Regions of different critical behavior of the two-flavor NJL model

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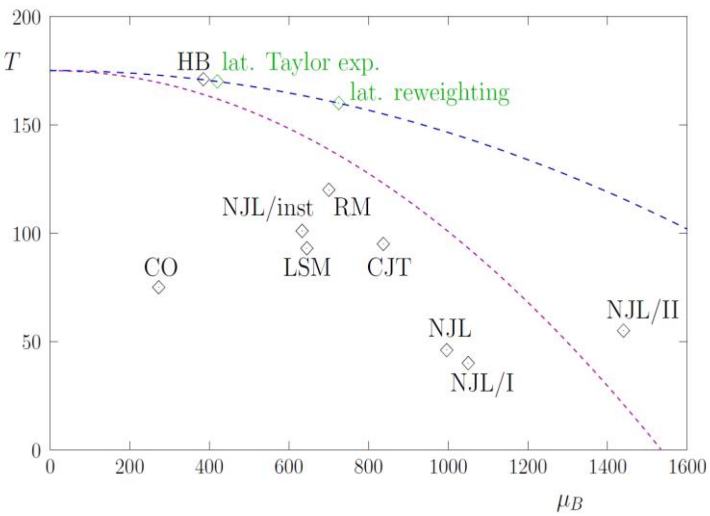
Juan Xiong and Jiarong Li, PRC83,025204(2011)

- **Background**
- **Regions of different critical behavior**
- **Summary**

Background



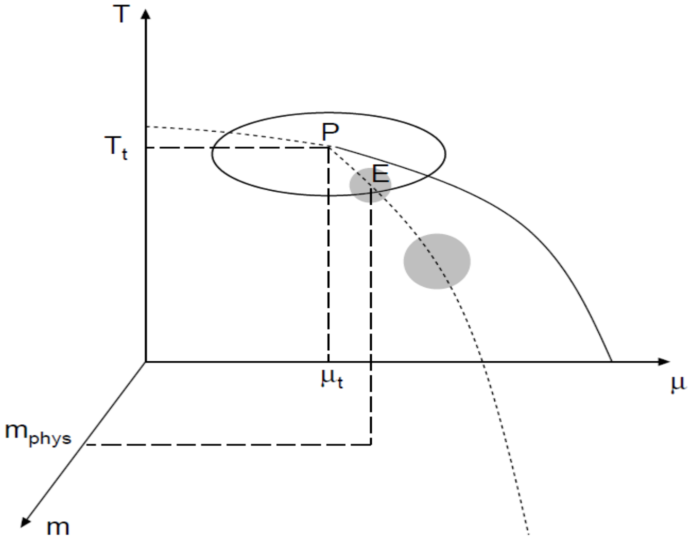
The position of the critical point

In experiment

collective flow, higher moments,…

In theory

Lattice QCD, effective model (defined from thermodynamics),…



The physics around the critical point

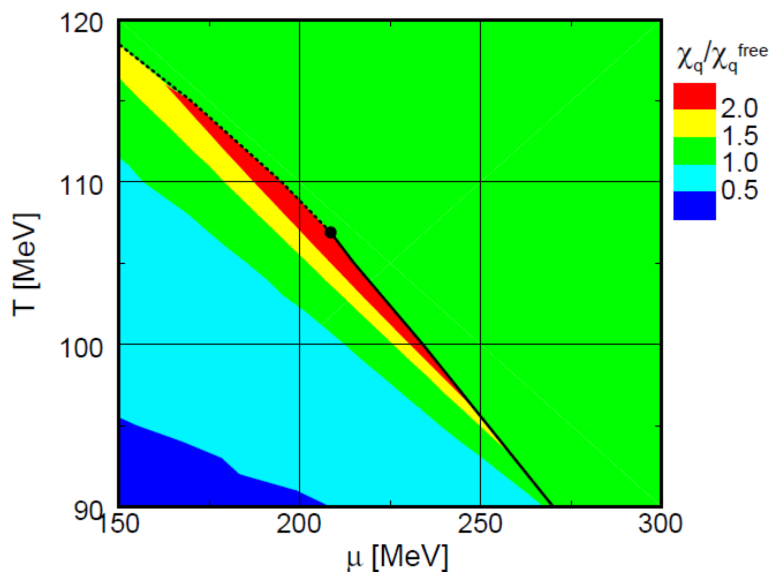
"The CP is not just pointlike, but has rich structure" (Y.Hatta, T.Ikeda, PRD(03))

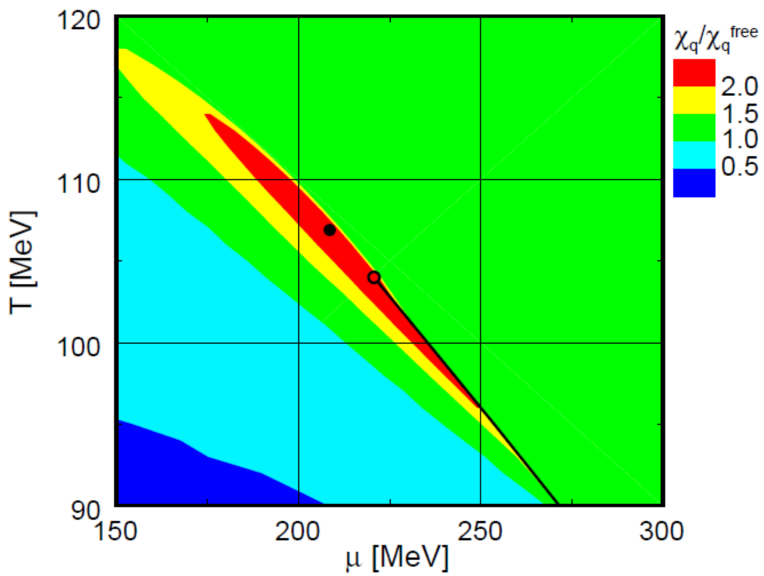
The critical region, The critical exponent

Critical region around critical point

Y. Hatta and T. Ikeda, Phys. Rev. D67,014028(2003)

CJT effective potential for QCD in the improved ladder approximation





Around the TCP $\chi_q = -\frac{1}{V}\frac{\partial^2\Omega}{\partial\mu^2}$ **Around the CEP**

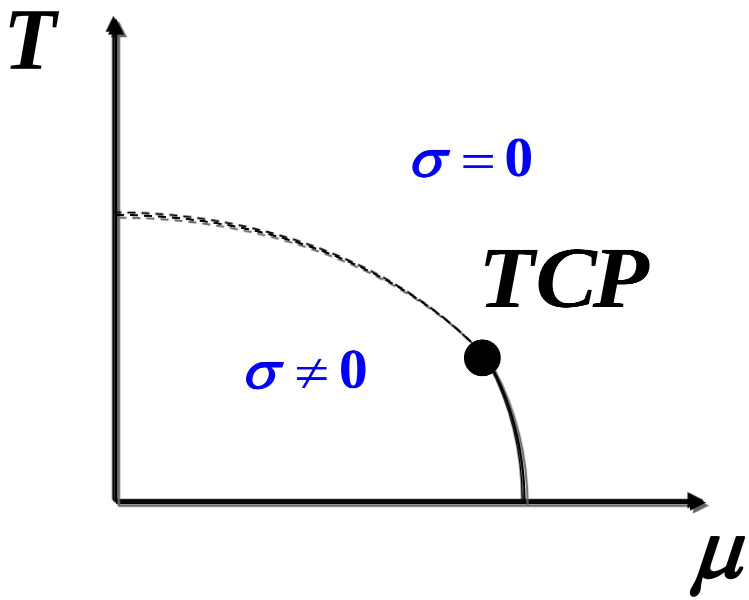
Seeing also Bernd-Jochen Schaefer's talk on plenary session 10

Two flavor NJL model

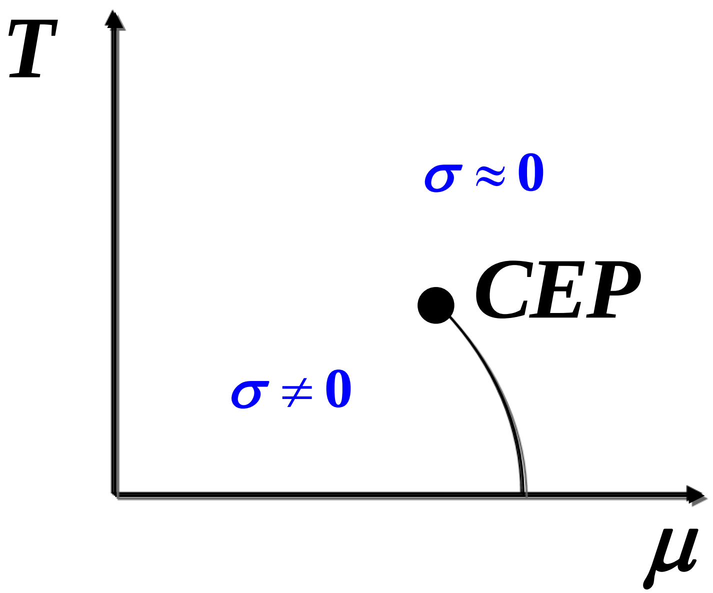
$$L = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m_0 + \mu\gamma^0\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\tau\psi\right)^2\right]$$

$$\Omega\left(T,\mu;\sigma\right) = G\sigma^2 - 2N_c N_f \int_{\Lambda} \frac{d^3\vec{p}}{(2\pi)^3}\left[E_p + T\ln\left(1 + e^{-(E_p-\mu)/T}\right) + T\ln\left(1 + e^{-(E_p+\mu)/T}\right)\right]$$

$$E_p = \sqrt{p^2 + (m_0 - 2G\sigma)^2} \qquad \sigma = \langle\bar{\psi}\psi\rangle$$



In the chiral limit



In the real world

Landau theory for NJL model

- In the vicinity of critical point, the chiral condensate is arbitrarily small

$$\Omega(T,\mu;\sigma) = \frac{1}{2} a\sigma^2 + \frac{1}{4} b\sigma^4 + \frac{1}{6} c\sigma^6 - h\sigma$$

- For example, in chiral limit ($m_0 = 0$)

$$a(T,\mu) = 2G - \frac{24G^2}{\pi^2} \int_\Lambda k\left(1 - f^+ - f^-\right) dk$$

$$b(T,\mu) = \frac{48G^4}{\pi^2} \int_\Lambda \frac{1}{k}\left(1 - f_+ - f_-\right) dk - \frac{48G^4}{\pi^2 T} \int_\Lambda \left(f_+ + f_- - f_+^2 - f_-^2\right) dk$$

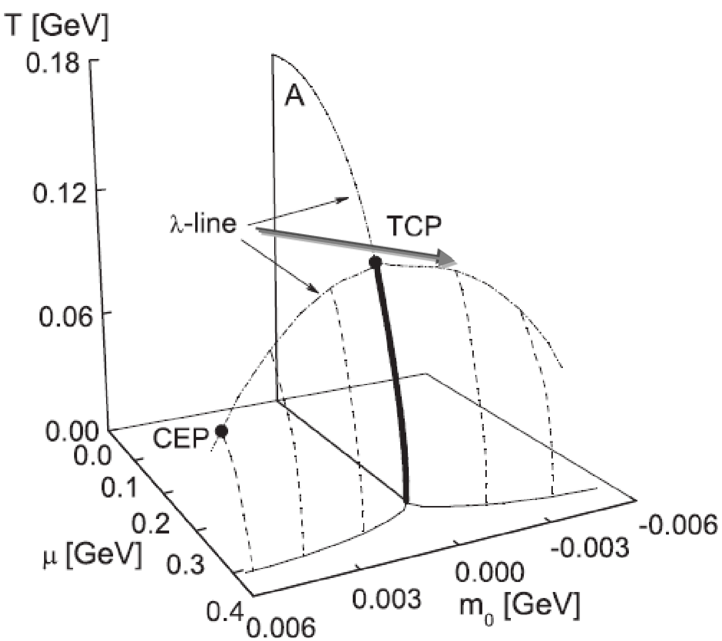
$$c(T,\mu) = -\frac{288G^6}{\pi^2} \int_\Lambda \frac{1}{k^3}\left(1 - f_+ - f_-\right) dk + \frac{96G^6}{\pi^2 T} \int_\Lambda \frac{1}{k^2}\left(3 f_+ + 3 f_- - f_+^2 - f_-^2\right) dk$$

$$+ \frac{96G^6}{\pi^2 T^2} \int_\Lambda \frac{1}{k}\left(f_+ + f_- - f_+^2 - f_-^2 + 2 f_+^3 + 2 f_-^3\right) dk$$

$$h(T,\mu) = -\frac{\partial\Omega}{\partial\sigma}$$

In NJL model, the current quark mass plays the role of the external field. Then h equal to zero in chiral limit.

The phase diagram(I)



The three λ lines connect at TCP, which defined by

$$\frac{\partial^2 \Omega}{\partial \sigma^2} = \frac{\partial^3 \Omega}{\partial \sigma^3} = 0$$

Solution I: $a = h = 0$

which gives the λ line in the $m_0 = 0$ plane

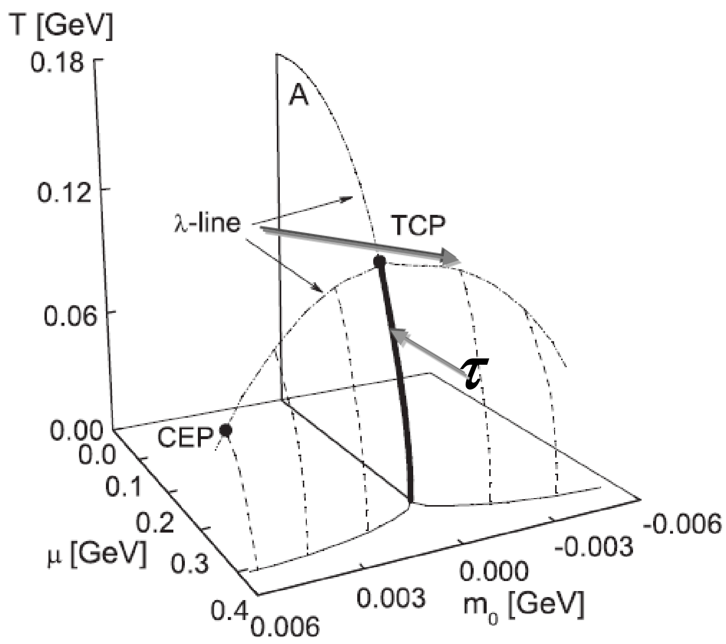
Solution II and III:

$$a = \frac{9b^2}{20c}, h = \pm \frac{8c}{3}\left(\frac{-3b}{10c}\right)^{5/2}$$

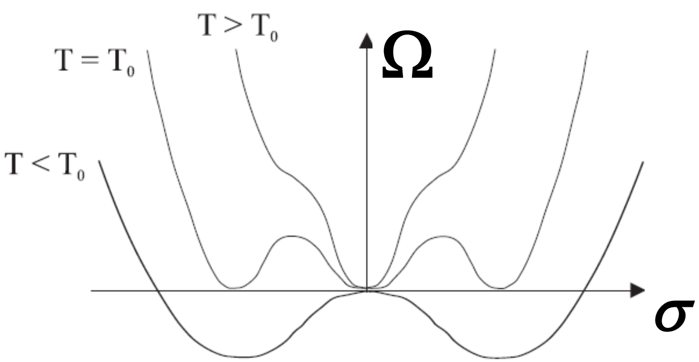
which gives the λ lines of the CEP for $m_0 \neq 0$

These three λ lines are the second order phase transition lines in landau theory

The phase diagram(II)



The first order phase transition still can be described by the landau theory. when h=0, the potential becomes a typical phi^6 one.

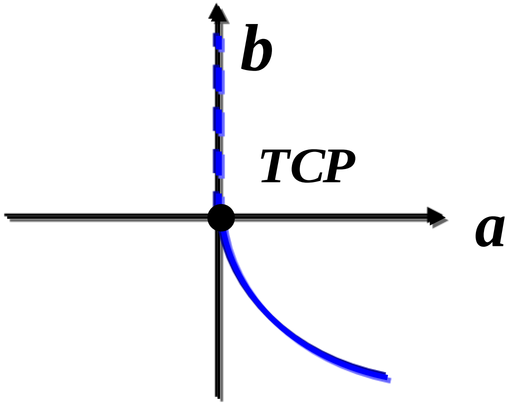


The potential has one minimum at $\sigma = 0$, and other two symmetric ones. When the three minima are equal, the first order phase transition happens,

$$b^2 = \frac{16}{3}ac, \ b < 0, \ h = 0$$

When approaching the TCP, the three local minima become one, which is given by

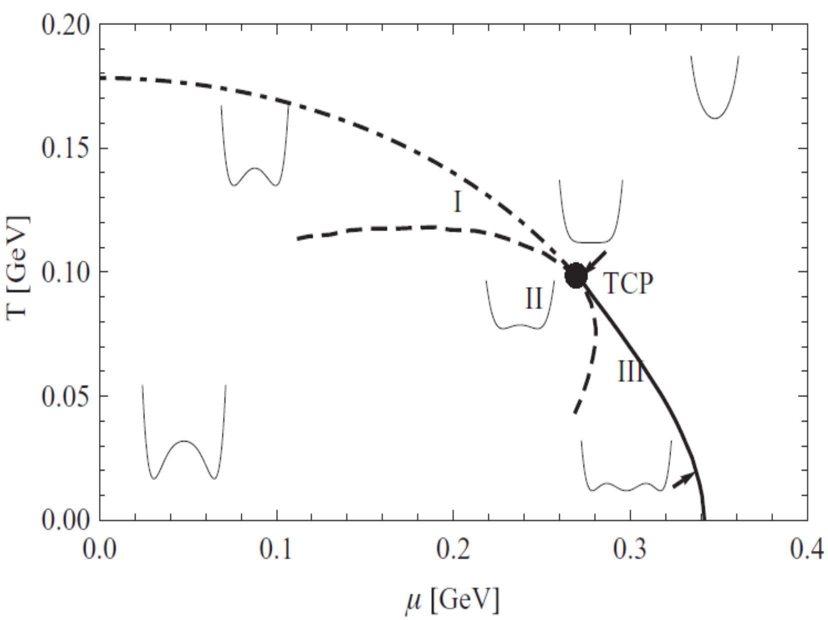
$$a = b = 0$$



Regions of different critical behavior(I)

In the chiral symmetry broken phase, all the critical exponent are determined by the order parameter, which minimize the thermodynamic potential

$$\frac{\partial\Omega}{\partial\sigma}=0,\ \frac{\partial^2\Omega}{\partial\sigma^2}>0 \quad \xrightarrow{\text{For } m_0=0} \quad \sigma_0^2=\frac{-b+\sqrt{b^2-4ac}}{2c}$$



In region I, defined by $b^2>>-4ac$, we have

$$\sigma_0^2\approx-\frac{a}{b} \qquad \beta_\lambda=\frac{1}{2}$$

Which is the typical critical exponent of phi4 theory

In region II, defined by $b^2<<-4ac$, we have

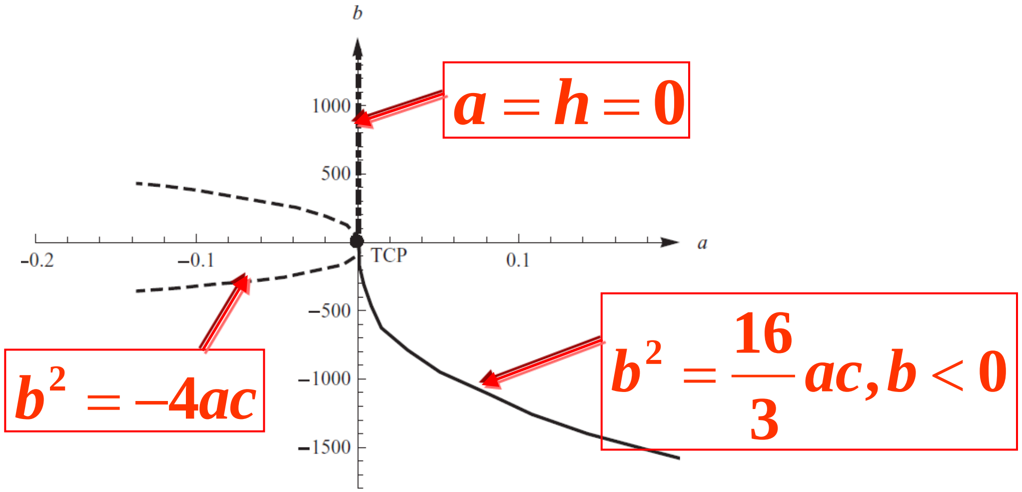
$$\sigma_0^2\approx\sqrt{-\frac{a}{c}} \qquad \beta_\lambda=\frac{1}{4}$$

Which is the typical critical exponent of phi6 theory

When $b^2\sim-4ac$, there is a crossover region where the critical exponent transfers from phi4 type to phi6 type.

Regions of different critical behavior(II)

Put this phase diagram on the a-b plane



$$a = h = 0$$

$$b^2 = -4ac$$

$$b^2 = \frac{16}{3}ac, b < 0$$

The dash-dotted line is the second order phase transition line

The solid line is the first order phase transition line

The dashed line indicate the crossover region of the critical exponent between phi4 and phi6 theory

$$c > 0$$

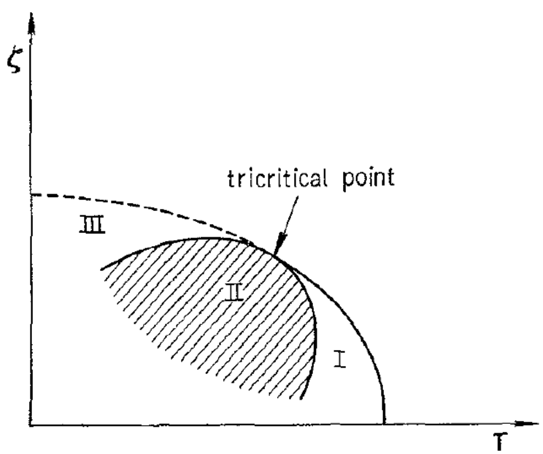


Fig. 3. Regions of different critical behaviour in the $T\zeta$ plane

R.bausch, Z.Phys.254(72)81-88

The phase diagram of He3-He4 mixtures

$$\xi \sim \mu$$

Regions of different critical behavior(III)

For $m_0 \neq 0$, the order parameter is still determined by

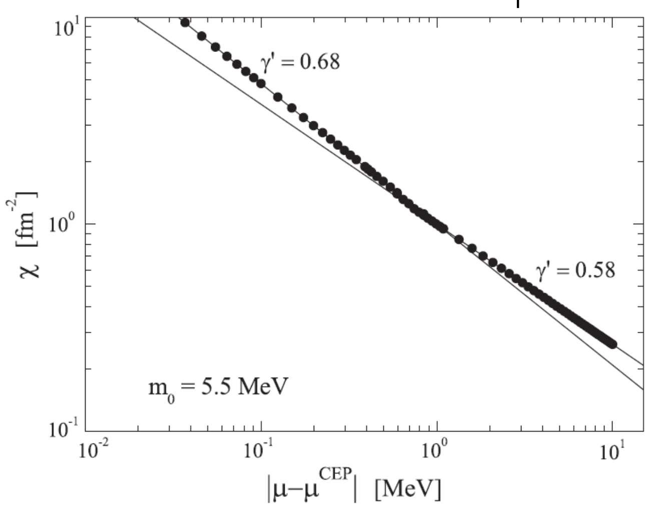
$$\frac{\partial \Omega(T,\mu;\sigma)}{\partial \sigma} = 0, \quad \frac{\partial^2 \Omega(T,\mu;\sigma)}{\partial \sigma^2} > 0$$

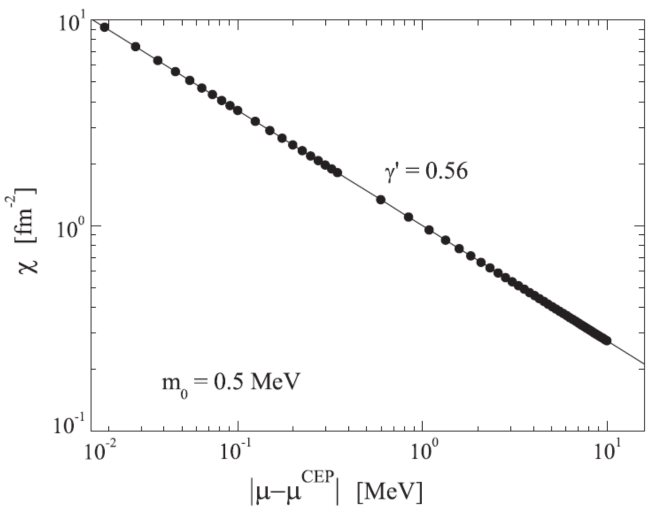
The solution this equation is expressed by the generalized hypergeometric function. We cannot analyze the critical region as the case of m_0=0.

We calculate the quark number susceptibility $\chi = -\partial^2\Omega / \partial\mu^2$ around the CEP and extract the critical exponent by the linear logarithmic fit

$$\ln\chi = -\gamma' \ln\left|\mu - \mu^{CEP}\right|$$

Fixed $T = T^{CEP}$





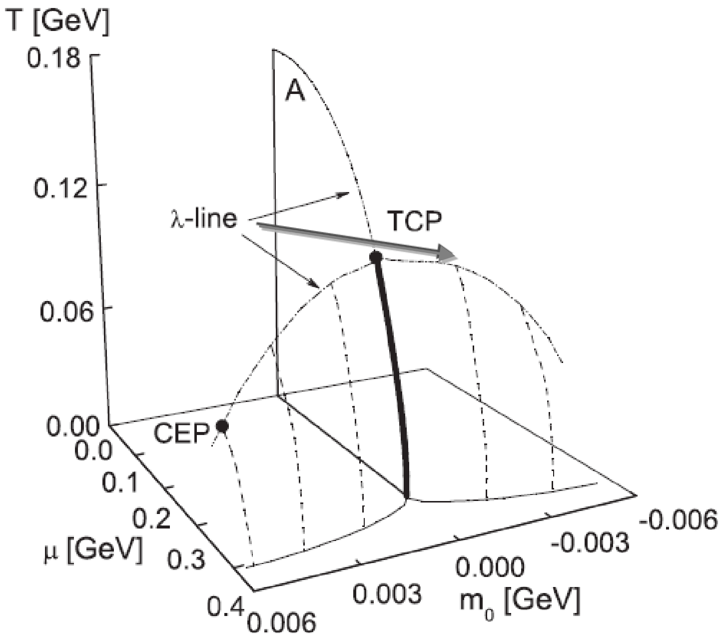
When one approaches the TCP along the line of CEPs, there exists a crossover region

summary

- **With the help of the landau theory, we analyze the phase diagram of the NJL model.**
- **We find the vicinity of the TCP can be divided into different regions according to the critical exponent.**
- **Along the lambda line of CEP, the critical exponent will change smoothly when approaching the TCP.**

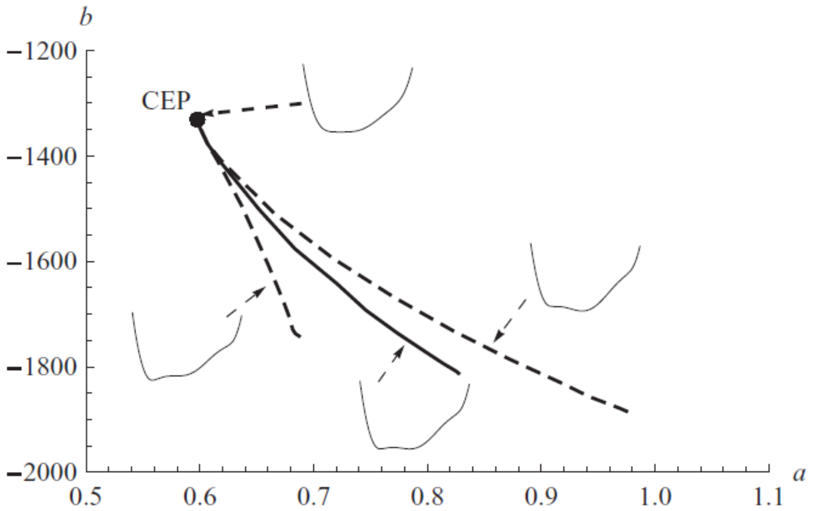
Thank you for your attention !

The phase diagram(III)



The first order phase transition for $m_0 \neq 0$

$$\frac{\partial \Omega}{\partial \sigma} = 0, \frac{\partial^2 \Omega}{\partial \sigma^2} > 0$$



$$\frac{\partial \Omega}{\partial \sigma} = 0, \frac{\partial^2 \Omega}{\partial \sigma^2} > 0$$

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$m_0 \neq 0$