



# Scale Invariance as a Solution for Mass without Mass

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# Physics Today, 52 (Nov. 1999); 53 (Jan. 2000)

**F. Wilczek**

## REFERENCE FRAME

### Mass without Mass I: Most of Matter

Frank Wilczek

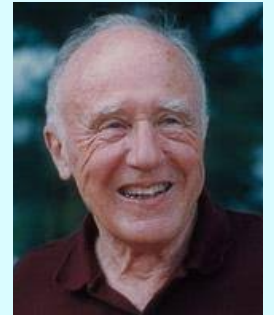
With his unique talent for the paradoxical profundity, John Wheeler coined the phrase “mass without mass” to advertise the goal of removing any mention of mass from the basic equations of physics.<sup>1</sup> Can we really hope to do this? How far have we come? Why should we try? In this piece, I answer the first question and part of the second; in my next column, I’ll round out the story and look ahead.



to reality while working with a truncated version of QCD, which contains only the color gluons plus up and down quark fields. The heavier quarks play an extremely minor role in the structure of the proton and neutron.

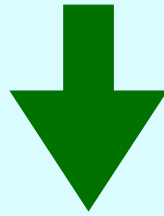
Our theory of the color gluons is derived from a powerful symmetry principle—non-Abelian, or Yang-Mills, gauge symmetry—similar in many respects to the general covariance of general relativity. Gauge sym-

**J. Wheeler, 1962**

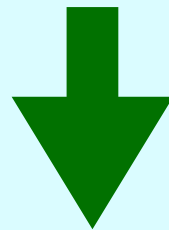


With his unique talent for the paradoxical profundity, John Wheeler coined the phrase “mass without mass” to advertise the goal of removing any mention of mass from the basic equations of physics.<sup>1</sup> Can we

**If we start with a theory containing a mass term from the beginning, we have no chance to explain its origin.**



**The Lagrangian should not contain any dimensionful parameter.**



**Scale Invariance**

# Fundamental energy scales

Electroweak scale  $E_{EW}$  ,

Planck scale  $M_{Pl}$  ,

Cosmological constant  $\Lambda$  ,

etc

Shift the question of why  $E_{EW} / M_{Pl}$   
and  $\Lambda / M_{Pl}$  are so small (hierarchy problem)



of where  $E_{EW}$ ,  $M_{Pl}$  and  $\Lambda$  come from (their origin)



Scale invariance is hardly broken  
by scale anomaly.

Callan, '70; Symanzik, '70

However, scale anomaly  
can not directly generate **mass gap.**

To generate a mass gap, scale invariance  
has to be **spontaneously broken.**



# Dynamical generation of $E_{EW}$

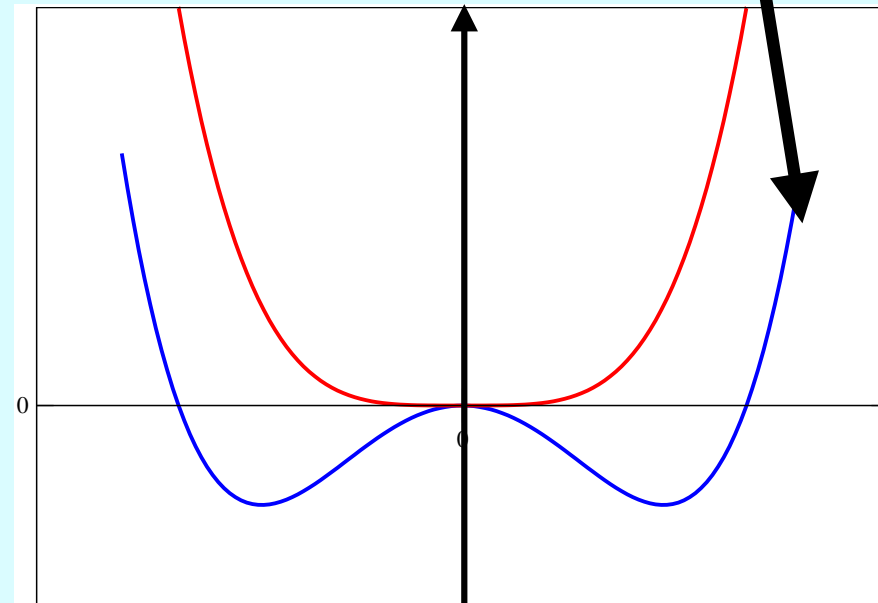
Coleman+Weinberg, '73

with  $\mu_H^2 = 0$  :

if  $m_{top} = 0$

Higgs potential

Scale anomaly



Unfortunately:  $m_H \sim 10 \text{ GeV}$



# Dynamical generation of $M_{Pl}$

## Induced gravity

with scalars

- \*Fujii '74
- \*Minkowski, '77
- \*Englert, Gunzig, Truffin+Windey, '75
- \*Minkowski, '77
- \*Chundnovsky, '78
- \*Fradkin+Vilkovisky, '78
- \*Zee, '79
- \*Smolin, '79
- \*Terazawa, '81
- \*Nieh, '82

.....

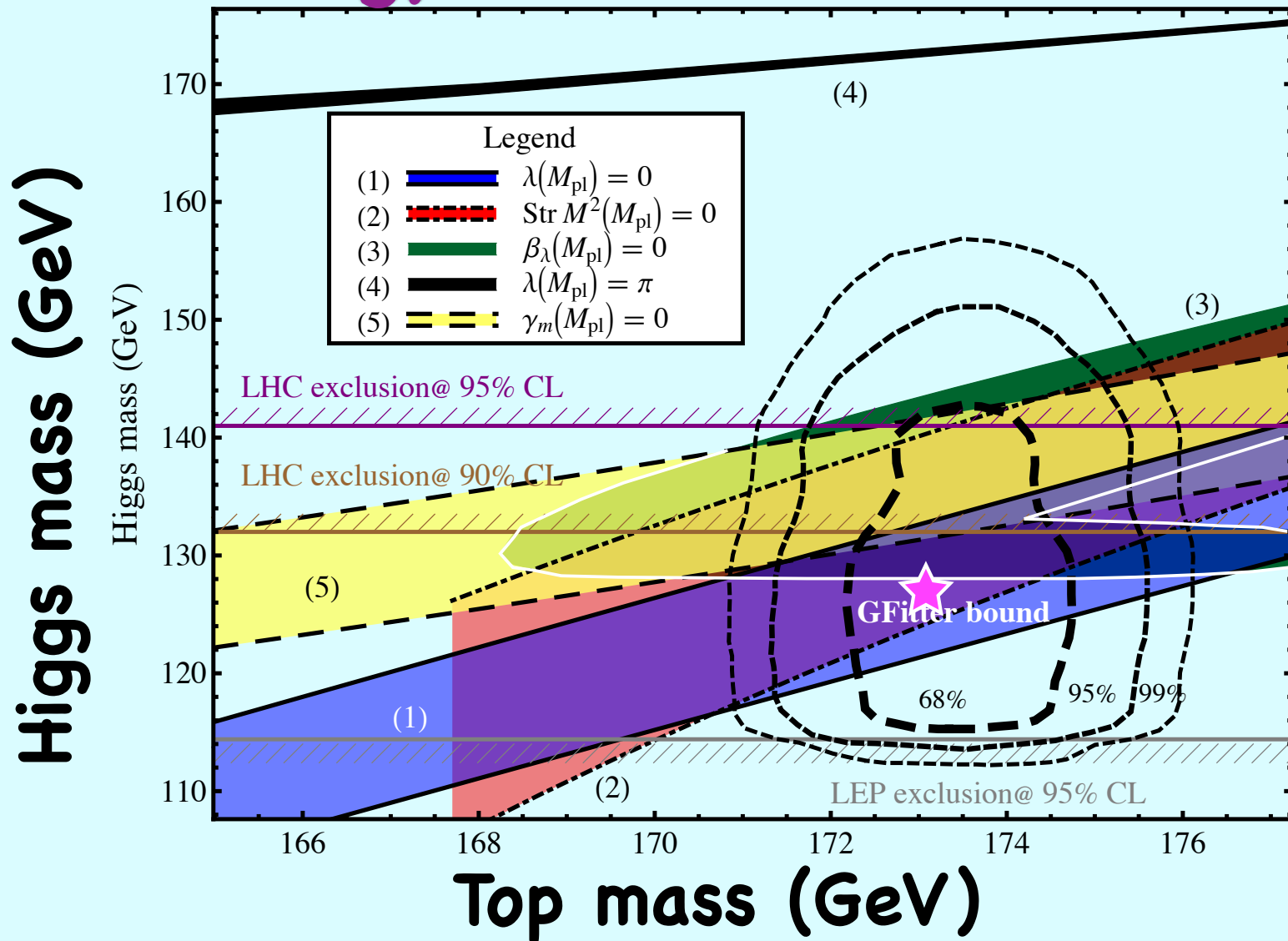
without scalars

- \*Akama, Chikashige+ Matsuki, '78
- \*Adler, '80
- \*Zee, '81

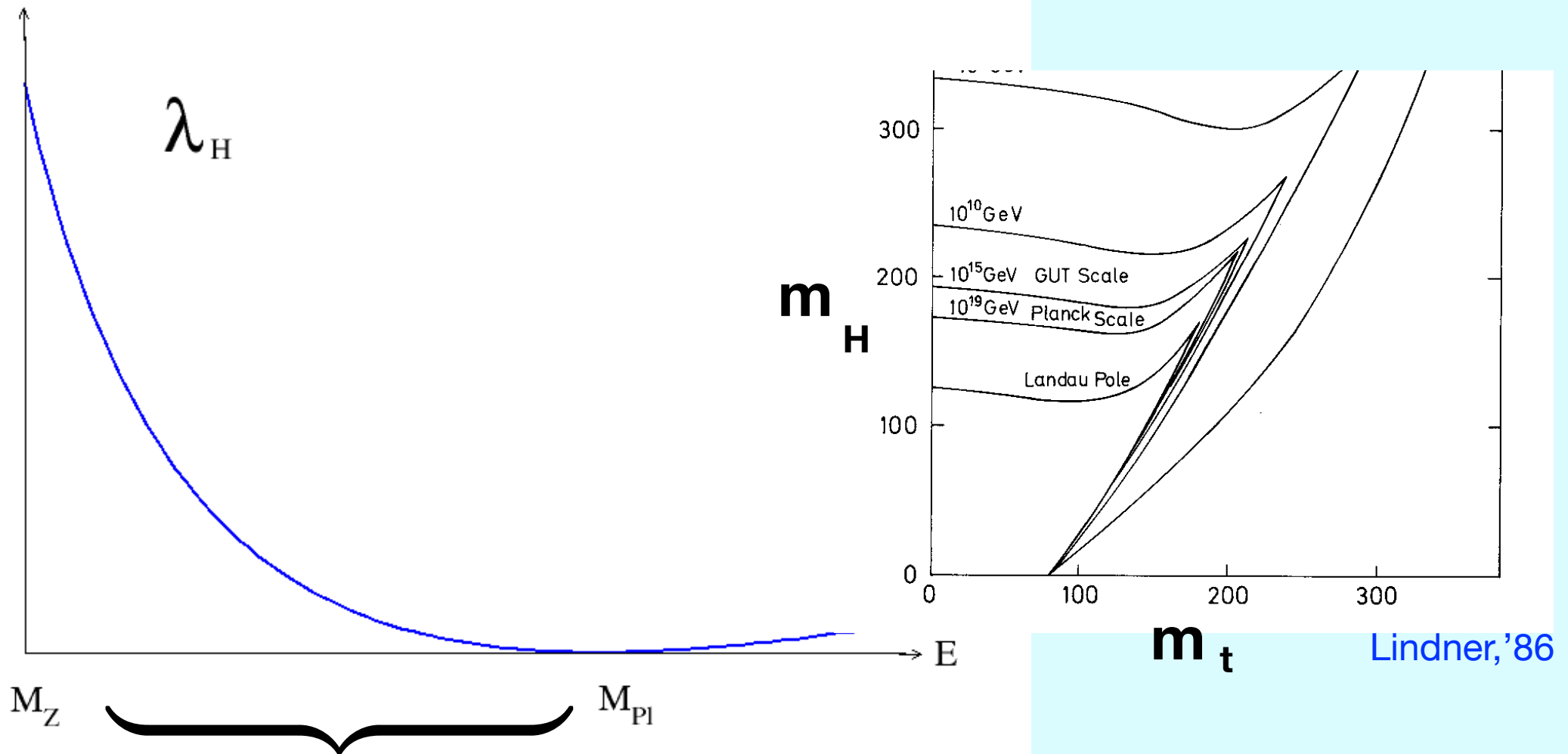
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# Indications for Scale Invariance: At low energy



Holthausen+Lee+Lindner, '12. See also  
Berzukov et al, '12; Degrassi et al '12, Buttazzo et al, '12



**Desert => Scale invariance is broken only by anomaly if  $\mu_H = 0$ .** Bardeen, '95

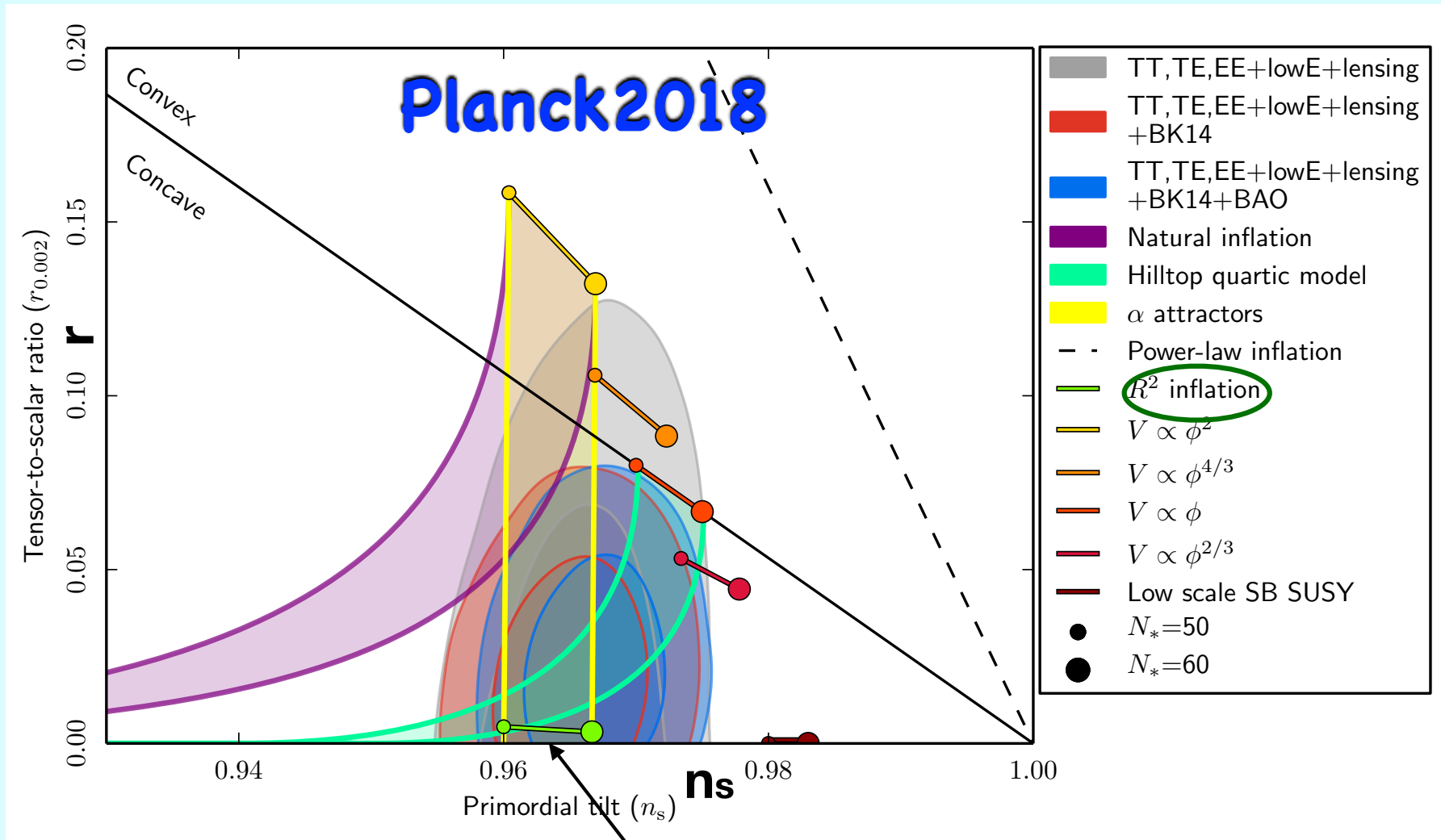
$\{\lambda_H, y_t, g_3\}$  - system is asymptotically free.

The max. value of  $y_t^2/\lambda_H \rightarrow m_t^2/m_H^2 \simeq 1.9$   $\left( [m_t^2/m_H^2]_{\text{exp}} \simeq 2.0 \right)$



# Indications for Scale Invariance: At high energy

Planck Collaboration: Constraints on Inflation

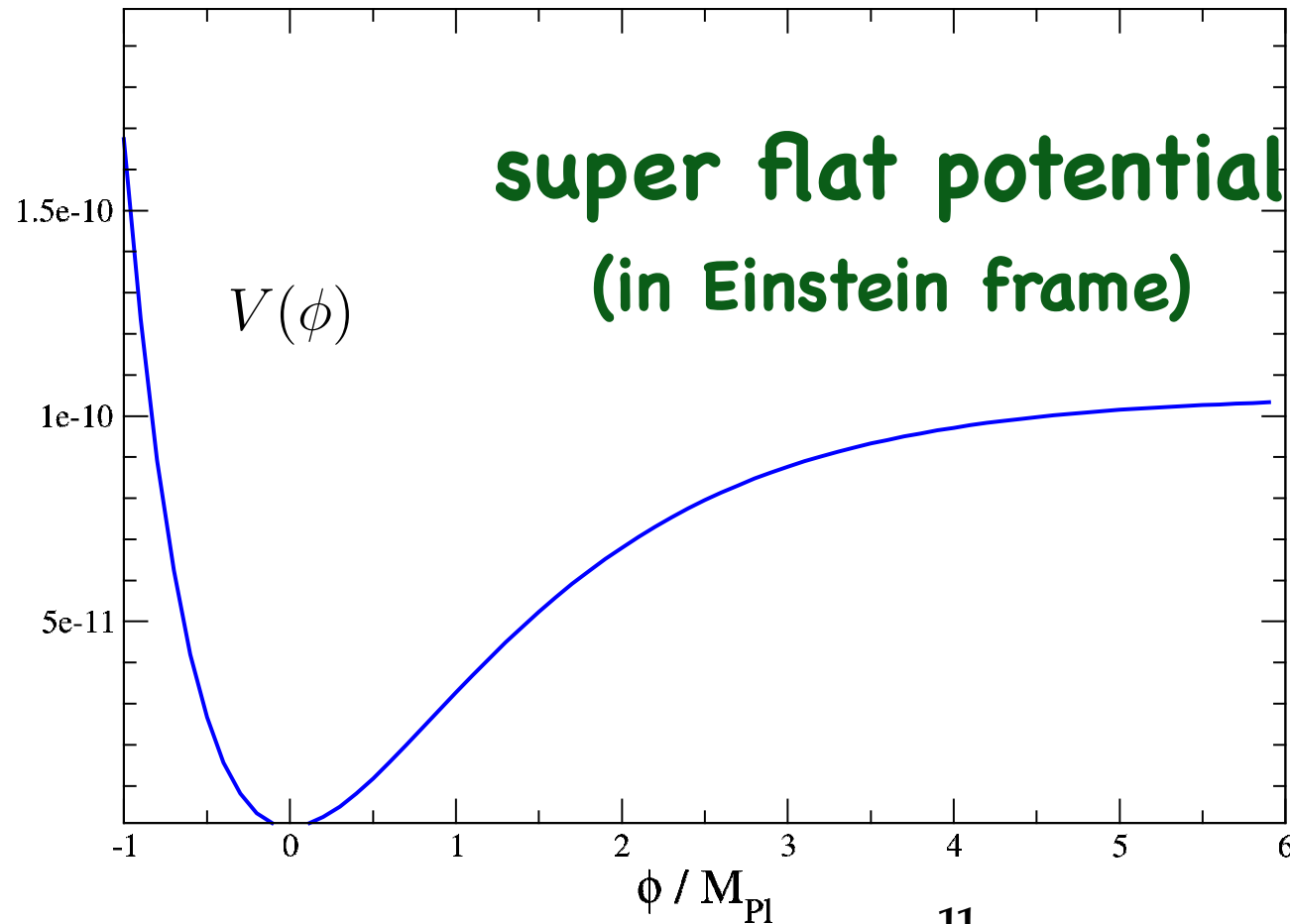


**$R^2$  inflation**

# scale invariant

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{M_{\text{Pl}}^2}{2} R + \left\{ \begin{array}{ll} \gamma R^2 & (\gamma \sim 10^9) \\ \beta |H|^2 R - \lambda_H |H|^4 & (\beta \sim 10^4) \end{array} \right.$$

for  $\left\{ \begin{array}{l} R^2 \text{ inflation, Starobinsky, '80} \\ \text{Higgs inflation, Bezrukov and Shaposhnikov, '08.} \end{array} \right.$



$$r \propto \left( \frac{V'(\phi)}{V(\phi)} \right)^2$$

# Scenarios for scalegenesis

## \* Perturbative scenario

$\langle \phi \rangle \neq 0$  by CW Coleman+Weinberg,'73; Gildener+Weinberg,'76

## \* Nonperturbative scenario

$\langle \bar{\psi}\psi \rangle \neq 0$  Nambu+Jona-Lasinio,'60;'61

$\langle S^\dagger S \rangle \neq 0$  Osterwalder+Seiler, 78; Fradkin+Shenker,'79

## \* By boundary condition

**(Inertial spontaneous scale symmetry breaking)**

Ferreira,Hill+Ross,'16;'18



## Nonperturbative scenario

- 
- 
- \*Weinberg, '76;79
- \*Susskind, '79

- 
- \*Hur, Jung, Ko, Lee, '07
- \*Hur, Ko, '11
- \*Heikinheimo et al, '13

- \*JK, Holthausen, Lim, Lindner, '13
- \*JK, Lim, Lindner, '14\*
- \*JK, Yamada, '15

\*

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## Perturbative scenario

- \*Coleman - Weinberg, '73
- \*Gildener - Weinberg, '76

- \*Hempfling, '96

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- \*Meissner, Nicolai, '07,
- \*Chang, Ng, Wu, '07
- \*Foot, Kobakhidze, Volkas, '07
- \*Espinosa, Quiros, '07
- \*Iso, Okada, Orisaka, '09
- \*Holthausen, Lindner, Schmidt, '09

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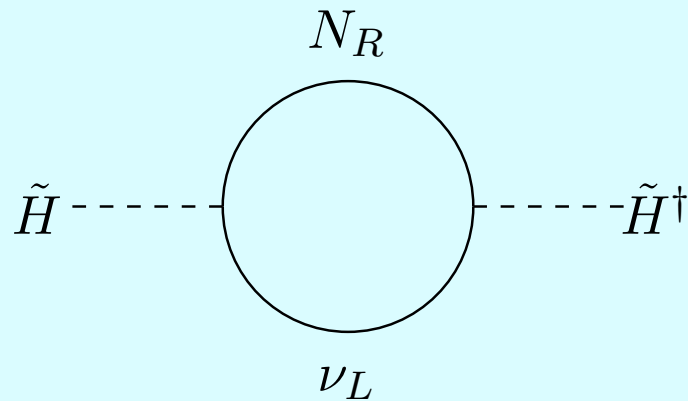
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- \*Jung, Lee+Nam, '19;
- \*Wetterich, 19;
- \*Salvio, 20 for reviews

# Application to the neutrino option

Brivo+Trott '17



$$\longrightarrow |\Delta\mu_H^2| \sim \frac{y_\nu^2 m_N^2}{4\pi^2}$$

Vissani,'97; Casa et al,'99;  
Clarke et al,'15; Bambhaniya et al,'16

## Use it to trigger EW symmetry breaking

Brivo+Trott '17

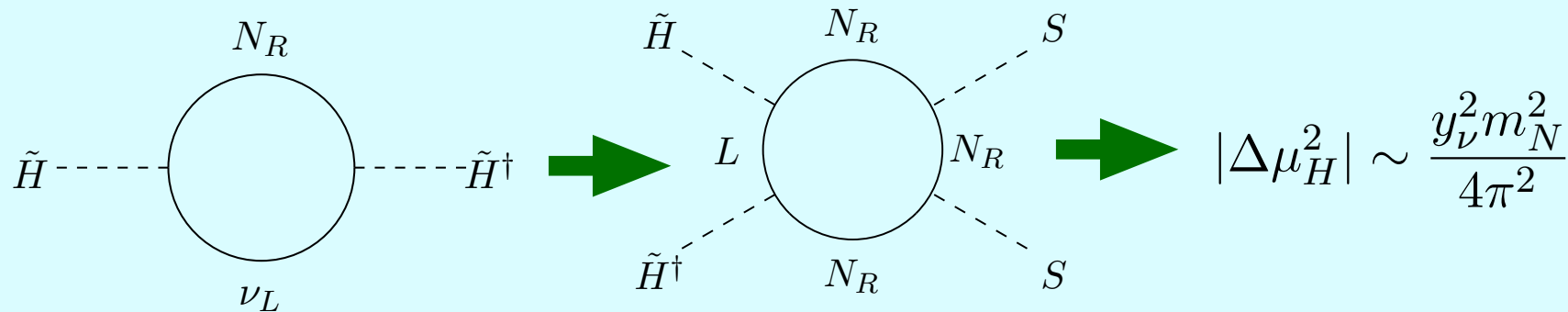
$$|\Delta\mu_H^2| \sim \frac{y_\nu^2 m_N^2}{4\pi^2} \sim (100 \text{ GeV})^2$$

if  $m_N \sim 10^7 \text{ GeV}$  and  
type I seesaw is used.

# Scale invariant extension by

Brdar, Emonds, Helmboldt+Lindner,'19

(talk by Rezaeck)



quadratic corr.  $\Rightarrow$  logarithmic corr.

to  $\lambda_{HS} S^2 H^\dagger H$  with  $\lambda_{HS} \ll \frac{y_\nu^2 y_N^2}{4\pi^2} \sim O(10^{-12})$  !!

**Higgs naturalness  $\Rightarrow$  Naturalness of  $\lambda_{HS}$**

Related topics: GW, Leptogenesis, etc

Brdar, Helmboldt+JK,'19;

Brivo+Trott,'19, '20; Brdar, Helmboldt, Iwamoto+Schmitz , '19;

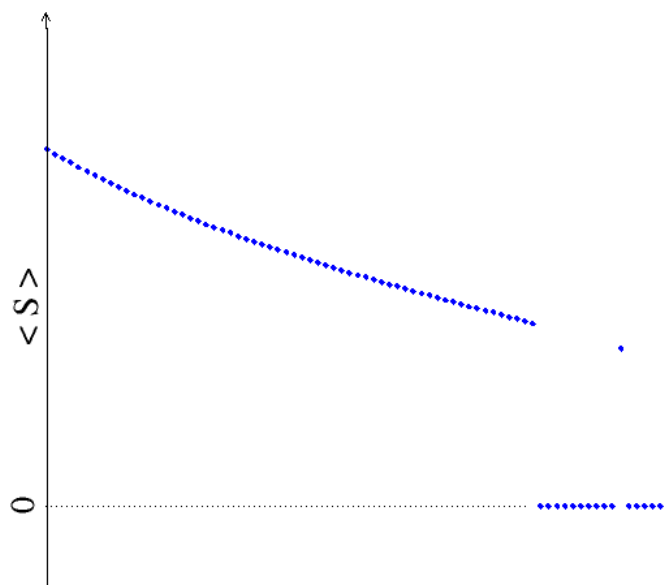
Brivo, Moflat, Pascoli+Turner,'20 Aoki, Brdar+JK,'20



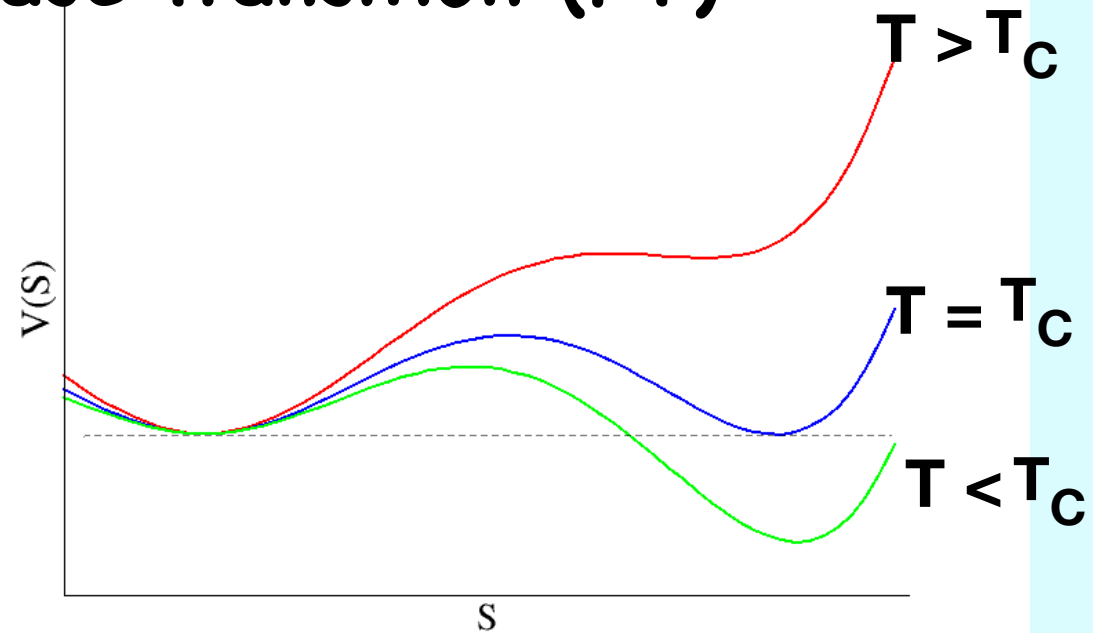
# Characteristics

of spontaneous scale symmetry breaking  
at finite temperature

## 1st Order Phase Transition (PT)



$T_C$



at the laboratory ( $H=0$ )

# In the expanding Universe

## Nucleation rate

$$\Gamma \simeq H^4 \quad \text{for PT to start} \Rightarrow T_n < T_C$$

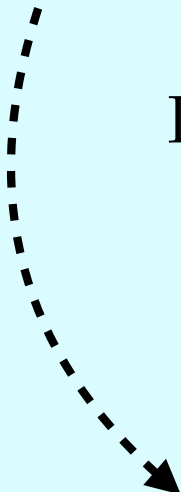
(n=nucleation)

$$\Gamma(T) \simeq T^4 \left( \frac{S_3}{2\pi T} \right)^{3/2} \exp(-S_3/T)$$

Linde, '83

(talk by Gonzalez)

$$\simeq \Gamma(T_n) e^{-\beta \Delta t} \quad \text{for } T \sim T_n \quad (S_3 = 3 - d \text{ Euclidean action})$$


$$\frac{S_3}{T_n} \sim 140$$

## $1/\beta$ duration of PT

$$\frac{\beta}{H} = T \frac{d}{dT} \left( \frac{S_3}{T} \right) \Big|_{T=T_n}$$

$$\frac{\beta}{H} \sim S_3/T_n \sim O(100) \quad \text{Hogan,'83; Witten,'84}$$

## In scale invariant theories:

$$\beta/H \left\{ \begin{array}{l} \sim O(10) \text{ for Coleman-Weinberg} \\ \text{Konstandin+Serpico,'11; Marzola,Racioppi+Vaskonen,'17;} \\ \text{Brdar,Helmboldt+JK,'18; Prokopec,Rezacek+Swiezewska,'19;....} \\ \sim O(10^{3\sim 4}) \text{ for } \langle \bar{\psi}\psi \rangle \neq 0 \text{ or } \langle S^\dagger S \rangle \neq 0 \end{array} \right.$$

JK+Yamada,'16; Helmboldt,JK+van der Woude,'19; Aoki+JK,'19



# Coleman-Weinberg scenario

$\beta/H$  is small  $\Rightarrow$  long lasting PT  $\Rightarrow T_n \ll T_C$

## Supercooled PT

If  $T_n < T_{EW}$ ,  $T_n >$  or  $< T_{QCD}$ ,  
a new cosmological history has to be drawn:

- \* DM production
- \* Baryogenesis
- \* EW PT
- \* Reheating etc

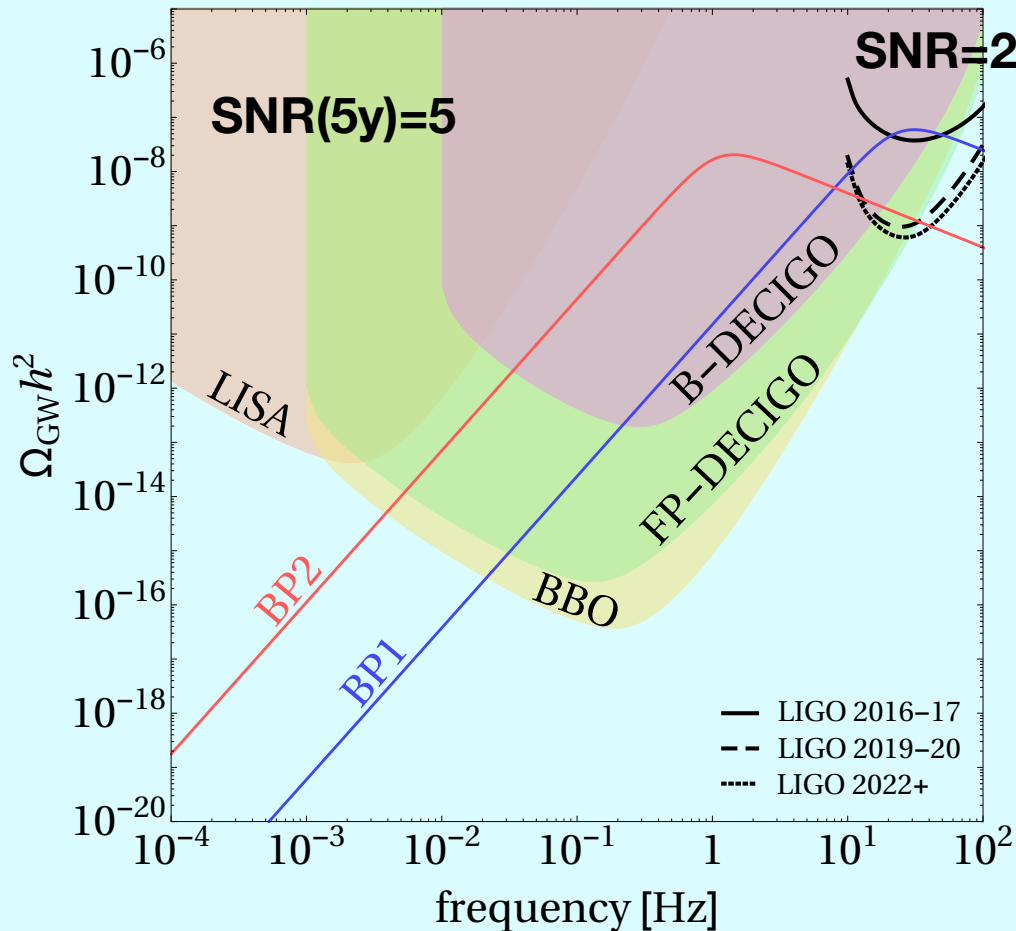
Konstandin+Servant,'11; Iso,Shimada+Serpico,'17;Hambye,Strumia+Teresi,'18,  
Jinno et al,'19; Brdar,Helmbolt+Lindner,'20; Döring et al,'21.....(talk by Döring)

# Gravitational Waves

$\beta / H$  is small  $\Rightarrow$  long lasting PT  $\Rightarrow T_n \ll T_C$

$\Rightarrow \langle S \rangle / T_n \gg 1 \Rightarrow$  strong PT  $\Rightarrow$  strong GW signal

Marzola, Racioppi+Vaskonen, '17;  
 Prokopec, Rezacek+Swiezewska, '19;  
 Marzo, Marzola+Vaskonen, '19;  
 Mohamednejad, '19; Ellis, Lewicki+No, '19;  
 Ellis, Lewicki+Vaskonen, '20, .....



**Scale invariant ext.  
of Neutrino Option**

Brdar, Helmboldt+JK, '18

$$T_C \simeq 2 \times 10^8 \text{ GeV}, \quad T_n \simeq 4 \times 10^4 \text{ GeV}, \quad \beta/H \simeq 10 \quad \text{for B1}$$

# Non-perturbative scenario

$$\frac{\beta}{H} = T \frac{d}{dT} \left( \frac{S_3}{T} \right) \Big|_{T=T_n} \sim \frac{S_3}{T} \sim O(100) \rightarrow f \sim 10^{-8} \text{ Hz}$$

Witten, '84

**No 1st principle calculation of  $S_3 / T$**

on going project ,USQCD, ,19

## Analysis based on EFTs

Helmboldt,JK+van der Woude,'19

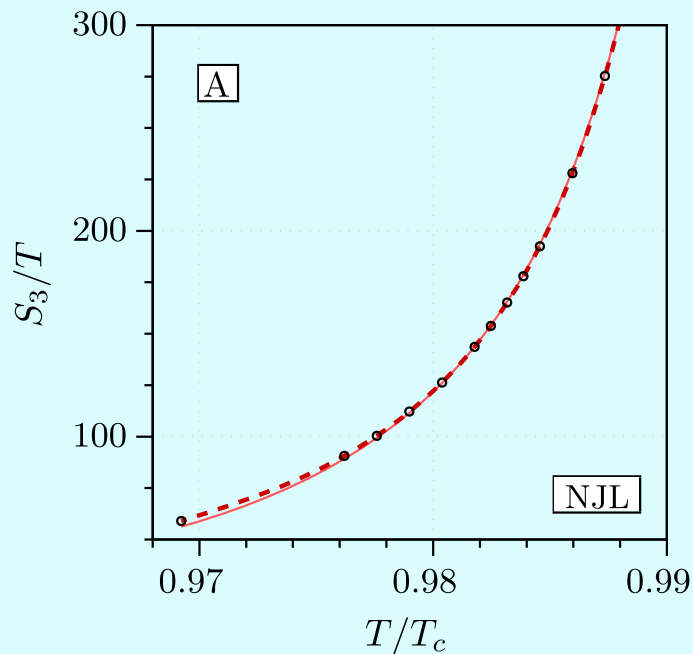
- Nambu-Jona-Lasinio (NJL)
  - Polyakov-NJL (PNJL)
  - Linear Sigma Model (LSM)
- Tsumura,Yamada+Yamaguchi,'17;  
Croon,Houtz+Sanz,'19

Holographic QCD, talk by Schwaller

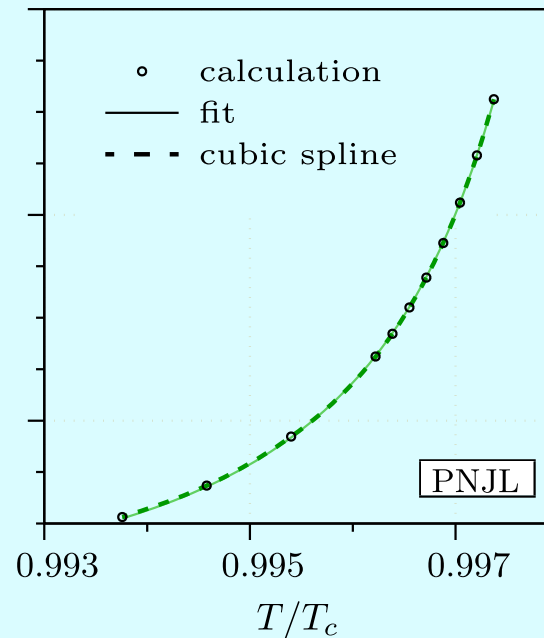
# Critical-Phenomena-like behavior of $S_3 / T$

$$\frac{S_3}{T} \propto \left(1 - T/T_C\right)^{-\nu} \quad \text{anticipated by Hogan, '83}$$

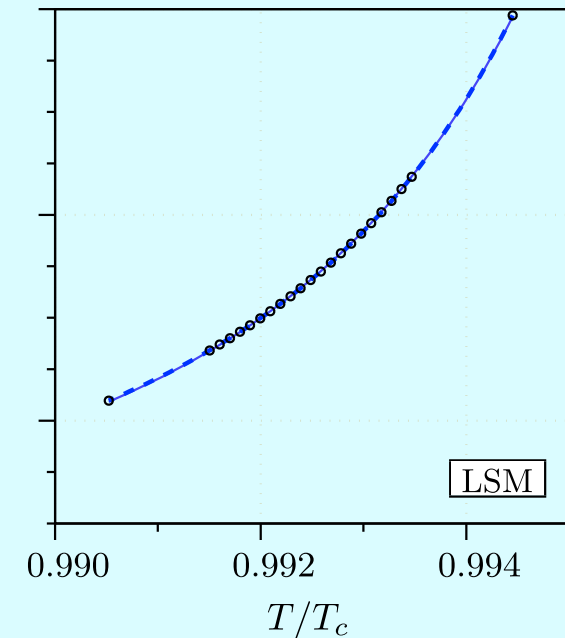
$\nu = 1.76$



$\nu = 1.82$



$\nu = 1.86$



Helmboldt, JK+van der Woude, '19



But  $\beta / H \sim 10^{(3 \sim 4)}$

$\Rightarrow f \sim 10^{-3}$  Hz

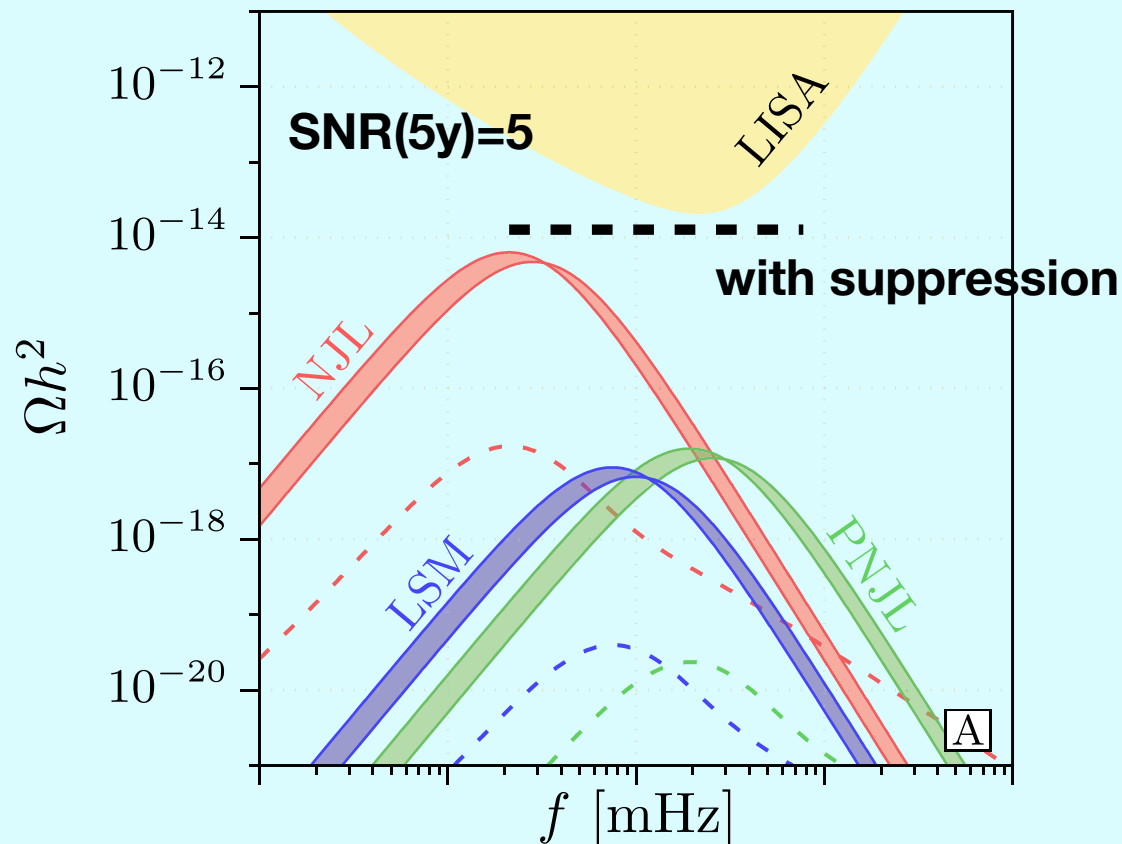
( $\sim 10^{-8}$  Hz by Witten,'84)

$\Rightarrow$  short PT

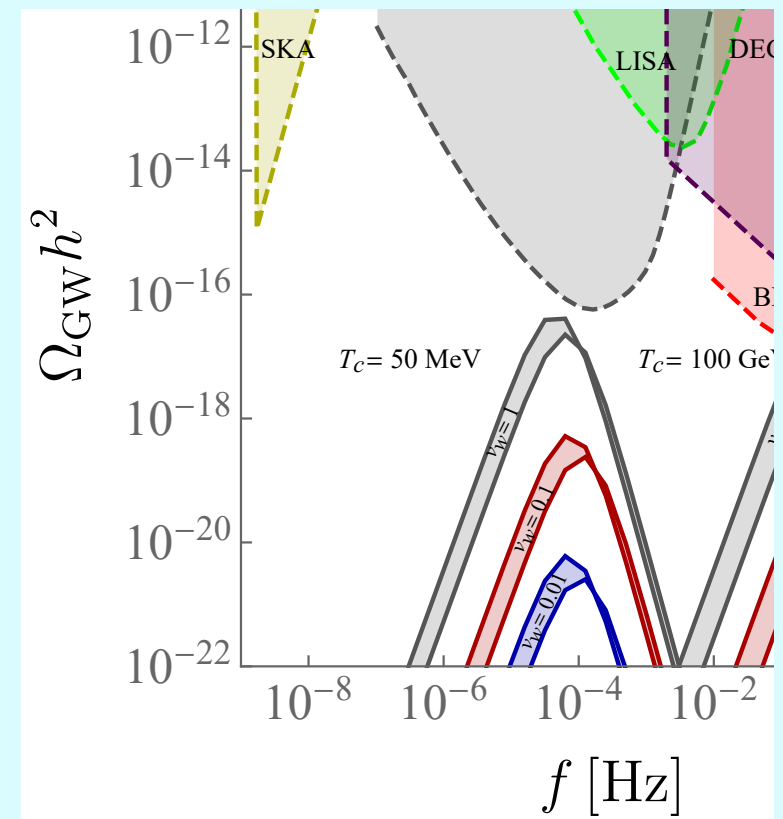
$\Rightarrow$  suppression of GW (sound wave)

The result is academic,  
because  $m_q = 0$ .

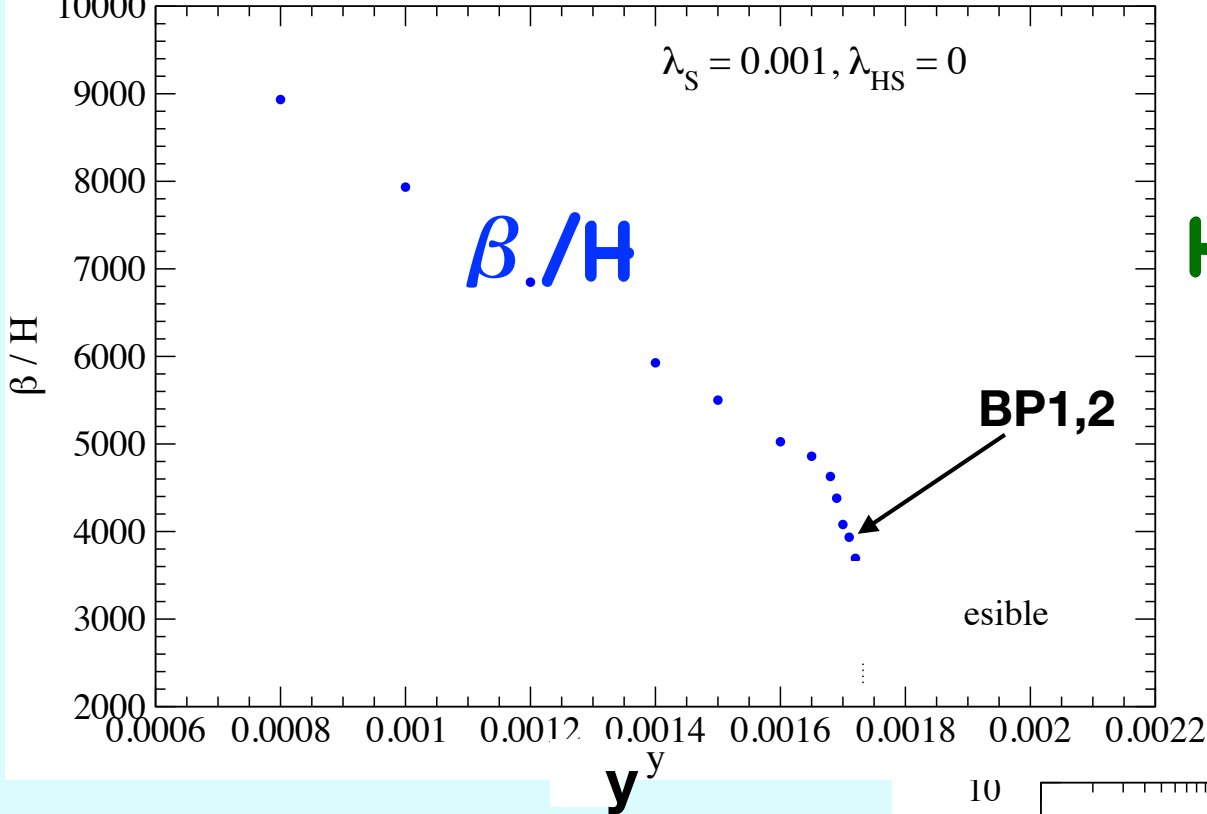
Ellis, Lewicki+No, '18; '19



Helmboldt, JK+van der Woude, '19



from Schwaller's talk



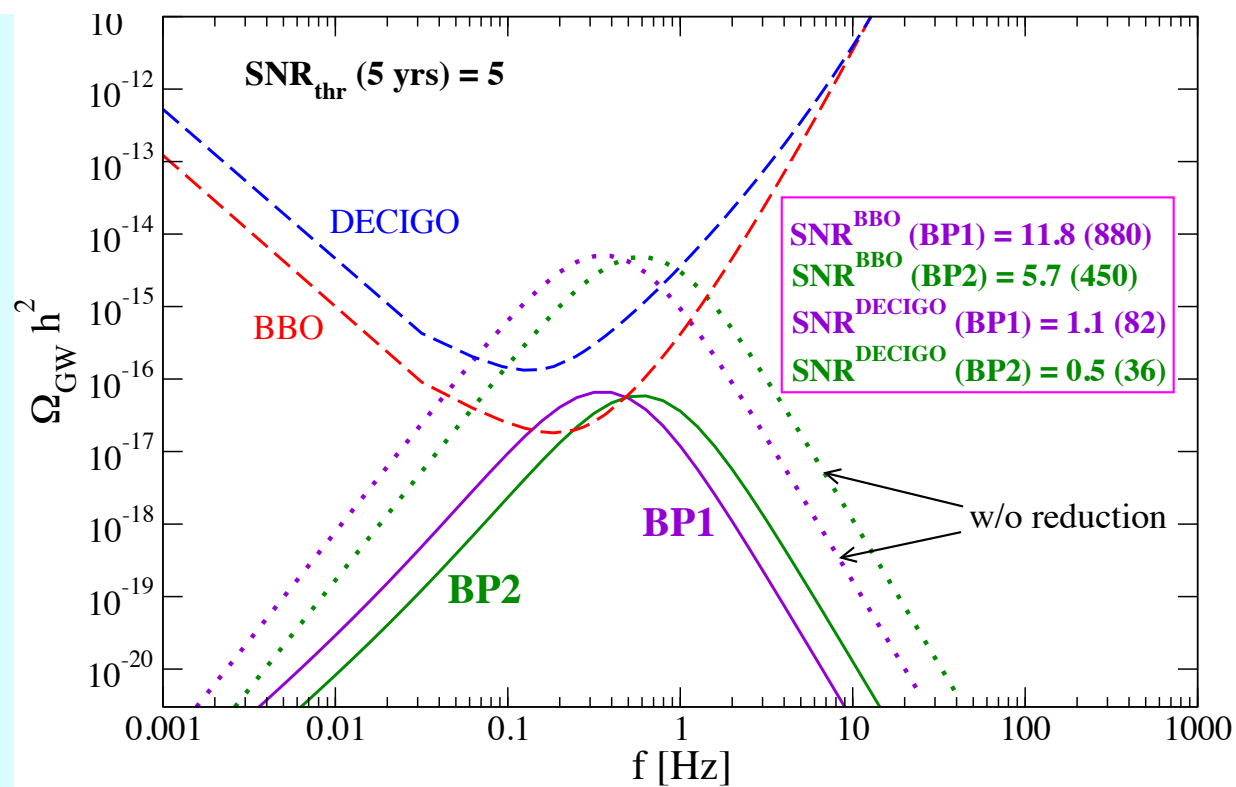
# Hidden QCD-like model

Aoki+JK, ,19

--- without suppression

$y S \bar{\psi} \psi$

## Hidden fermion



# Including Gravity

Spontaneous generation (SG) of  $M_{\text{Pl}}$   
= SG of Einstein-Hilbert theory

**The economic and simple way:**

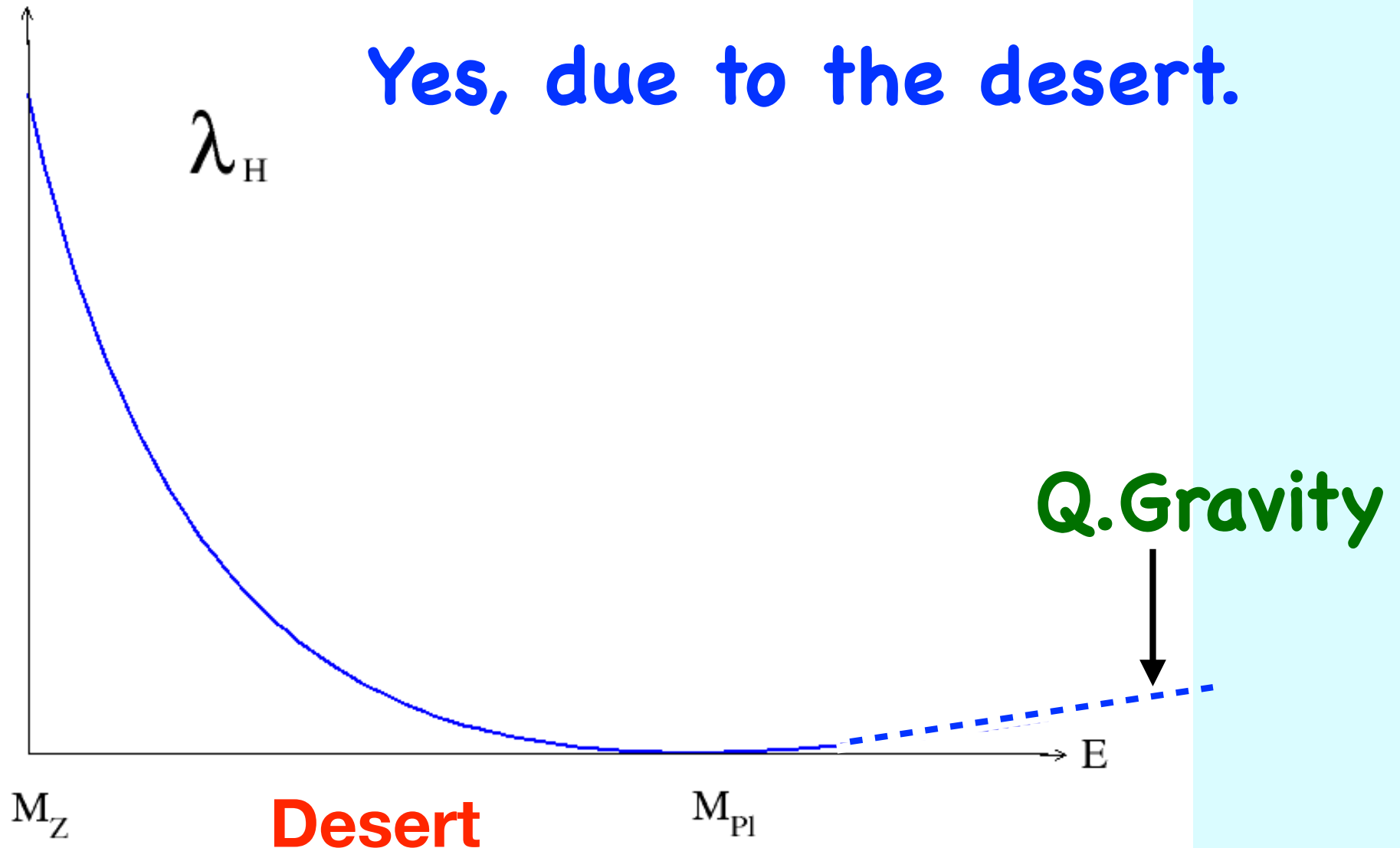
$$\frac{\xi_S}{2} S^2 R \rightarrow \frac{\xi_S}{2} \langle S \rangle^2 R \rightarrow \frac{M_{\text{Pl}}^2}{2} R$$

$$M_{\text{Pl}} = \sqrt{\xi_S} \langle S \rangle$$

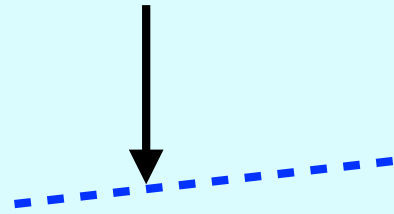
Brans+Dicke,'61; Fujii,'74; Englert+Truffin+Grastmans,'76; Minkowsky,'77;.....

Is the spontaneous generation of  $M_{Pl}$  related to SM?

Yes, due to the desert.



# Q. Gravity



Non-perturbative scenario: Fixed Point (asymptotic safety)

Weinberg,'78; Reuter,98; Don+Percacci,'98; Souma,'99;.....;  
Shapshnikov+Wetterich,'10; ..,Wetterich,'19... (talk by Gutierrez)

Perturbative senario

't Hooft+Veltman,'74; Deser+van Nieuwenhuizen,'74;  
Stelle,'77;Fradkin+Tseytlin,'82; .....;Salvio+Strumia,'14; '18;  
Salvio,'20

Strings ..... (talk by Dvali, Ibanez)

## Perturbative scenario

Salvio+Strumia,'14; '18; Salvio,'20

based on renormalizability  
of Quadratic Gravity (QG)

Stelle,'77

Quantum corrections can be controlled.

QG includes:

$$-\frac{1}{2}\xi_{ab}\phi_a\phi_b R + \gamma R^2 - \kappa W^2$$

$W$  = Weyl tensor

Origin of massive spin-two ghost (discussed by Koshelev)



# Naturalness of $\lambda_{HS}$

$$M_{\text{Pl}}^2 = \xi_S \langle S \rangle^2 \rightarrow \lambda_{HS} \langle S \rangle^2 H^\dagger H \rightarrow \lambda_{HS} \langle S \rangle^2 \lesssim O(m_H^2)$$



$$\lambda_{HS}/\xi_S \lesssim M_{\text{Pl}}^2/m_H^2 \sim 2.5 \cdot 10^{-33} !!$$

**Renormalizability + scale invariance**



**The naturalness may be achieved**

$$\mathcal{L} \supset -(\beta_H H^\dagger H + \frac{1}{2}\beta_S S^2) R + \gamma R^2 - \kappa W^2$$

$$\mu \frac{d\lambda_{HS}}{d\mu} = -\frac{\xi_S}{(4\pi)^2} \left( \frac{5\xi_H}{4} \kappa^{-2} + \frac{\xi_H}{36} \gamma^{-2} (1 + 6\xi_H)(1 + 6\xi_S) \right) + O(\lambda_{HS})$$

Salvio+Strumia, '14;'18

$$\lesssim 2.5 \cdot 10^{-33} \xi_S \quad \mathbf{0} \quad \infty \times 0 = \mathbf{??}$$

$$\left\{ \begin{array}{l} \kappa^{-1} \rightarrow 0 \quad \text{Fradkin+Tseytlin, '82;} \\ \gamma^{-1} \rightarrow \infty \quad \text{as } \mu \rightarrow \infty \text{ Avramidi, '95} \\ \xi_{H,S} \rightarrow -1/6 \end{array} \right.$$

Elizalde et al, '96; Yoon+Yoon, '97; Buchbinder, '92; Shapiro, '20

**Even  $\mu$  around  $M_{pl}$**

$$\Delta\lambda_{HS} \lesssim 2.5 \cdot 10^{-33} \xi_S$$

**Strong constraints on  $\xi_{H,S}$ ,  $\gamma$  and  $\kappa$**

$$\xi_{H,S} , \gamma , \kappa$$

I. responsible for the naturalness of  $\lambda_{HS}$

and at the same time

II. control the slow-roll parameters of inflation

$\gamma$  for  $R^2$  Inflation

Starobinsky,'80

$\xi_H$  for Higgs inflation

Bezrukov+Shaposhnikov,'07

$\kappa$  contributes to  $r$

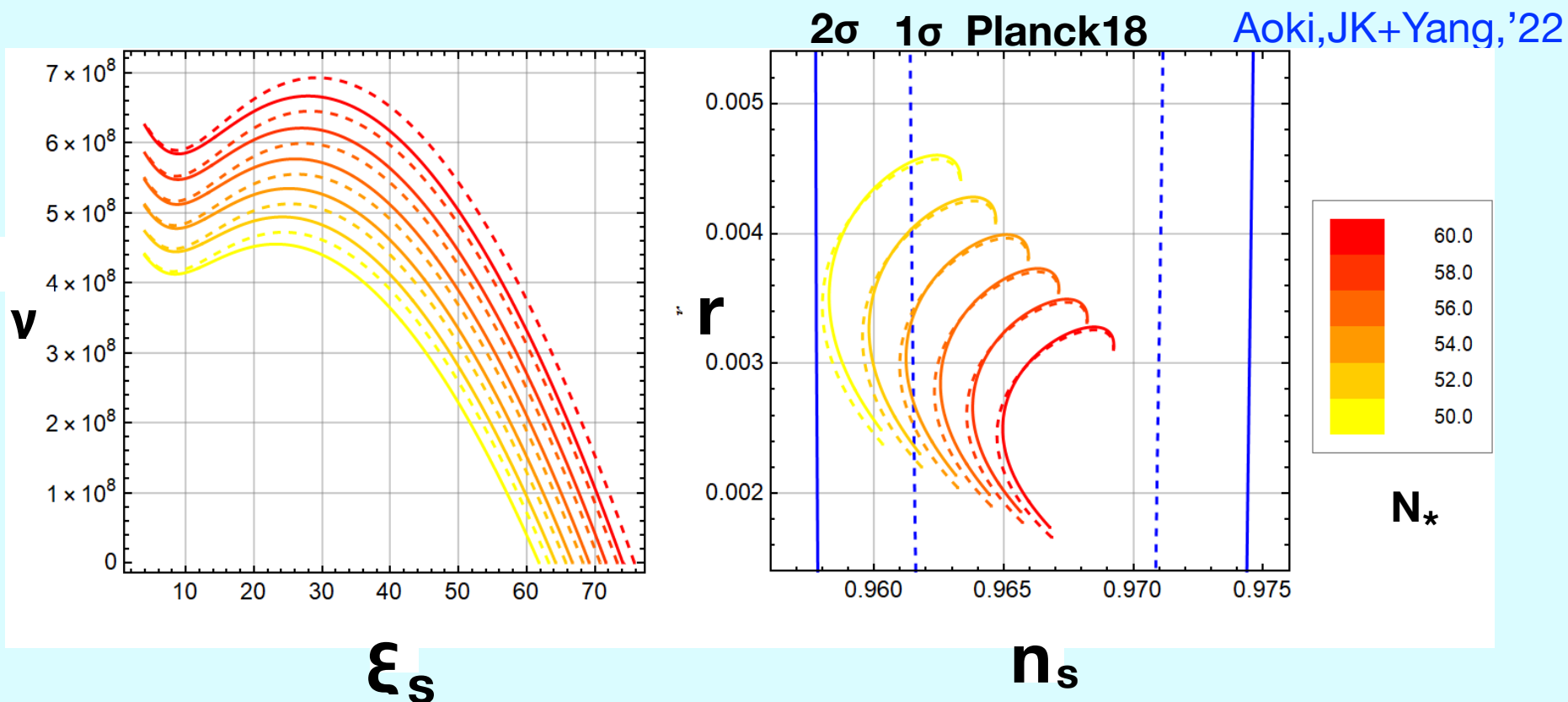
Baumann,H.Lee+Pimentel,'15; Salvio,'19

triggers SSB of scale invariance

JK,Kuntz,Rezacek+Saake, to appear

# Scale invariant extension of the Starobinsky model

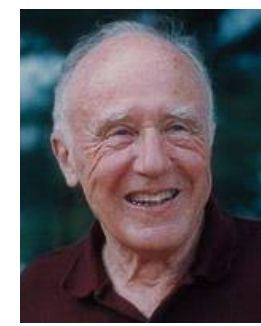
Salvio+Strumia,'14; Kannike et al,'15; Rinaldi et al,'14; Farzinnia+Kouwn,'15;  
 Ghilenea +H.M.Lee,'18; Karam+Pappas+Tamvakis,'18;JK,Lindner,Schmitz+Yamada,'18;  
 JK,Kuntz,Lindner,Rezacek,Saake+Trautner,'20; Ghoshal+Mukhejee+Rinaldi,'22;...  
 see e.g. Gialamas et al, 2104.04550 for more references (talk by Rezacek)



**Naturalness**  $\Rightarrow (\xi_H \lesssim 10^{-14}$  and  $\kappa \gtrsim 10^8$ ) OR  $(\xi_H \simeq -1/6$  and  $\kappa \gtrsim 10^{19})$

Theoretical aspect:

# Summary



1.  $E_{EW}$  and  $M_{Pl}$  can be spontaneously generated, both perturbatively and non-perturbatively
2. Naturalness of  $m_H \Rightarrow$  naturalness of  $\lambda_{HS} \Rightarrow$  multiplicative solution
3. The desert  $\Rightarrow$  the SM and Planck scale physics are directly related  
Naturalness of  $\lambda_{HS} \Leftrightarrow$  Inflationary parameters

Phenomenological aspect:

1. For CW breaking: Supercooling  $\Rightarrow$  new history of the Universe, strong GW
2. For non-perturbative scale symmetry breaking:  
Short duration of PT  $\Rightarrow$  suppression of GW  
should be confirmed by 1st principle calculations
3. Scale invariant extensions of the  $R^2$  model of Starobinsky  
 $\Rightarrow r \gtrsim r_{R^2} \sim O(10^{-3})$  and more than two scalars during inflation



Is it possible to distinguish these models by measuring small  $r$  and also non-Gaussianity in the CMB anisotropy and large scale structure by future experiments such as LiteBIRD, CMB-S4, Simons Observatory DESI, Euclid, LSST, etc ?



