Determining the CEP via the Dyson-Schwinger Equation Approach of QCD

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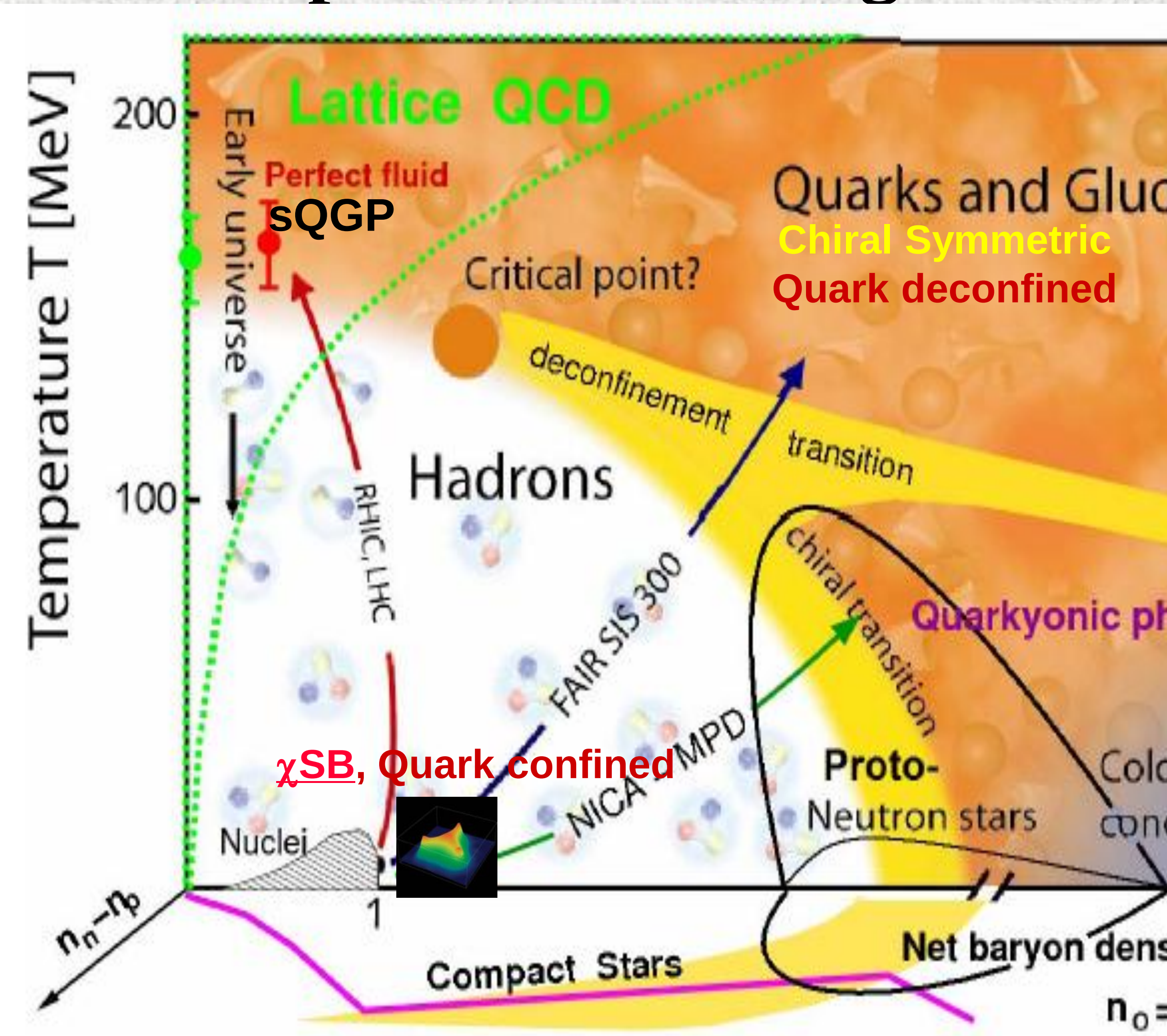
Outline

I. Introduction
Ⅱ. Dyson-Schwinger Eq. Approach
Ⅲ. Sign. Quantity & Numerical results
Ⅳ. Summary & Remarks

7th International Workshop on CPOD, Wuhan, Nov.7-11, 2011

I. Introduction

QCD Phase Diagram: Phase Boundary, Specific States, e.g., CEP, sQGP, Quakyonic,



Phase Transitions involved:

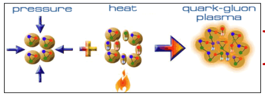
Deconfinement–confinement

DCS – DCSB

Flavor Sym. – FSB

Items Influencing the Phase Transitions:

Medium: Temperature T ,



Density ❒ (or μ)

Size

Intrinsic: Current mass,

Coupling Strength,

Color-flavor structu

… …

♣ Critical Endpoint (CEP) is one of the most concerned item

The Position of CEP is a highly debated problem!

♠ **(p)NJL model & others give quite large μ_E/T_E (> 3.0)**

Sasaki, et al., PRD 77, 034024 (2008); Costa, et al., PRD 77, 096001 (2008);

Fu & Liu, PRD 77, 014006 (2008); Ciminale, et al., PRD 77, 054023 (2008);

♠ **Lattice QCD gives smaller μ_E/T_E (0.4 ~ 1.1)**

Fodor, et al., JHEP 4, 050 (2004); Gavai, et al., PRD 71, 114014 (2005);

♠ **RHIC Experimental observables**

R.A. Lacey, et al., nucl-ex/0708.3512 =➔ $\mu_E/T_E \cong 1$; ……

M. Stephanov, et al., PRL 81, 4816 ('98); PRL 102, 032301 ('09);

♠ **Simple DSE Calculations with Different Effective Gluon Propagators Generate Different Results (0.0, 1.3)**

- **What can sophisticated DSE calculation produce ?**
- **Why different models give distinct results ?**

II. Dyson-Schwinger Equation Approach of QCD

Theory The Frontiers of Nuclear Science — A LONG RANGE PLAN — December 2007

The primary goal of the RHIC scientific program in the coming years is to progress from qualitative statements to rigorous quantitative conclusions. Quantitative conclusions require sophisticated modeling of relativistic heavy-ion collisions and rigorous comparison of such models with

Thus, an essential requirement for the field as a whole is strong support for the ongoing theoretical studies of QCD matter, including finite temperature and finite baryon density lattice QCD studies and phenomenological modeling, and an increase of funding to support new initiatives enabled by experimental and theoretical breakthroughs. The success of this effort mandates significant additional investment in theoretical resources in terms of focused collaborative initia-

General view of Theor. Aps.

♠ **Lattice QCD:**

Running coupling behavior,
Vacuum Structure,
Temperature effect,
"Small chemical potential";
…

♠ **Continuum:**

(1) **Phenomenological models**
(p)NJL、(p)QMC、QMF、

(2) **Field Theoretical**
Chiral perturbation,
Renormalization Group,
QCD sum rules,

The approach should manifest simultaneously:

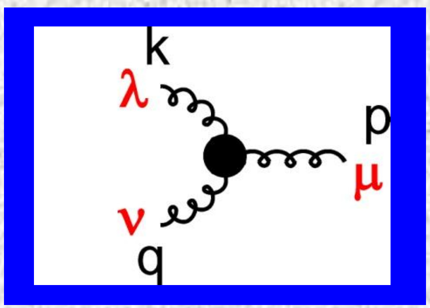
(1) DCSB & its Restoration ,

(2) Confinement & Deconfinement .

♠ The Dyson-Schwinger Equation Approach

Dyson-Schwinger Equations

Slavnov-Taylor Identity

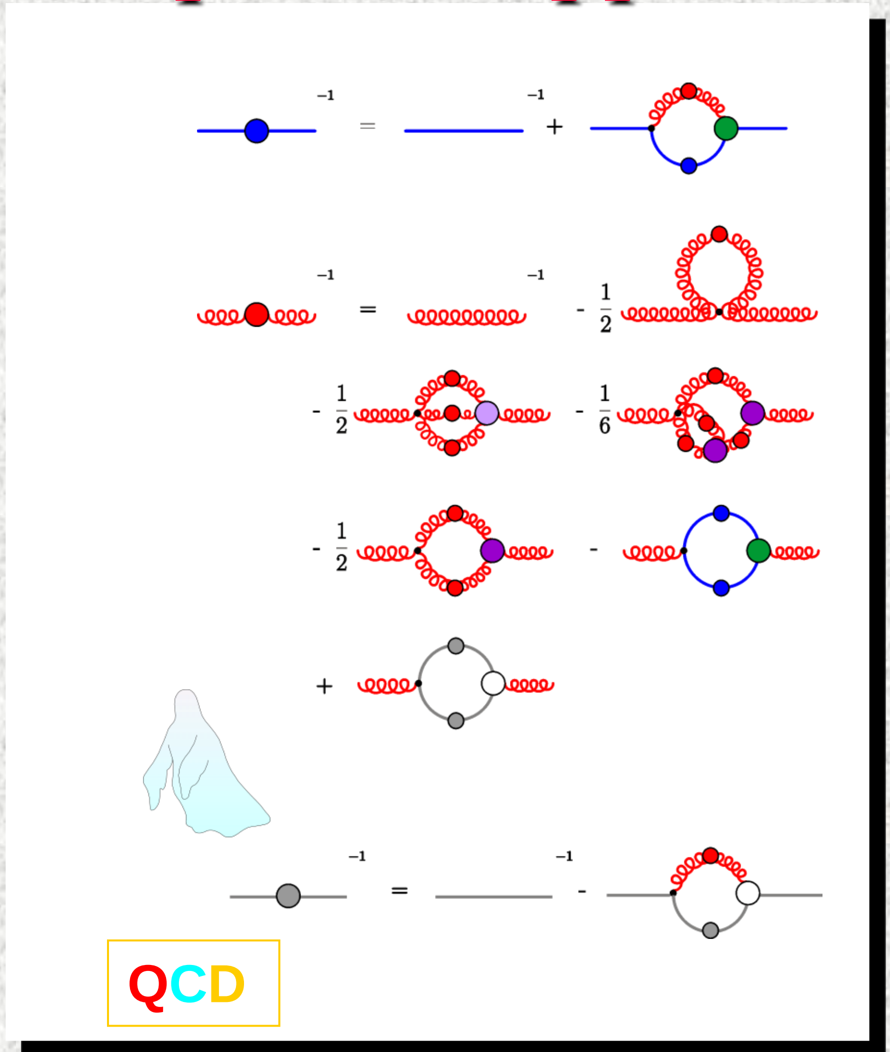


axial gauges **BBZ**

$$k_\lambda \, \Gamma^{\lambda\mu\nu}(k,p,q) \;=\; \Pi^{\mu\nu}(p) \;-\; \Pi^{\mu\nu}(q)$$

covariant gauges

$$k_\lambda \, \Gamma^{\lambda\mu\nu}(k,p,q) \;=\; H(k^2) \left[G_{\mu,\sigma}(q,-k) \, \Pi^T_{\sigma,\nu}(p) \;-\; G_{\nu\sigma}(p,-k) \, \Pi^T_{\sigma\mu}(q) \right]$$



C. D. Roberts, et al, PPNP 33 (1994), 477; 45-S1, 1 (2000); EPJ-ST 140(2007), 53;
R. Alkofer, et. al, Phys. Rep. 353, 281 (2001); C.S. Fischer, JPG 32(2006), R253; …… .

♣ Practical Algorithm at Present Stage

● Quark equation at zero chemical potential

$$G^{-1}(p) = Z_2(i\gamma \cdot p + m_{bar}) + \frac{4}{3}\int_q^{\Lambda} 4\pi\alpha(p-q) D_{\mu\nu}^{free}(p-q)\gamma_\mu G(q)\Gamma_\nu, \quad (1)$$

where $D_{ab}^{free}(p-q)$ is the effective gluon propagator,

$G^{-1}(p)$ can be conventionally decomposed as

$$G^{-1}(p) = i\gamma \cdot pA(p^2) + B(p^2), \quad M(p^2) = \frac{B(p^2)}{A(p^2)}$$

● Quark equation in medium

$$G^{-1}(p) \quad \Longrightarrow \quad \mathcal{G}^{-1}(p, \omega_n, \mu)$$

♠ with

$$\mathcal{G}^{-1}(p, \omega_n, \mu) = iA(p, \omega_n, \mu)\vec{\gamma} \cdot \vec{p} + iC(p, \mu)\gamma_4(\omega_n + i\mu) + B(\tilde{p}) + \cdots \quad (3)$$

♣ Models of the Vertex

$$\Gamma^a_\mu(q,p) = t^a \Gamma_\mu(q,p)$$

(1) Bare Vertex

$$\Gamma_\mu(q,p) = \gamma_\mu$$

(Rainbow-Ladder Approx.)

(2) Ball-Chiu Vertex

$$\begin{aligned}\Gamma^{BC}_\mu(p,q) &= \frac{A(p^2)+A(q^2)}{2}\gamma_\mu + \frac{(p+q)_\mu}{p^2-q^2}\{[A(p^2)-A(q^2)]\frac{(\gamma\cdot p+\gamma\cdot q)}{2} \\ &\quad -i[B(p^2)-B(q^2)]\},\end{aligned}$$

(3) Curtis-Pennington Vertex

$$\begin{aligned}\Gamma^{CP}_\mu(p,q) &= \Gamma^{BC}_\mu(p,q) + \frac{1}{2}(A(p^2)-A(q^2))\frac{\gamma_\mu(p^2-q^2)-(k+p)_\mu\gamma\cdot(p+q)}{d(p,q)}, \\ d(p,q) &= \frac{(p^2-q^2)^2+[M^2(p^2)+M^2(q^2)]^2}{p^2+q^2} \circ\end{aligned}$$

(4) BC+ACM Vertex **(Chang, Liu, Roberts, PRL 106, 072001 ('11)**

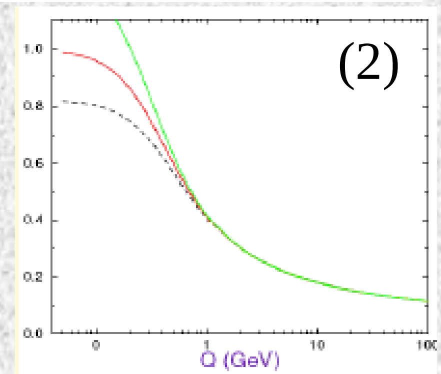
$$\Gamma^{\rm acm}_\mu(p_f,p_i) = \Gamma^{\rm acm_4}_\mu(p_f,p_i) + \Gamma^{\rm acm_5}_\mu(p_f,p_i),$$

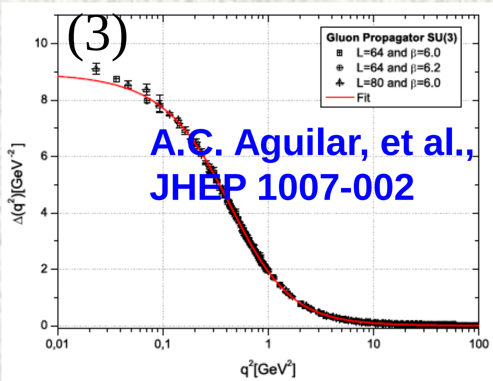
♣ Effective Gluon Propagators

$$g^2 D_{\rho\sigma}(k) = 4\pi \frac{\mathcal{G}(k^2)}{k^2} \left(\delta_{\rho\sigma} - \frac{k_\rho k_\sigma}{k^2}\right)$$

(1) MN Model $g^2 \mathrm{D}(p-q) = \frac{3}{16}\eta^2 \delta(p-q),$

(2) $(q^4 + \Delta)^{-1}$ Model





(3) More Realistic model

(4) An Analytical Expression of the Realistic Model:

Maris-Tandy Model

$$\frac{\mathcal{G}(t)}{t} = \frac{4\pi^2}{\omega^6} D\, t e^{-t/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln\left[\tau + \left(1 + t/\Lambda^2_{\rm QCD}\right)^2\right]} \frac{1 - \exp(-t/[4m_{\mathcal{F}}^2])}{t}$$

(5) Point Interaction: (P) NJL Model

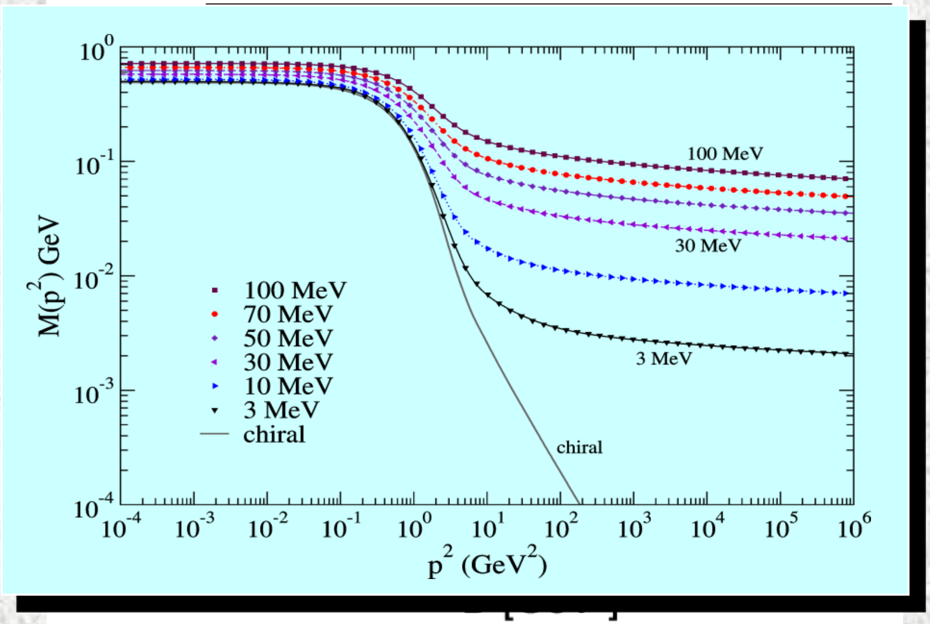
Dynamical chiral symmetry breaking (χSB) generates the mass of Fermion

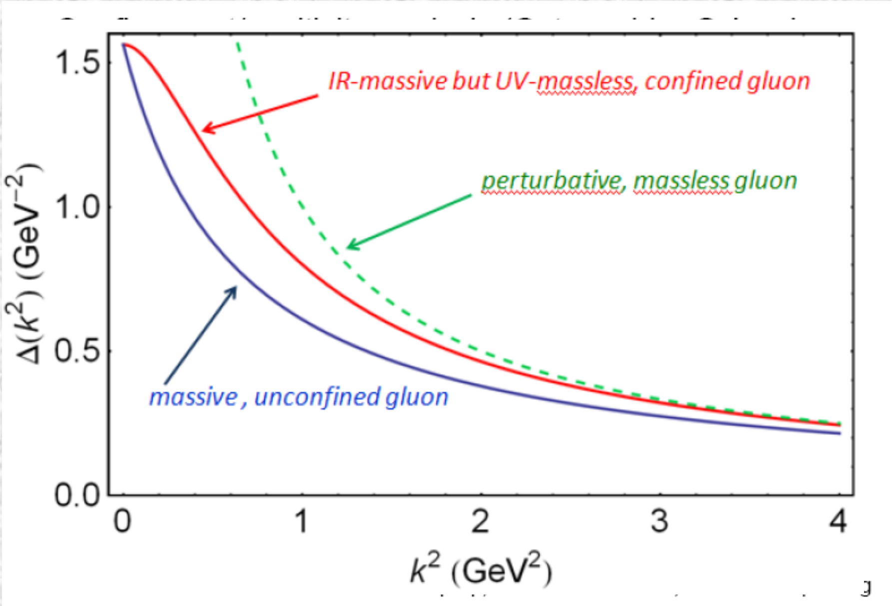
$$\mathcal{M}(p) \simeq m_0 \left[\ln p/\Lambda_{QCD}\right]^d + C \frac{-\langle\bar{q}q\rangle}{p^2 \left[\ln p/\Lambda_{QCD}\right]^d}$$ ⟹ χSB

$<\bar{q}q>_0 \sim -(240\ \mathrm{MeV})^3$ ⟹ **Dynamical Mass**

In DSE approach

$$M(p^2) = \frac{B(p^2)}{A(p^2)}$$





DSE approach meets the requirements!

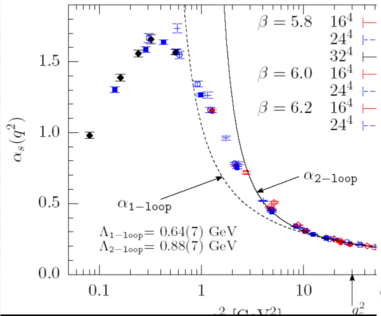
♠ Effect of the Running Coupling Strength on the Chiral Phase Transition

(W. Yuan, H. Chen, Y.X. Liu, Phys. Lett. B 637, 69 (2006))

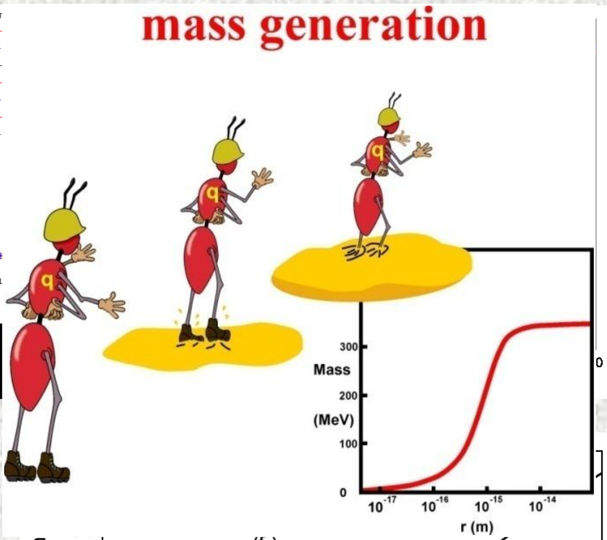
$$\alpha_{tue}(k^2) = g_k^2 \frac{k^2}{4\pi} \mathcal{D}(k^2) = \pi \xi \frac{k^4}{\omega^2} e^{-k^2/\omega^2}$$

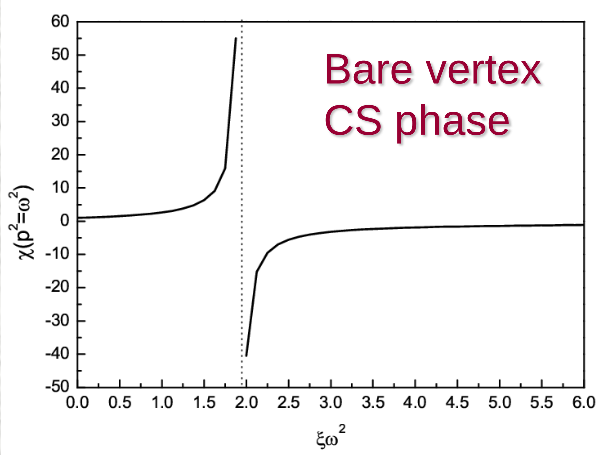
$$V(r) = \frac{\sqrt{\pi}\xi\omega^3}{8}(\omega^2 r^2 - 6) e^{-\frac{\omega^2 r^2}{4}}$$

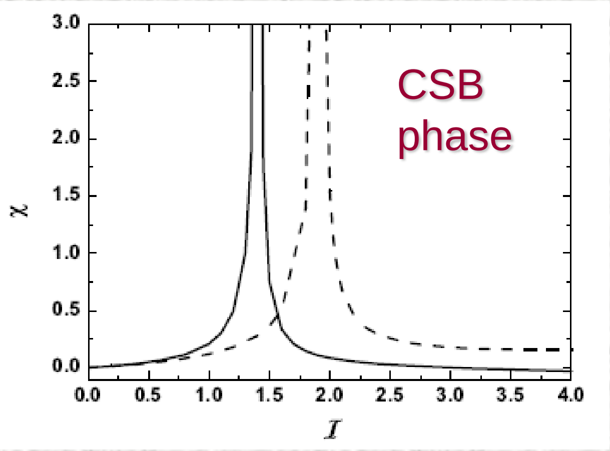
parameters are taken from Phys. Rev. D 65, 094026 (1997), with f_π fitted as $f_\pi = 93 MeV$

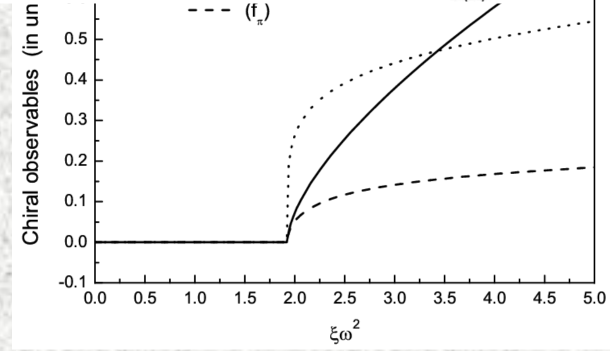


Lattice QCD result
PRD 72, 014507 (2005)



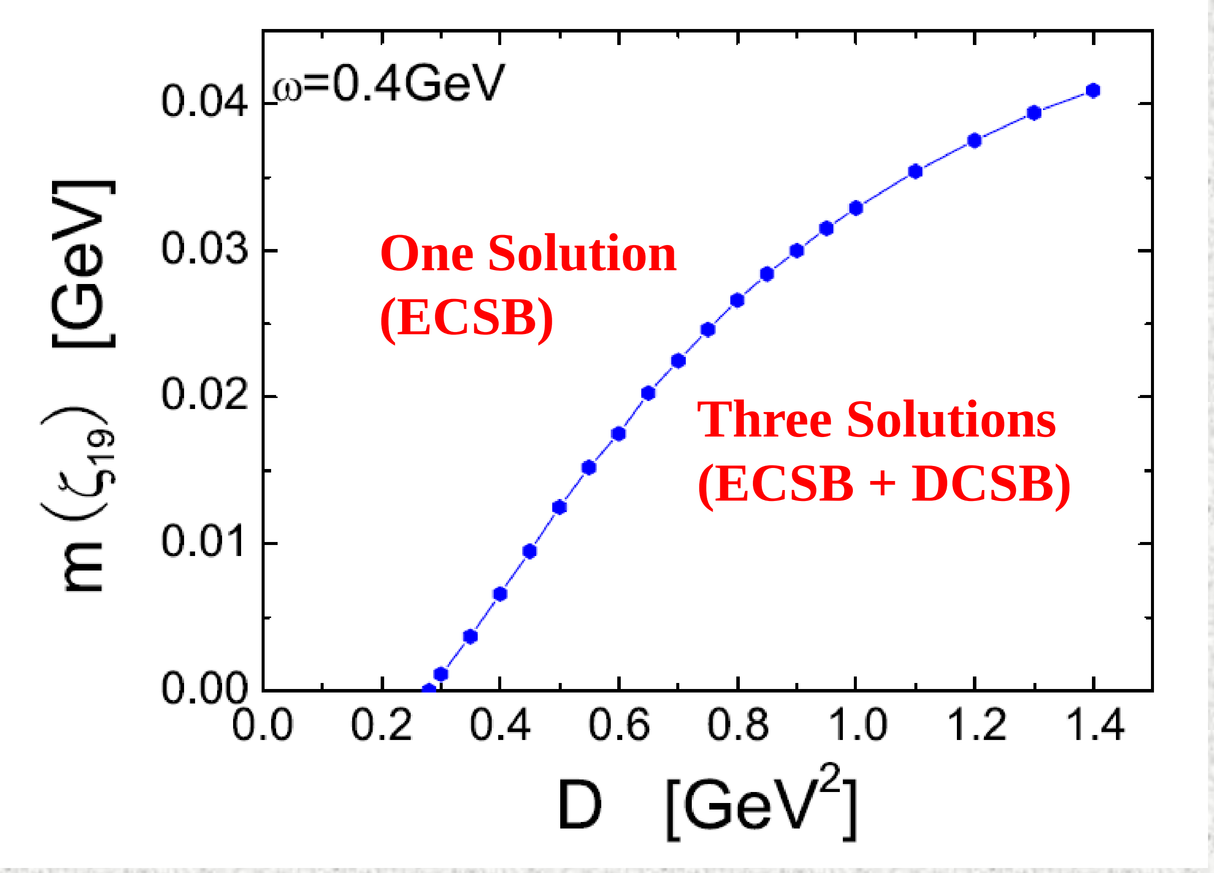






(BC Vertex: L. Chang, Y.X. Liu, R.D. Roberts, et al., Phys. Rev. C 79, 035209 (2009))

♣ Part of the QCD Phase Diagram in terms of the Current Mass and Coupling Strength



♣ **The one with multi-node solutions is more complicated and more interesting.**

♠ A comment on the DSE approach of QCD

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NUCLEAR PHYSICS A

Nuclear Physics A 796 (2007) 83–100

Phases of dense quarks at large N_c

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One way of computing the properties of a quarkyonic phase is to use approximate solutions of Schwinger–Dyson equations [23]. These are, almost uniquely, the one approximation scheme which includes both confinement and chiral symmetry breaking. They do have features reminiscent of large N_c: at low momentum, if chiral symmetry breaking occurs, the gluon propagator for $N_f = 3$ is numerically close to that for $N_f = 0$. At present, solutions at $\mu \neq 0$ assume a Fermi surface dominated by quarks; if quark screening is not too large at moderate μ, these models should exhibit a quarkyonic phase.

III. A Quantity Identifying the CEP & Numerical Results in DSE Approach

♠ Phase transitions result from non-perturbation effects;

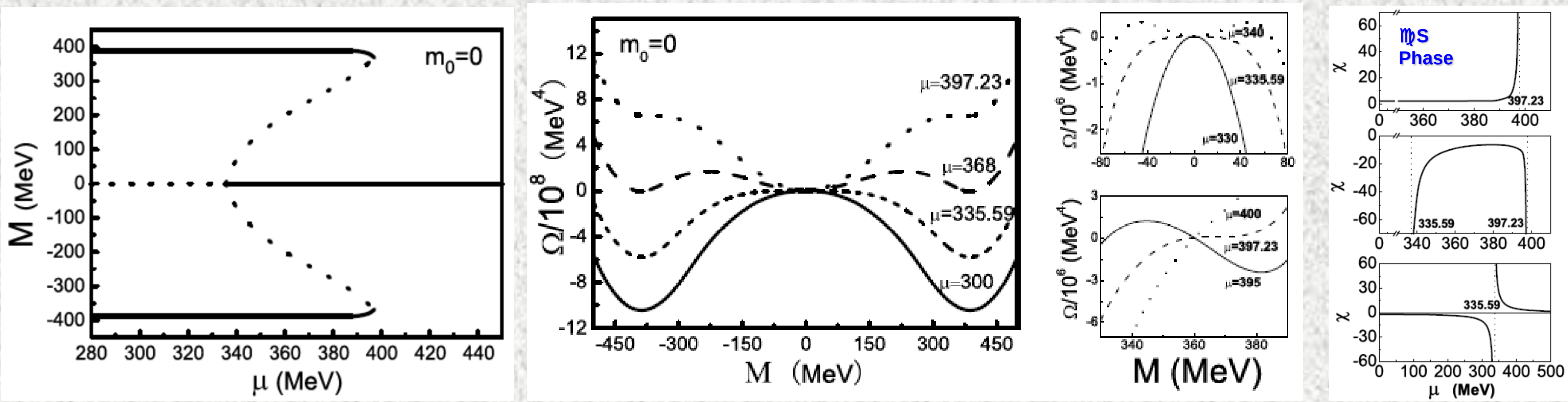
one can then not get the thermal potential of the strongly interacting (quark, gluon) system.

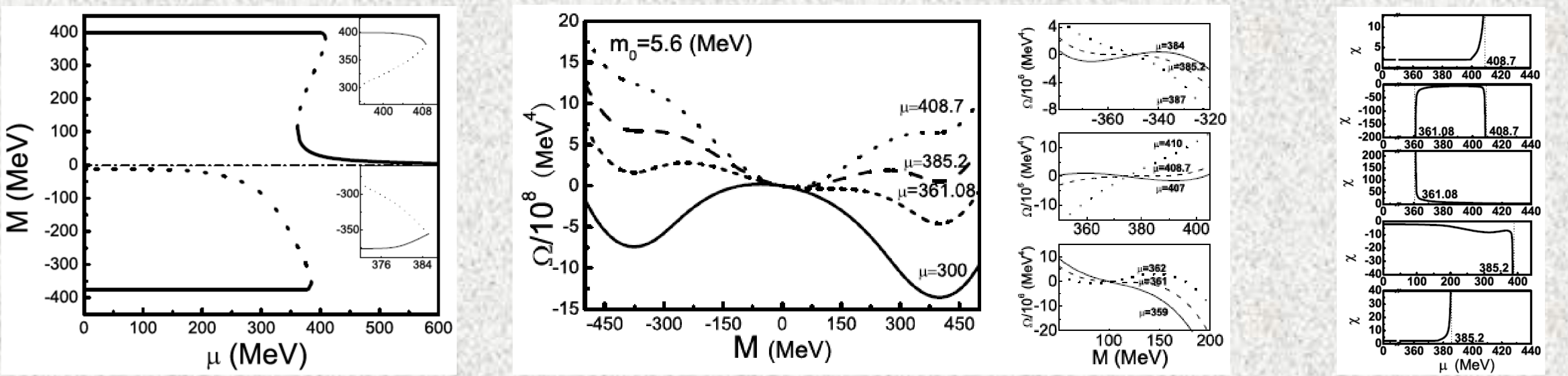
♠ Usual way identifying a phase transition that analyzes the thermal potential gets invalid !!

♠ We propose the chiral susceptibility $\chi = \frac{\partial M}{\partial m}$ or $\chi = \frac{\partial B}{\partial m}$ can be the signature to identify not only the phase transition but also the critical point (tricritical point).

✦ Example 1 for the Chiral Susceptibility (χS & χSB phases simultaneously) to be a Signature of the Chiral Phase Transition

Point Interaction ➔ $$\chi = \frac{\partial M}{\partial m} = \frac{1}{1 - 4GN_cN_f\frac{\partial}{\partial M}\left\{\int\frac{d^3p}{(2\pi)^3}\frac{M}{E_p}[1-n_p(T,\mu)-\bar{n}_p(T,\mu)]\right\}} = \frac{1}{G\frac{\partial^2\Omega}{\partial M^2}\Big|_{\frac{\partial\Omega}{\partial M}=0}}.$$





Y. Zhao, L. Chang, W. Yuan, Y.X. Liu, Eur. Phys. J. C 56, 483 (2008)

✦ Example 2: The Chiral Phase Evolution is a second order phase transition in chiral limit, but a crossover beyond chiral limit

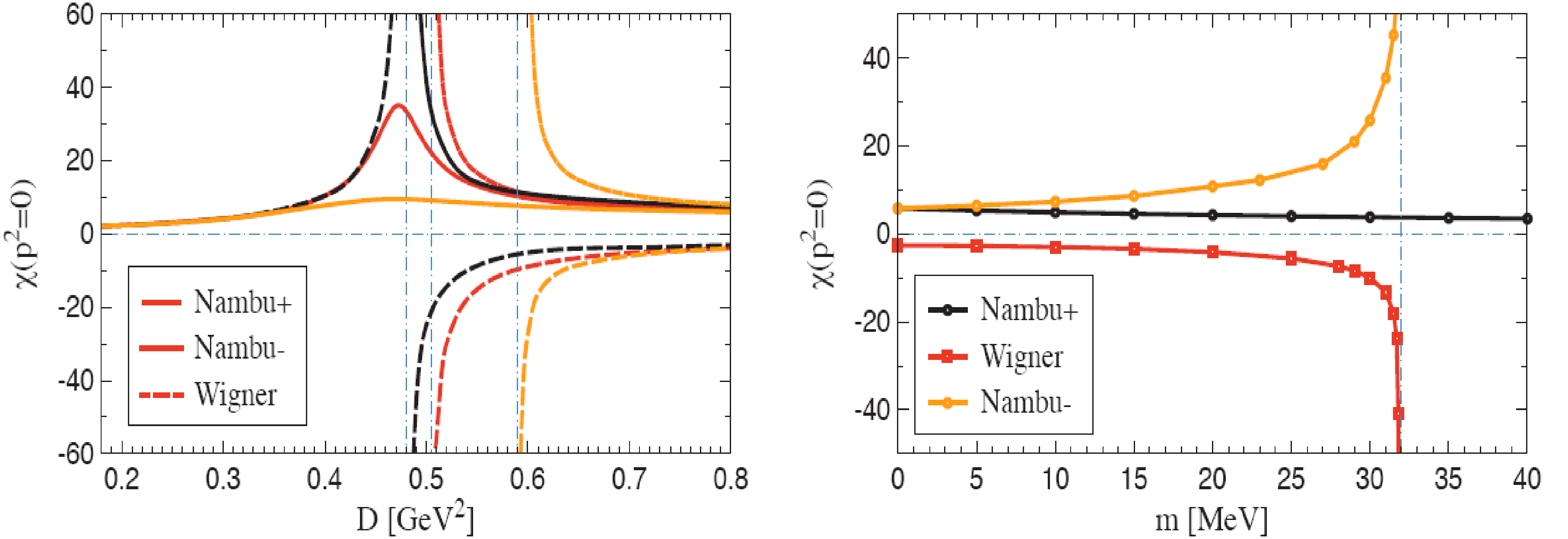


FIGURE 2. (color online) Calculated (with bare vertex approximation, Gaussian-like dressed gluon propagator with $\xi = 0.5$ GeV) variation behaviors of the chiral susceptibility at and beyond the chiral limit (with $m_0 = 0.5$ MeV(in red), 5.0 MeV(in yellow), respectively) with respect to the coupling strength D (left panel) and those against the current-quark mass with a fixed $D = 1.0\ \mathrm{GeV}^2$ (right panel).

Y.X. Liu, S.X. Qin, L. Chang, & C.D. Roberts, AIP Conf. Proc. 1354, 91 (2011)

✦ Example 3: In Ginzberg-Landau Theory

Thermal Potential:

$$\Omega(T,\mu,\eta)=\Omega_0(T,\mu)+\tfrac{1}{2}\alpha(T,\mu)\eta^2+\tfrac{1}{4}\beta(T,\mu)\eta^4 + \tfrac{1}{6}\gamma(T,\mu)\eta^6+\cdots$$

Stable ♒ 🖫 0 corresponds symmetric phase,
Stable ♒ 🕒 0 stands for symmetry broken phase.

Second order phase transition:

$$\alpha=\alpha_0(T-T_c),\ \beta>0,\ \gamma=\cdots=0;$$

$$\left.\frac{\partial^2\Omega}{\partial\eta^2}\right|_{\eta=0}=\alpha_0(T-T_c),\qquad \left.\frac{\partial^2\Omega}{\partial\eta^2}\right|_{\eta\neq0}=-2\alpha_0(T-T_c);$$

The $\left(\frac{\partial^2\Omega}{\partial\eta^2}\right)^{-1}$ s of the two phases diverge at the same T_c, but in opposite direction.

First order phase transition:

Thermal Potential:

$$\Omega(T,\mu,\eta) = \Omega_0(T,\mu) + \tfrac{1}{2}\alpha(T,\mu)\eta^2 + \tfrac{1}{4}\beta(T,\mu)\eta^4 + \tfrac{1}{6}\gamma(T,\mu)\eta^6 + \cdots$$

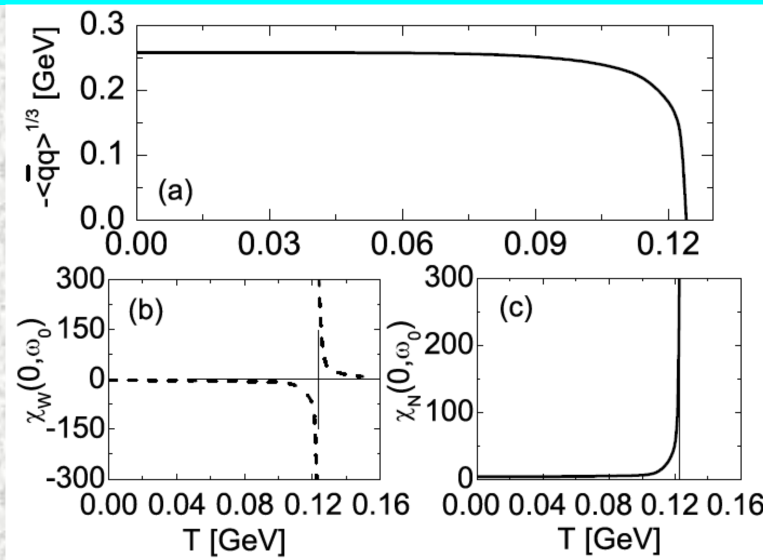
$$\gamma > 0,\ \beta = \beta_0(\mu - \mu_{c,2}),\ \alpha \le \frac{\beta^2}{4\gamma};$$

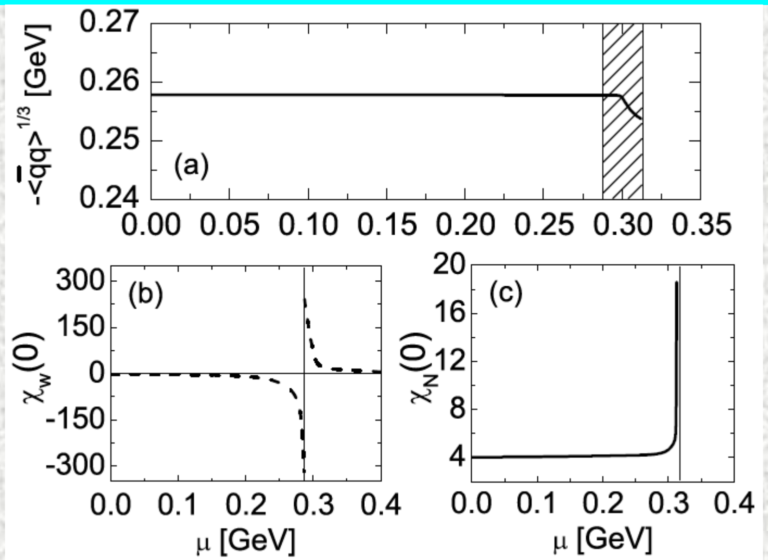
$$\left.\frac{\partial^2\Omega}{\partial\eta^2}\right|_{\eta=0} = \alpha, \qquad \left.\frac{\partial^2\Omega}{\partial\eta^2}\right|_{\eta\neq 0} = \frac{\sqrt{\beta^2-4\alpha\gamma}\,(\sqrt{\beta^2-4\alpha\gamma}-\beta)}{\gamma};$$

The $\left(\frac{\partial^2\Omega}{\partial\eta^2}\right)^{-1}$ s of the two phases diverge at different O_c, even in opposite direction.

The state at which the chiral susceptibilities of the two phases begin to diverge at different O_c is just the critical endpoint (tricritcal point).

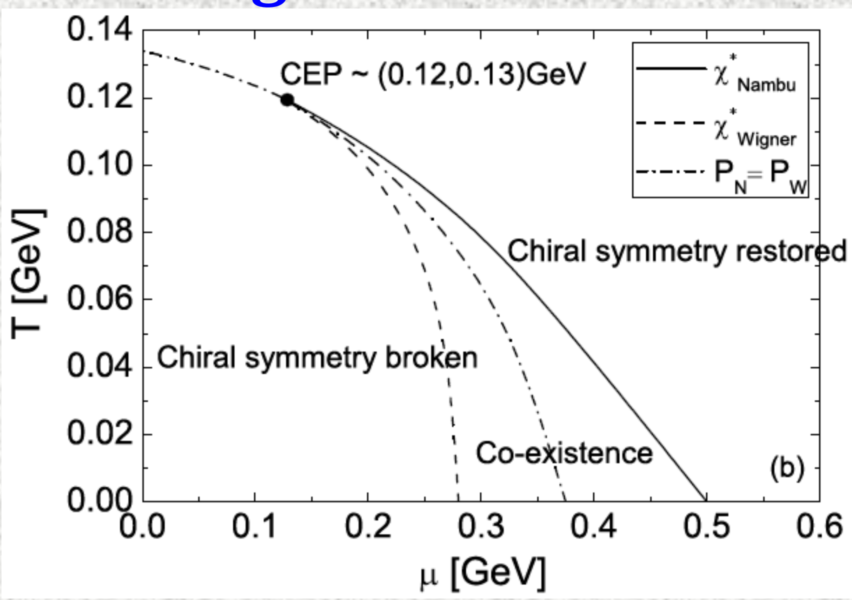
♠ Numerical Results: Phase Diagram & CEP

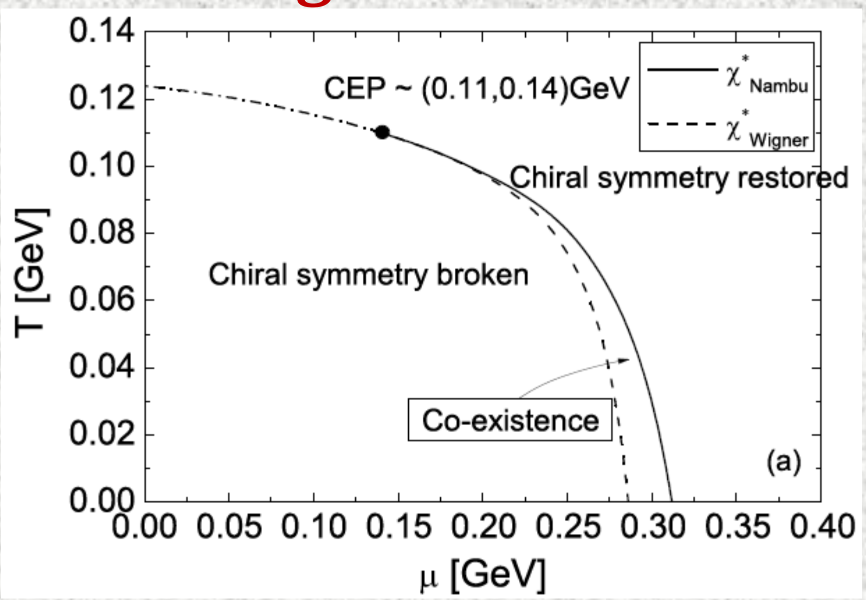




Phase diagram in bare vertex

Phase diagram in BC vertex





S.X. Qin, L. Chang, H. Chen, Y.X. Liu, C.D. Roberts, PRL 106, 172301('11)

♠ Unifying previous results in diff. approaches

model			result			
vertex	$\hat{D}^{1/2}$	σ	T_c	Δ_C	$(\mu_E, T_E)/T_c$	μ_E/T_E
BC	0.7	0.50	0.124	0.026	(1.13, 0.89)	1.27
BC	"	0.45	0.128	0.048	(0.69, 0.92)	0.75
BC	"	0.40	0.139	0.076	(0.16, 0.96)	0.17
Bare	1.0	0.50	0.133	0.220	(0.98, 0.90)	1.08
Bare	"	0.45	0.136	0.280	(0.81, 0.89)	0.91
Bare	"	0.40	0.148	0.360	(0.17, 0.95)	0.18

Small σ ➔ short range in momentum space

➔ long range in coordinate space

MN model ➔ infinite range in r-space

NJL model ➔ "zero" range in r-space

Longer range Int. ➔ Smaller μ_E/T_E

IV. Summary & Remarks

♠ Discussed some aspects of the Early Universe Matter Evolution in view of the QCD phase transitions in the DS equation approach of QCD

- Dynamical Mass is Generated by DCSB;
- Phase Diagram is given;
- CEP is fixed & the different results are unified;
- Coexisting Phase is discussed.

♠ Far from Well Established !

♣ Observables ?!

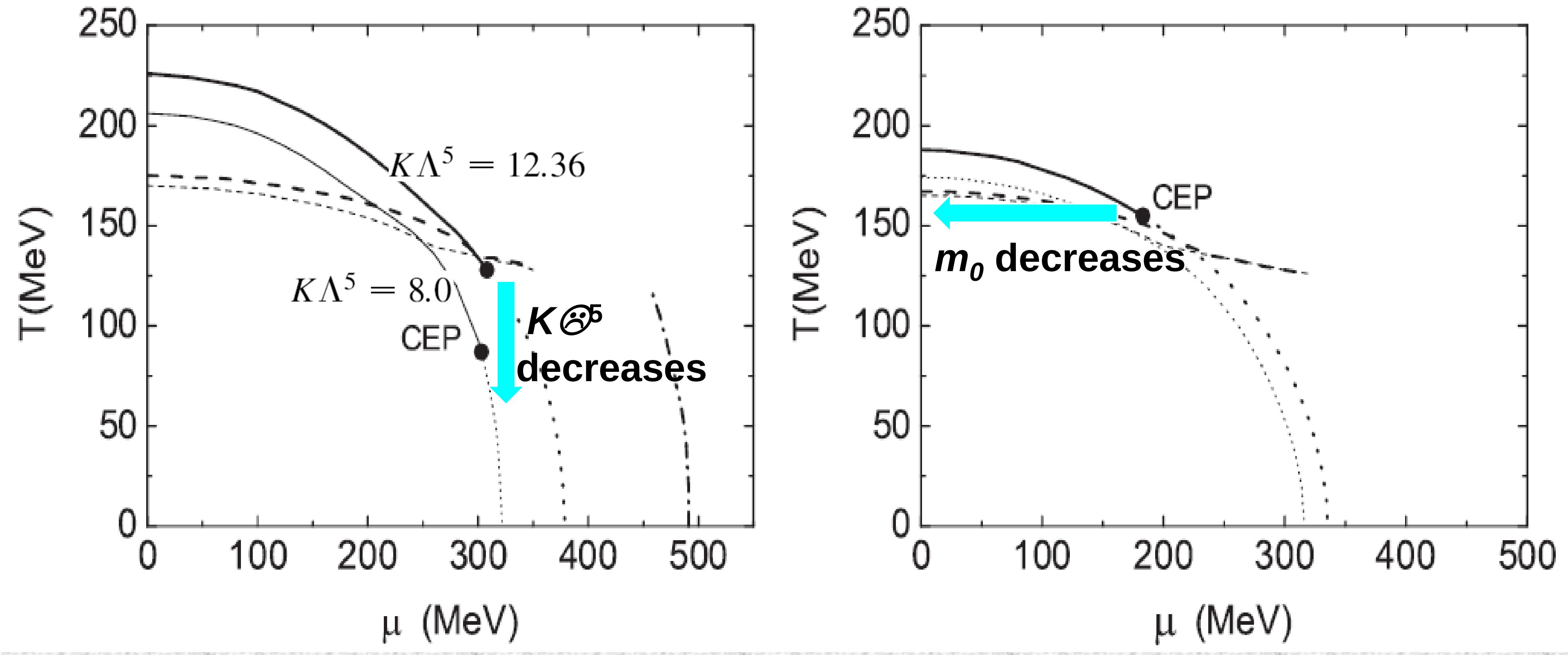
♣ Mechanism ?! Process ?!

● ● ● ● ● ●

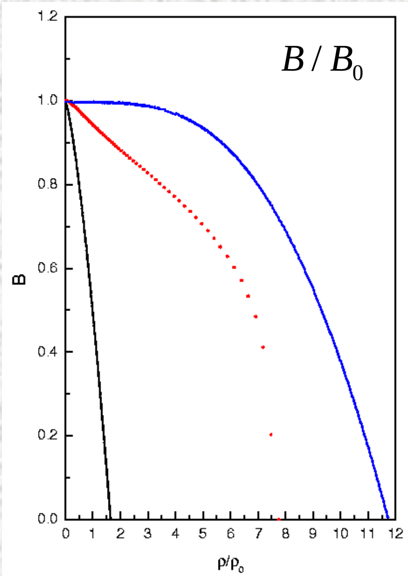
♠ The location of the CEP depends on the flavor mixing interaction strength and the current quark mass

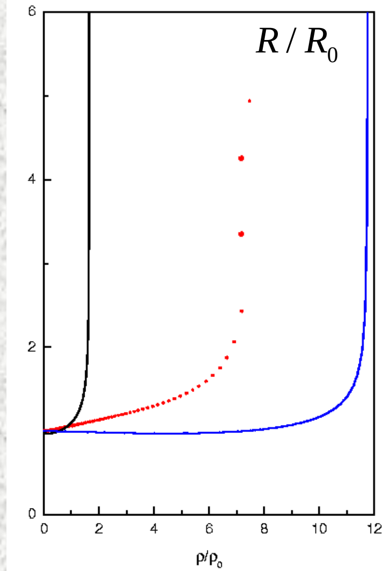
(W.J. Fu, Z. Zhao, Y.X. Liu, Phys. Rev. D 77, 014006 (2008) ((2+1) flavor pNJL model);

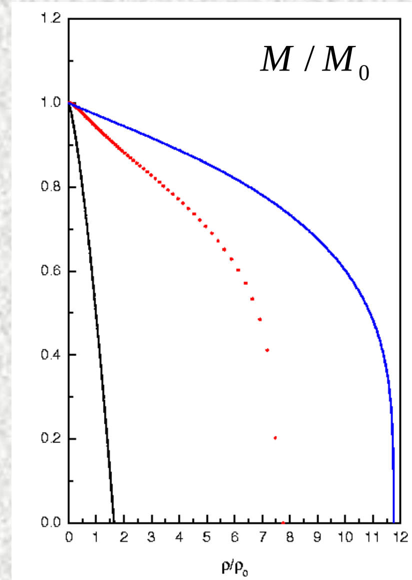
more simple case: 2-flavor, Z. Zhang, Y.X. Liu, Phys. Rev. C 75, 064910 (2007))



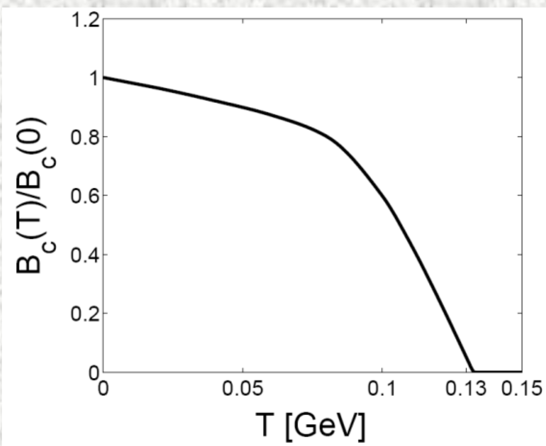
♠ Density & Temperature Dependence of some Properties of Nucleon in DSE Soliton Model

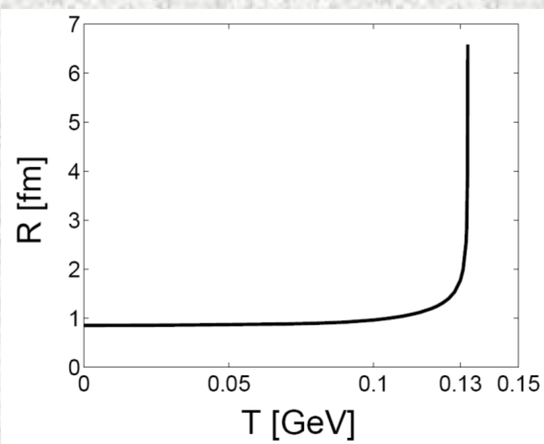


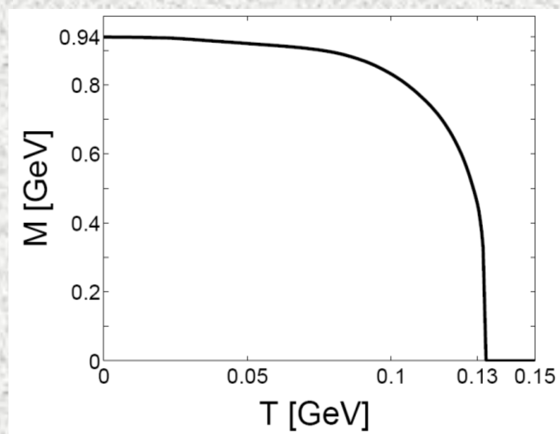




(Y. X. Liu, et al., NP A 695, 353 (2001); NPA 725, 127 (2003); NPA 750, 324 (2005))

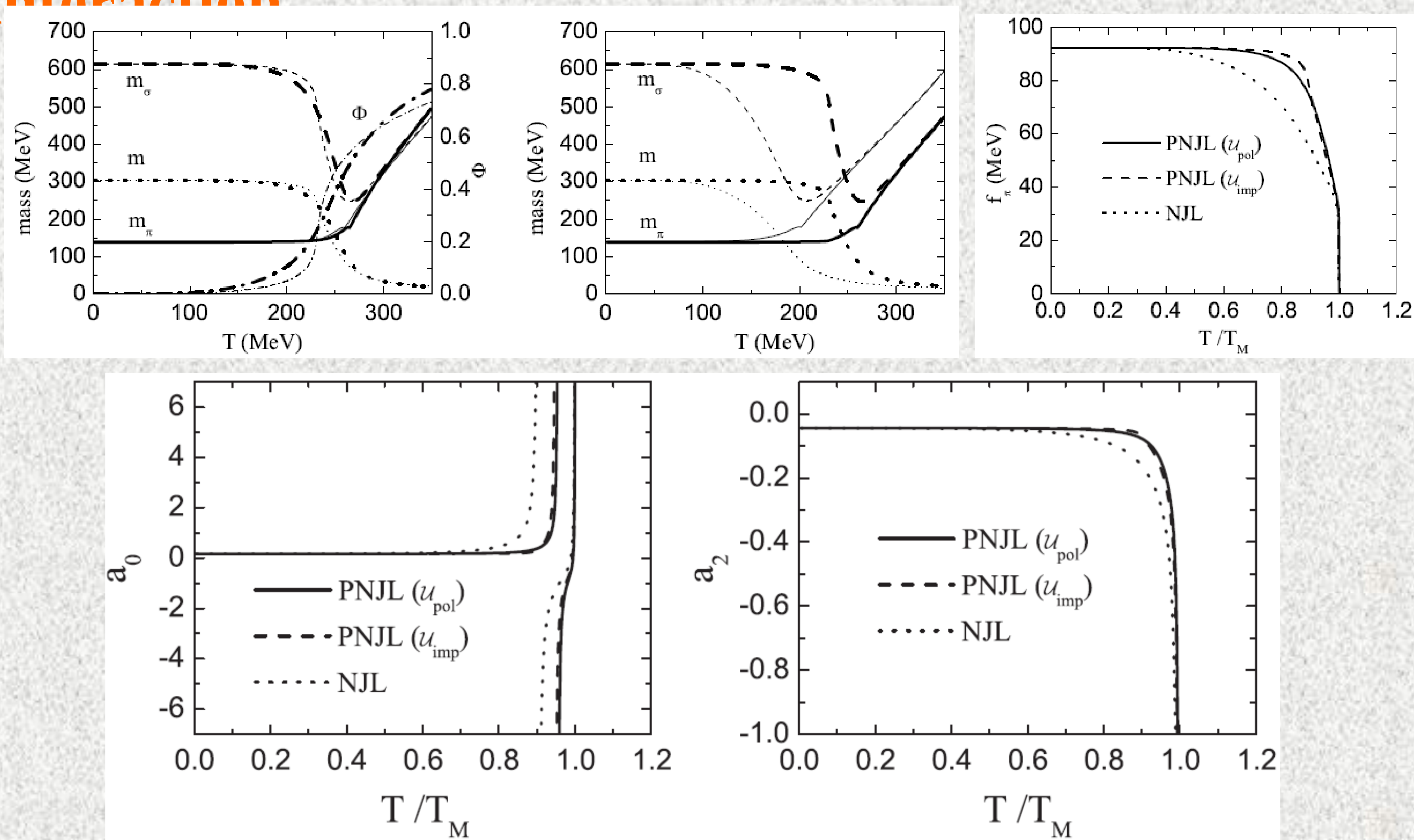






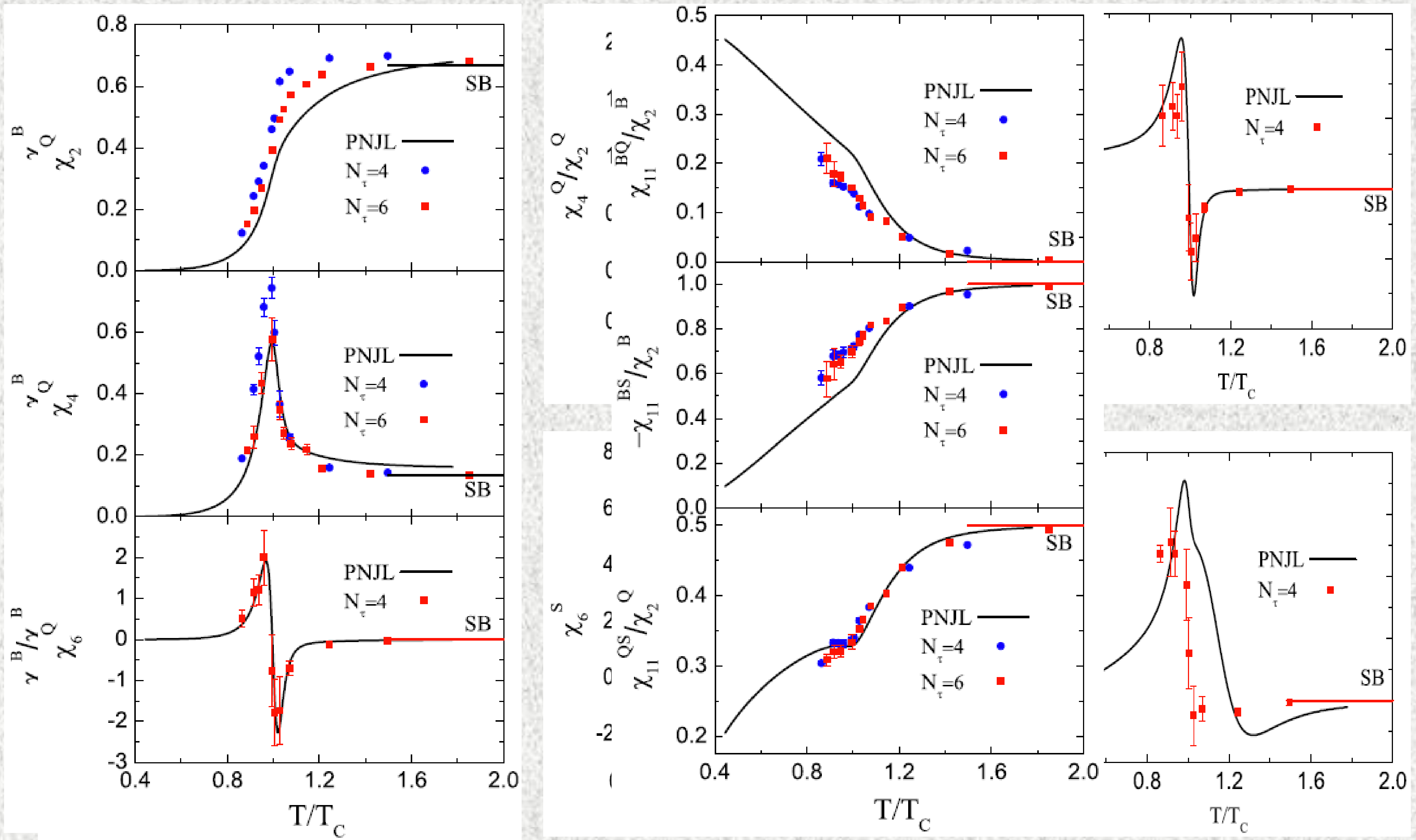
(Y. Mo, S.X. Qin, and Y.X. Liu, Phys. Rev. C 82, 025206 (2010))

T-dependence of some properties of □ & σ-mesons and □-□ S-L. in the model with contact interaction



(Wei-jie Fu, and Yu-xin Liu, Phys. Rev. D 79, 074011 (2009))

♠ Fluctuation & Correlation of Conserved Charges in the model with contact interaction



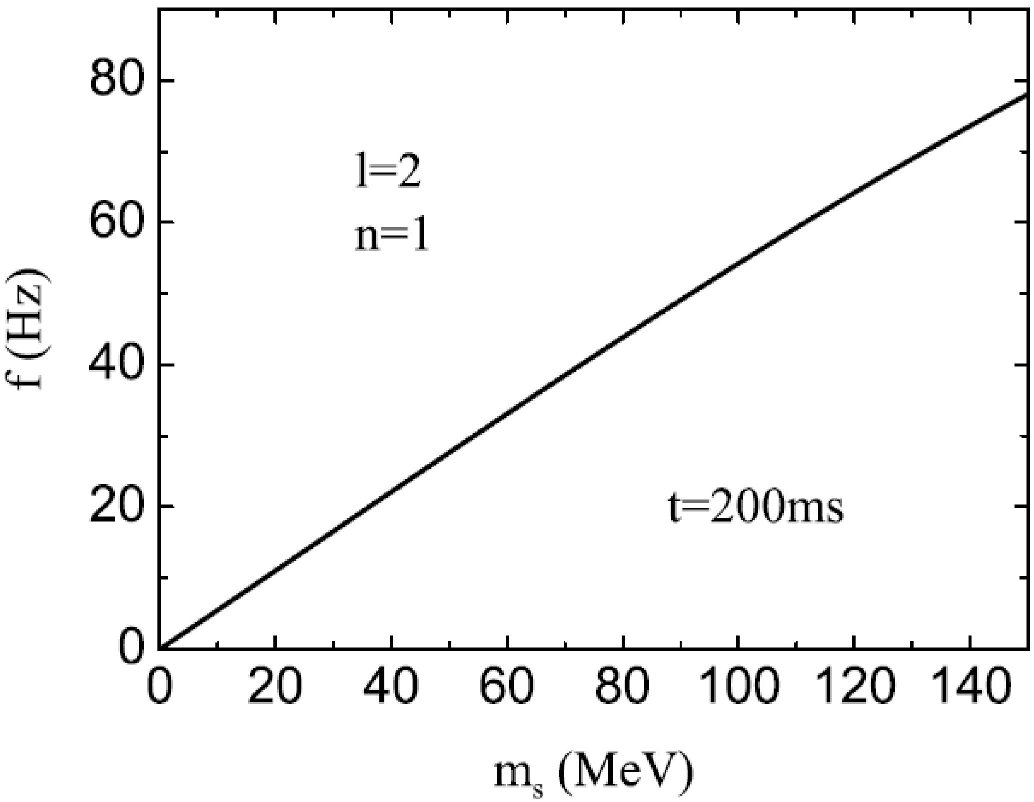
(W.J. Fu, Y.X. Liu, & Y.L. Wu, Phys. Rev. D 79, 014028 (2010))

♠ Distinguishing Strange Quark Matter from Hadron Matter in Compact Stars

Neutron Star: RMF, **Quark Star: Bag Model**

→ **Frequency of g-mode oscillation**

Radial or of g-moc	Quark Star	
	$= 200$	$t = 300$
$n = 1$	78.0	63.1
$n = 2$	45.5	40.0
$n = 3$	30.8	27.8



W.J. Fu, H.Q. Wei, and Y.X. Liu, arXiv: 0810.1084, Phys. Rev. Lett. 101, 181102 (2008)

Taking into account the χSB effect

Newly obtained results for QS in NJL Model

Radial order of g-mode	Neutron Star			Strange Quark Star		
	$t = 100$	$t = 200$	$t = 300$	$t = 100$	$t = 200$	$t = 300$
$n = 1$	717.6	774.6	780.3	100.2	115.4	107.4
$n = 2$	443.5	467.3	464.2	60.1	57.0	51.8
$n = 3$	323.8	339.0	337.5	42.9	40.9	40.2

Ott et al. have found that these g-mode pulsation of supernova cores are very efficient as sources of g-waves (PRL **96**, 201102 (2006))

DS Cheng, R. Ouyed, T. Fischer, ·····

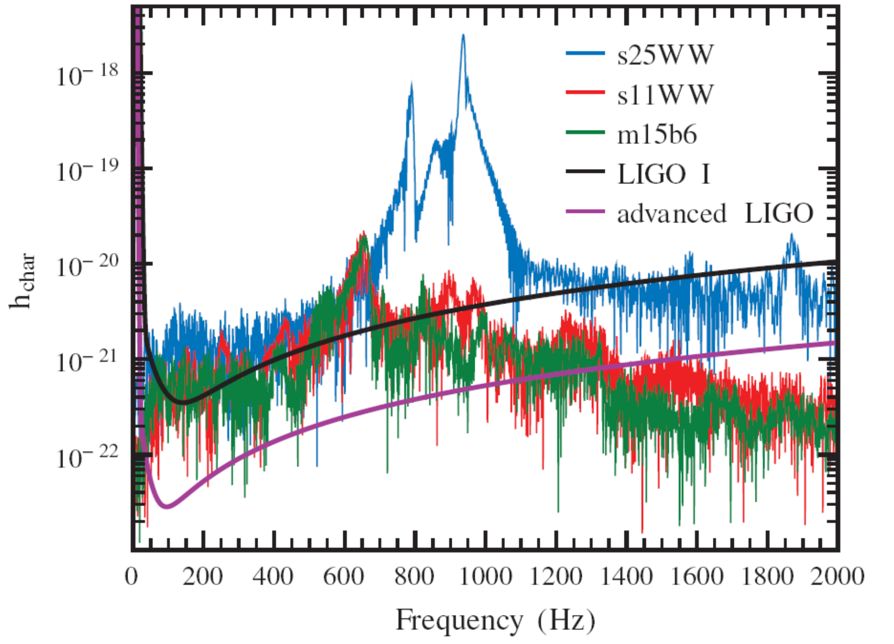


FIG. 4 (color). Characteristic strain spectra contrasted with initial and advanced LIGO (optimal) rms noise curves.

The g-mode pulsation frequency can be a signal to identify the deconfinement phase transition in compact stars.

Thanks !!

♠ Analytic Continuation from Euclidean Space to Minkowskian Space

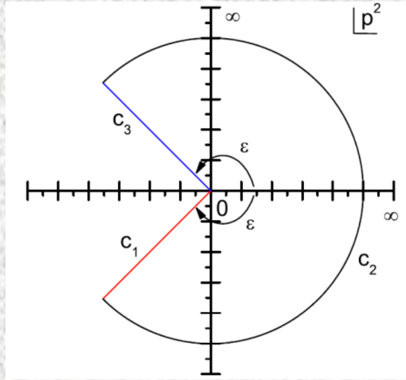
$$g_{\mu\nu}(\varepsilon) = \begin{pmatrix} e^{i\varepsilon} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

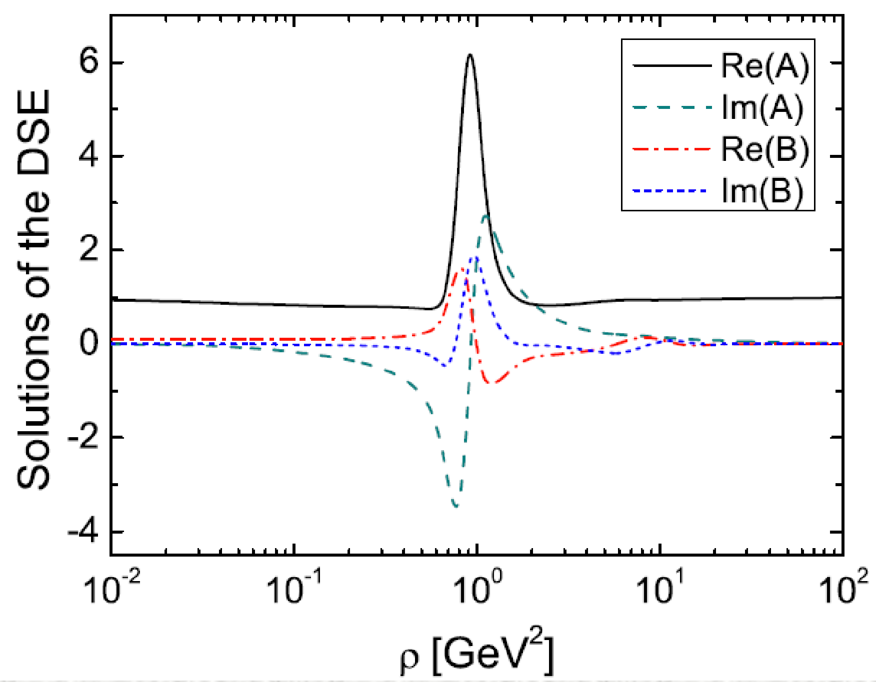
$\varepsilon = 0$, $e^{i\varepsilon} = 1$,

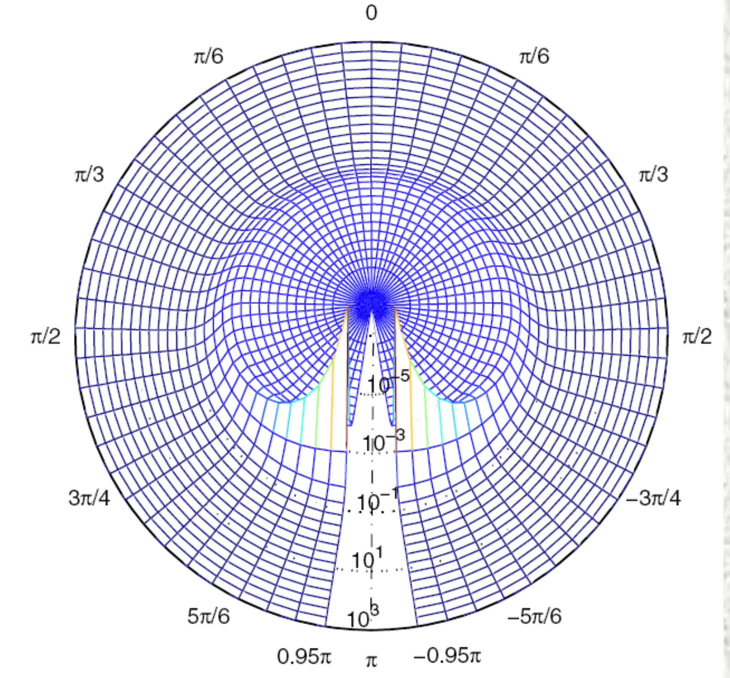
==> E.S.

$\varepsilon = \pi$, $e^{i\varepsilon} = -1$,

==> M.S.





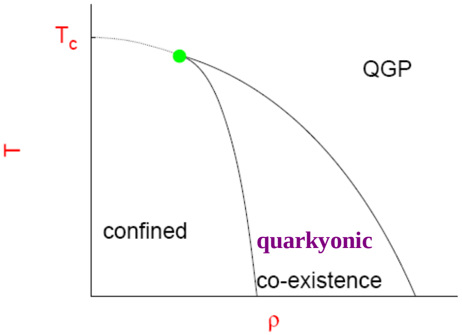


(W. Yuan, S.X. Qin, H. Chen, & YXL, PRD 81, 114022 (2010))

♣ Special topic (2): Coexistence region (Quarkyonic ?)

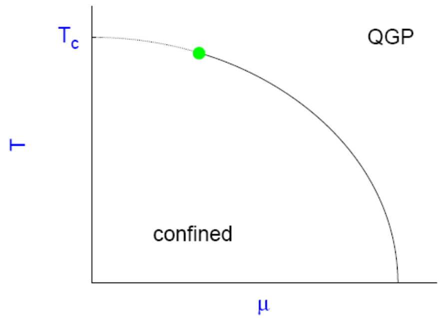
♠ **Lattice QCD Calculation**

de Forcrand, et al.,
Nucl. Phys. B Proc. Suppl. 153, 62 (2006); …



and **Generaal (large-N_c) Analysis**

McLerran, et al., NPA 796, 83 ('07);
NPA 808, 117 ('08);
NPA 824, 86 ('09), …



claim that there exists a quarkyonic phase.

♠ **Inconsistent with Coleman-Witten Theorem !!**

♠ **Can sophisticated continuous field approach of QCD give the coexistence (quarkyonic) phase ?**

♠ **What can we know more for the coexistence phase?**

♣ Special Topic (3): Quark Matter at T above but near T_c

- **HTL Cal.** (Pisarski, PRL 63, 1129('89); Blaizot, PTP S168, 330('07)),
  Lattice QCD (Karsch, et al., NPA 830, 223 ('09); PRD 80, 056001 ('09))
  NJL (Wambach, et al., PRD 81, 094022(2010))
  & Simple DSE Cal. (Fischer et al., EPJC 70, 1037 (2010)) **show:**
  there exists thermal & Plasmino excitations in hot QM.
- **Other Lattice QCD Simulations**
  (Hamada, et al., Phys. Rev. D 81, 094506 (2010)) **claims:**
  No qualitative difference between the quark propagators in the deconfined and confined phases near the T_c.
- **RHIC experiments** (Gyulassy, et al., NPA 750, 30 (2005); Shuryak, PPNP 62, 48 (2009); Song, et al., JPG 36, 064033 (2009); … …) **indicate:**
  the matter is in sQGP state.

♠ **What is the nature of the matter in DSE?**

♠ Property of the matter above but near the T_c

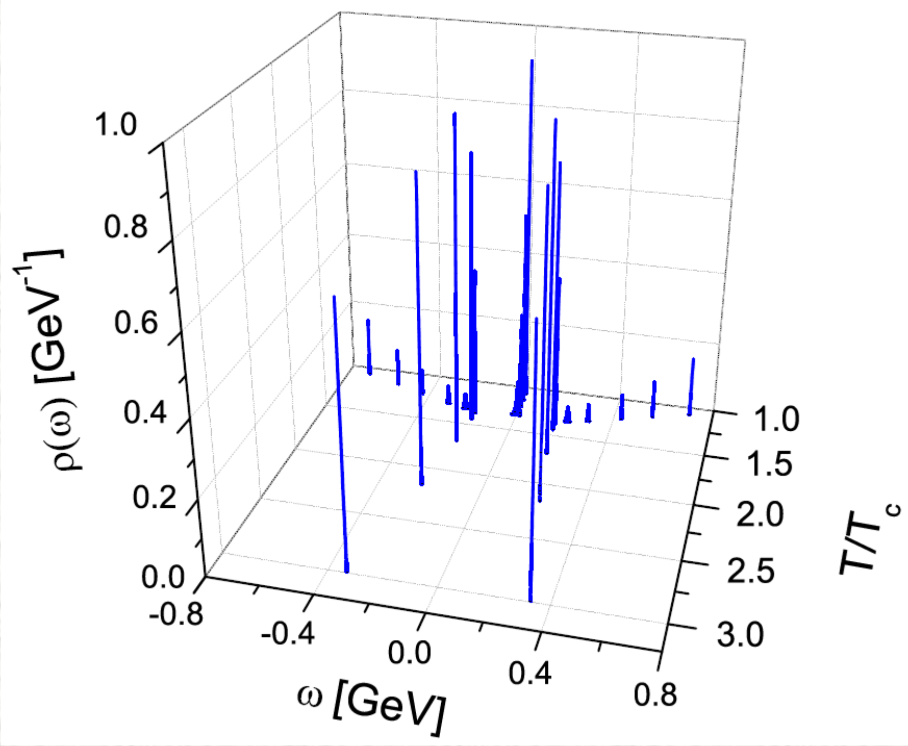
Solving quark's DSE → Quark's Propagator

In M-Space, only Yuan, Liu, etc, PRD 81, 114022 (2010).
Usually in E-Space, Analytical continuation is required.

Maximum Entropy Method

(Asakawa, et al., PPNP 46,459 (2001); Nickel, Ann. Phys. 322, 1949 (2007))

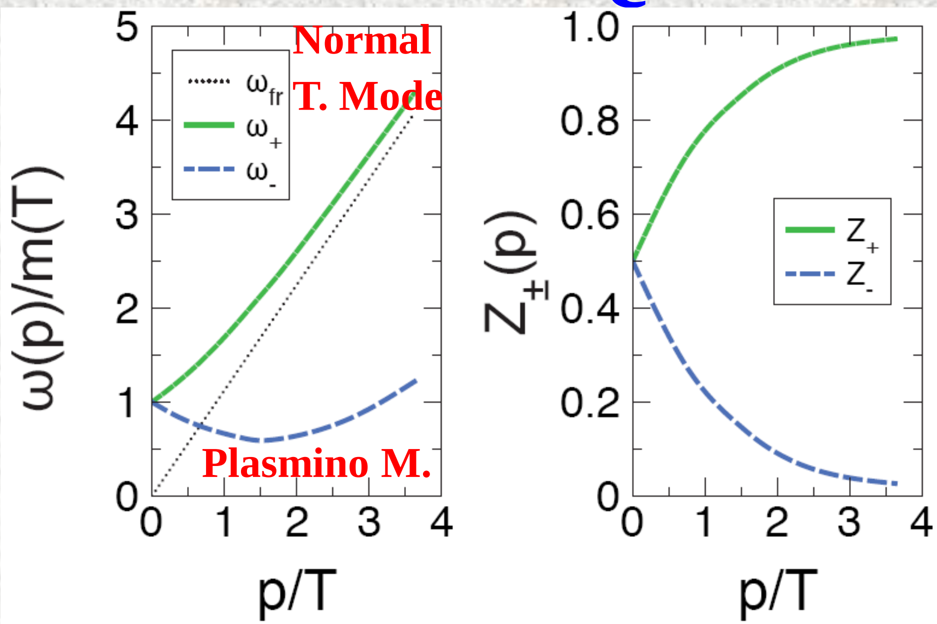
→ **Spectral Function**



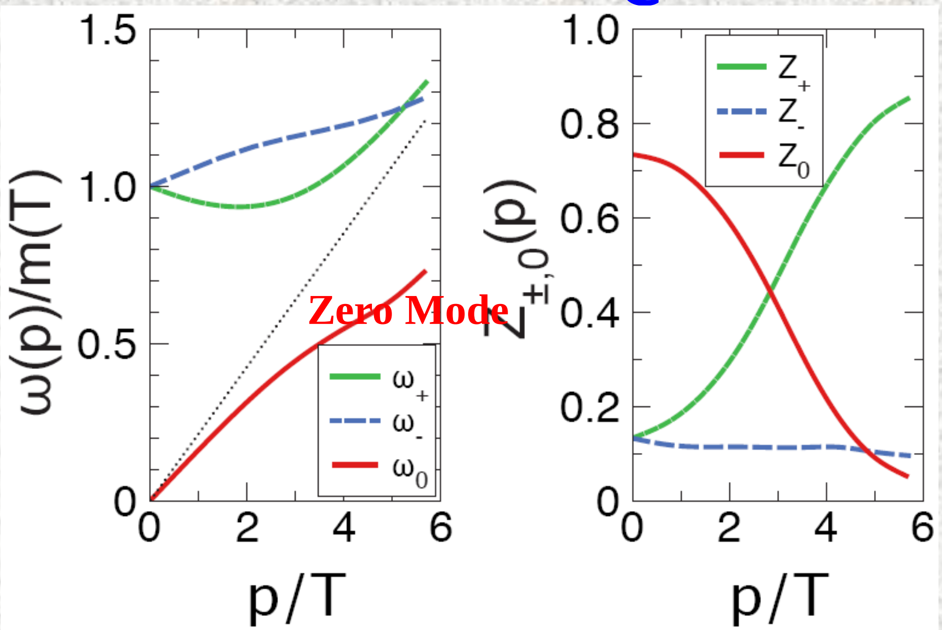
Qin, et al., PRD 84, 014017(2011)

Disperse Relation and Momentum Dependence of the Residues of the Quasi-particles' poles

$T = 3.0T_C$



$T = 1.1T_C$



- ♣ **The zero mode exists at low momentum ($<7.0T_C$), and is long-range correlation ($\lambda \sim \omega^{-1} > \lambda_{FP}$) .**
- ♣ **The quark at the *T* where χS is restored involves still rich phases. And the matter is sQGP.**

S.X. Qin, L. Chang, Y.X. Liu, C.D. Roberts, PRD 84, 014017('11)

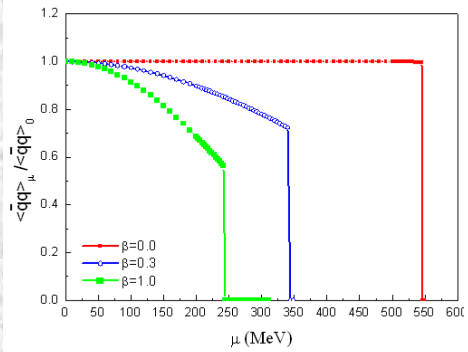
♠Effect of the Chemical Potential on the Chiral Phase Transition

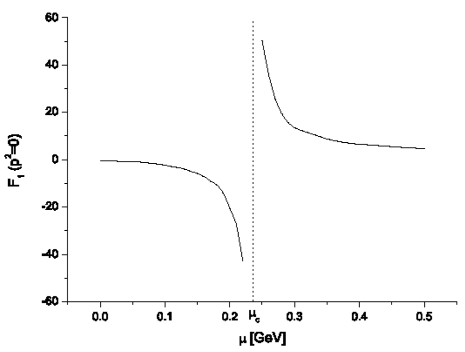
Chiral channel:

(L. Chang, H. Chen, B. Wang, W. Yuan, and Y.X. Liu, Phys. Lett. B 644, 315 (2007))

$$D'_{\mu\nu}(k) = t_{\mu\nu} 4\pi^2 d \frac{\chi^2}{(k^2+\beta\mu^2)^2+\Delta},$$

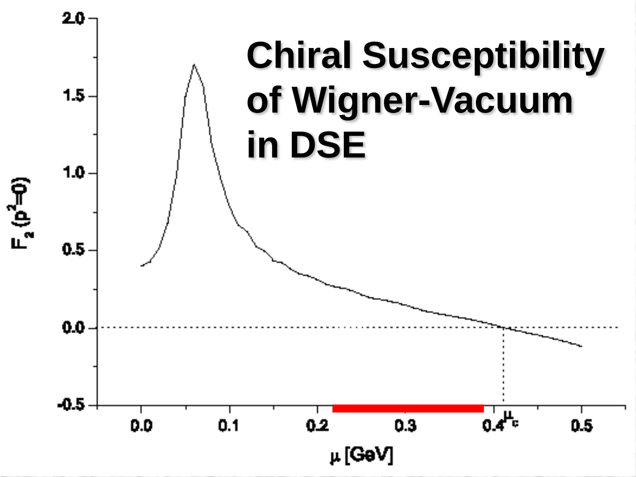
$$V'(r) = -\frac{d\pi\chi^2}{r\sqrt{\Delta}} e^{-r\sqrt{\frac{\sqrt{\beta^2\mu^4+\Delta}+\beta\mu^2}{2}}} \sin[r\sqrt{\frac{\sqrt{\beta^2\mu^4+\Delta}-\beta\mu^2}{2}}]$$





Diquark channel:

(W. Yuan, H. Chen, Y.X. Liu, Phys. Lett. B 637, 69 (2006))



Some Refs. of DSE study on CSC

1. D. Nickel, et al., PRD 73, 114028 (2006);
2. D. Nickel, et al., PRD 74, 114015 (2006);
3. F. Marhauser, et al., PRD 75, 054022 (2007);
4. 5. D. Nickel, et al., PRD 77, 114010 (2008);

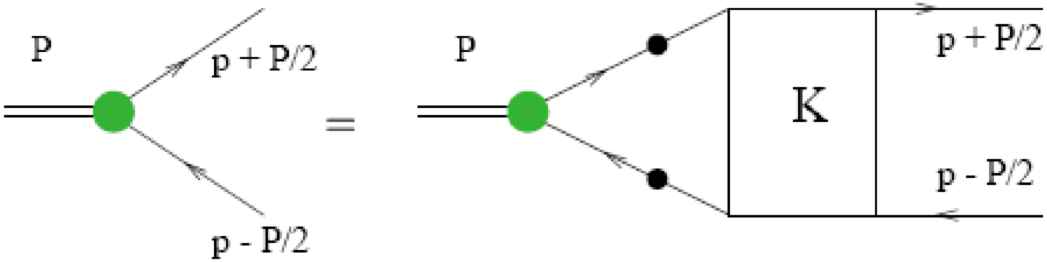
…………

Hadron Structure

Meson Bethe-Salpeter Eqn

- Quantum field theory bound states: BSE

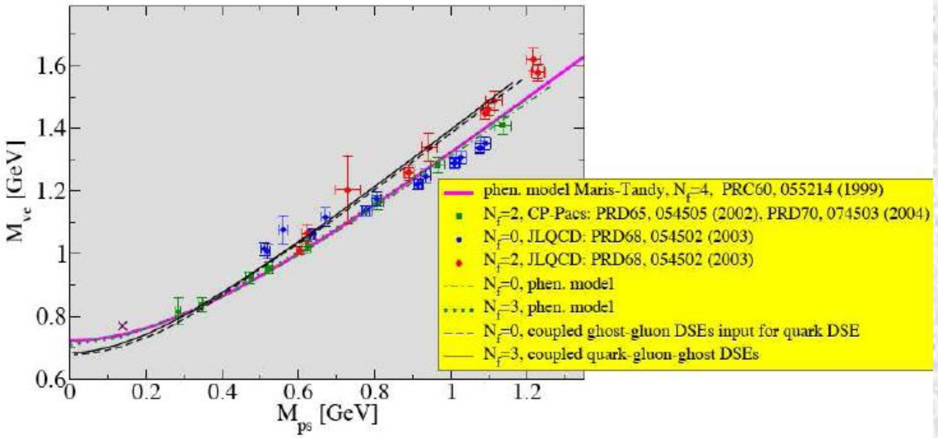
$$\Gamma_M(p;P) = \int_k^\Lambda K(p,k;P)\, S(k_+)\, \Gamma_M(k;P)\, S(k_-)$$



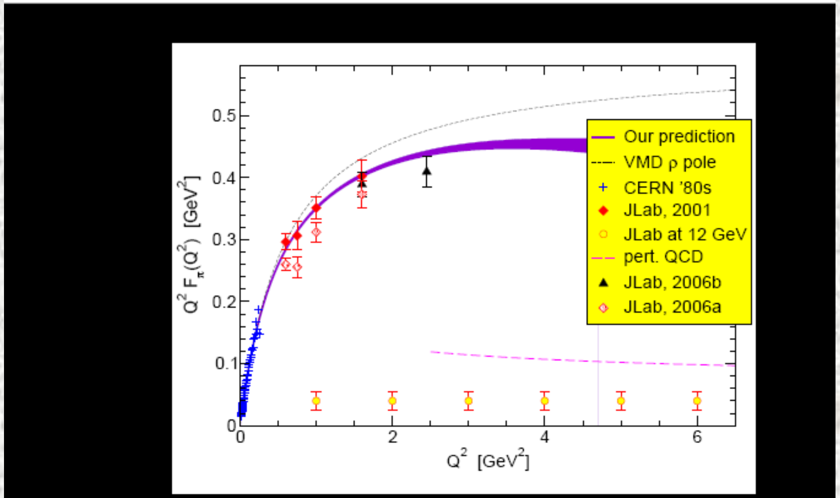
- Light quark propagator $\frac{1}{i\gamma\cdot p A(p^2)+B(p^2)}$
- Heavy quark propagator $\frac{1}{i\gamma\cdot p+M^{\rm cons}}$
- Fit $M^{\rm cons}$ to lightest ps meson

Some Numerical Results

DSE and Lattice results for M_V and M_{ps}



Pion electromagnetic form factor



PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]
2006a: V. Tadevosyan *et al*, [nucl-ex/0607007], 2006b: T. Horn *et al*, [nucl-ex/0607005]

Summary of light meson results
$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle^0_\mu$	(0.236 GeV)^3	$(0.241^\dagger)^3$
m_π	0.1385 GeV	$0.138^\dagger$
f_π	0.0924 GeV	$0.093^\dagger$
m_K	0.496 GeV	$0.497^\dagger$
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

γπγ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6$ s^{-1}	7.38
$\Gamma(K_{\mu3})$	$5.2 \cdot 10^6$ s^{-1}	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
m_{K^*}	0.892 GeV	0.936
f_{K^*}	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^*K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

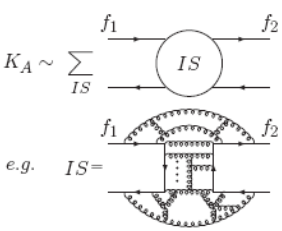
$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^*K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^*K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

Axial anomaly and $\eta - \eta'$ states

- Ch symm: $\partial_\mu(z)\langle j^\alpha_{5\mu}(z)\, q(x)\bar{q}(y)\rangle$ involves $2\,\mathrm{tr_f}(\mathcal{F}^\alpha)\langle Q_t(z)q(x)\bar{q}(y)\rangle$
- Matrix elements, amputated ⇒ AV-WTI



$$P_\mu \Gamma^\alpha_{5\mu}(k;P) = -2i\, \mathcal{M}^{\alpha\beta}\Gamma^\beta_5(k;P) - \delta_{\alpha,0}\,\Gamma_A(k;P)$$
$$+S^{-1}(k_+)\, i\gamma_5 \mathcal{F}^\alpha + i\gamma_5 \mathcal{F}^\alpha S^{-1}(k_-)$$

- Residues at PS poles ⇒ PS mass formula for arbitrary m_q, any flavor:

$$m^2_p f^\alpha_p = 2\,\mathcal{M}^{\alpha\beta}\rho^\beta_p + \delta^{\alpha,0}\, n_p \quad , \quad n_p = 2\,\mathrm{tr_f}(\mathcal{F}^0)\,\langle 0|Q_t|p\rangle$$

$$i\rho^\alpha_p(\mu) = Z_4\,\mathrm{tr}\int^\Lambda_q \mathcal{F}^\alpha \gamma_5\, \chi_p(q;P)\ , \quad p = \text{any PS state}$$

——[Bhagwat, Chang, Liu, Roberts, PCT, PRC **(76), 2007; arXiv:0708.1118**]

♠ Effect of the F.-S.-B. (m_0) on Meson's Mass

Solving the 4-dimenssional covariant B-S equation with the kernel being fixed by the solution of DS equation and flavor symmetry breaking, we obtain

	Expt. (GeV)	Calc. (GeV)	Th/
"ρ^0"	0.7755	0.7704	
$\rho^\pm$	0.7755	0.7755	
"ω"	0.7827	0.7806	
$K^{*\pm}$	0.8917	0.8915	
K^{*0}	0.8960	0.8969	
ϕ	1.0195	1.0195	
D^{*0}	2.0067	1.8321	
$D^{*\pm}$	2.0100	1.8387	
$D_s^{*\pm}$	2.1120	1.9871	
J/ψ	3.0969	3.0969	
$B^{*\pm}$		4.8543	
B^{*0}		4.8613	
B_s^{*0}		5.0191	
$B_c^{*\pm}$		6.2047	
Υ	9.4603	9.4603	

	Expt. (GeV)	Calc. (GeV)	Th/Ex-1 (%)
π^0	0.13498	0.13460	-0.3
$\pi^\pm$	0.13957	0.13499	-3.3
$K^\pm$	0.49368	0.41703	-15.5
K^0	0.49765	0.42662	-14.3
η	0.54751	0.45499	-16.9
η'	0.95778	0.91960	-4.0
D^0	1.8645	1.6195	-13.1
$D^\pm$	1.8693	1.6270	-13.0
$D_s^\pm$	1.9682	1.7938	-8.9
η_c	2.9804	3.0171	1.2
$B^\pm$	5.2790	4.7747	-9.6
B^0	5.2794	4.7819	-9.4
B_s^0	5.3675	4.9430	-7.9
$B_c^\pm$	6.286	6.1505	-2.2
η_b	9.300	9.4438	1.5

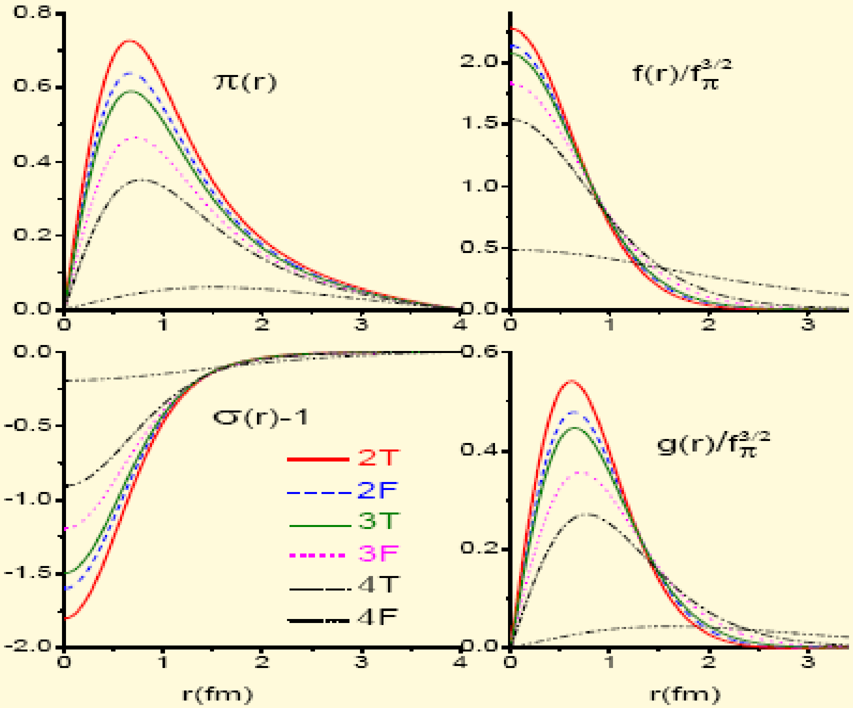
(L. Chang, Y. X. Liu, C. D. Roberts, et al., Phys. Rev. C 76, 045203 (2007))

DSE Soliton Description of Nucleon

Maris-Tandy模型有效胶子传播子

$$g^2D(q) = \frac{4\pi^2 d}{\omega^6} q^2 e^{-q^2/\omega^2} + \frac{8\pi^2\gamma_m\pi}{\ln[\tau + (1+\frac{q^2}{\Lambda_{QCD}^2})^2]} \frac{1-\exp(-\frac{q^2}{4m_t^2})}{q^2},$$

$\gamma_m = 12/25$、$\tau = e^2 - 1$、$\Lambda_{QCD} = 0.234$ GeV和$m_t = 0.5$ GeV。介子性质对于参数的约束$\omega d = (0.72GeV)^3$ 。



	2T	2F	3T	3F	4T	4F
ω (GeV)	0.401		0.450		0.472	
d (GeV^2)	0.930		0.830		0.790	
ε_{val} (MeV)	107	162	189	237	270	300
E_p(MeV)	109	123	126	106	93	15
E_k(MeV)	766	608	527	355	224	18
E_{tot} (MeV)	1196	1217	1220	1172	1127	933
M_{ncr}(MeV)	1060	1094	1084	1044	996	890
M_{rec}(MeV)	956	909	892	814	761	618
R_{ncr}(fm)	0.67	0.71	0.79	0.93	1.15	2.74
R_{rec}(fm)	0.60	0.61	0.68	0.77	0.95	2.22

B. Wang, H. Chen, L. Chang, & Y. X. Liu, Phys. Rev. C 76, 025201 (2007)

Collective Quantization: Nucl. Phys. A790, 593 (2007).

Compositions and Phase Structure of Compact Stars and their Identification

Radio Pulses =➔ "Neutron" Stars

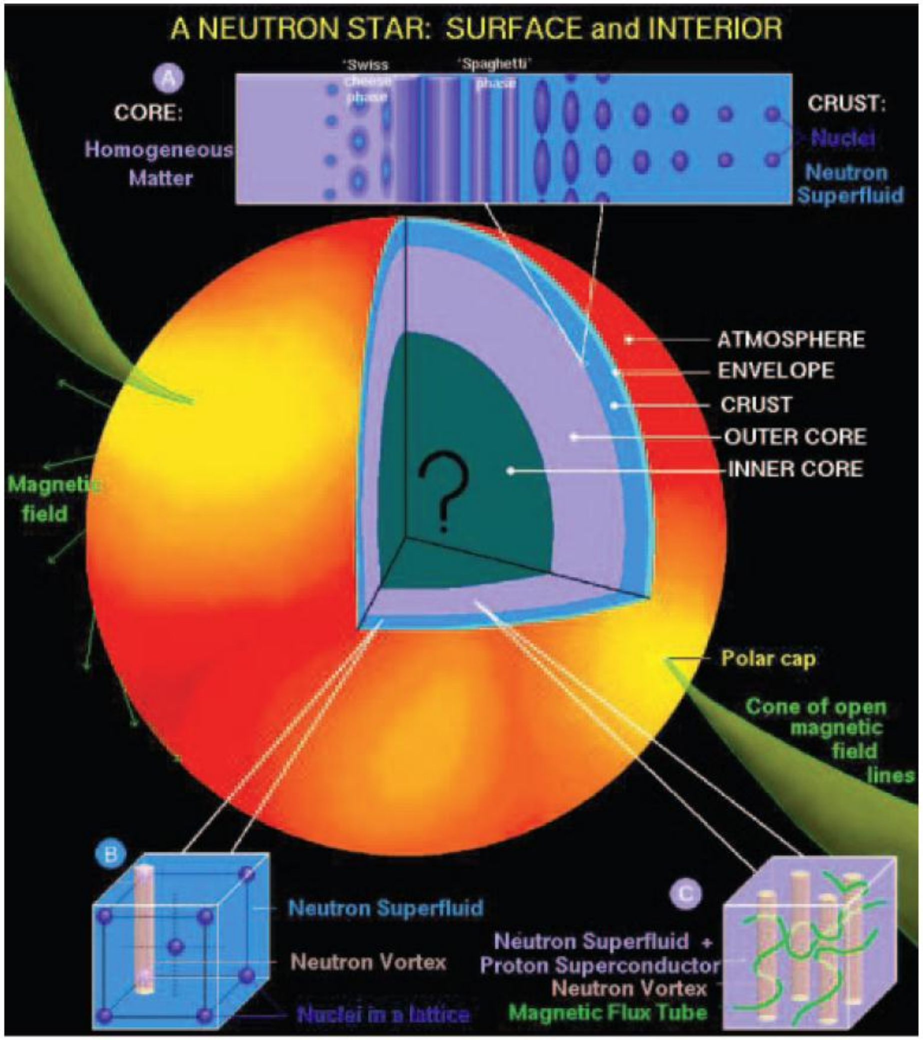


Fig. 3. The major regions and possible composition inside a normal-matter neutron star. The top bar illustrates expected geometric transi-

Composition & Structure of NS are Still Under Study !

J. M. Lattimer, *et al.* *Science* **304**, 536 (2004)

Conjecture of the Composition of Compact Stars

(F.Weber, PPNP 54, 193 (2005))

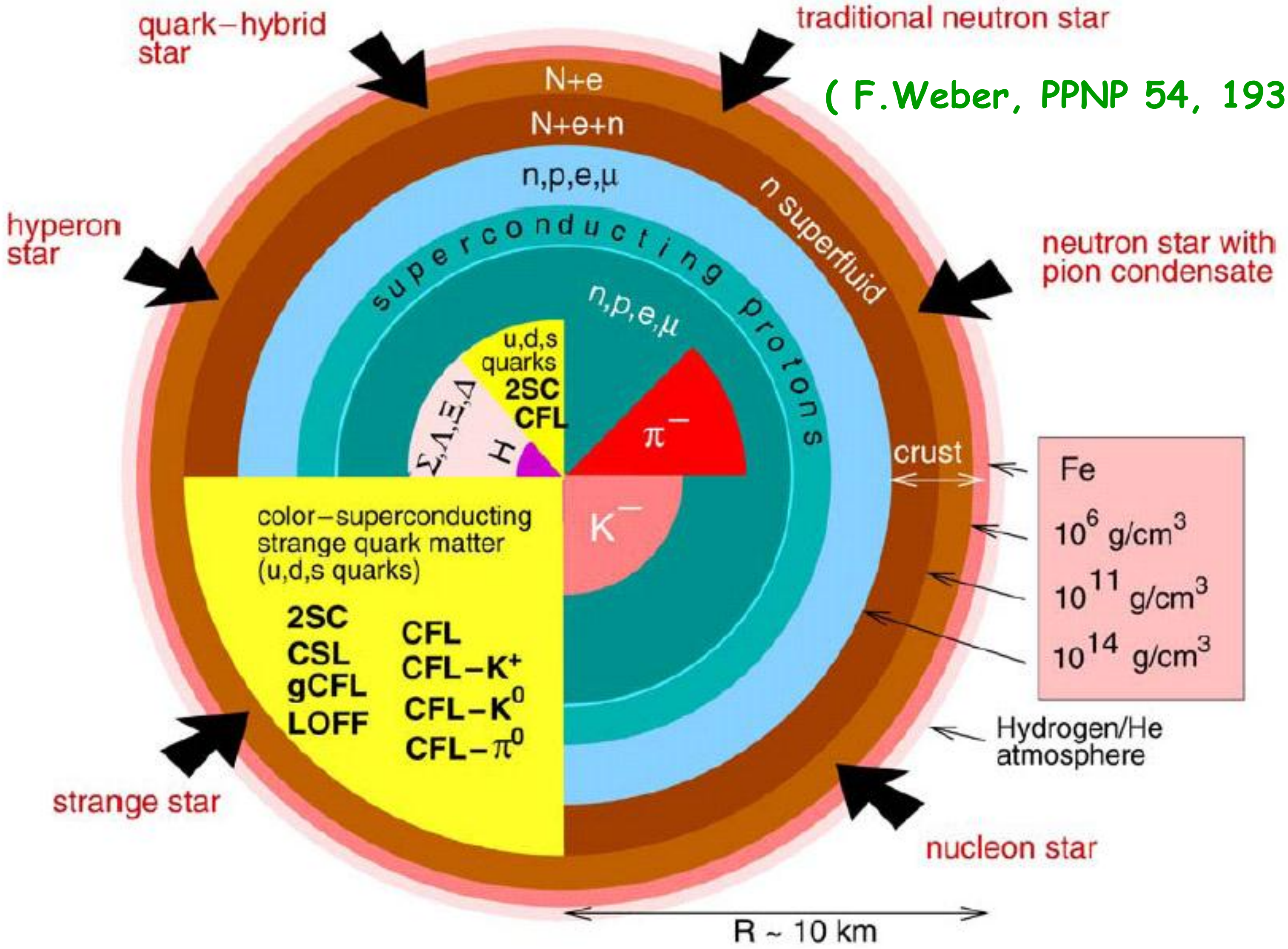


Fig. 1. Competing structures and novel phases of subatomic matter predicted by theory to make their appearance in the cores ($R \lesssim 8$ km) of neutron stars [1].