# BARYON & ELECTRIC CHARGE FLUCTUATIONS NEAR CHIRAL CROSSOVER

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# Conclusions

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• Do we see it in experiment?
Yes...

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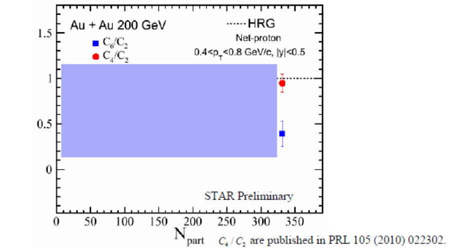
L. Chen, BNL workshop 2011, month ago:



# Conclusions

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or smaller than HRG values

- $\chi_6^B < 0$ and $\chi_6^Q < 0$ $\nrightarrow$ transition close to freeze-out.

Non-equilibrium effects, conservation laws, limited acceptance effect (V. Koch's talk)

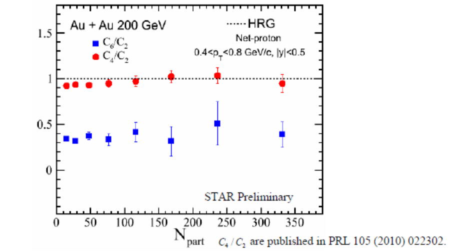
- Do we see it in experiment?

Yes...

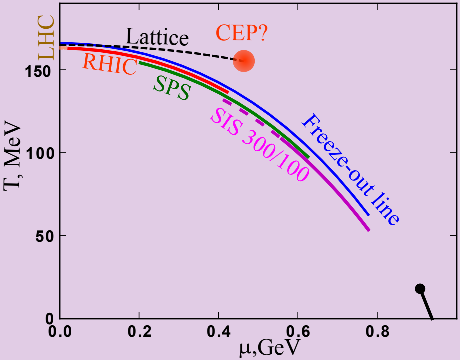
- Can we interpret it?

Presently, No...

L. Chen, BNL workshop 2011, month ago:



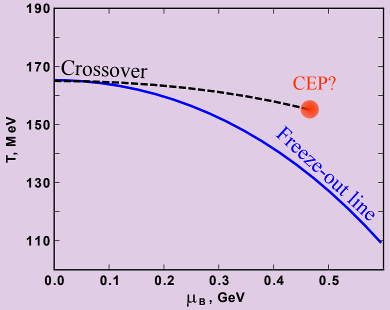
# Not QCD PHASE DIAGRAM




Structure of the phase diagram

- crossover at small $\mu_B$, underlying O(4) universality class
- (Expected) critical end point, 3d Ising model universality class
- (Expected) first-order transition

# Curvature of transition line and freeze-out




- LGT QCD: curvature of crossover line $\mu_B$

$$T/T_c \approx 1 - \mathbf{0.0066}\left(\frac{\mu_B}{T}\right)^2$$

- HRG model: freeze-out curve

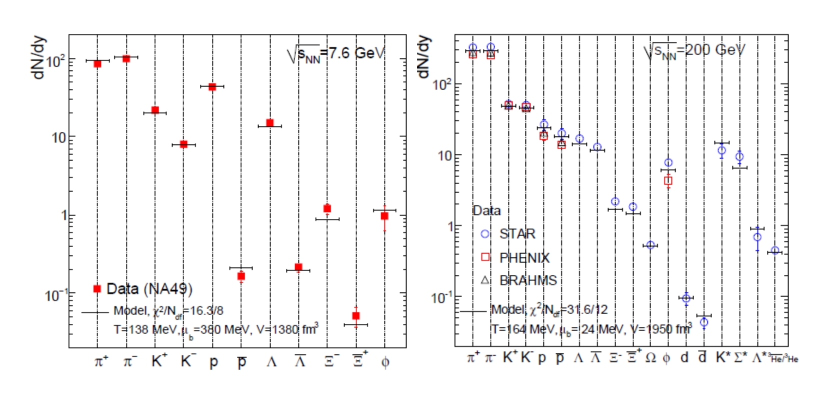
$$T/T_c \quad \approx \quad 1 - \mathbf{0.023}(\mu_B/T)^2$$

But:

- only curvature. Lattice: $\mu^2$. FO: error bars
- freeze-out line at high $\mu_B$: can be refined from LGT calculations (F. Karsch talk)

# Hadron resonance gas model vs experiment

**Hadron Resonance Gas (HRG) model:**

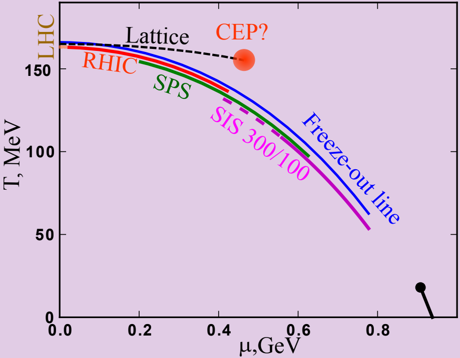



A. Andronic et al. HRG including heavy resonances, excluded volume corrections and etc.

Particle yields are well described by HRG model

**Where is the transition?**

# Where is transition?




Can we learn something beyond this with fluctuations of baryon/electric charge?

- 1st order phase transition?
- critical end point?
- proximity of freeze-out and crossover line? this talk

**Experiment:**

$\mathcal{P}(N) \rightsquigarrow \langle N^k \rangle = \sum_N N^k P(N) \rightsquigarrow$ cumulants

**Theory:**

$p(T,\mu) \rightsquigarrow \partial^n/\partial\mu^n \;\; p(T,\mu) \rightsquigarrow \chi_n \cdot (VT^3) \equiv$ cumulants

**HRG** ($\mu_S = \mu_Q = 0$):

Baryon number fluctuations:

- $T << m_p \rightsquigarrow$ Boltzmann approximation:
$p/T^4 = \sum_i f(m_i/T) \cosh(\mu_B/T) + g(T)$

- $\chi_{2n} \propto \cosh(\mu_B/T) \quad \chi_{2n+1} \propto \sinh(\mu_B/T)$

- $\chi_{2n}/\chi_2 = 1 \quad \chi_{2n+1}/\chi_1 = 1$

- $\chi_{2n} > 0$

- Effect of statistics $\chi_6/\chi_2 =$ 0.95 at $\sqrt{s} = 10$ GeV

- Electric charge fluctuations: ratios > 1 due to Bose statistics of pions and multiple charged hadrons

**Properties:**

- At CEP: for $n \geq 2$, $\chi_n \propto \xi^{n\beta\delta/\nu-3} \approx \xi^{5n/2-3}$, e.g. $\chi_4 \sim \xi^7$ (M. Stephanov '09)
- Diverging $\chi_2$ can signal spinodal decomposition of a non-equilibrium 1st order transition (C. Sasaki et. al. '07)

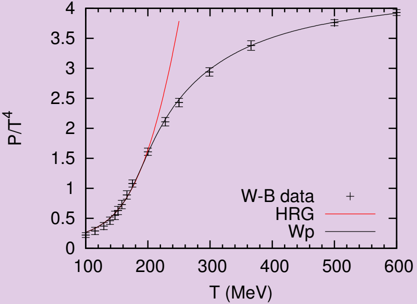
- Negative values of high order cumulants ($\chi_6^B$ and $\chi_6^Q$ ) close to crossover even at $\mu_B = 0$ (B. Friman et. al. '11)

**Why negative?**

**Why sixth order?**

# WHY NEGATIVE?

**Pressure (Borsanyi et al., 2010):**




- Close to transition scaling field $a = T - T_{trans.} + \kappa\mu_B^2$ (F. Karsch and K. Redlich)
- At $\mu_B = 0$: $\partial^2/\partial\mu_B^2\big|_{\mu_B=0} \sim \partial/\partial T$
- $\exists$ $n$ such that $\frac{\partial^n(p/T^4)}{\partial T^n} < 0 \quad \rightsquigarrow \chi_{2n}^B < 0.$

**Why sixth order?**

**Zero baryon chemical potential $\mu_B = 0$**

- $\partial^2/\partial\mu_B^2\big|_{\mu_B=0} \sim \partial/\partial T$
- $\chi_2^B(\mu_B = 0) \sim (\partial^2/\partial\mu_B^2)p \sim (\partial/\partial T)p \sim s$

  Thus, $\chi_2^B(\mu_B = 0) > 0$
- $\chi_3^B = 0$ at $\mu_B = 0$

**Zero baryon chemical potential $\mu_B = 0$**

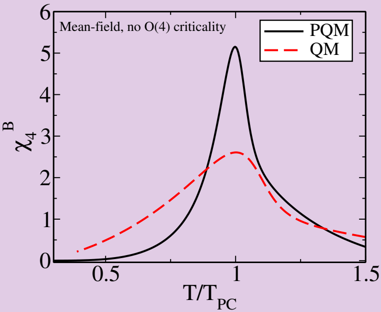
- $\partial^2/\partial\mu_B^2\big|_{\mu_B=0} \sim \partial/\partial T$
- $\chi_4^B(\mu_B = 0) \sim c_V$

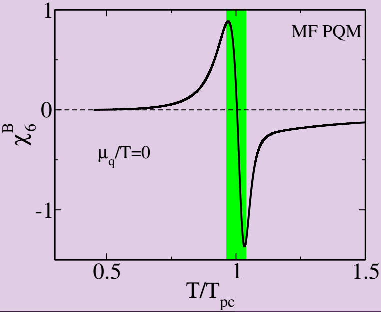
  $c_V$ is positive for thermodynamically stable systems $\rightsquigarrow$ $\chi_4^B(\mu_B = 0) > 0$
- $\chi_5^B = 0$ at $\mu_B = 0$

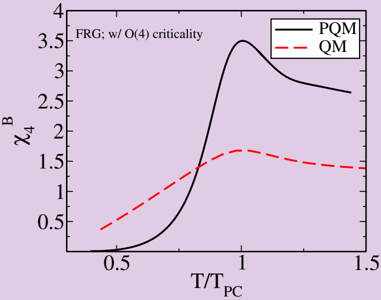
**Zero baryon chemical potential $\mu_B = 0$**

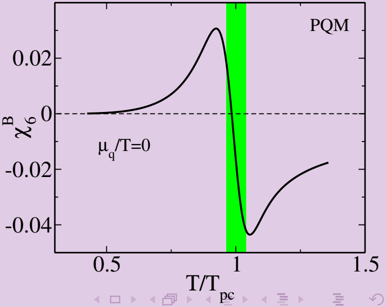
- Sign of $\chi_6^B$ is not constraint.
  **Sign change: Sixth order cumulant is the lowest possible one.**
- Sign of $\chi_6^B \sim \frac{\partial^2 \chi_B^4}{\partial \mu^2} \sim \frac{\partial \chi_B^4}{\partial T} \sim \frac{\partial c_V}{\partial T}$
  $\chi_6^B \sim \frac{\partial c_V}{\partial T}$
- Not universal, but general argument:
  energy $\rightarrow$ phase change, not $\rightarrow \Delta T$:
  $c_V$ has peak structure on transition $\rightsquigarrow$ negative $\chi_6^B$

# Temperature dependence of $\chi_4^B$ ($c_V$) in models



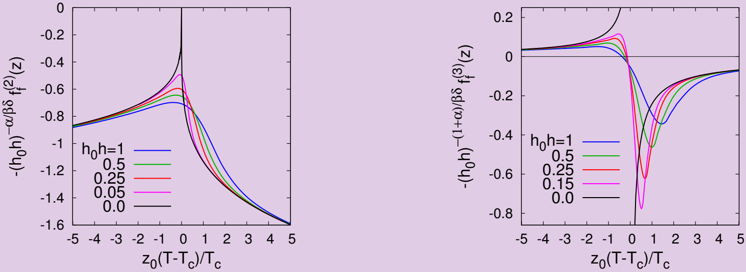



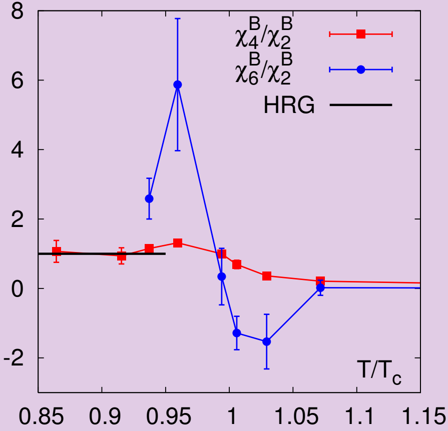
Based on: J. Engels, F. Karsch, arXiv:1105.0584 and
B. Friman et. al., arXiv:1103.3511

O(4) model singular part of $p/T^4 \propto -f(a,h)/T^4, \quad h \propto m_q$

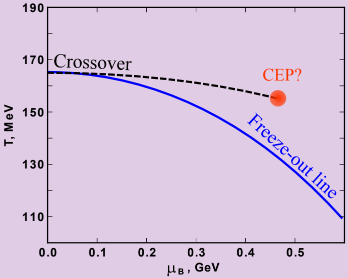



$\chi_4(\mu = 0)$ or $c_V$ $\qquad$ $\chi_6(\mu = 0)$ or $dc_V/dT$

# LATTICE




C. Schmidt, 2010




## CEP

- **Good:** $\chi_4 \sim \xi^7 \rightsquigarrow$ strong signal
- **Bad:** CEP is off from FO line $\rightsquigarrow$ signal may be washed out by
- **Bad:** low energy of collision $\rightsquigarrow$ conservation laws dominate the scene (M. Nahrang's talk)

## Crossover

- **Bad:** Signal is not strong, but independent on $\xi$
- **Good:** FO line and crossover are close to each other
- **Good:** high energy of collision $\rightsquigarrow$ reasonable cuts remove impact from cons. laws

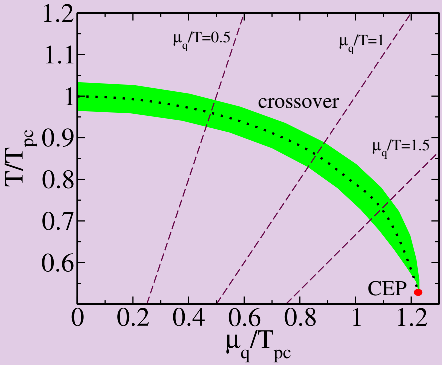
Lattice QCD restrictions

- continuum limit for cumulants
- non-zero chemical potential

QCD inspired model

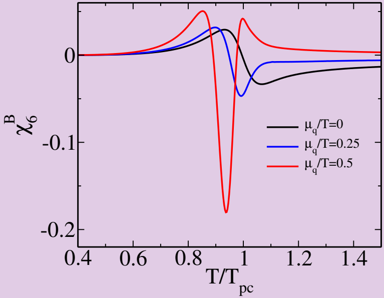
- O(4) symmetry in limit of vanishing mass for light quarks
- simulation of confinement properties (ratios of cumulants are sensitive to degrees of freedom)

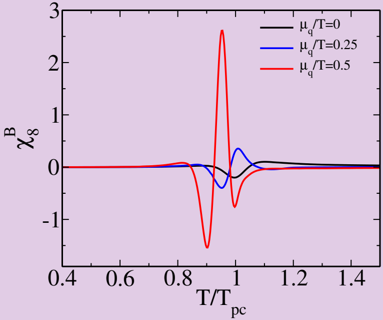
- **accounts for universal critical behaviour near chiral transition**
- reproduces scaling properties and critical exponents (Berges '00, B. Stokic et. al. '10)
- respects symmetries (Goldstone theorem fulfilled, second-order phase transition in $O(4)$ model)




**Crossover**: $|\partial\sigma/\partial T| > 0.95 \cdot \max(|\partial\sigma/\partial T|)$

# High-order cumulants of the baryon number density

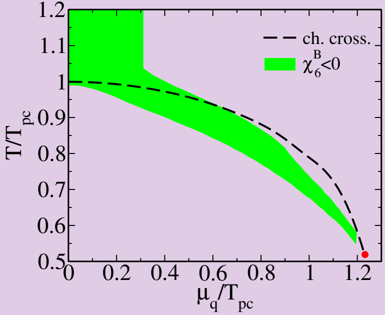


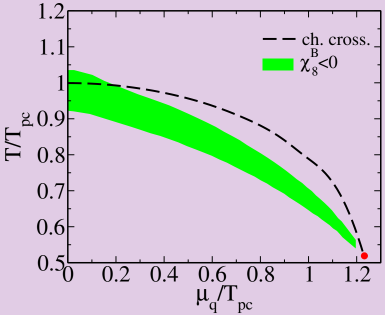




- Negative also at $\mu_q = 0$
- Temperature range of negative cumulants correlates with crossover temperature
- Many other constraints from O(4) scaling: B. Friman et. al. '11

# High-order baryon cumulant

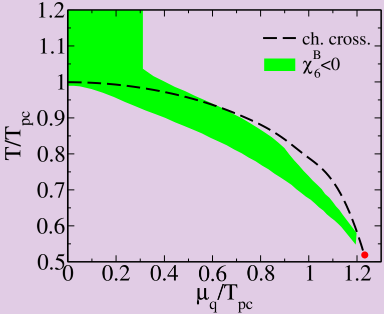
Temperature interval of negative cumulants closest to hadronic phase:



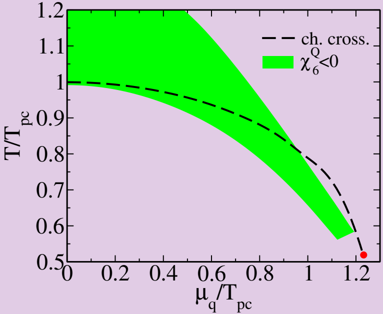




- Negative values (in broken phase!) of high-order cumulants: indicates proximity of freeze-out to crossover
- Accessible experimentally

Baryon charge:

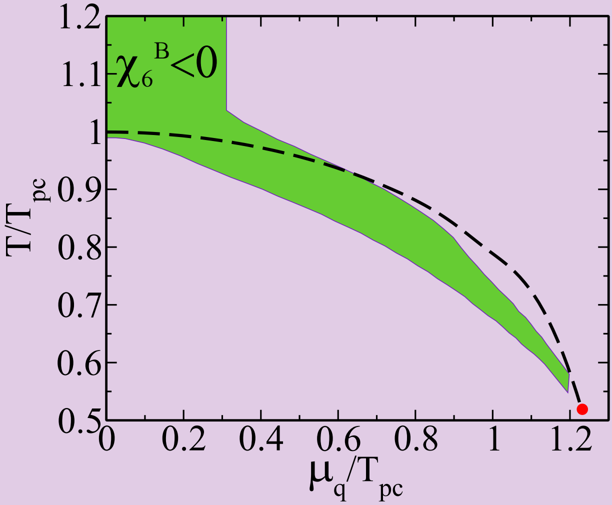



Electric charge:

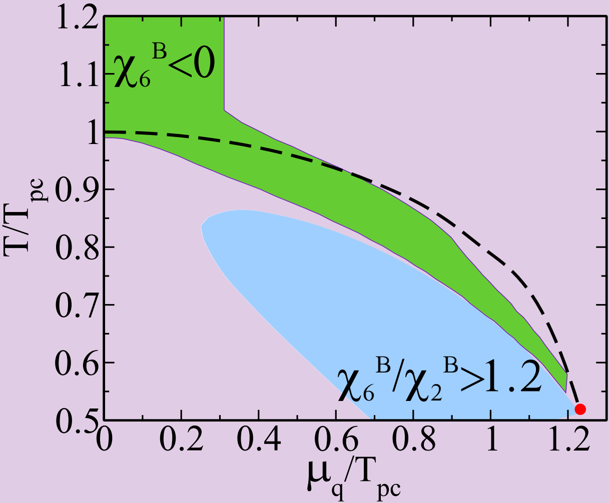



Electric charge fluctuations follow similar pattern as baryon fluctuations

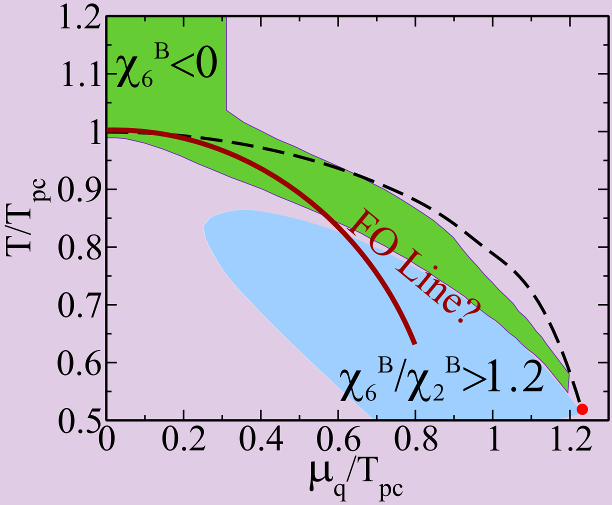
# FO scenarios

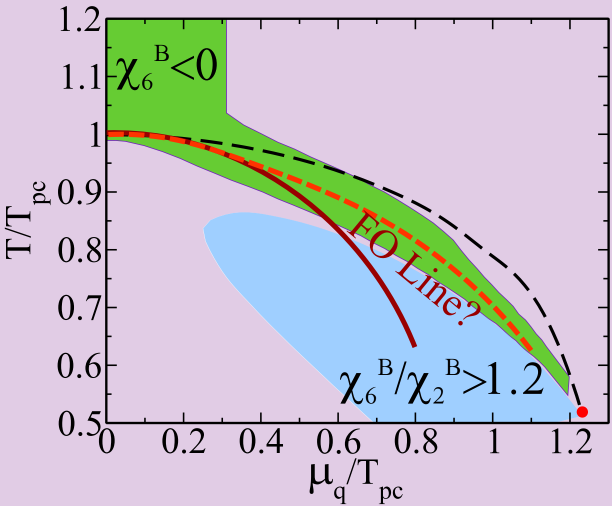


# FO scenarios



# FO scenarios

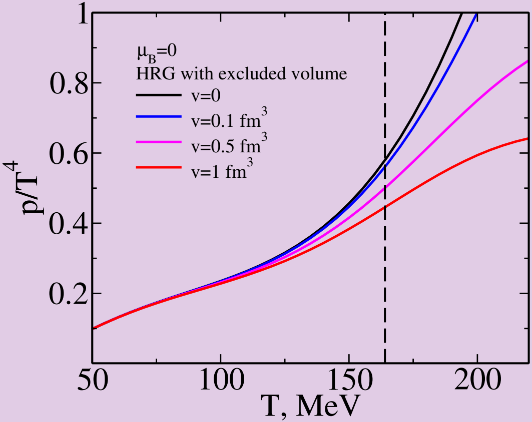





- Models are unable relate FO line and PT line. $\chi_6/\chi_2(\sqrt{s}) = f(\sqrt{s}) = ?$
- Low energies: cumulants might be affected by conservation laws

# Grain of salt

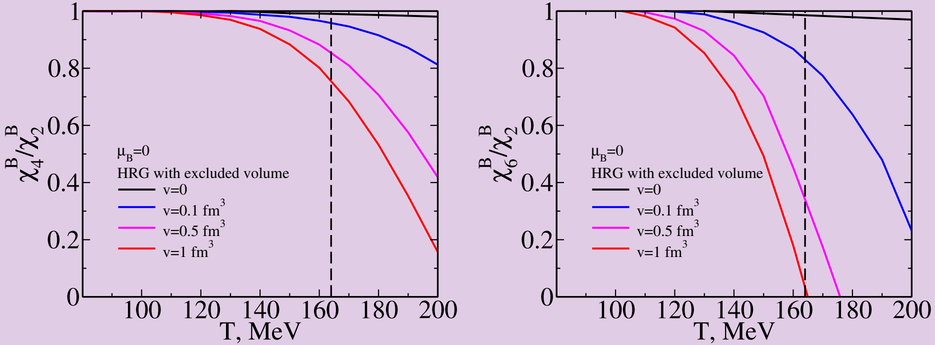
Hadron resonance gas with excluded volume $p(T,\mu) = \sum_i p_i(T, \mu_i - vp)$




A. Friesen

Pressure bends. . .

Hadron resonance gas with excluded volume



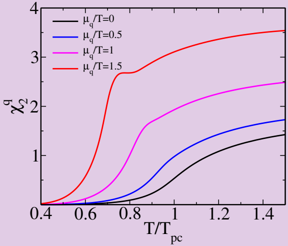

A. Friesen

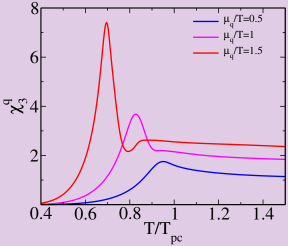
A. Andronic: FO fit with $v = 0.45$ fm$^3$

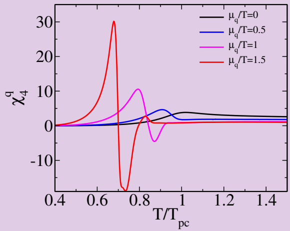
# Thank you for attention

Collaborators: B. Friman, F. Karsch and K. Redlich

# Net-quark number density fluctuations $\delta N_q = N_q - \langle N_q \rangle$




V.S., B. Friman and K. Redlich PRC'11

- $\chi_2^q$: non-monotonic structure (diverges at CEP)
- $\chi_4^q$: negative for nonzero $\mu_q$

# Cumulants ∝ susceptibilities

- Fluctuations of net-quark number $\chi_n^q$ and net-baryon charge $\chi_n^B$

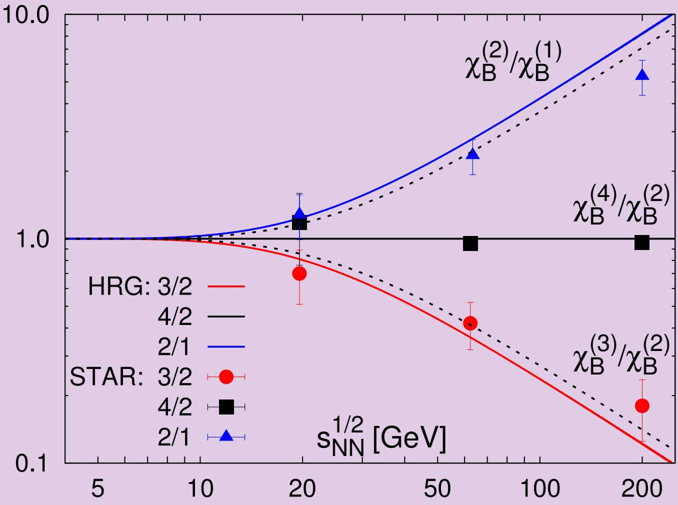
$$\chi_n^q = \frac{\partial^n (p/T^4)}{\partial(\mu_q/T)^n} \quad | \quad \chi_n^B = \frac{\partial^n (p/T^4)}{\partial(\mu_B/T)^n} = \left(\frac{1}{3}\right)^n \chi_n^q$$

- Fluctuations of electric charge $\chi_n^Q$

$$\chi_n^Q = \frac{\partial^n (p/T^4)}{\partial(\mu_Q/T)^n}$$

- Fluctuations of net-strange number…

# Comparison of the HRG model with experiment




F. Karsch and K. Redlich, ’10

**Kurtosis** $R^q_{4,2} = \frac{\chi^q_4}{\chi^q_2} = \frac{\langle(\delta N_q)^4\rangle}{\langle(\delta N_q)^2\rangle} - 3\langle(\delta N_q)^2\rangle$

(S. Ejiri, F. Karsch and K. Redlich '05):

quark content of effective degrees of freedom that carry baryon number

- **Low temperature phase**: dominance of effective three-quark states:

  $P_{\text{baryons}}/T^4 \approx \sum_i F(m_i/T)\cosh(3\mu_q/T)$

  $\rightsquigarrow R^q_{4,2} = 9$

- **High-temperature phase**:

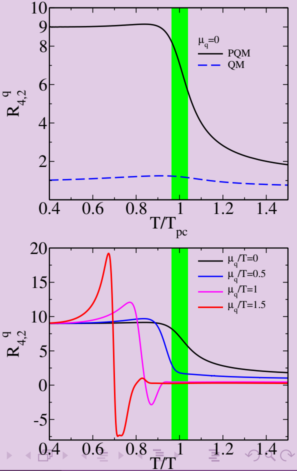
  $P_{q\bar{q}}/T^4 \approx N_f N_c \left[\frac{1}{12\pi^2}\left(\frac{\mu_q}{T}\right)^4 + \frac{1}{6}\left(\frac{\mu_q}{T}\right)^2 + \frac{7\pi^2}{180}\right]$

  $\rightsquigarrow R^q_{4,2} = (6/\pi^2) \approx 1$

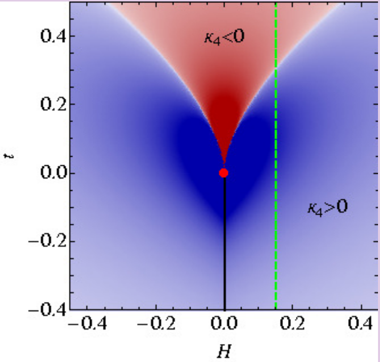
- *PQM: statistical confinement*

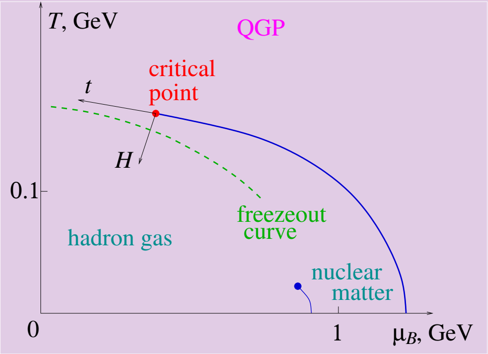
- $m_\pi = 0, \mu_q \neq 0$: kurtosis diverges

  $R^q_{4,2} \sim \left(\frac{\mu_q}{T}\right)^4 / t^{2+\alpha}$ ($t \propto$ distance to chiral critical line)



M. Stephanov '11: 3d Ising universality class $\rightsquigarrow$ kurtosis is negative close to CEP

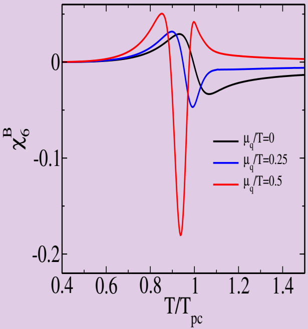


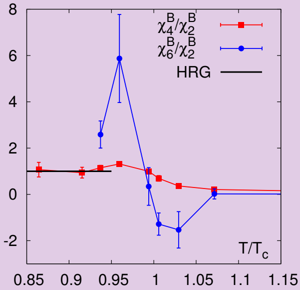



# Does negative kurtosis signal CEP?

Negative kurtosis is necessary, but not sufficient condition of CEP.

$$\text{CEP} \rightarrow R^B_{4,2} < 0, \text{ but } R^B_{4,2} < 0 \nrightarrow \text{CEP}$$




$$\text{sign}(R^B_{4,2}) = \text{sign}\chi^B_4, \quad \chi^B_4(\mu/T) \approx \chi^B_4(0) + \frac{1}{2}\chi^B_6(0)\cdot(\mu/T)^2 + O((\mu/T)^4)$$

$$R^B_{4,2} < 0 \rightarrow \text{non-trivial phase diagram}$$

**Functional Renormalization Group**

- $p(T,\mu,k)$, $k$ defines IR cut off $\rightsquigarrow$
  $p(T,\mu,k)$ includes modes with momentum $> k$.
- Functional renormalization group equation (exact and general):

$$p(T,\mu,k-dk) = p(T,\mu,k) + \boxed{\text{Exact FRG flow}}$$

- Iterating towards $k \to 0$: $p(T,\mu,k=0)$ includes all momentum modes
- Exact FRG is useless, approximations (leading order in gradient expansion):

$$p(T,\mu,k-dk) = p(T,\mu,k) + \boxed{\text{Approximate FRG flow}}$$

FRG review: J. Berges, N. Tetradis & C. Wetterich, Phys.Rept.363:223-386, '02
FRG formulation of PQM model: V. S., B. Stokic, B. Friman & K. Redlich, PRC, '10

# FUNCTIONAL RENORMALIZATION GROUP

The general flow equation for the effective action

$$\partial_k \Gamma_k[\Phi,\psi] = \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_{kB}\left(\Gamma_k^{(2,0)}[\Phi,\psi] + R_{kB}\right)^{-1}\right\} - \mathrm{Tr}\left\{\partial_k R_{kF}\left(\Gamma_k^{(0,2)}[\Phi,\psi] + R_{kF}\right)^{-1}\right\}$$

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The flow equation for the PQM model

$$\partial_k \Omega(k, \rho \equiv \frac{1}{2}[\sigma^2 + \boldsymbol{\pi}^2]) = \frac{k^4}{12\pi^2}\left\{\frac{3}{E_\pi}\left[1 + 2n_B(E_\pi;T)\right] + \frac{1}{E_\sigma}\left[1 + 2n_B(E_\sigma;T)\right] - \frac{4N_f N_c}{E_q}\left[1 - N(\ell,\ell^*;T,\mu_q) - \bar{N}(\ell,\ell^*;T,\mu_q)\right]\right\}$$

$n_B(E;T)$ is the boson distribution functions
$N(\ell,\ell^*;T,\mu_q)$ are fermion distribution function modified owing to coupling to gluons
$E_\sigma$ and $E_\pi$ are the functions of $k$, $\partial\Omega/\partial\rho$ and $\rho\partial^2\Omega/\partial\rho^2$
$E_q = \sqrt{k^2 + 2g\rho}$

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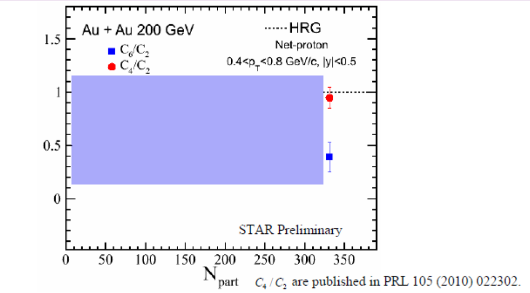
FRG defines $\Omega(k,\rho;T,\mu_Q,\mu_B)$.
**Physically relevant quantity** is the thermodynamical potential
$\bar{\Omega}(T,\mu_Q,\mu_B) \equiv \Omega(k\to 0,\rho\to\rho_0;T,\mu_Q,\mu_B)$, where $\rho_0$ is the minimum of $\Omega$.

# Preliminary data for $\chi_6/\chi_2$

L. Chen, one month ago, BNL workshop 2011:

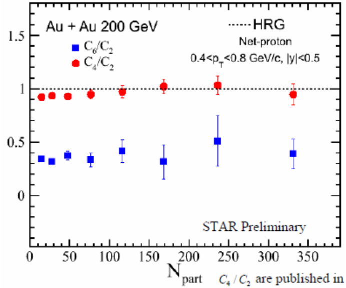



**Red points:** $\chi_4/\chi_2$.

Reminder: $\chi_4/\chi_2$ is not influenced by O(4) criticality

**Blue points:** $\chi_6/\chi_2$. Not negative, but **suppressed**!

L. Chen, one month ago, BNL workshop 2011:



$C_4/C_2$ are published in PRL 105 (2010) 022302.

**Red points:** $\chi_4/\chi_2$. Reminder: $\chi_4/\chi_2$ is not influenced by O(4) criticality

**Red points:** $\chi_6/\chi_2$. Not negative, but **suppressed**!

**Centrality dependence?**