

# Elliptic and Triangular Flow and their Correlations in ultrarelativistic High Multiplicity Proton Proton Collisions at 14 TeV

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for Advanced Studies

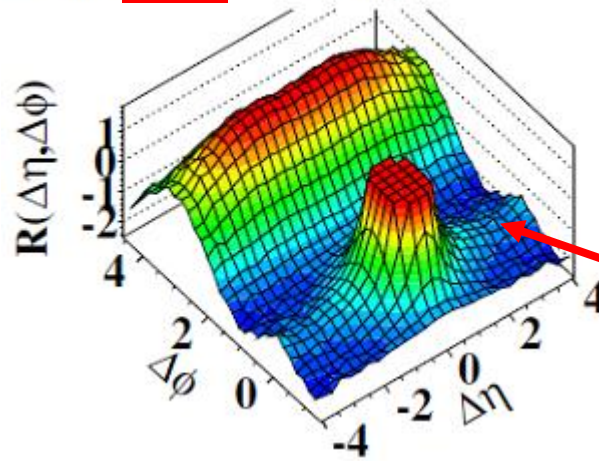


HIC  
for FAIR  
Helmholtz International Center

# Near side "ridge" in p-p Collisions at 7 TeV

experiment

(d) CMS  $N \geq 110$ ,  $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$

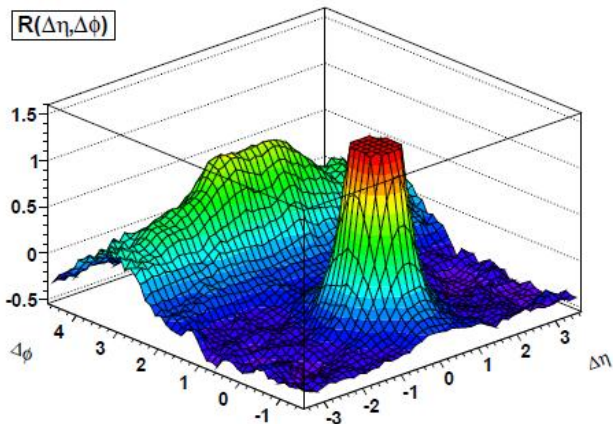


ridge

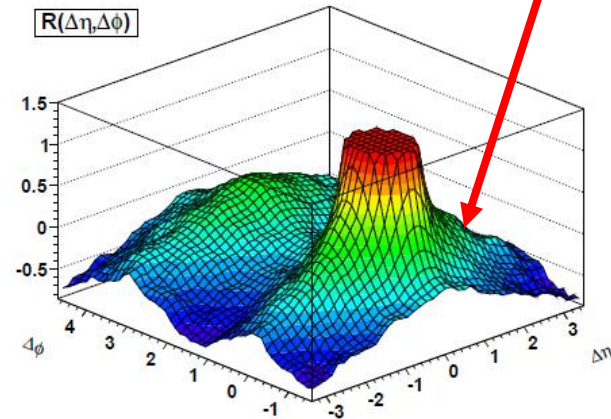
CMS Collaboration, JHEP 1009, 091 (2010)

theory

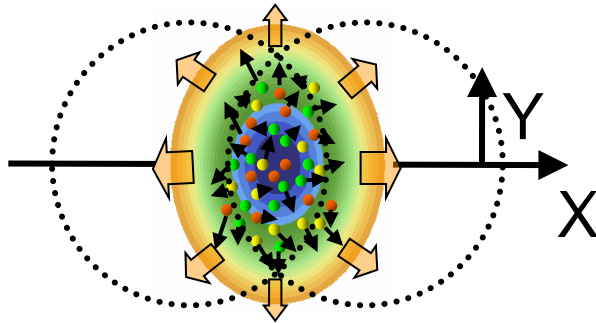
EPOS without hydro



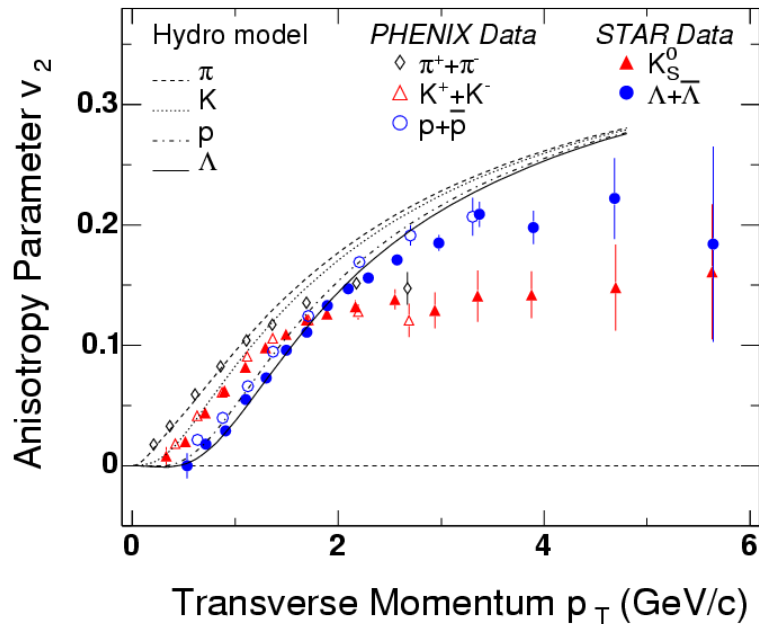
EPOS with hydro



# Au+Au Collisions at RHIC-200 GeV



eccentricity  $\rightarrow$  elliptic flow



QGP at RHIC is a nearly perfect fluid.

# p+p Collisions at LHC-14 TeV



collective flow ?

**NO:** too small volume

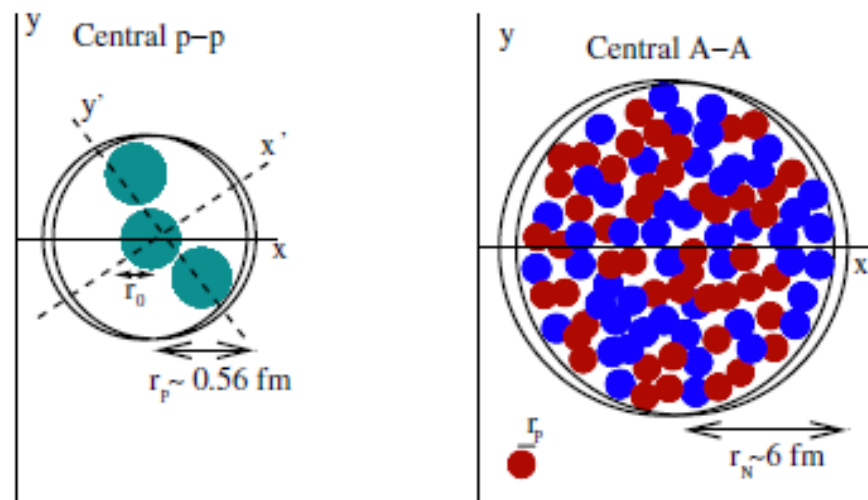
**YES:** very high energy density  
high multiplicity events

**NO:** symmetry in central collisions

**YES:** initial fluctuations

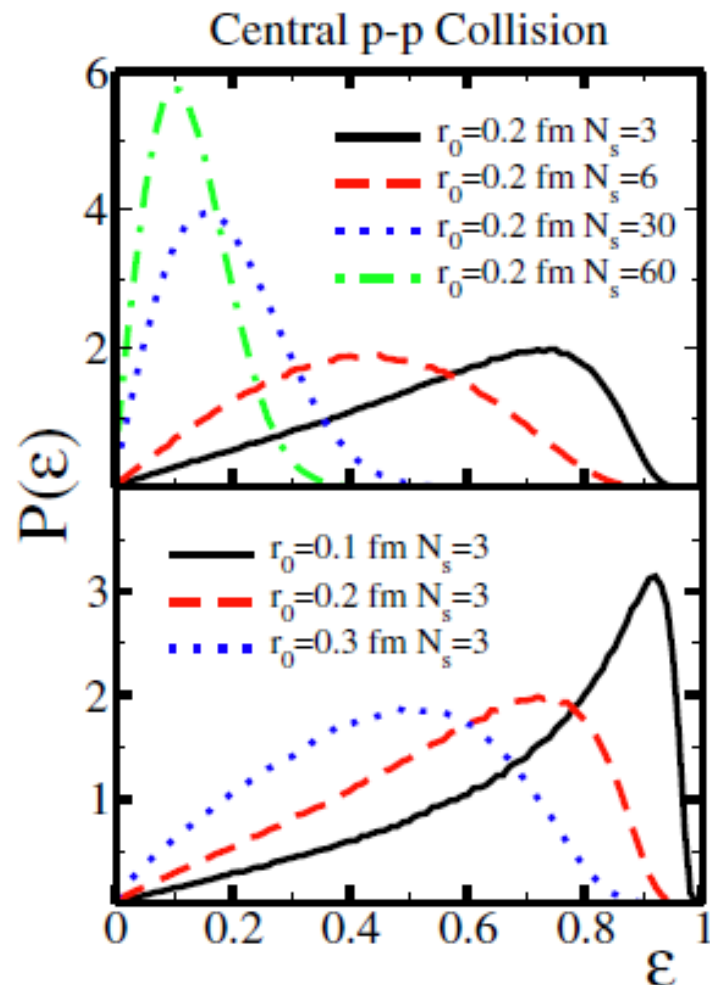
# Eccentricity Fluctuations Make Flow Measurable in High Multiplicity $p$ - $p$ Collisions

Jorge Casalderrey-Solana and Urs Achim Wiedemann



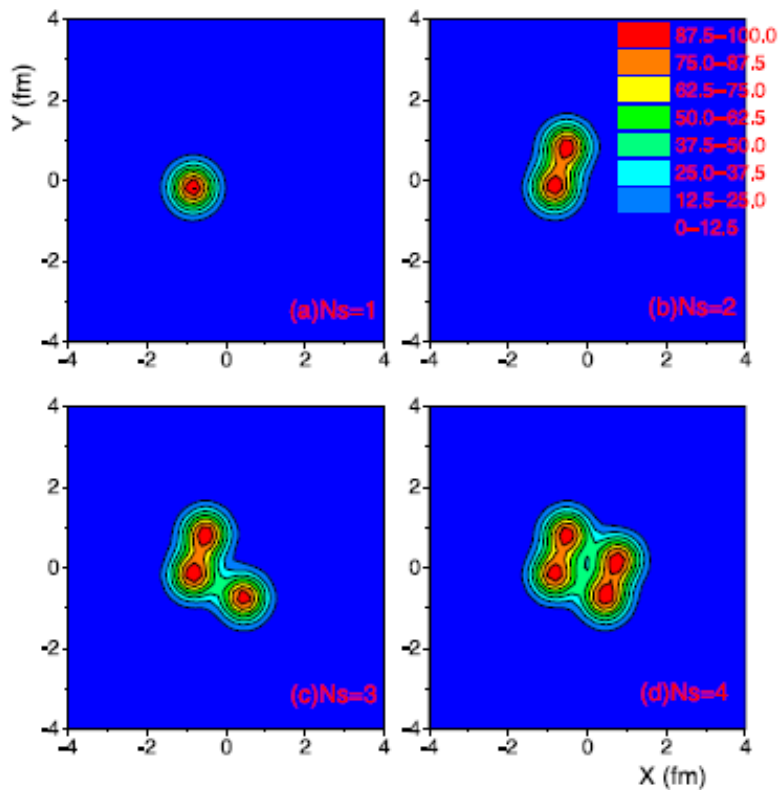
$$\epsilon = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}$$

Here  $\sigma_x^2 = \{x^2\} - \{x\}^2$ ,  $\sigma_y^2 = \{y^2\} - \{y\}^2$ ,  $\sigma_{xy} = \{xy\} - \{x\}\{y\}$  and the event-by-event average  $\{\dots\}$  is taken



# ideal hydrodynamic calculations: Chaudhuri, PLB 692 (2010) 15

p+p @ 14 TeV



$N_S$	$\epsilon$	$\langle n_{\text{mult}} \rangle$	$\langle p_T \rangle$ (GeV)	$\langle v_2 \rangle$
1	0	$4.97 \pm 0.02$ ( $4.97 \pm 0.02$ )	$0.722 \pm 0.001$ ( $0.722 \pm 0.001$ )	$0.003 \pm 0.001$ ( $0.003 \pm 0.001$ )
2	$0.532 \pm 0.052$	$7.75 \pm 1.17$ ( $7.88 \pm 1.11$ )	$0.634 \pm 0.054$ ( $0.632 \pm 0.054$ )	$0.147 \pm 0.071$ ( $0.152 \pm 0.068$ )
<b>3</b>	<b><math>0.536 \pm 0.051</math></b>	<b><math>9.68 \pm 2.24</math></b> <b>(<math>9.87 \pm 2.12</math>)</b>	<b><math>0.599 \pm 0.037</math></b> <b>(<math>0.601 \pm 0.040</math>)</b>	<b><math>0.160 \pm 0.053</math></b> <b>(<math>0.158 \pm 0.056</math>)</b>
4	$0.457 \pm 0.048$	$11.05 \pm 2.58$ ( $11.39 \pm 2.67$ )	$0.582 \pm 0.029$ ( $0.581 \pm 0.026$ )	$0.161 \pm 0.050$ ( $0.160 \pm 0.049$ )
EI		$8.36 \pm 2.91$	$0.634 \pm 0.065$	$0.118 \pm 0.019$
EII		$8.45 \pm 2.36$	$0.627 \pm 0.057$	$0.138 \pm 0.022$

$v_2 \approx 0.16$  for 3 hot spots, even in low multiplicity ( $n_{\text{mult}} \sim 10$ ) events

- geometrical overlap in p+p like in A+A (small  $v_2 \sim 3\%$ )

**hydro:** M. Luzum, P. Romatschke, PRL103 (2009).

S. K. Prasad, V. Roy, S. Chattopadhyay, A. K. Chaudhuri, PRC82 (2010).

G. Ortona, G. S. Denicol, P. Mota, T. Kodama, arXiv:0911.5158.

$\epsilon_2$ - $v_2$  scaling:

$$v_2\{4\} = \epsilon\{4\} \left( \frac{v_2}{\epsilon} \right)^{\text{hydro}} \frac{1}{1 + \frac{\bar{\lambda}}{K_0} \frac{\langle S \rangle}{dy}}$$

L. Cunqueiro, J. Dias de Deus, C. Pajares, Eur. Phys. J. C65 (2010).

D. d'Enterria, et al., Eur. Phys. J. C66 (2010).

- initial fluctuations (hot spots)

**hydro:** P. Bozek, Acta Phys. Polon. B41 (2010).

A. K. Chaudhuri, Phys. Lett. B692 (2010).

$\epsilon_2$ - $v_2$  scaling:

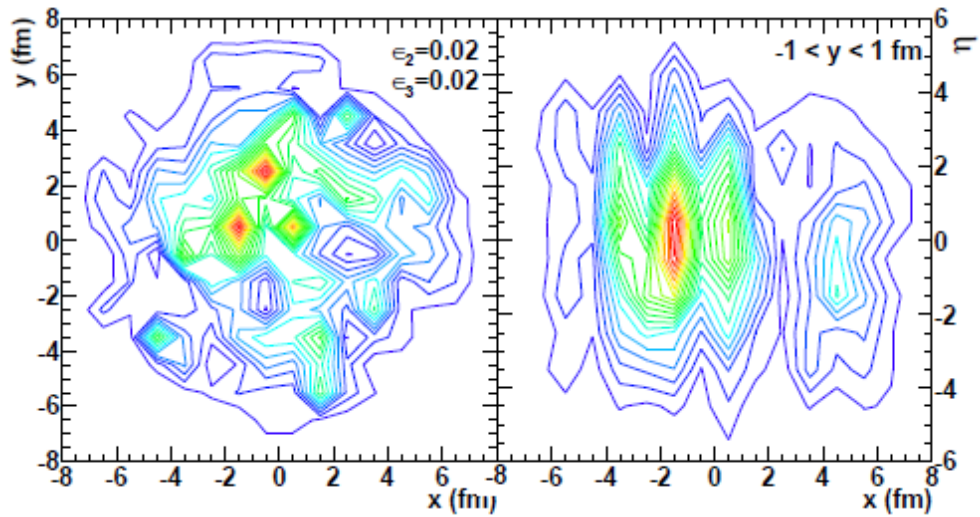
J. Casalderrey-Solana, U. A. Wiedemann, PRL104 (2010).

E. Avsar, et al., Phys. Lett. B702 (2011).

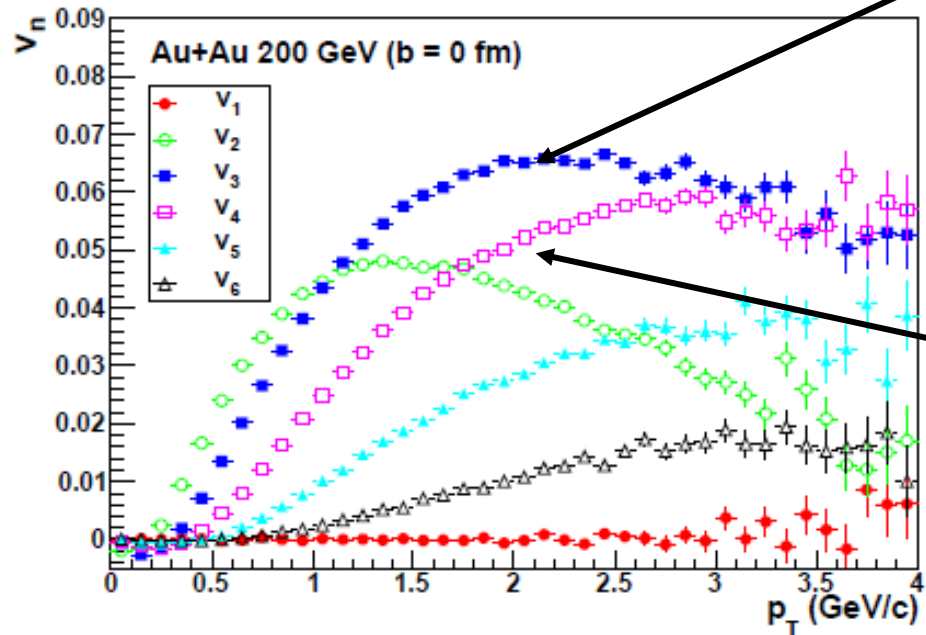
**transport:**

D. -M. Zhou, et al., Nucl. Phys. A860 (2011).

# Hot spots and harmonic flow



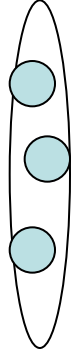
**AMPT for central Au+Au**  
G.-L. Ma and X.-N. Wang,  
PRL 106, 162301 (2011)



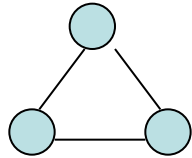
$V_3$

$V_2$

p+p @ LHC



$\varepsilon_2$  dominant  $\Rightarrow v_2$

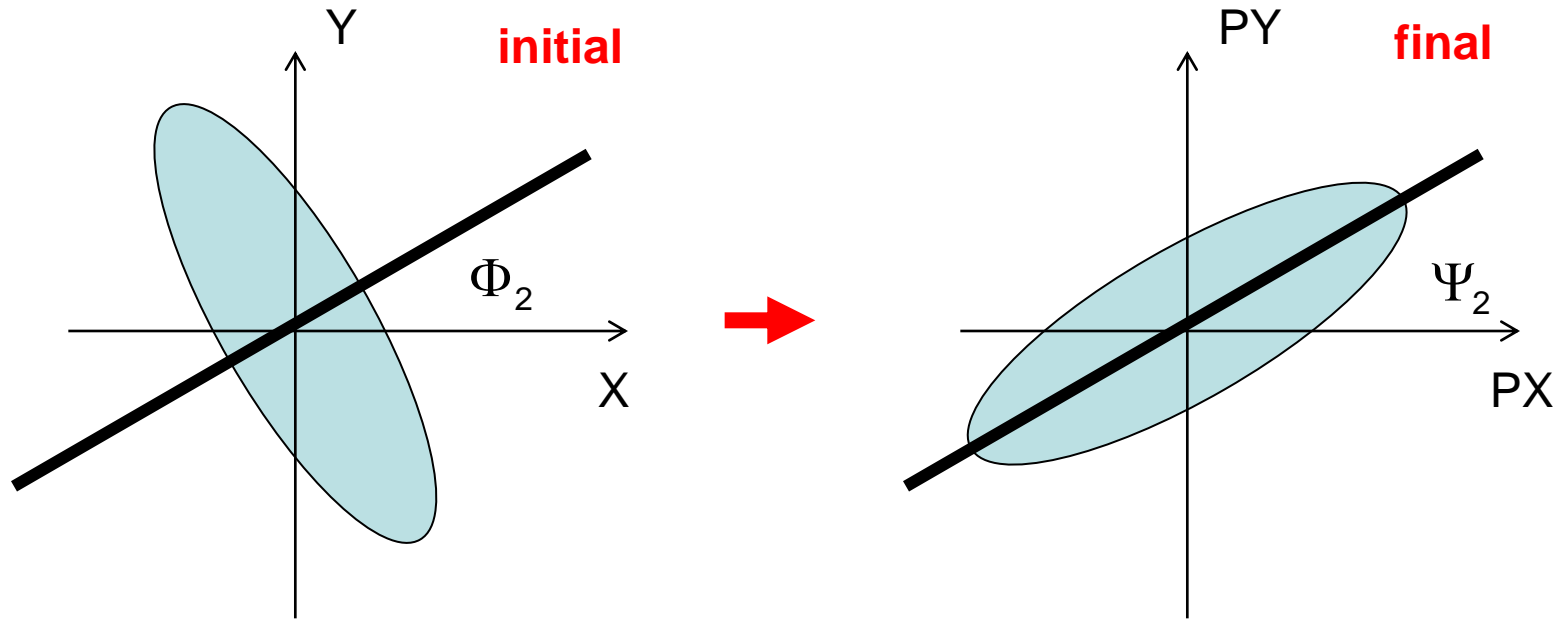


$\varepsilon_3$  dominant  $\Rightarrow v_3$

In contrast to Au+Au  $b=0$  at RHIC there may be  $\varepsilon_2$ - $\varepsilon_3$  event-by-event correlation in p+p at LHC.



# Definitions of event-plan angles



Initial eccentricity  $\varepsilon_2$   
Initial event-plan angle  $\Phi_2$

Elliptic flow  $v_2$   
Final event-plan angle  $\Psi_2$

$$\Phi_2 = \Psi_2$$

eccentricities

collective flow

$$\epsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi) \rangle^2 + \langle r^n \sin(n\phi) \rangle^2}}{\langle r^n \rangle}$$

$$v_n(p_T) = \langle \cos n(\psi - \Psi_n) \rangle$$

initial event-plane angle

final event-plane angle

$$\Phi_n = \frac{1}{n} \arctan \frac{\langle r^n \sin(n\phi) \rangle}{\langle r^n \cos(n\phi) \rangle}$$

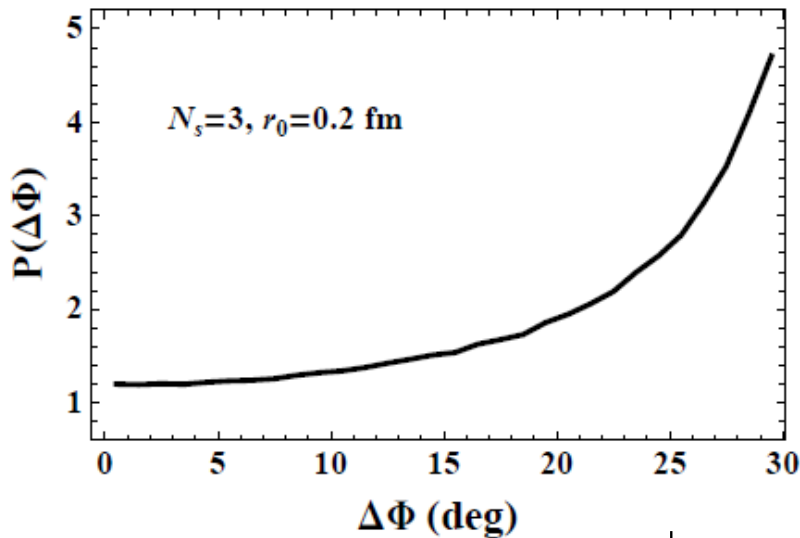
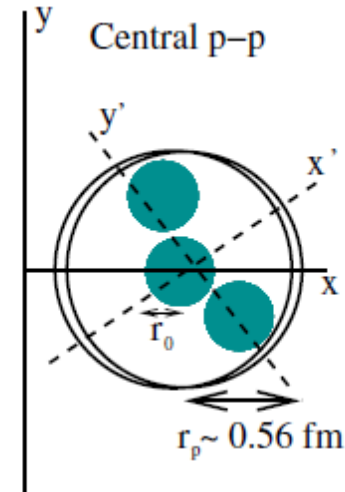
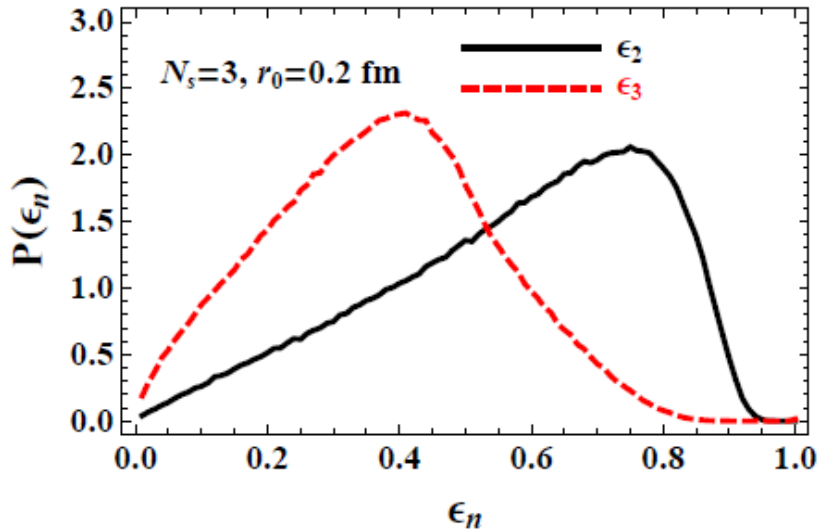
$$\Psi_n = \frac{1}{n} \arctan \frac{\langle \sin(n\psi) \rangle}{\langle \cos(n\psi) \rangle}$$

If the translations from  $\epsilon_n$  to  $v_n$  ( $n=2,3,\dots$ ) are completely independent

$$\Rightarrow \Phi_n = \Psi_n$$

$$\Phi_2 - \Phi_3 = \Psi_2 - \Psi_3$$

# $\epsilon_2, \epsilon_3$ and their correlation in the hot spots scenario



If  $\epsilon_n$  to  $v_n$  translations are independent ?

$$\Rightarrow P(\Delta\Psi) = P(\Delta\Phi)$$

From final event-plane correlations one can extract informations about initial conditions.

$$\Delta\Phi = |\Phi_2 - \Phi_3|$$

$$\Delta\Psi = |\Psi_2 - \Psi_3|$$

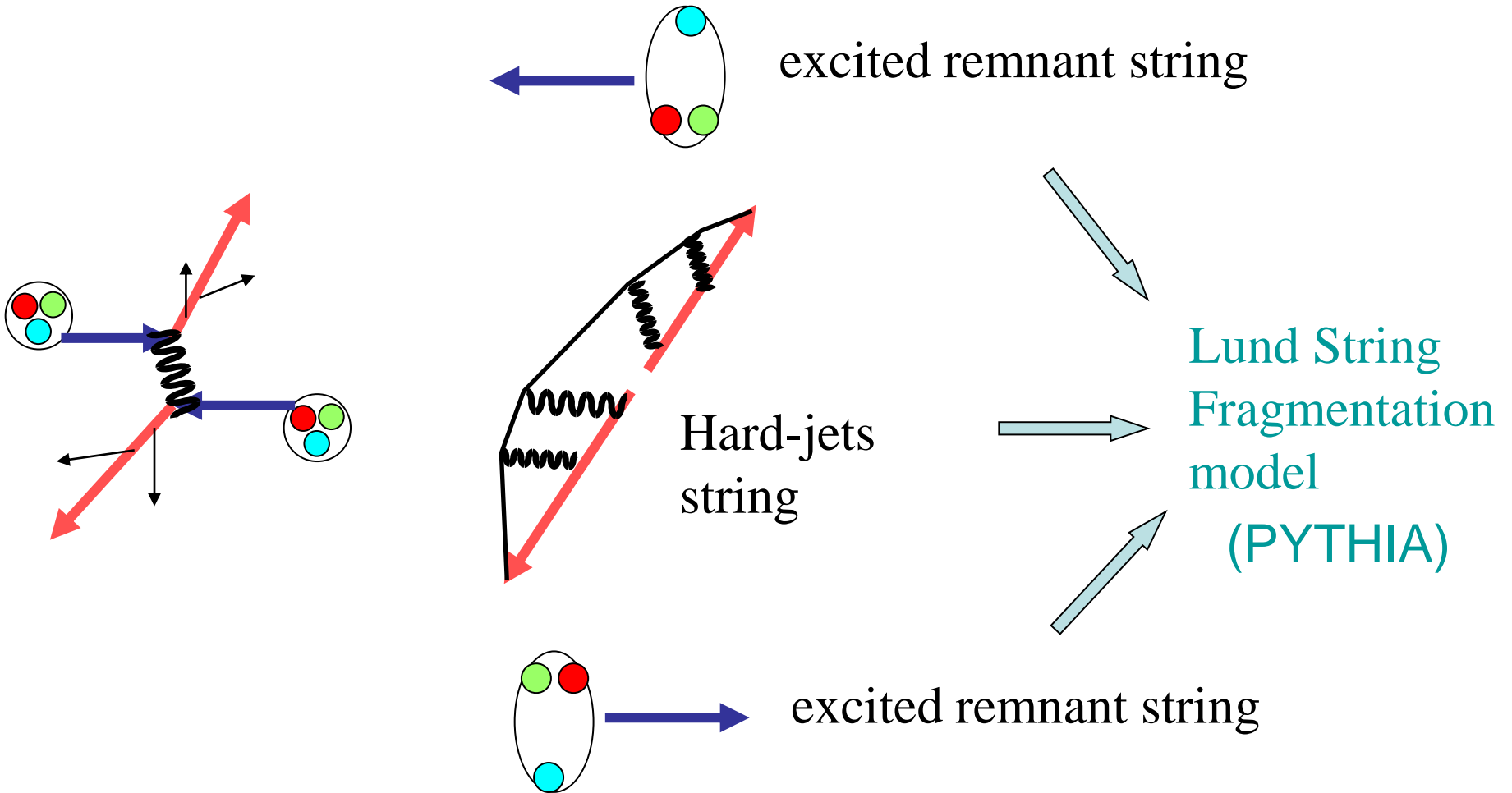
## Our Model:

Hot Spots + HIJING + Parton Transport(BAMPS)

# HIJING

X.N. Wang and M. Gyulassy, Phys. Rev. D44, 3501 (1991).

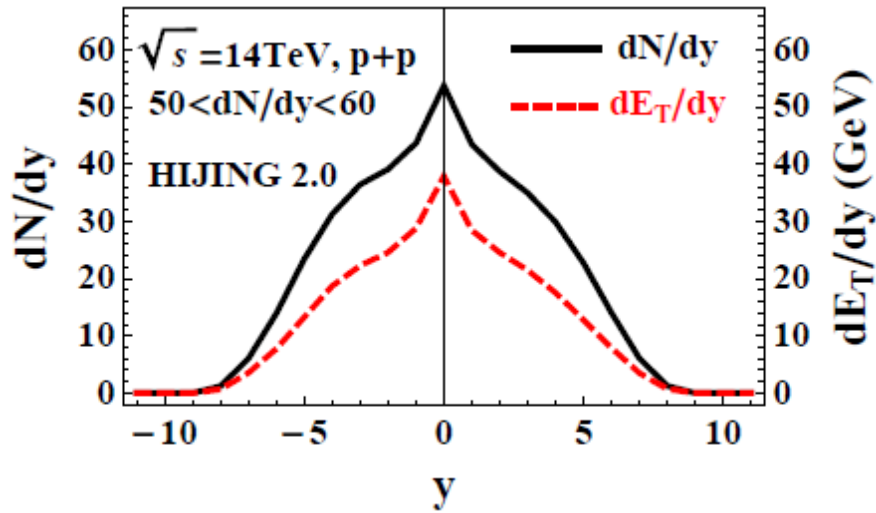
W.T. Deng, X.N. Wang and R. Xu, Phys. Rev. C83, 014915 (2011).



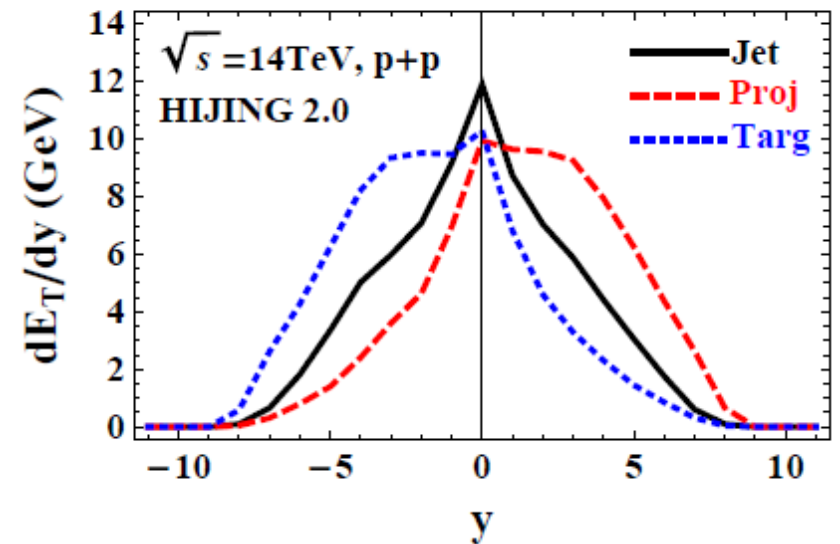
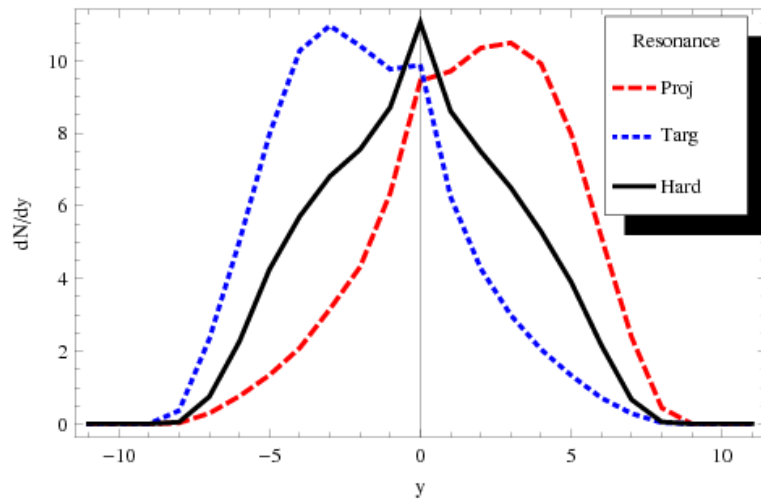
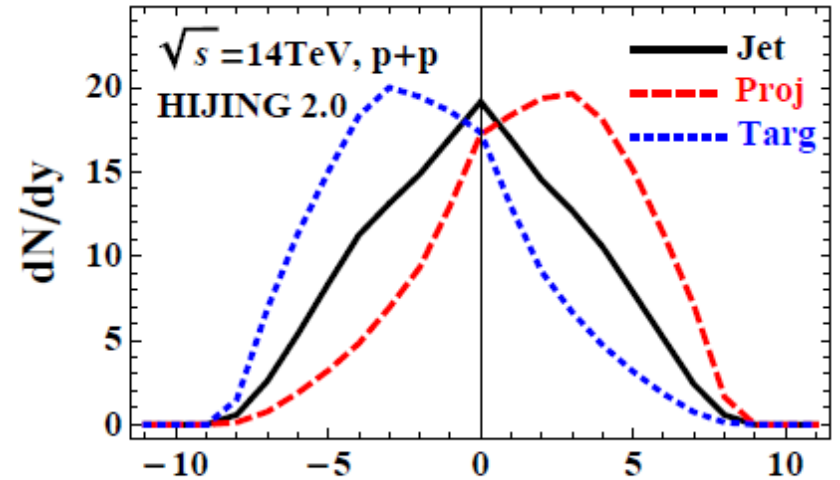
**3 strings**

# HIJING

at  $\sqrt{s} = 14$  TeV



Resonances break to quark-antiquark pairs.



# Parton Transport Model

**BAMPS**: Boltzmann Approach of MultiParton Scatterings

ZX and C. Greiner, PRC 71, 064901 (2005)

A transport algorithm solving the Boltzmann-Equations for on-shell partons with pQCD interactions

$$\left(\partial_t + \frac{\vec{p}}{E} \vec{\nabla}\right) f(x, p) = C_{gg \rightarrow gg} + C_{gg \leftrightarrow ggg}$$

**new development ggg → gg**

(Z)MPC, VNI/BMS, AMPT, PACIAE

2 ↔ 3 are essential for fast thermalization and the buildup of elliptic flow due to large open angle.

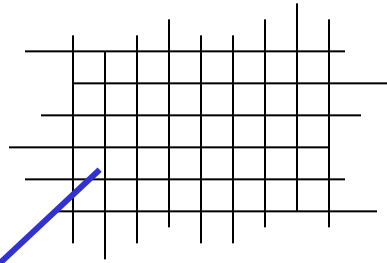
ZX, Greiner, Stöcker, PRL 101, 2008

# Stochastic algorithm

A.Lang et al., J. Comp. Phys. 106, 391(1993)

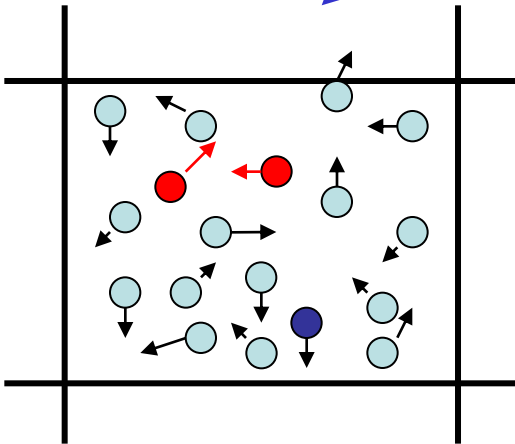
$$f(\bar{p}, \bar{x}, t) = \sum_i^N \delta^{(3)}(\bar{p}_i - \bar{p}) \delta^{(3)}(\bar{x}_i(t) - \bar{x})$$

Space is divided into small cells !



$\Delta^3x$

collision probability -- **stochastic**



$$\text{for } 2 \rightarrow 2 \quad P_{22} = v_{rel} \frac{\sigma_{22}}{N_{test}} \frac{\Delta t}{\Delta^3x}$$

$$\text{for } 2 \rightarrow 3 \quad P_{23} = v_{rel} \frac{\sigma_{23}}{N_{test}} \frac{\Delta t}{\Delta^3x}$$

$$\text{for } 3 \rightarrow 2 \quad P_{32} = \frac{1}{8E_1E_2E_3} \frac{I_{32}}{N_{test}^2} \frac{\Delta t}{(\Delta^3x)^2}$$

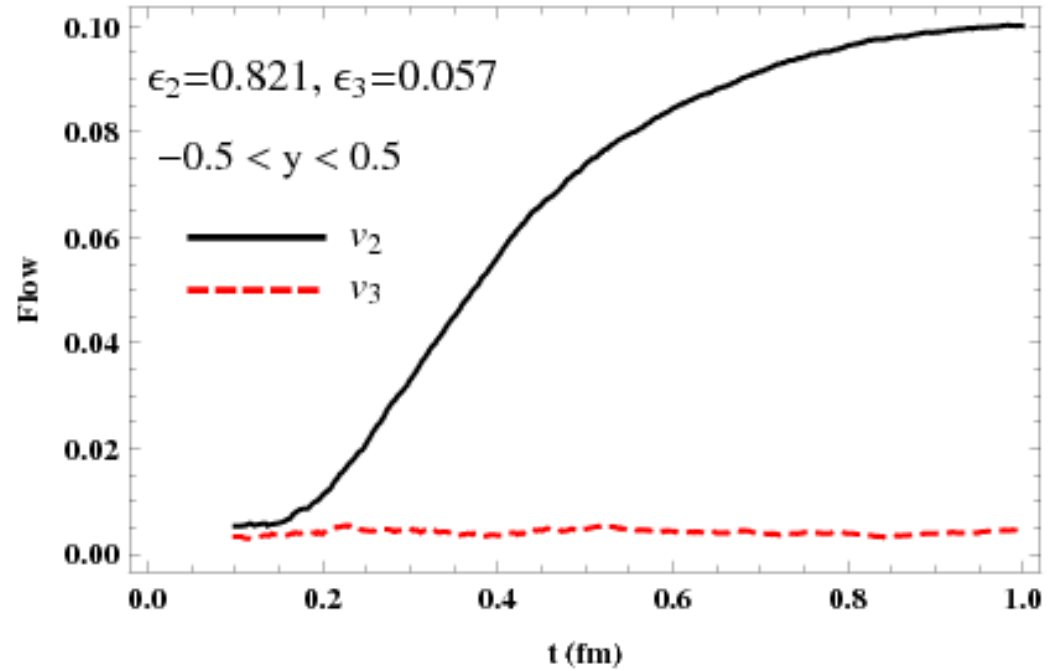
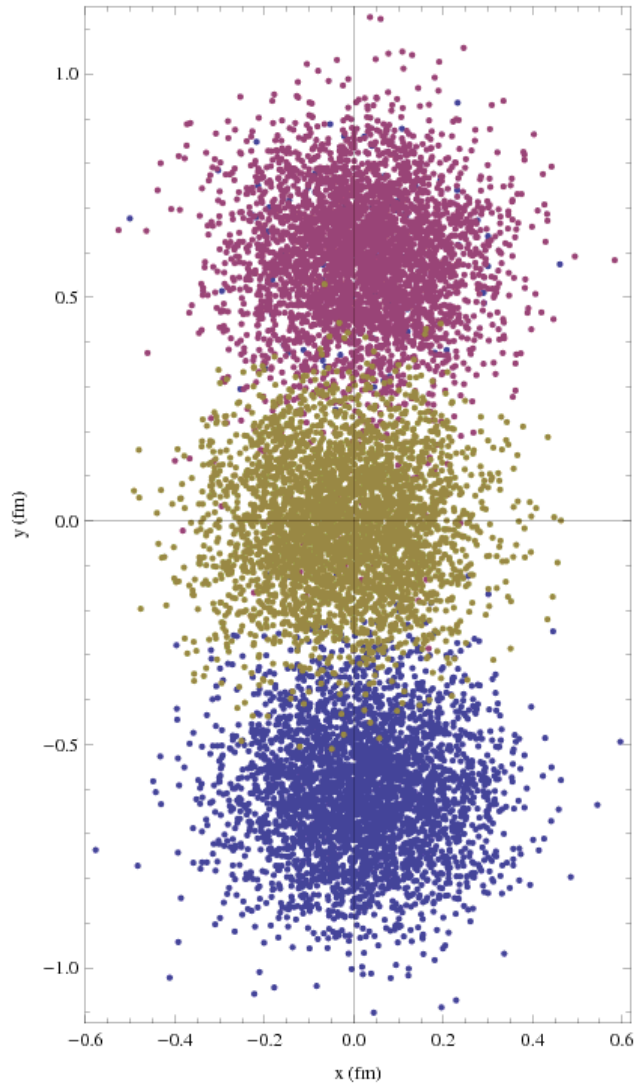
$$I_{32} = \frac{1}{2} \int \frac{d^3p'_1}{(2\pi)^3 2E'_1} \frac{d^3p'_2}{(2\pi)^3 2E'_2} |M_{123 \rightarrow 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3 - p'_1 - p'_2)$$

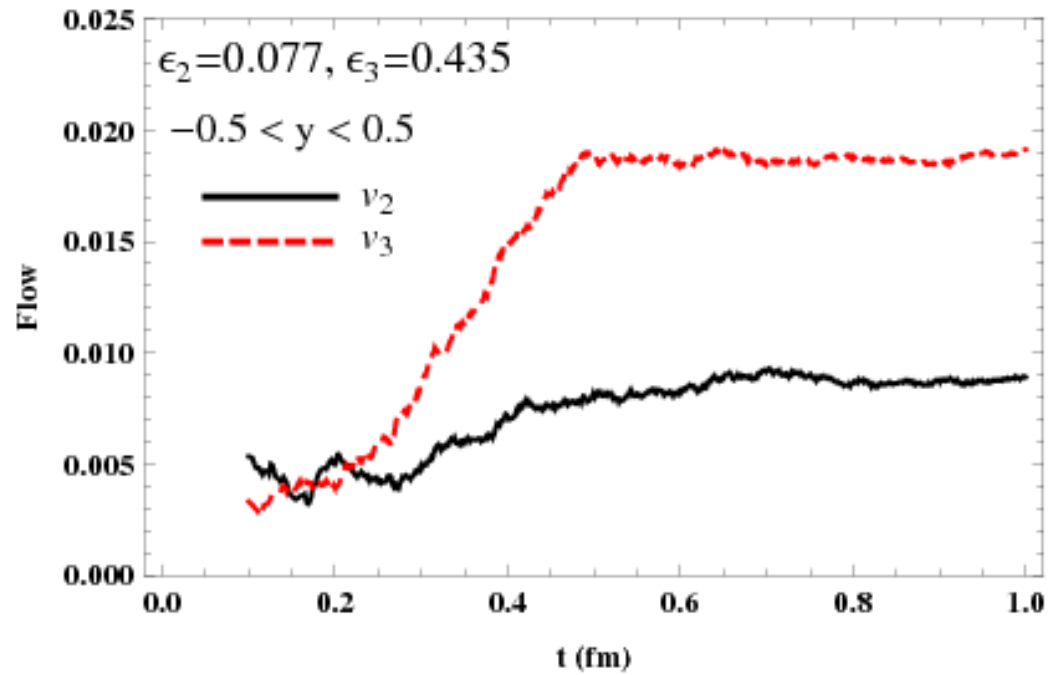
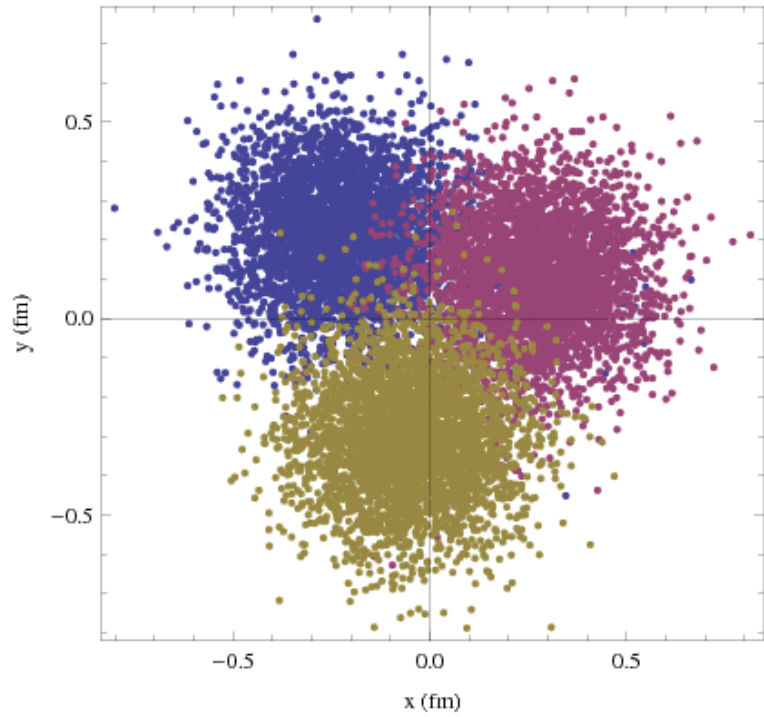


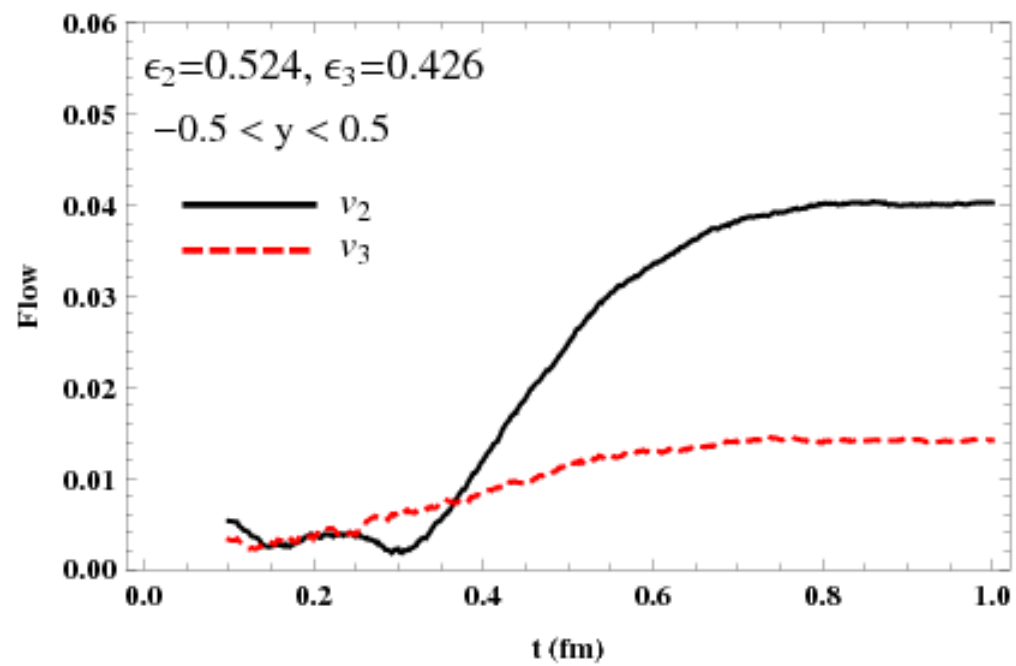
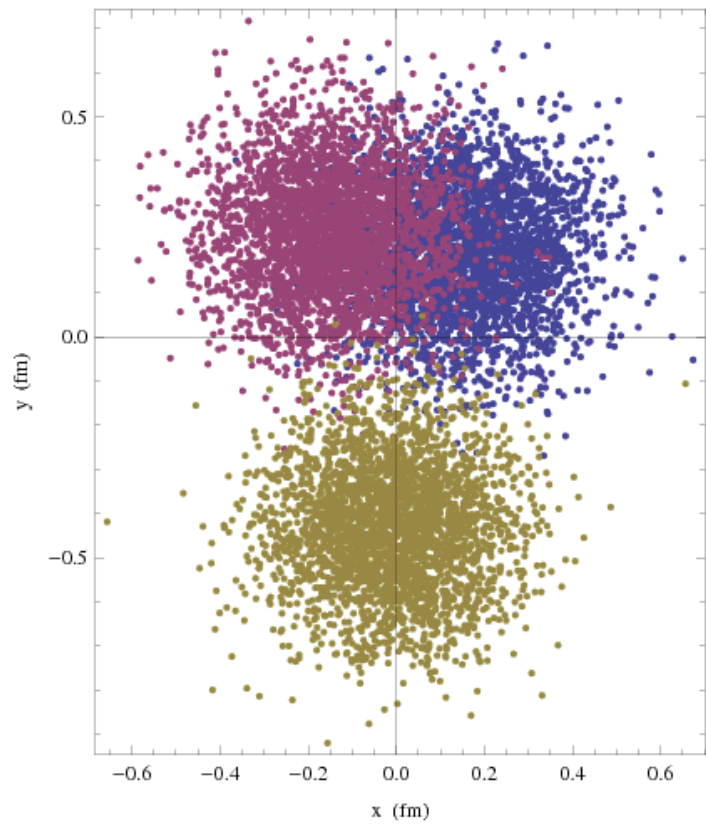
# Setups of BAMPS

- initial time:  $\tau_0=0.1$  fm/c
- interactions:
  - 2→2, isotropic distribution of the collision angle
  - mean freepath  $\lambda_{mfp} = (n\sigma)^{-1}$
  - mean partice distance  $d = n^{-1/3}$
  - $\lambda_{mfp} / d = 2 \Rightarrow \eta/s \approx 0.4$
- freeze-out:
  - Partons stop interacting when  $e < 1.0$  GeV/fm<sup>3</sup>.
- technique details:
  - cell length  $\Delta x=\Delta y=0.02$  fm,  $\Delta\eta=0.1$
  - 3000 test particles per real particle

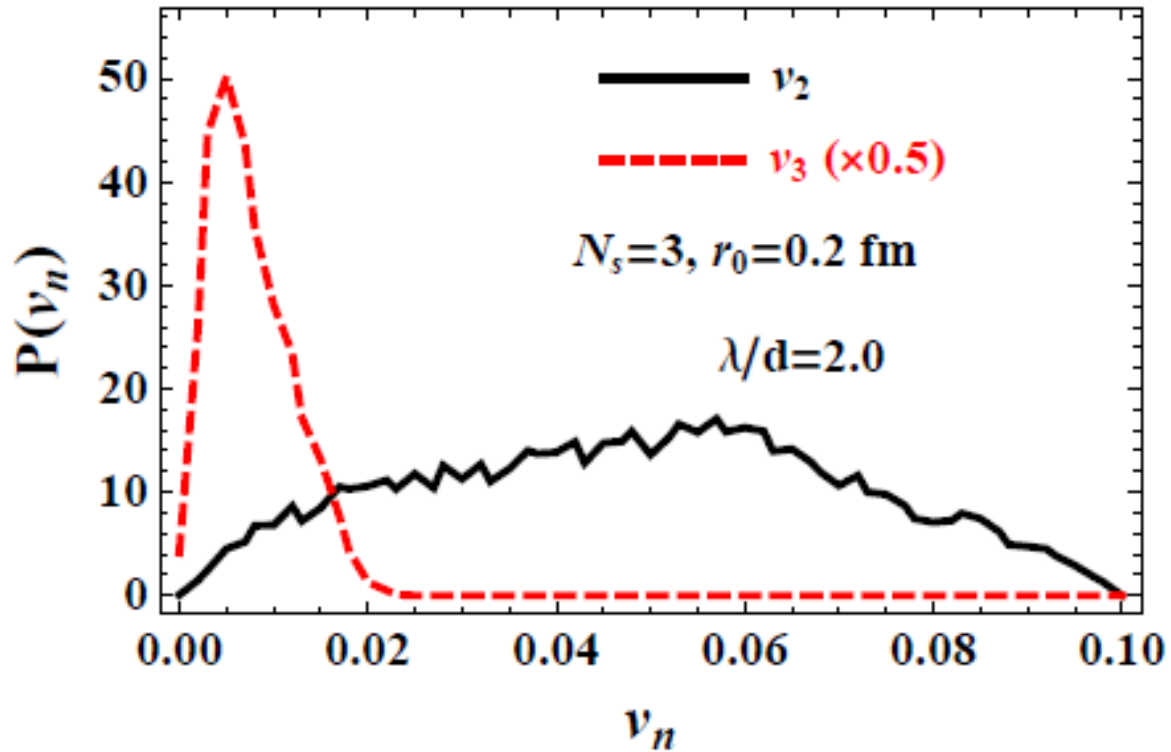
# Results of $v_2$ and $v_3$ at midrapidity



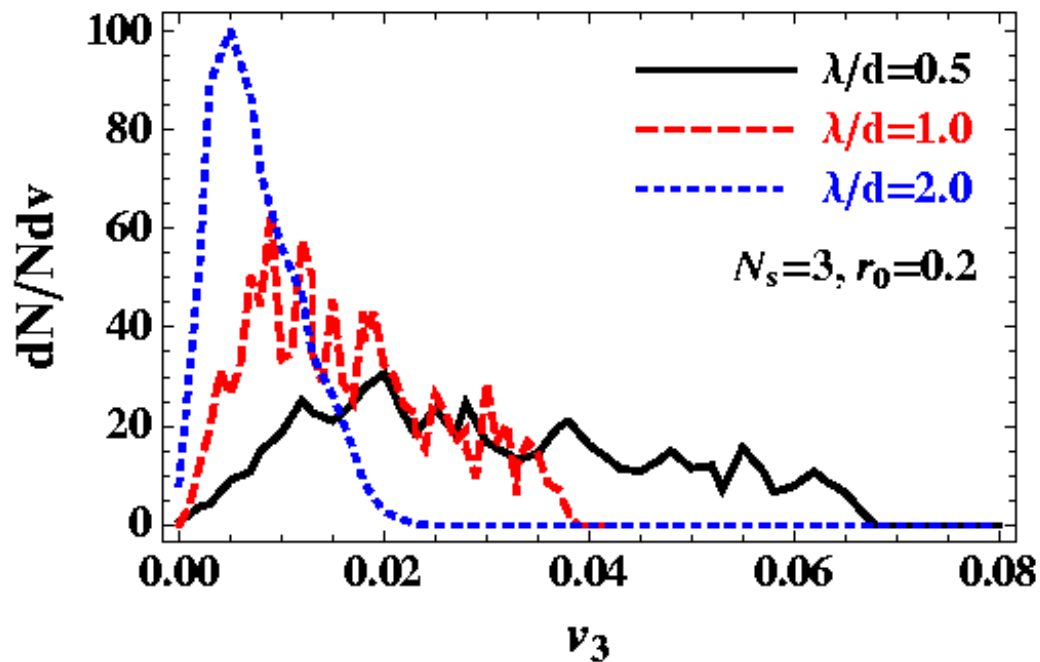
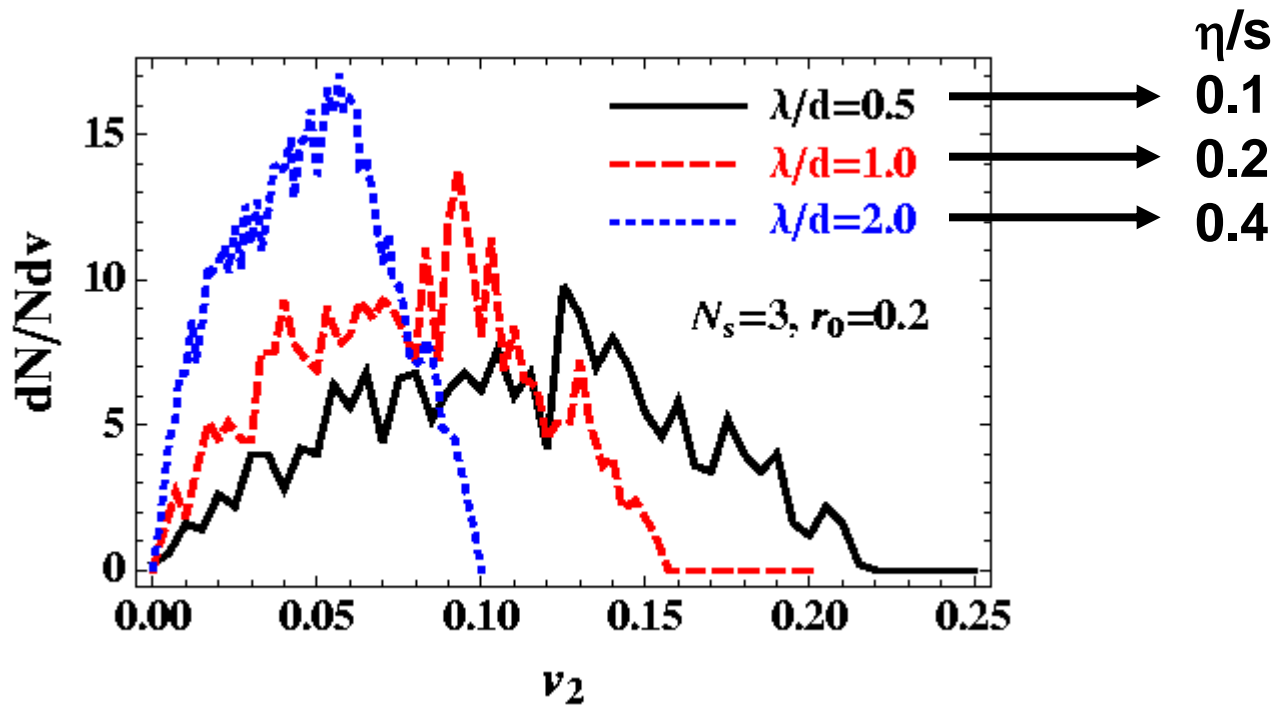




# Distributions of $v_2$ and $v_3$

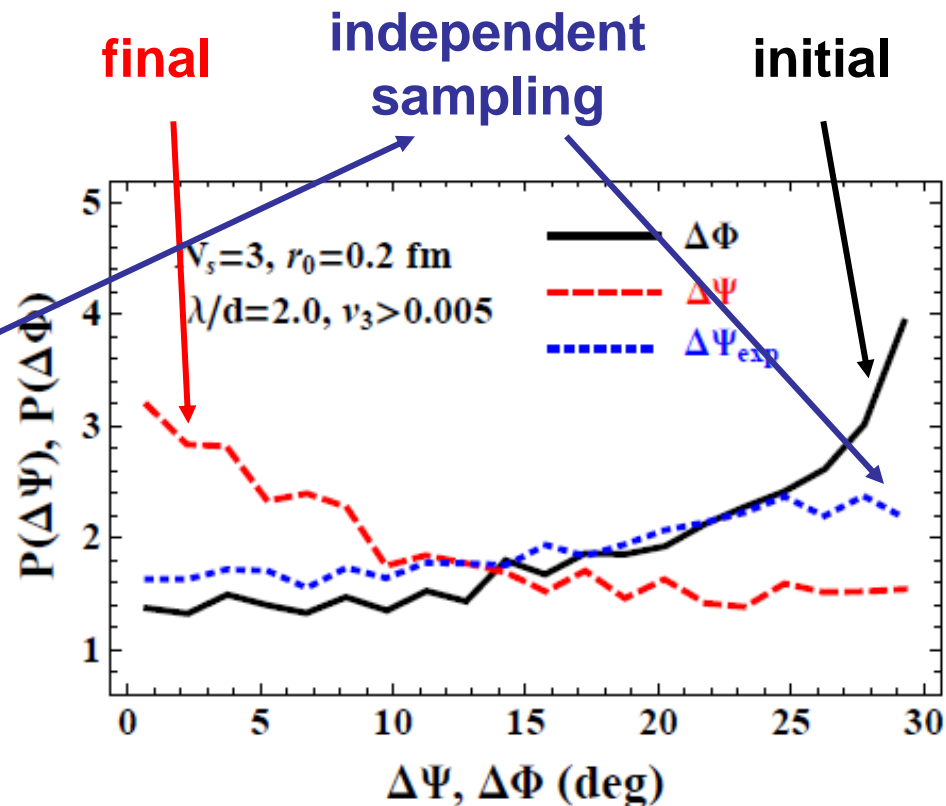
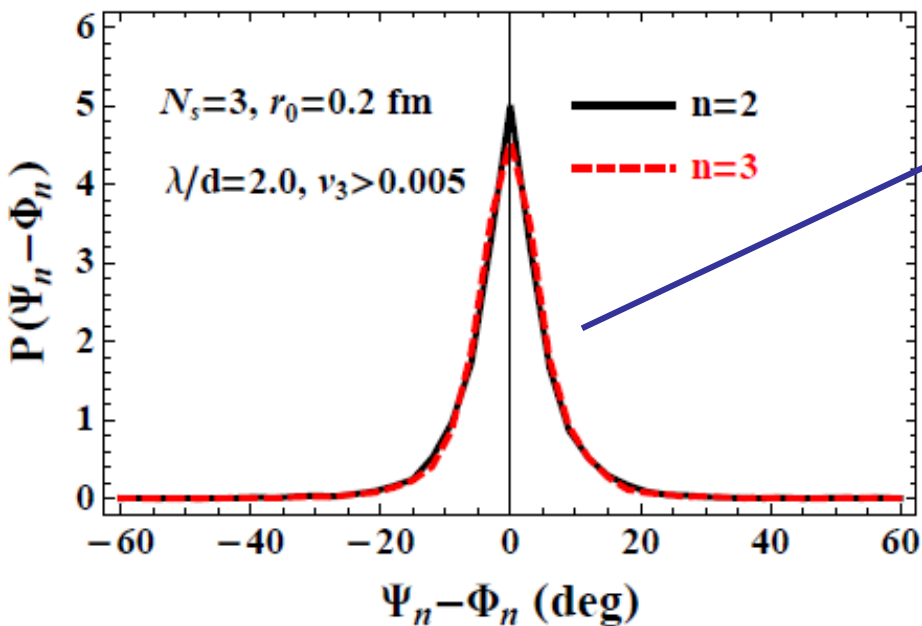


$10^4$  runs



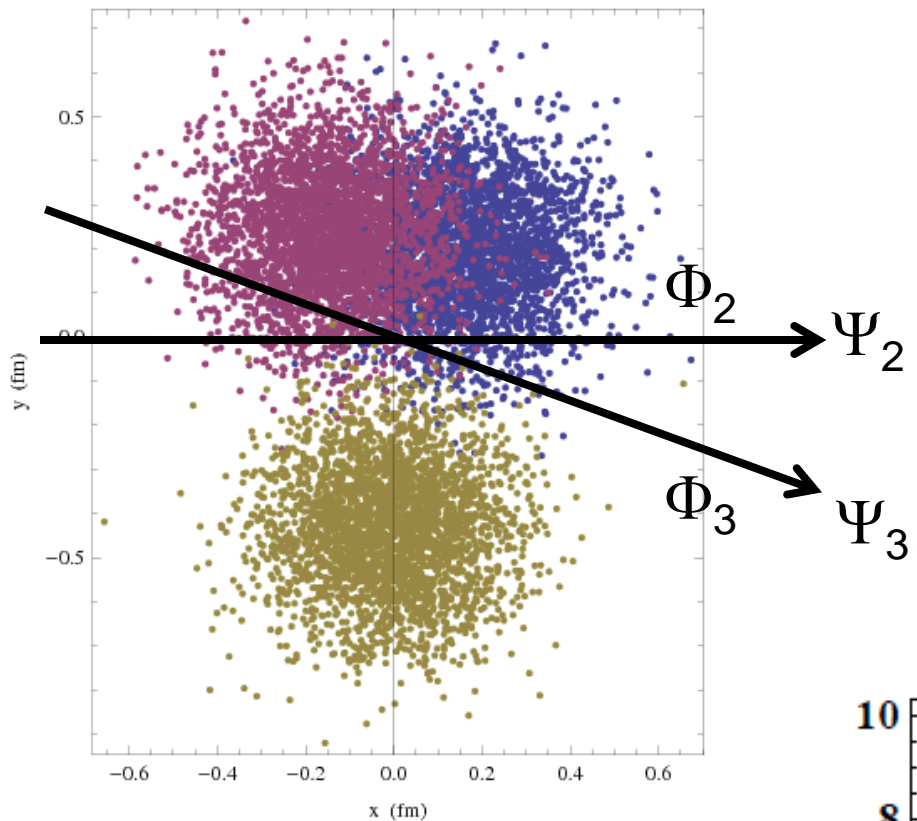
Elliptic and triangular flow are measurable quantities for  $\eta/s=0.1-0.4$  in high multiplicity events of p+p at 14 TeV.

# event-plane angular correlations



It seems **independent** translations from  $\Phi_2, \Phi_3$  to  $\Psi_2, \Psi_3$ .

Elliptic and triangular flow are **correlated** during the dynamical expansion.

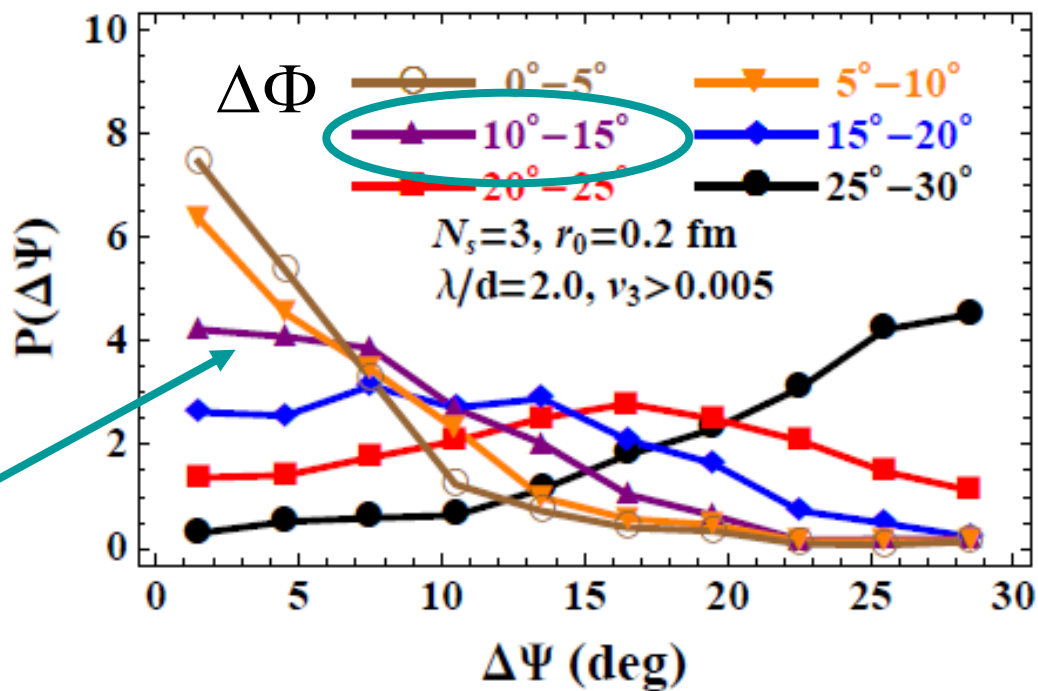


rotation of different event-planes to a unified event-plane

$$\Delta\Phi = |\Phi_2 - \Phi_3| = 10^\circ - 15^\circ$$

$P(\Delta\Psi)$  is broad and peaks

$$\text{at } \Delta\Psi = |\Psi_2 - \Psi_3| = 0^\circ$$





# Summary and Outlook

- Hot spots initial condition in high multiplicity pp events at LHC may generate measurable  $v_2$  and  $v_3$  for  $\eta/s=0.1-0.4$ .
- Dynamical correlation of  $v_2$  and  $v_3$  during the expansion
- $v_2$ - $v_3$  correlation in A+A at RHIC and LHC ?
- study  $v_2$ - $v_3$  correlation with smooth initial conditions