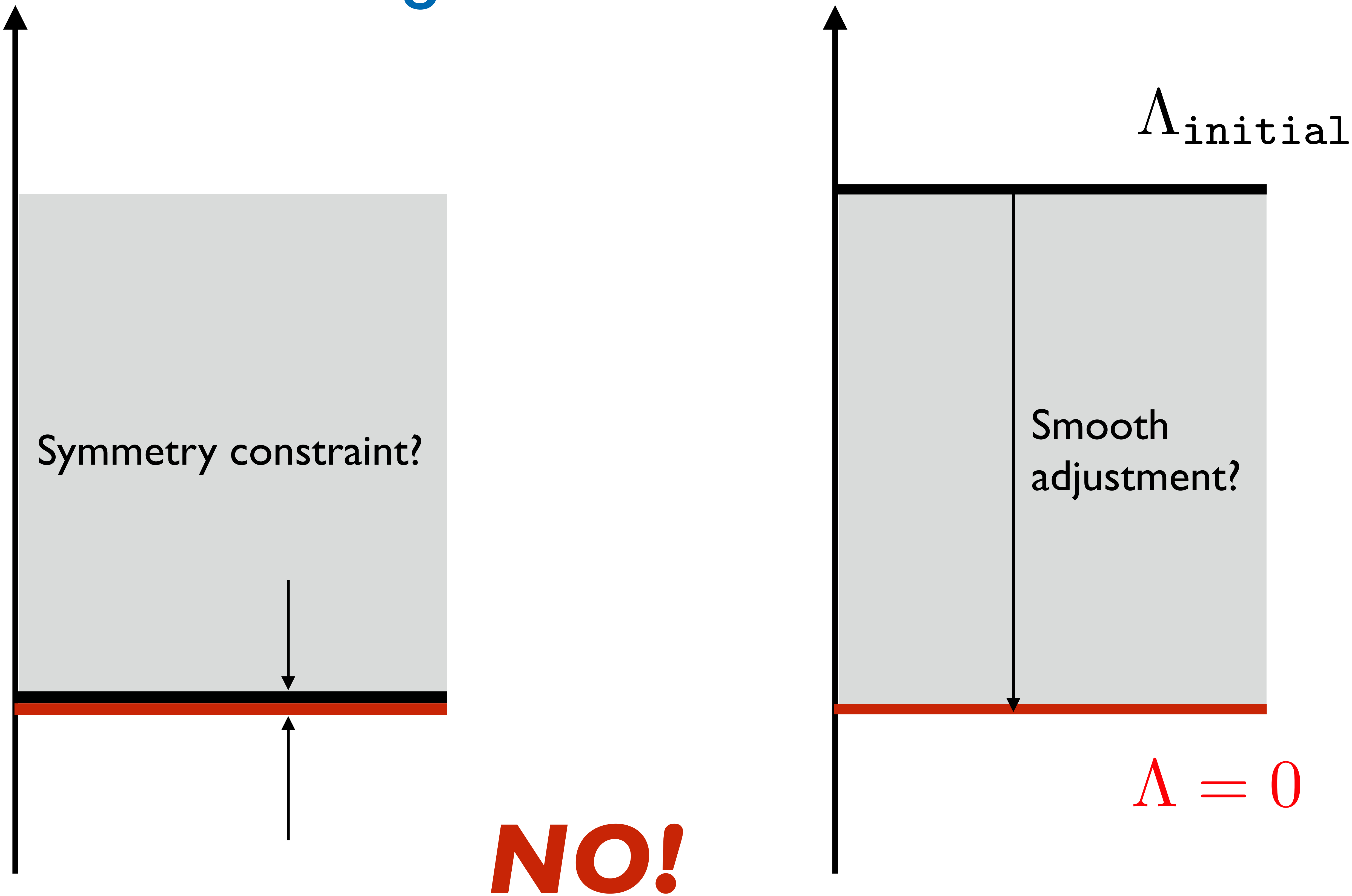
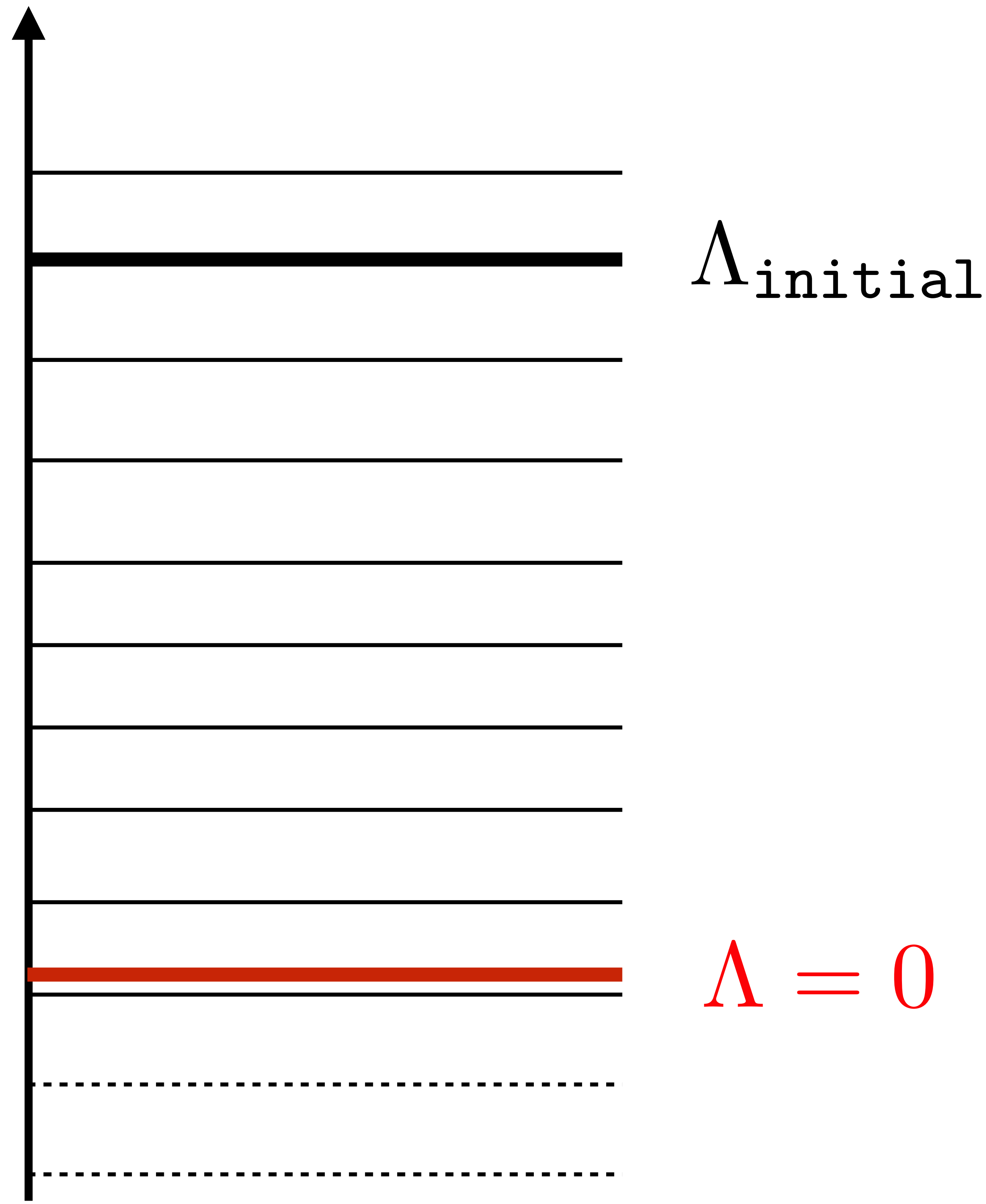


[Weinberg '89]

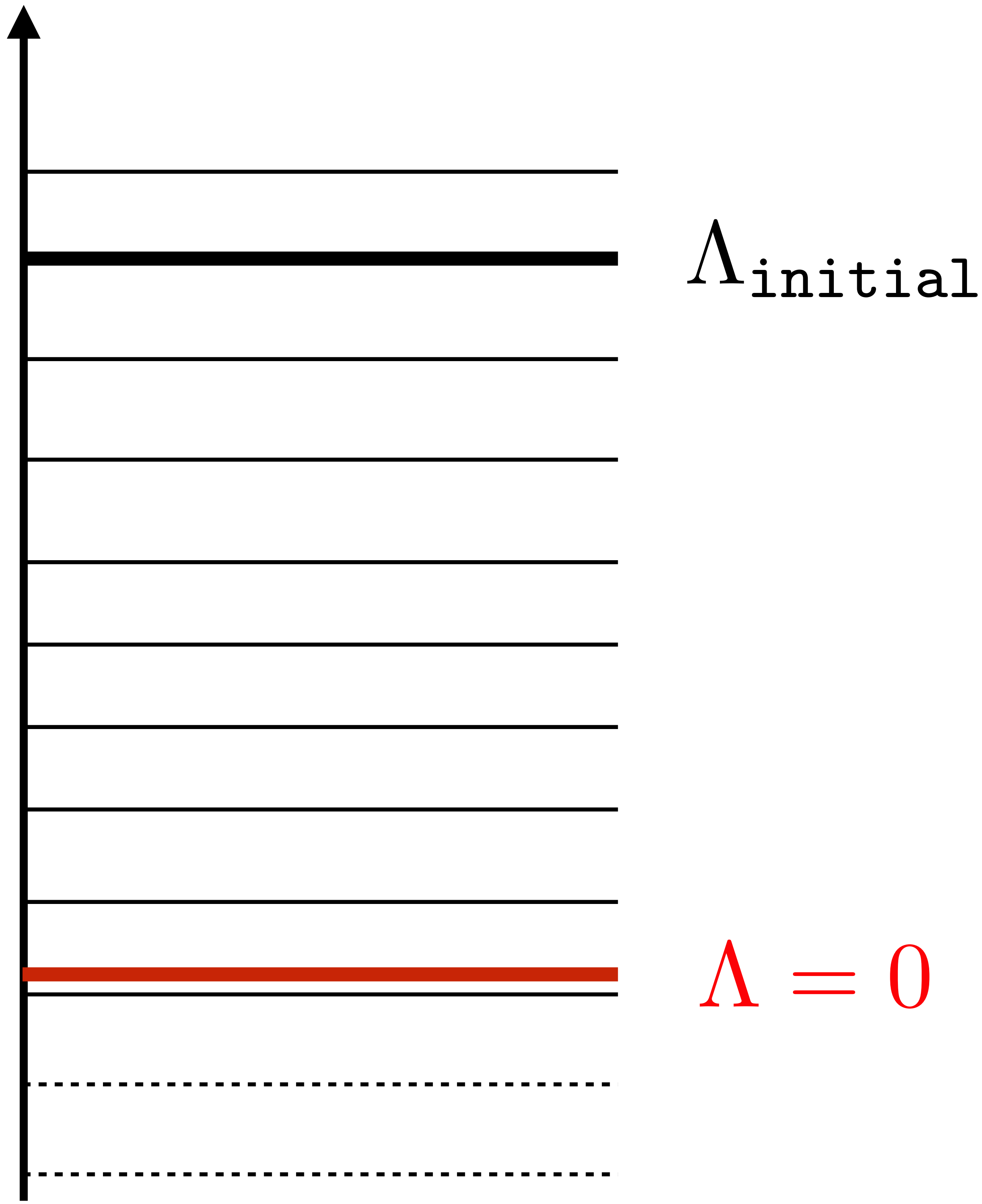
Stairway in Heaven



Stairway in Heaven

CC is unstable, it decays …

As long as the gaps are wide enough, we can fit the "real universe" inside it, all 60ish efolds of inflation, reheating, BBN, etc etc



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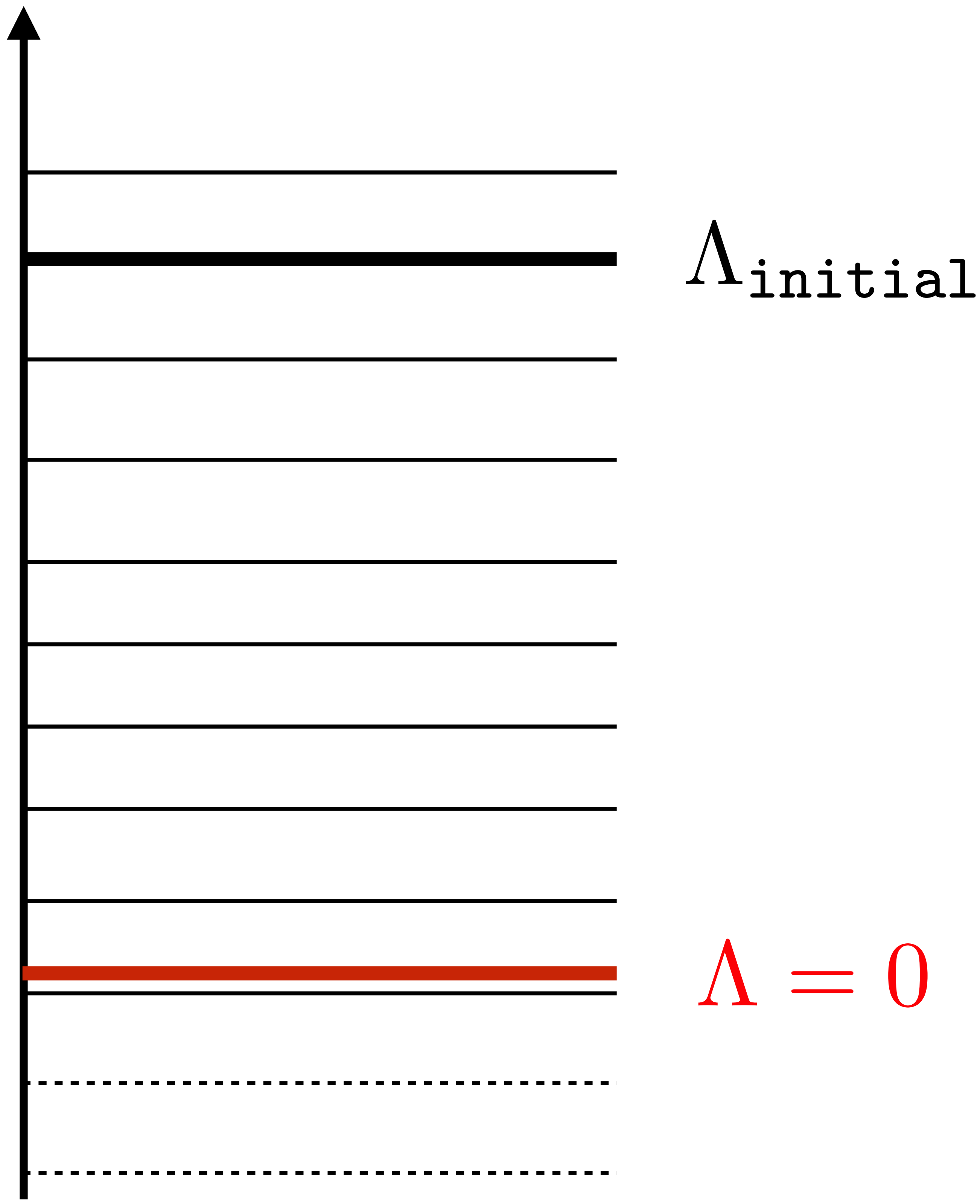

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for a single stairway, steps too tiny [Abbott '85]

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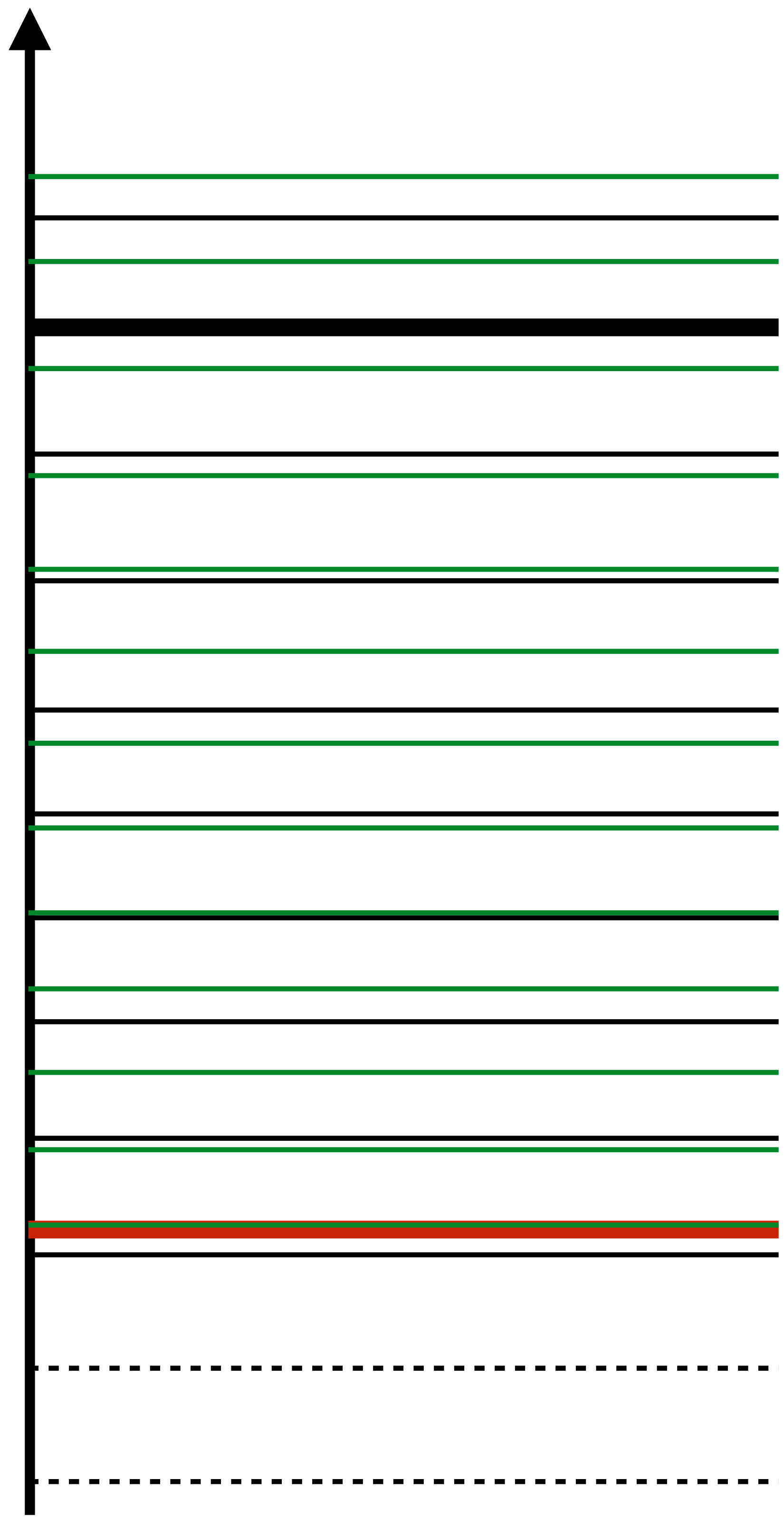

$\Lambda_{\mathtt{initial}}$

to accommodate small CC,

need $\geq$ 2 stairways somewhat out of step

… *a landscape*

$\Lambda = 0$

[Brown-Teitelboim '87 & '88] and [Bousso-Polchinski '00][Feng, March-Russell, Sethi & Wilczek '00]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm P}^2}{2} R - \Lambda - |F_{\mu\nu\rho\sigma}|^2 \right]$$

$$+ S_{\rm boundary} - \boxed{\mathcal{T}_A \int d^3\xi \sqrt{\gamma_A} - \mathcal{Q}_A \int A_3}$$

membrane charged under A_3

$$F_4 = dA_3$$

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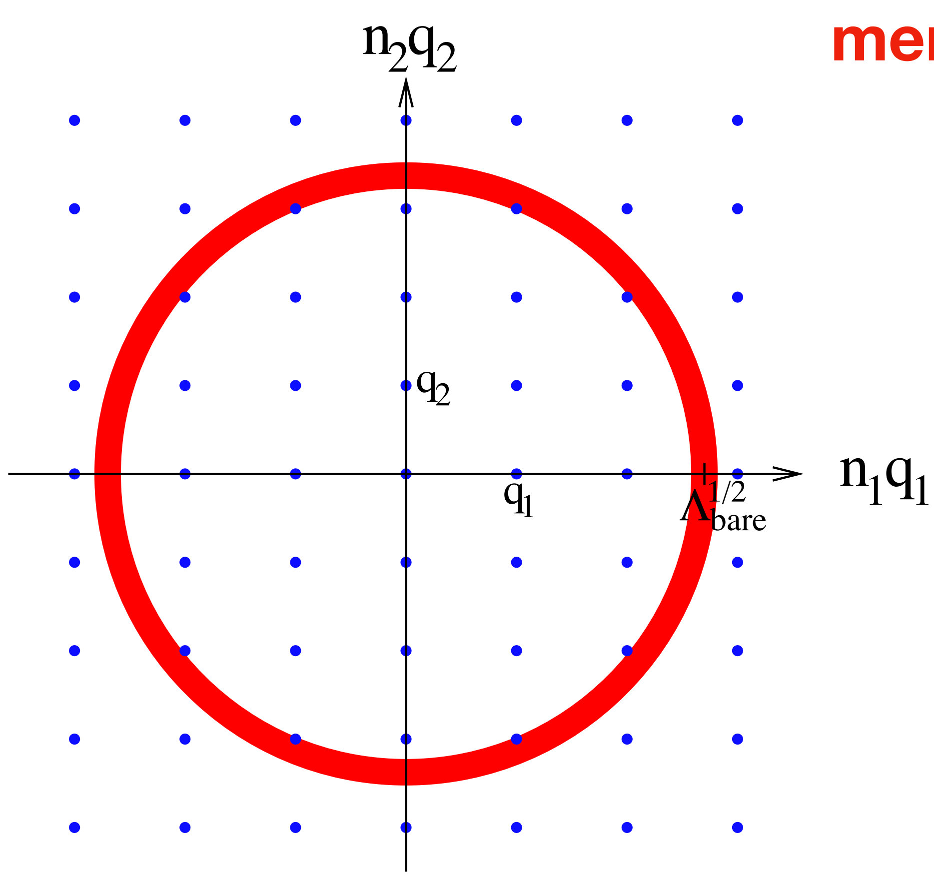

picture from [Bousso & Polchinski '00]

BP:

$$\Lambda = -\mathcal{O}(M_{\rm P}^4) + \sum_i q_i^2 N_i^2$$

compare to covariant unimodular GR [Henneaux-Teitelboim '89]:

enters as a Lagrange multiplier scalar field!

$$S = \int d^4x \left[\sqrt{-g}\frac{M_{\rm P}^2}{2} R - \Lambda \left(\sqrt{-g} - \frac{1}{M_{\rm P}^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} \right) \right]$$

$$F_4 = dA_3$$

$$\frac{\delta S}{\delta g^{\mu\nu}} \Rightarrow G_{\mu\nu} = -\frac{1}{M_{\rm P}^2}\left(T_{\mu\nu} + \Lambda g_{\mu\nu}\right)$$

$$\frac{\delta S}{\delta \Lambda} \Rightarrow \sqrt{-g}\epsilon_{\mu\nu\rho\sigma} = \frac{1}{M_{\rm P}^2} F_{\mu\nu\rho\sigma}$$

$$\frac{\delta S}{\delta A_3} \Rightarrow d\Lambda = 0 \ \Rightarrow \ \Lambda = const.$$

from now on: $M_{\rm P} = 1$

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[Kaloper; Kaloper & AW '22]

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Euclidean Field Eqs

[Kaloper; Kaloper & AW '22]

Euclidean Field Eqs

- Bulk:

$$ds_E^2 = dr^2 + a^2(r)\, d\Omega_3$$

$$\left(\frac{a'}{a}\right)^2 - \frac{1}{a^2} = -\frac{\Lambda}{3}$$

[Kaloper; Kaloper & AW '22]

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- Bulk solutions are sections of (horo)spheres

$$a(r) = a_0 \sin(\frac{r+\delta}{a_0})\,, \quad \text{for} \quad \Lambda > 0\,; \qquad a(r) = r + \delta\,, \quad \text{for} \quad \Lambda = 0\,;$$

$$a(r) = a_0 \sinh(\frac{r+\delta}{a_0})\,, \quad \text{for} \quad \Lambda < 0$$

[Kaloper; Kaloper & AW ’22]

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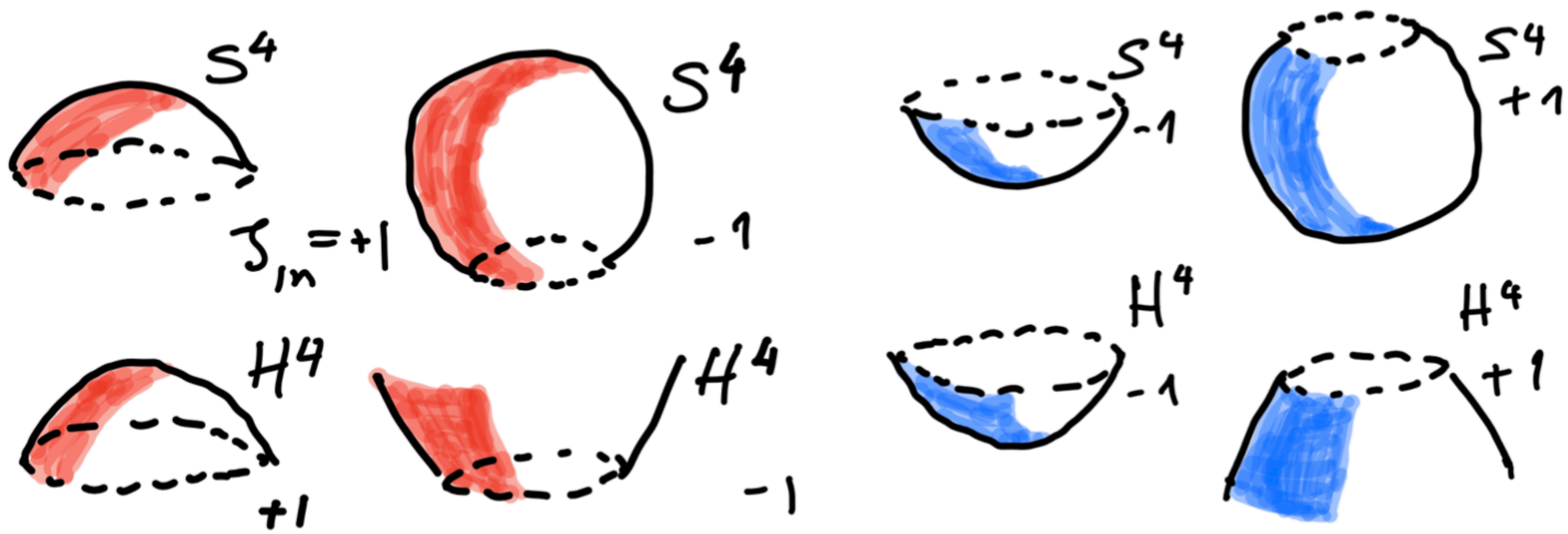
$\mathcal{T}_A, \mathcal{Q}_A \neq 0$

- Bulk sections:

$\Lambda > 0$ $\Lambda > 0$

$\Lambda < 0$ $\Lambda < 0$



inside outside

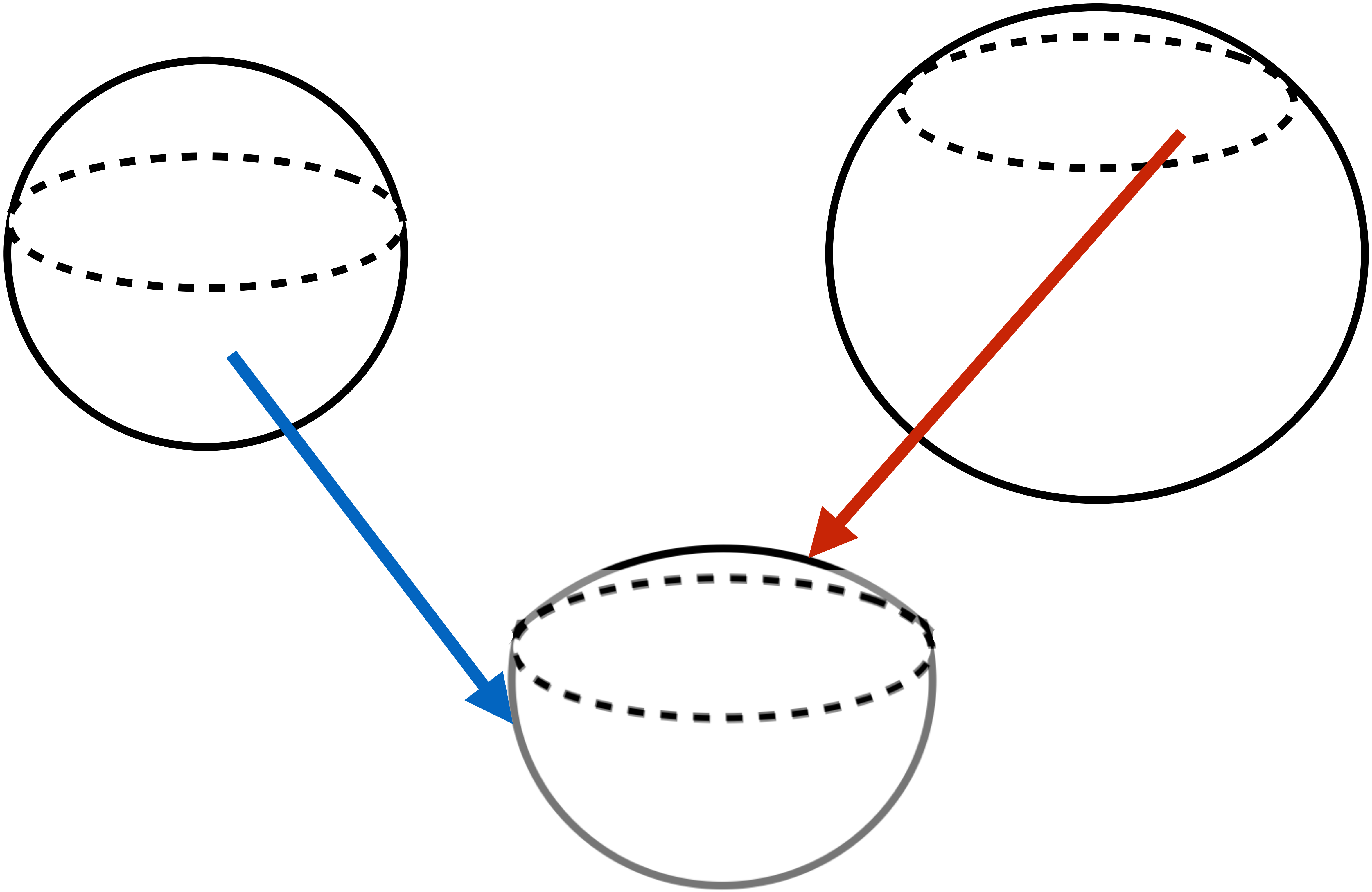
- Junction conditions: massaging the eqs, can rewrite them as

$$\zeta_{out}\sqrt{1-\frac{1}{3}\Lambda_{out}a^2} = -\frac{\mathcal{T}_A}{4}\,(1-q)\ a$$

$$\zeta_{in}\sqrt{1-\frac{1}{3}\Lambda_{in}a^2} = \frac{\mathcal{T}_A}{4}\,(1+q)\ a$$

$$q \equiv \frac{2\mathcal{Q}_A}{3\mathcal{T}_A^2}$$

glueing de Sitter Instantons



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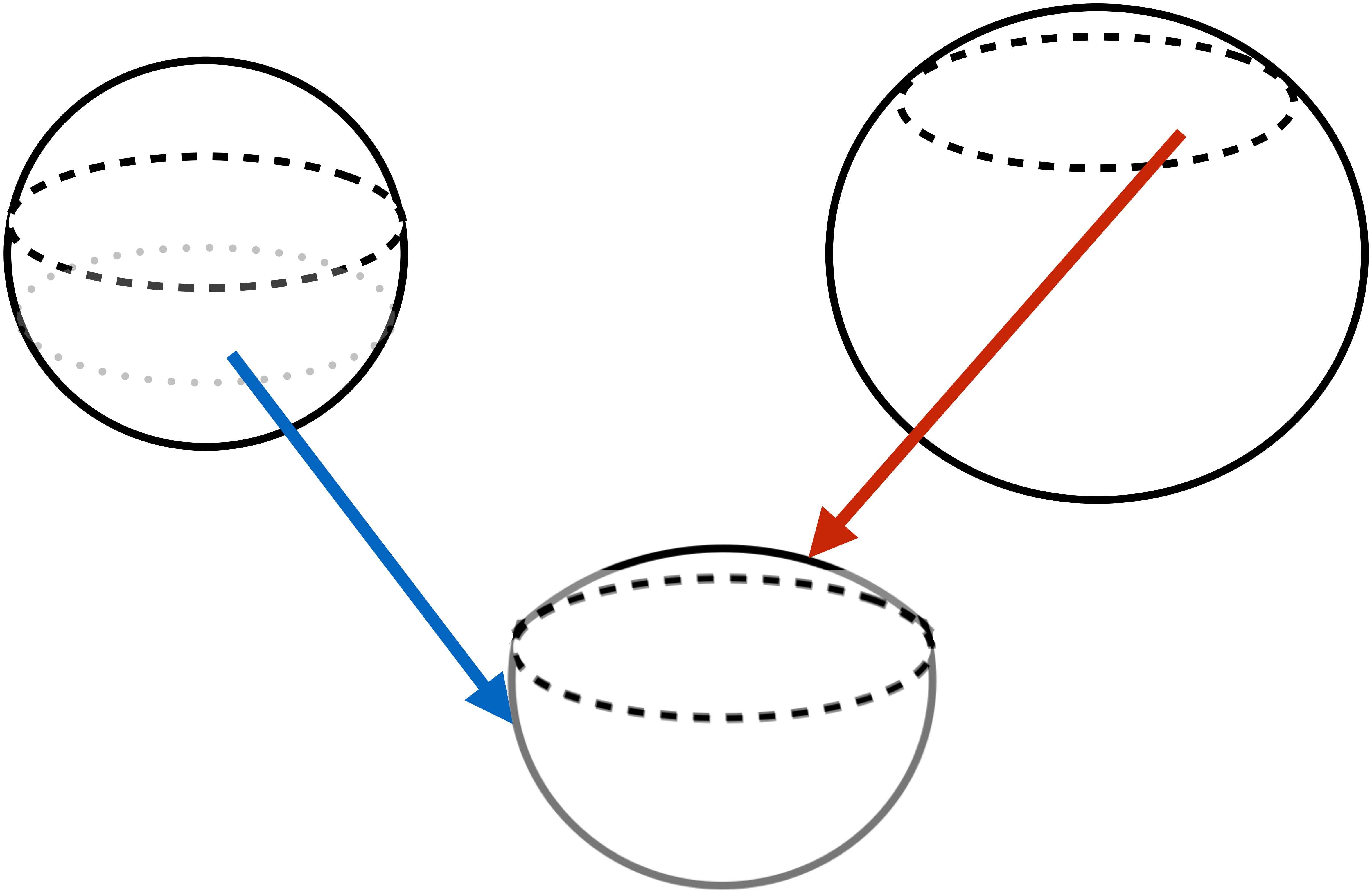

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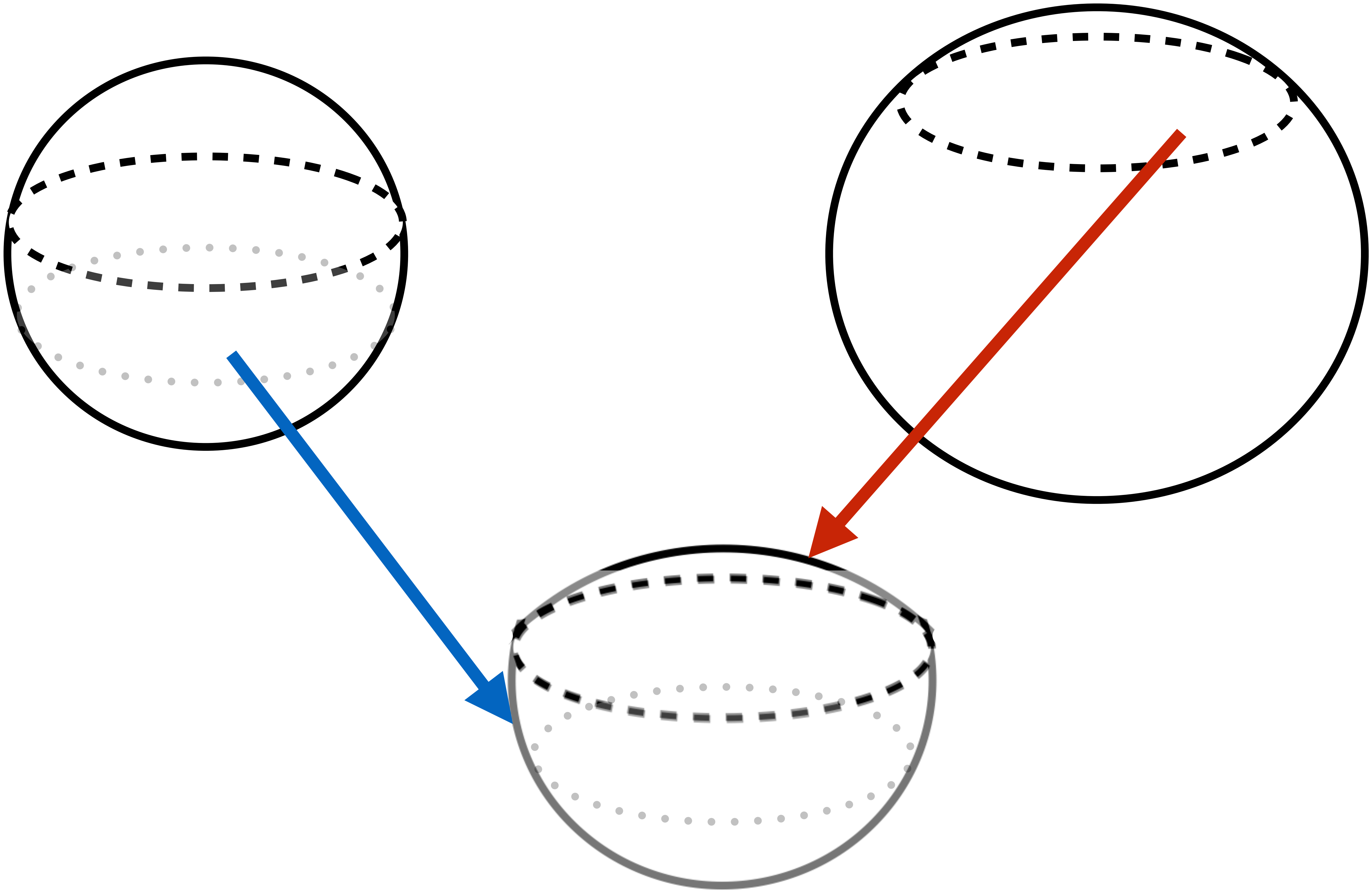
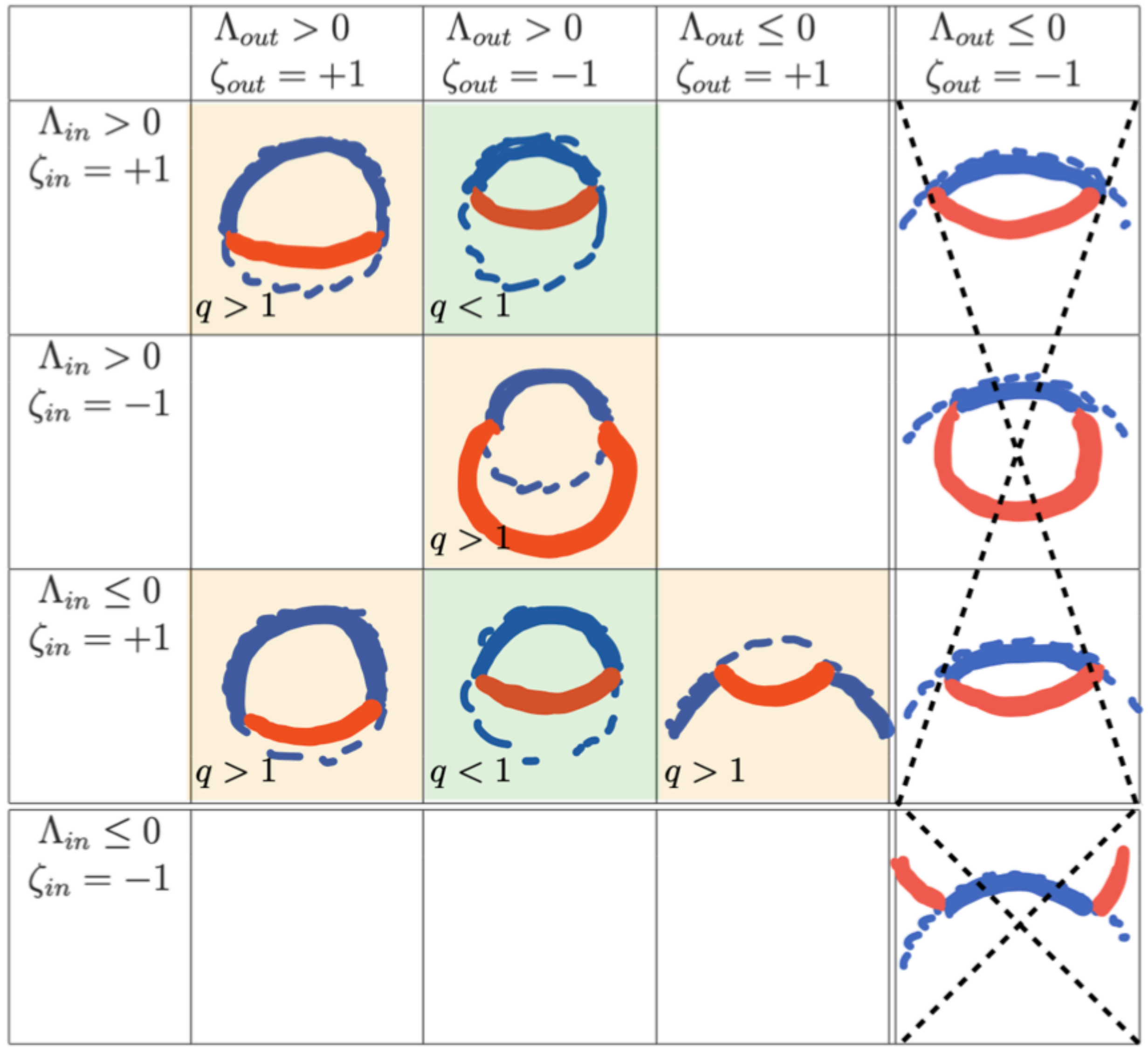


menu of instantons

[Brown & Teitelboim ’87/’88]



- white: kinematically forbidden (no valid j.c. pairing)
- pale gold: $q > 1$
- pale green: $q < 1$
- crossed-out: divergent bounce action

$$q \equiv \frac{2\mathcal{Q}_A}{3\mathcal{T}_A^2}$$

the crucial difference ...

- Junction conditions controlled by

here: $$\Big(1 \mp \underbrace{\frac{2\,M_{\rm P}^4\,\mathcal{Q}_A}{3\mathcal{T}_A^2}}_{q}\Big)$$

BP/BT: $$\Big(1 \mp \underbrace{\frac{2M_{\rm P}^2 \cdot 2Q_A\,\mathcal{Q}_A}{3\mathcal{T}_A^2}}_{q}\Big)$$

- in BP/BT ratio *q* changes with decreasing background Q_A …

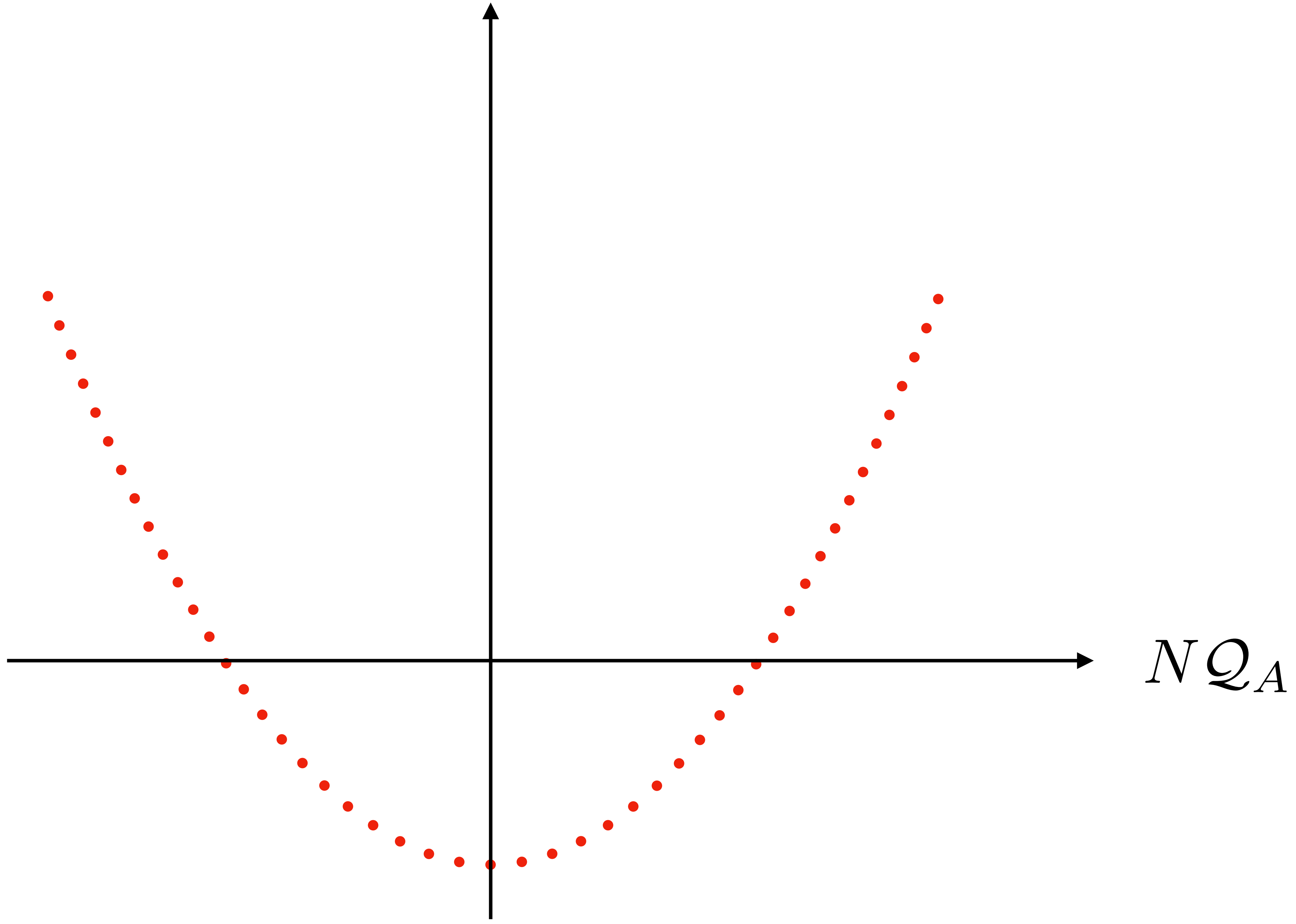
- here, *q* is constant - we can choose!

$$\frac{2\,M_{\rm P}^4\,\mathcal{Q}_A}{3\mathcal{T}_A^2} = q > 1 \quad or \quad < 1$$

[Kaloper; Kaloper & AW ’22]

BP/BT:

$$V = \Lambda_0 + N^2 \mathcal{Q}_A^2 \quad , \quad \Lambda_0 < 0$$



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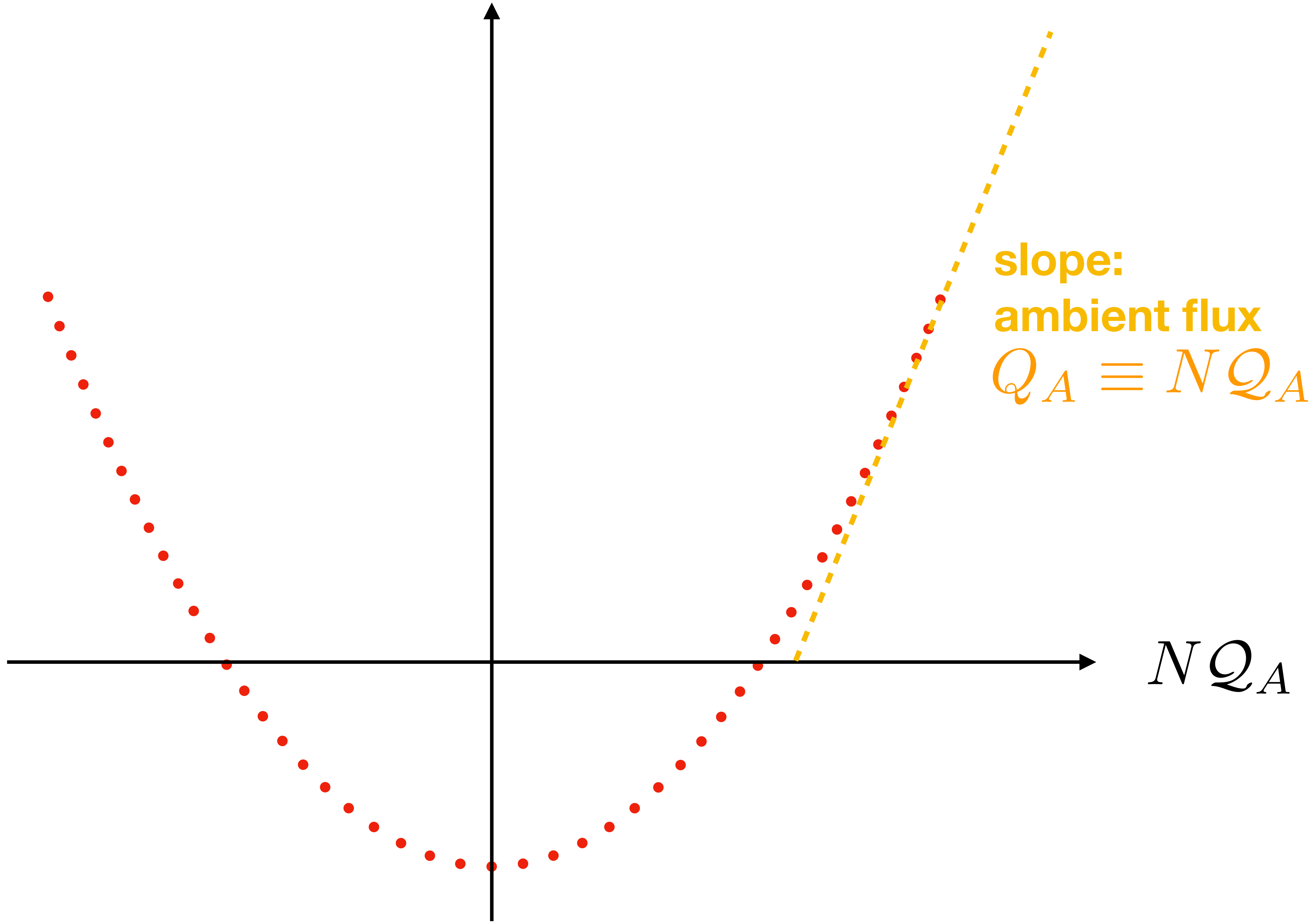

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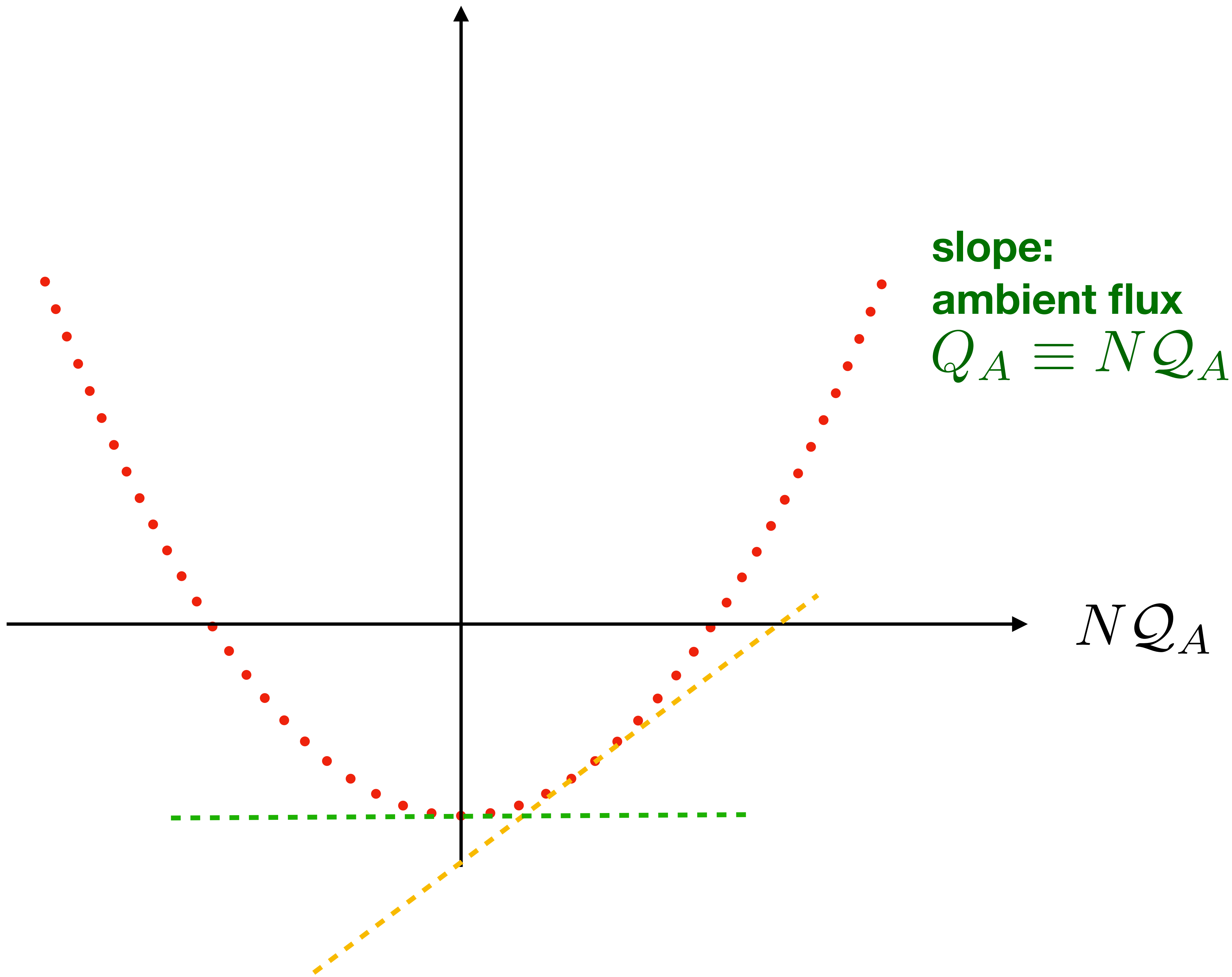


Bounce Action and Decay Rate

- tunneling rate & bounce action:

$$\Gamma \sim e^{-S(\texttt{bounce})} \qquad S(\texttt{bounce}) = S(\texttt{instanton}) - S(\texttt{parent})$$

- on-shell bounce action - evaluated at critical radius:

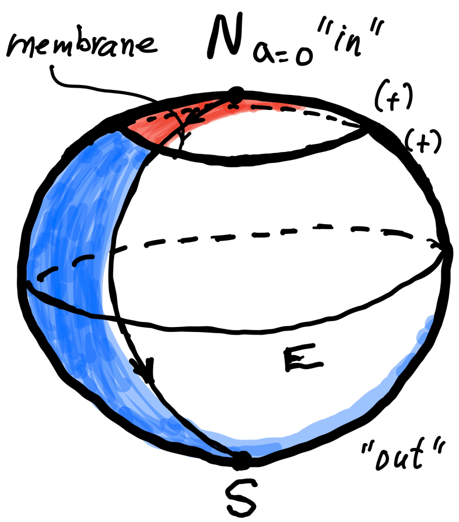
$$S(\texttt{bounce}) = 2\pi^2 \Big\{ \Lambda_{out} \int_{North\ \ Pole}^{a} da \Big(\frac{a^3}{a'}\Big)_{out} - \Lambda_{in} \int_{North\ \ Pole}^{a} da \Big(\frac{a^3}{a'}\Big)_{in} \Big\} - \pi^2 a^3 \mathcal{T}_A$$

$$2\pi^2 \Lambda_{in/out} \int_{North\ \ Pole}^{a} da \Big(\frac{a^3}{a'}\Big) = 18\pi^2 \frac{M_{\rm P}^4}{\Lambda_{in/out}} \Big(\frac{2}{3} - \zeta_{in/out} \big(1 - \frac{\Lambda_{in/out} a^2}{3M_{\rm P}^4}\big)^{1/2} + \frac{\zeta_{in/out}}{3} \big(1 - \frac{\Lambda_{in/out} a^2}{3M_{\rm P}^4}\big)^{3/2}\Big)$$

- rate calculable for instanton menu;
  divergent case are crossed out
- eq.s identical to Brown-Teitelboim;
  final rates depend on junction condition signs

Comparison of Decay Rates

[Brown & Teitelboim ’87/’88; Bousso & Polchinski ’00]



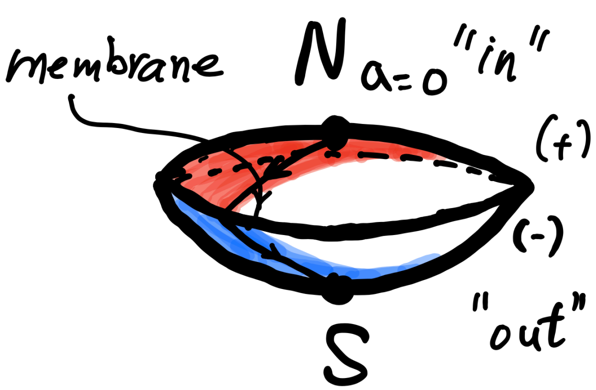
gold

$$S_{\text{bounce}} \simeq \frac{27\pi^2}{2}\frac{\mathcal{T}_A^4}{(\Delta\Lambda)^3} \simeq 108\pi^2\frac{\mathcal{T}_A^4}{M_{\rm P}^6 Q_A^3} \qquad for \quad q > 1$$

- overshoots $\Lambda = 0$ into AdS

- process absent for $q < 1$ green

[Kaloper; Kaloper & AW ’22]



$$S_{\text{bounce}} \simeq \frac{24\pi^2 M_{\rm P}^4}{\Lambda_{out}}\left(1 - \frac{8}{3}\frac{M_{\rm P}^2\,\Lambda_{out}}{\mathcal{T}_A^2}\right) \qquad for \quad q < 1$$

- dependence on parent Λ persists for dS $\longrightarrow$ AdS transitions

→ this “brakes” the evolution

Cosmological Constant: No Problem!

- Define the problem first

$$\Lambda_{\mathtt{total}} = M_{\mathrm{P}}^2 \left(\frac{\mathcal{M}_{\mathtt{UV}}^4}{\mathcal{M}^2} + \frac{V}{\mathcal{M}^2} + \lambda \right), \qquad \lambda = \lambda_0 + N \frac{\mathcal{Q}_A}{2},$$

- So:

$$\Lambda_{\mathtt{total}} = M_{\mathrm{P}}^2 \left(\frac{\Lambda_0}{\mathcal{M}^2} + N \frac{\mathcal{Q}_A}{2} \right),$$

- Thus the CC is unstable - BUT - to make it arbitrarily small eventually we must either take a tiny membrane charge or fine tune initial value

- This is the problem.

The Fix: add ≥ 1 extra flux & charge

$$S = S[g, A] + \int d^4x \frac{\Lambda}{M_{\rm P}^2} \epsilon^{\mu\nu\rho\sigma} \hat{F}_{\mu\nu\rho\sigma} - \mathcal{T}_{\hat{A}} \int d^3\xi \sqrt{\gamma_{\hat{A}}} - \mathcal{Q}_{\hat{A}} \int \hat{A}_3$$

$$\frac{\mathcal{Q}_{\hat{A}}}{\mathcal{Q}_A} = \omega \in (\textbf{nearly})\textbf{Irrational Numbers}$$

[Banks, Dine & Seiberg '88]

- As a result: $\Lambda_{\tt total} = M_{\rm P}^2 \Big(\frac{\Lambda_0}{\mathcal{M}^2} + \frac{\mathcal{Q}_A}{2} (N + \hat{N}\omega) \Big) \,.$
- $N, \hat{N}$ are integers; there exist $N, \hat{N}$, such that CC <<< 1
- long tunneling sequences:
  `green' instantons 'jump' CC down as long as CC > 0
- slow-down near zero CC

$$S_{\tt bounce} \simeq \frac{24\pi^2 M_{\rm P}^4}{\Lambda_{out}} \to \infty \quad \Rightarrow \quad \Gamma \to 0$$

[Kaloper & AW '22]

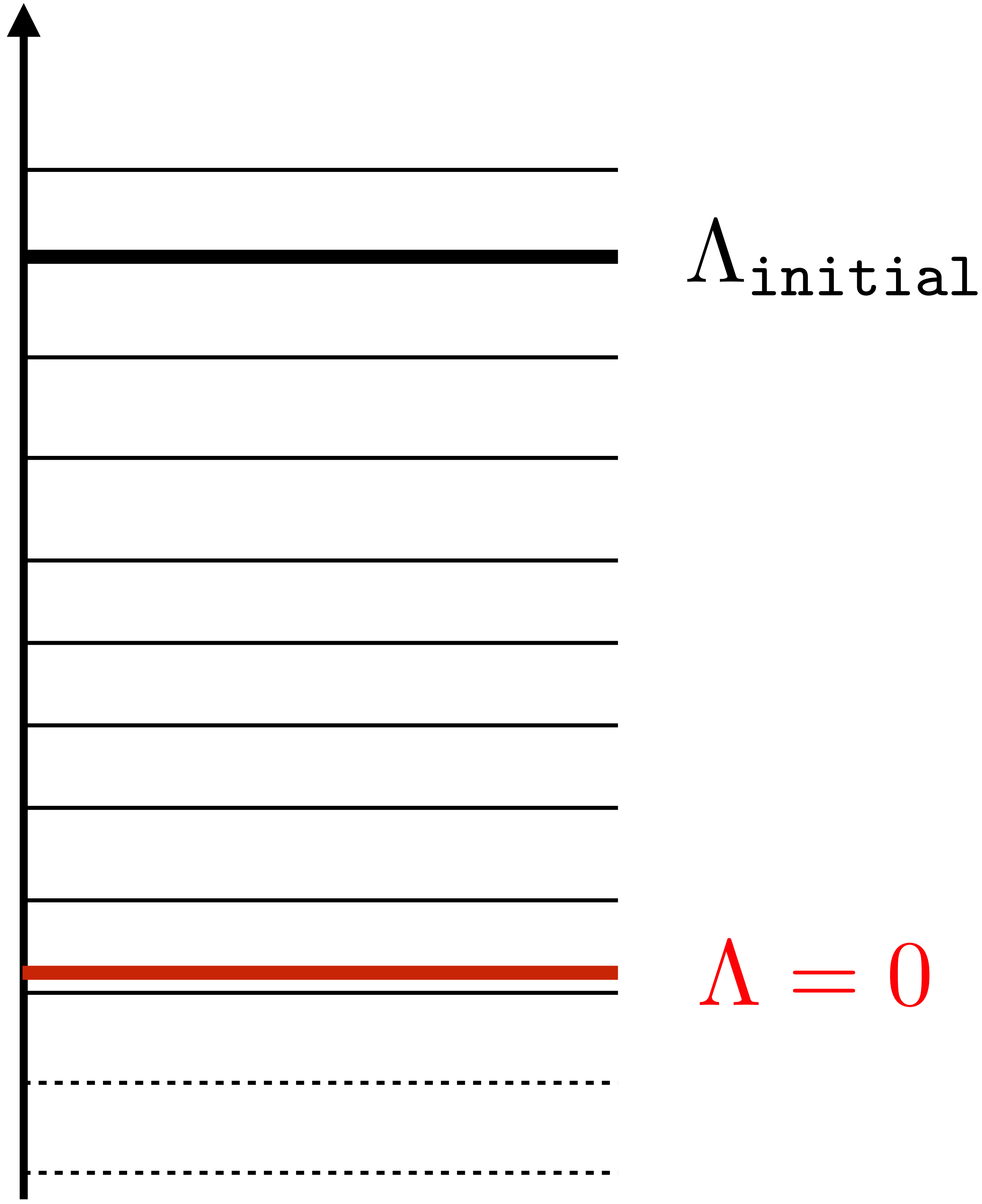
Stairway in Heaven

to dynamically get small CC,

need ≥ 2 stairways somewhat out of step

… *a landscape*

+ jumps stopping at zero CC (green instantons)



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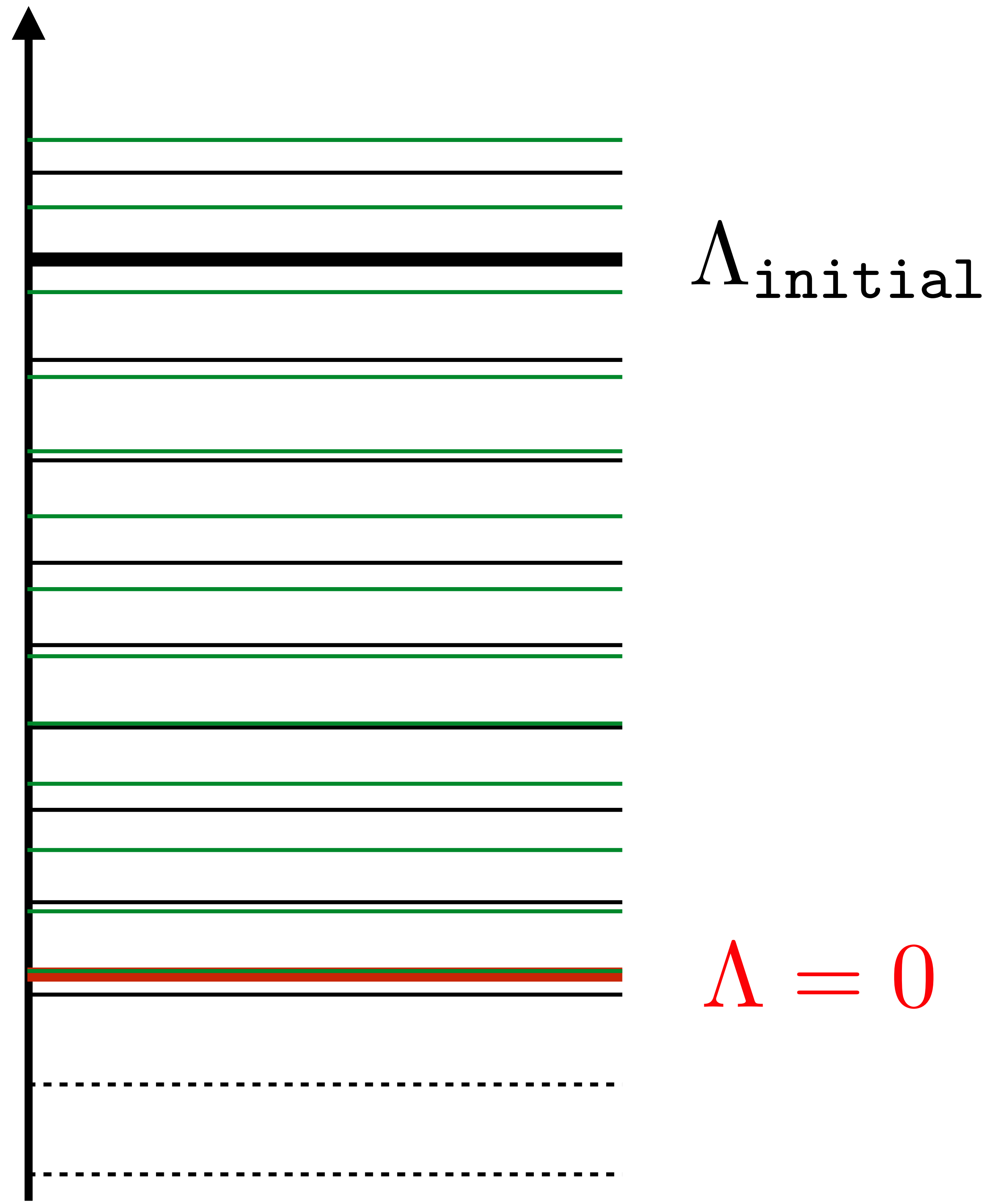

[Kaloper & AW ’22]

Approximate Density of States

- discrete evolution ~ Hawking-Baum CC-distro ['84]

$$Z = \int e^{-S_E} \simeq e^{-S_{classical}} = \begin{cases} e^{24\pi^2 \frac{M_{\rm P}^4}{\Lambda}} = e^{\frac{A_{\tt horizon}}{4G_N}}\,, & \Lambda > 0\,; \\ e^{\Lambda \int d^4x\sqrt{g}} = 1\,, & \Lambda = 0\,; \\ e^{-|\Lambda| \int d^4x\sqrt{g}} \to 0\,, & \Lambda < 0\,, \quad \text{noncompact}\,. \end{cases}$$

- The conclusion is:
  - with irrational charge ratio *or* many fluxes/charges
  - \`green instanton' dominance — $q < 1$

$$\frac{\Lambda}{M_{\rm P}^4} \to 0 \qquad \text{without anthropics!}$$

[Kaloper & AW '22]

Summary

- GR with CC linearly coupled to 4-form fields with membranes discharges CC via tunneling jumps;

  dynamically stops at CC = 0^+

  need several almost mutually irrational charges to approach zero CC close enough with finitely large CC jumps

- dS is unstable and decays towards Minkowski - this is desirable, since it can relax CC & removes eternal dS

- small-CC dS may be pretty long lived - a good thing, too

- CC jumps are large — inflation possible in principle: work in progress

- SM parameters may also be subject to such discrete variations, is there a connection?

stringy musings …

$$S^{(10)}_{MIIA} = \int -\tfrac{1}{2}\hat{R} * \mathbf{1} - \tfrac{1}{4} d\hat{\phi} \wedge * d\hat{\phi} - \tfrac{1}{4} e^{-\hat{\phi}} \hat{H}_3 \wedge * \hat{H}_3 - \tfrac{1}{2} e^{\frac{3}{2}\hat{\phi}} \hat{F}_2 \wedge * \hat{F}_2$$

$$-\tfrac{1}{2} e^{\frac{1}{2}\hat{\phi}} \hat{F}_4 \wedge * \hat{F}_4 - \tfrac{1}{2} e^{\frac{5}{2}\hat{\phi}} (m^0)^2 * \mathbf{1} + \mathcal{L}_{\text{top}} \ ,$$

$$\mathcal{L}_{\text{top}} = -\tfrac{1}{2}\Big[\hat{B}_2 \wedge d\hat{C}_3 \wedge d\hat{C}_3 \ - (\hat{B}_2)^2 \wedge d\hat{C}_3 \wedge d\hat{A}_1 + \tfrac{1}{3} (\hat{B}_2)^3 \wedge (d\hat{A}_1)^2$$

$$-\tfrac{m^0}{3} (\hat{B}_2)^3 \wedge d\hat{C}_3 + \tfrac{m^0}{4} (\hat{B}_2)^4 \wedge d\hat{A}_1 + \tfrac{(m^0)^2}{20} (\hat{B}_2)^5\Big] \ ,$$

$$\hat{H}_3 = d\hat{B}_2 \ , \quad \hat{F}_2 = d\hat{A}_1 + m^0 \hat{B}_2 \ , \quad \hat{F}_4 = d\hat{C}_3 - \hat{A}_1 \wedge \hat{H}_3 - \tfrac{m^0}{2} (\hat{B}_2)^2$$

speculation:

- **generate minimum at small VEV for the axions in B_2 using NP effects …**
- **generates hierarchically smaller coeff.s for $(m_0)^2$-terms than for terms linear in m_0**
- **$m_0\, dC_3$ provides a ΛF_4 coupling, & D8-branes on 6-manifold provide 4D membranes**