Three-loop HTL QCD Trace Anomaly

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Phys. Rev. Lett. 104, 122003 (2010); JHEP 08, 113 (2010);

Phys. Lett. B 696, 468 (2011); JHEP 08 ,053 (2011); Phys. Rev. D 84, 087703 (2011)

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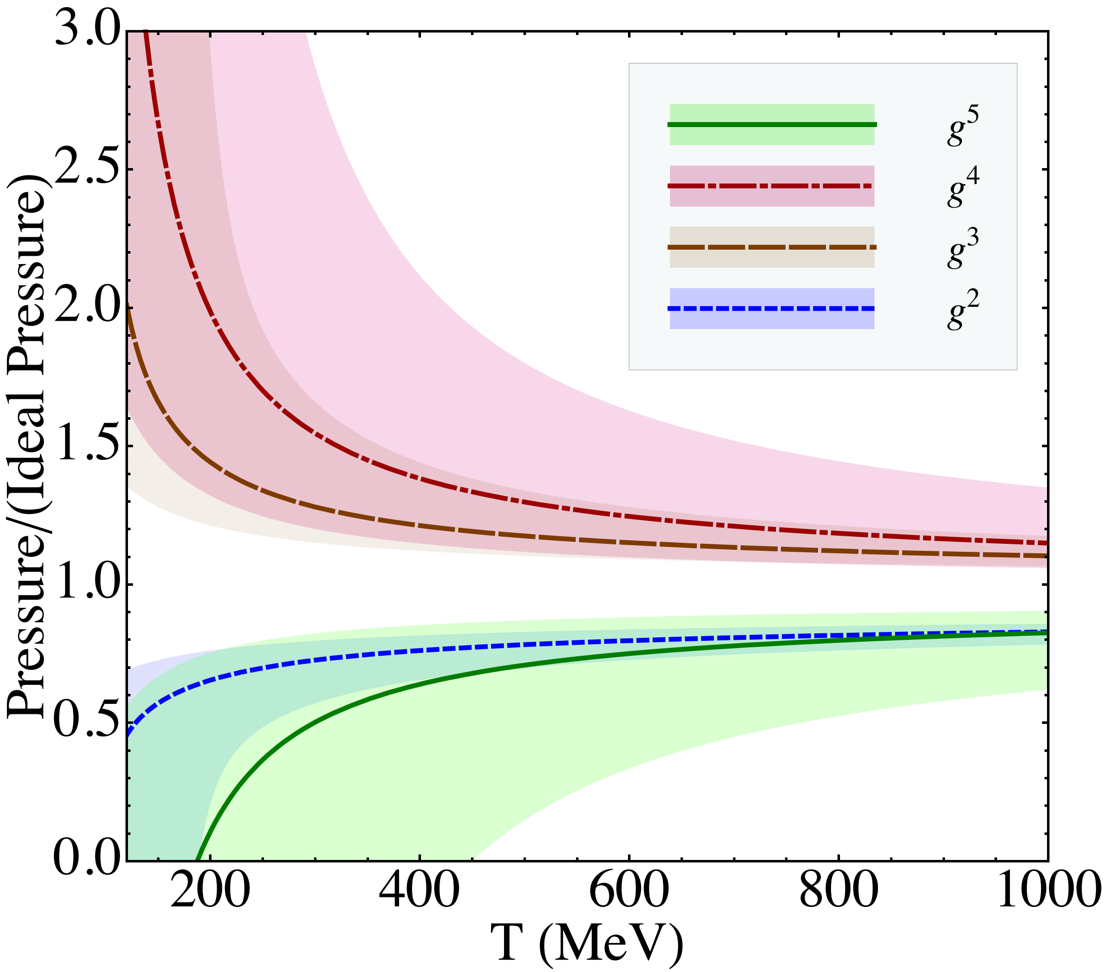
Introduction: Heavy ion collisions $\to$ QGP or QGL?

- RHIC: $T_0 \sim$ 350 MeV $\sim 2\,T_c$.
- LHC: $T_0 \sim$ 800 - 1000 MeV ~ 4 - $6\,T_c$.
- Quark Gluon Plasma (QGP) or Quark Gluon Liquid (QGL) at LHC?
- Running coupling expected is $g_s \sim 2$ or $\alpha_s = g_s^2/4\pi \sim 0.4$.
- Neither infinitesimally small, nor infinitely large: intermediate coupling.
- Can pQCD methods reproduce lattice data for thermodynamic functions at such “intermediate” couplings ($g_s \sim 2$)?
- More importantly the resulting machinery should be able to address real-time dynamics as well.

Intro: Non-convergence of canonical thermal QCD



Weak-coupling expansion QCD free energy with $N_c = 3$ and $N_f = 3$ vs temperature.

$(\pi T \leq \mu \leq 4\pi T)$

- The weak-coupling expansion of the QCD free energy, $\mathcal{F}$, is known to order $g_s^6 \log g_s$. [1,2,3,4,5,6,7]
- At temperatures expected at RHIC energies, $T \sim 0.35$ GeV, the running coupling $g_s \sim 2$ or $\alpha_s \sim 0.4$.
- The successive terms contributing to $\mathcal{F}$ can strictly only form a decreasing series if $\alpha_s \lesssim 1/20$ which corresponds to $T \sim 10^5$ GeV.

[1] Shuryak, 78.

[2] Kapusta, 79.

[3] Toimela, 85.

[4] Arnold and Zhai, 94/95.

[5] Kastening and Zhai, 95.

[6] Braaten and Nieto, 96.

[7] Kajantie, Laine, Rummukainen and Schröder, 02.

Hard-Thermal-Loop perturbation theory (HTLpt)

- HTLpt is a reorganization of the QCD perturbative series (Andersen, Braaten and Strickland, 99)

$$\mathcal{L}_{\rm HTLpt} = \left(\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL} - \delta\mathcal{L}_{\rm HTL}\right)\Big|_{g\to\sqrt{\delta}g}$$

- The Hard-Thermal-Loop (HTL) effective action reads

$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}m_D^2 {\rm Tr}\left(G_{\mu\alpha}\left\langle\frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y G^\mu{}_\beta\right)$$

- δ counts the number of HTL dressed loops.

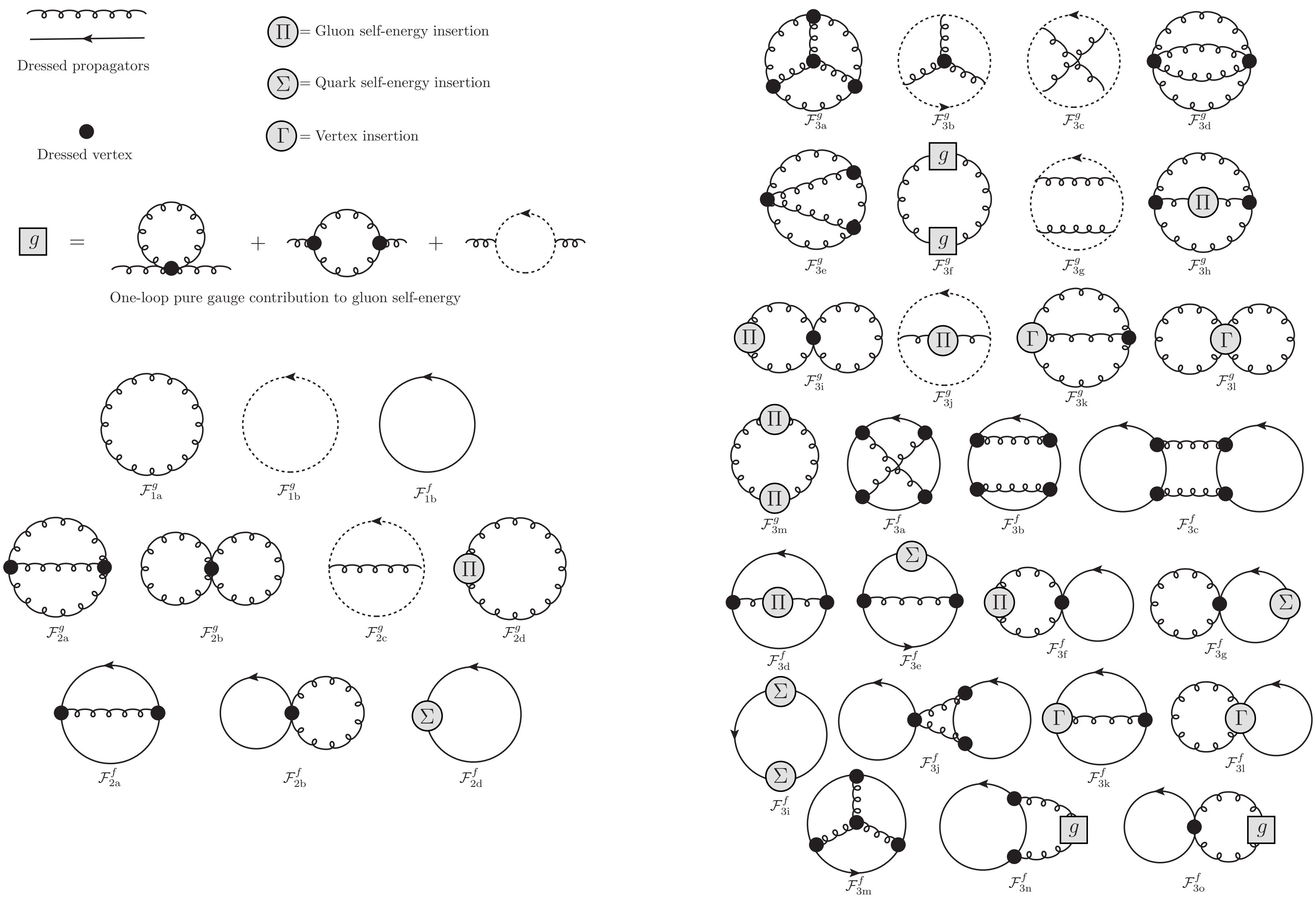
- Adding $\mathcal{L}_{\rm HTL}$ shifts the expansion to an ideal gas of massive quasiparticles, which are the appropriate d.o.f. at high T.

Hard-Thermal-Loop perturbation theory (HTLpt)

- Interested in $T > 2 - 3\, T_c$ where a quasiparticle description is more reliable than below where nonperturbative effects are significant.

- HTLpt is an extension of variational perturbative theory or linear δ expansion to thermal gauge theory. (Yukalov, 76; Caswell, 79; Halliday and Suranyi, 79; Seznec and Zinn-Justin, 79; Barnes and Ghandour, 80; Killingbeck, 81; Stevenson, 81/82/84/85; Le Guillou and Zinn-Justin, 83; Shaverdyan and Usherveridze, 83; Yamazaki, 84; Mitter and Yamazaki, 84; Feynman and Kleinert, 86; Stevenson and Tarrach, 86; Okopinska, 87; Duncan and Moshe, 88; Namgung, Stevenson and Reed, 89; Ritschel, 89/91; Jones and Moshe, 90; Stancu and Stevenson, 90; Neveu, 91; Yukalov, 91; Gandhi, Jones and Pinto, 91; Thoma, 91; Tarrach, 91; Haugerud and Raunda, 91; Bender, Cooper, Milton, Moshe, Pinsky and Simmons, 92; Gandhi and Pinto, 92; Duncan and Jones, 93; Yamada, 93; Klimenko, 93; Sissakian, Solovtsov and Shevchenko, 93; Duncan, 93; Bender, Duncan and Jones, 94; Sissakian, Solovtsov and Solovtsova, 94; Kleinert, 95; Arvanitis, Jones and Parker, 95; Guida, Konishi and Suzuki, 95/96; Buchmuller and Philipsen, 95; Alexanian and Nair, 95; Janke and Kleinert, 95/97; Bellet, Garcia and Neveu, 96; Arvanitis, Geniet, Kneur and Neveu, 97; Jackiw and Pi, 97; Karsch, Patkos and Petreczky, 97; Cornwall, 98; Chiku and Hatsuda, 98; $\cdots\cdots$)

HTLpt: QCD diagrams through NNLO



HTLpt: NNLO thermodynamic potential for QCD

- For QCD with general N_c and N_f ($\mathcal{F}_{\rm ideal} \equiv -\frac{(N_c^2-1)\pi^2 T^4}{45}$, $\hat{x}_D \equiv \frac{x}{2\pi T}$)

$$
\begin{aligned}
\frac{\Omega_{\rm NNLO}}{\mathcal{F}_{\rm ideal}} = \; & 1+\frac{7}{4}\frac{d_F}{d_A}-\frac{15}{4}\hat{m}_D^3+\frac{c_A\alpha_s}{3\pi}\left[-\frac{15}{4}+\frac{45}{2}\hat{m}_D-\frac{135}{2}\hat{m}_D^2-\frac{495}{4}\left(\log\frac{\hat{\mu}}{2}+\frac{5}{22}+\gamma_E\right)\hat{m}_D^3\right] \\
& +\frac{s_F\alpha_s}{\pi}\left[-\frac{25}{8}+\frac{15}{2}\hat{m}_D+15\left(\log\frac{\hat{\mu}}{2}-\frac{1}{2}+\gamma_E+2\log 2\right)\hat{m}_D^3-90\hat{m}_q^2\hat{m}_D\right] \\
& +\left(\frac{c_A\alpha_s}{3\pi}\right)^2\left[\frac{45}{4}\frac{1}{\hat{m}_D}-\frac{165}{8}\left(\log\frac{\hat{\mu}}{2}-\frac{72}{11}\log\hat{m}_D-\frac{84}{55}-\frac{6}{11}\gamma_E-\frac{74}{11}\frac{\zeta'(-1)}{\zeta(-1)}\right.\right. \\
& \left.\left.+\frac{19}{11}\frac{\zeta'(-3)}{\zeta(-3)}\right)+\frac{1485}{4}\left(\log\frac{\hat{\mu}}{2}-\frac{79}{44}+\gamma_E+\log 2-\frac{\pi^2}{11}\right)\hat{m}_D\right] \\
& +\left(\frac{c_A\alpha_s}{3\pi}\right)\left(\frac{s_F\alpha_s}{\pi}\right)\left[\frac{15}{2}\frac{1}{\hat{m}_D}-\frac{235}{16}\left(\log\frac{\hat{\mu}}{2}-\frac{144}{47}\log\hat{m}_D-\frac{24}{47}\gamma_E+\frac{319}{940}+\frac{111}{235}\log 2\right.\right. \\
& \left.\left.-\frac{74}{47}\frac{\zeta'(-1)}{\zeta(-1)}+\frac{1}{47}\frac{\zeta'(-3)}{\zeta(-3)}\right)+\frac{315}{4}\left(\log\frac{\hat{\mu}}{2}-\frac{8}{7}\log 2+\gamma_E+\frac{9}{14}\right)\hat{m}_D+90\frac{\hat{m}_q^2}{\hat{m}_D}\right] \\
& +\left(\frac{s_F\alpha_s}{\pi}\right)^2\left[\frac{5}{4}\frac{1}{\hat{m}_D}+\frac{25}{12}\left(\log\frac{\hat{\mu}}{2}+\frac{1}{20}+\frac{3}{5}\gamma_E-\frac{66}{25}\log 2+\frac{4}{5}\frac{\zeta'(-1)}{\zeta(-1)}-\frac{2}{5}\frac{\zeta'(-3)}{\zeta(-3)}\right)\right. \\
& \left.-15\left(\log\frac{\hat{\mu}}{2}-\frac{1}{2}+\gamma_E+2\log 2\right)\hat{m}_D+30\frac{\hat{m}_q^2}{\hat{m}_D}\right]+s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{15}{64}(35-32\log 2)-\frac{45}{2}\hat{m}_D\right]
\end{aligned}
$$

PURELY ANALYTIC!!!

HTLpt: Mass prescriptions

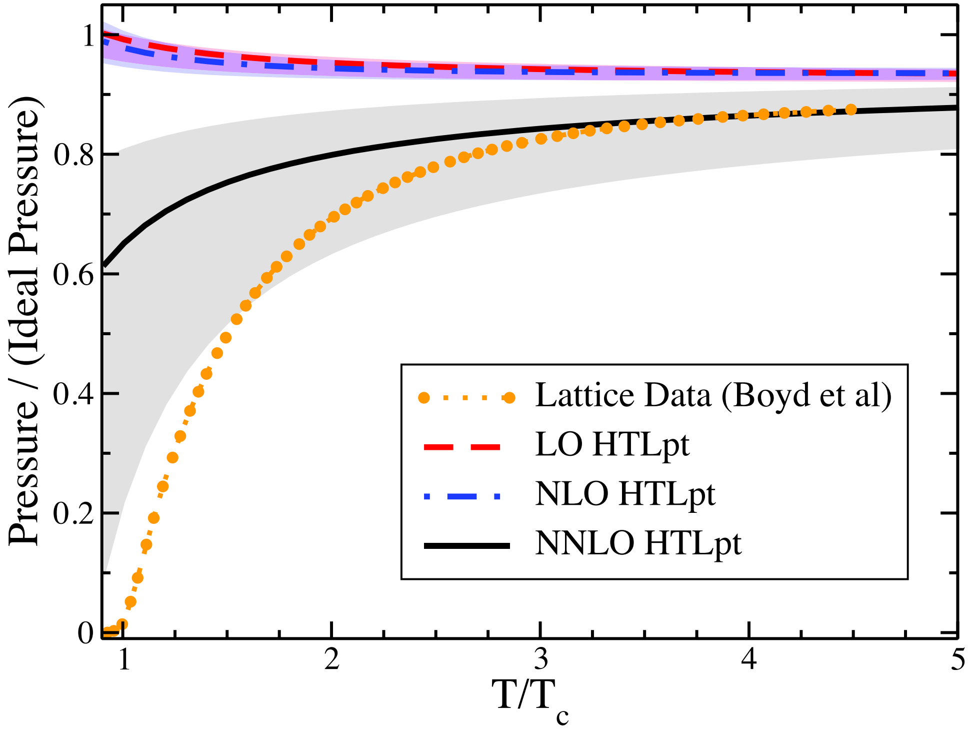
- Use the gauge-invariant NLO electric mass from dimensional reduction for m_D: hard contribution (from the scale T) to Debye mass and well defined to all orders (Braaten and Nieto, 96)

$$
\begin{aligned}
m_D^2 \quad = \quad & \frac{4\pi\alpha_s}{3}T^2 \left\{ c_A + s_F + \frac{c_A^2\alpha_s}{3\pi}\left(\frac{5}{4} + \frac{11}{2}\gamma_E + \frac{11}{2}\log\frac{\hat{\mu}}{2}\right)\right. \\
& + \frac{c_A s_F \alpha_s}{\pi}\left(\frac{3}{4} - \frac{4}{3}\log 2 + \frac{7}{6}\gamma_E + \frac{7}{6}\log\frac{\hat{\mu}}{2}\right) \\
& \left. + \frac{s_F^2\alpha_s}{\pi}\left(\frac{1}{3} - \frac{4}{3}\log 2 - \frac{2}{3}\gamma_E - \frac{2}{3}\log\frac{\hat{\mu}}{2}\right) - \frac{3}{2}\frac{s_{2F}\alpha_s}{\pi}\right\}
\end{aligned}
$$

- Set $m_f = 0$ for simplicity since fermions are IR safe.

HTLpt: Free energy through NNLO

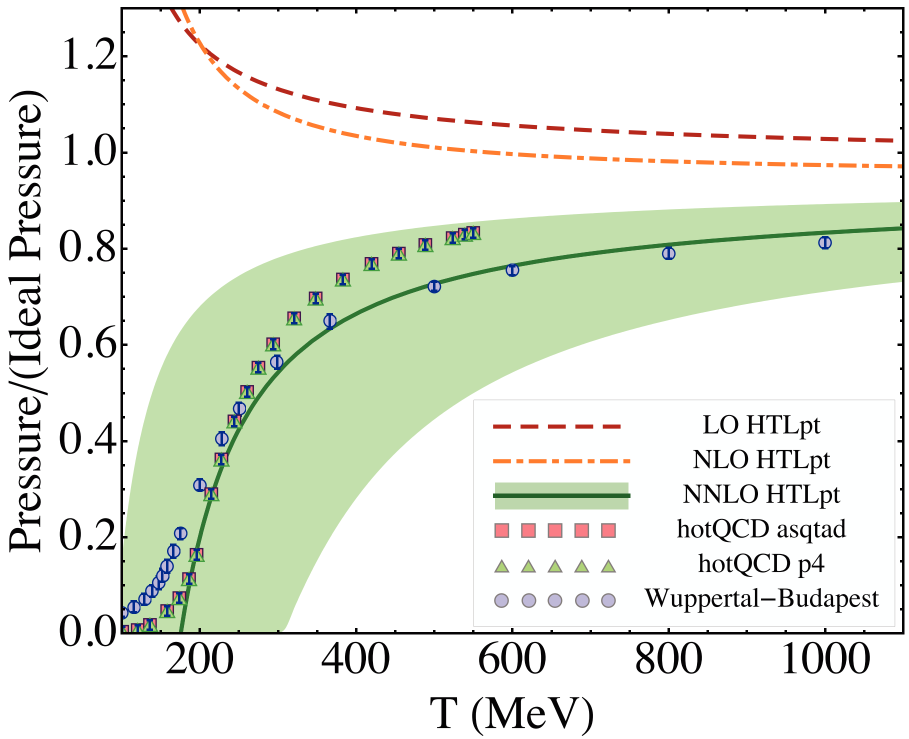
Pure-glue QCD



Andersen, Strickland and Su,

PRL 104, 122003 (2010) & JHEP 1008, 113 (2010)

QCD with $N_f = 3$

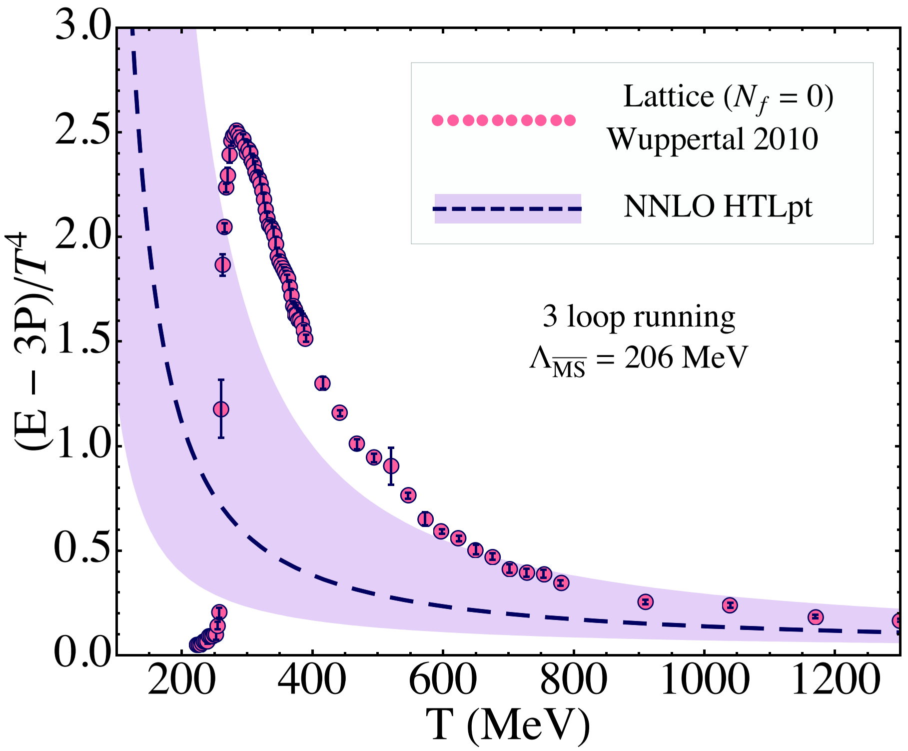


Andersen, Leganger, Strickland and Su,

PLB 696, 468 (2011) & JHEP 08 ,053 (2011)

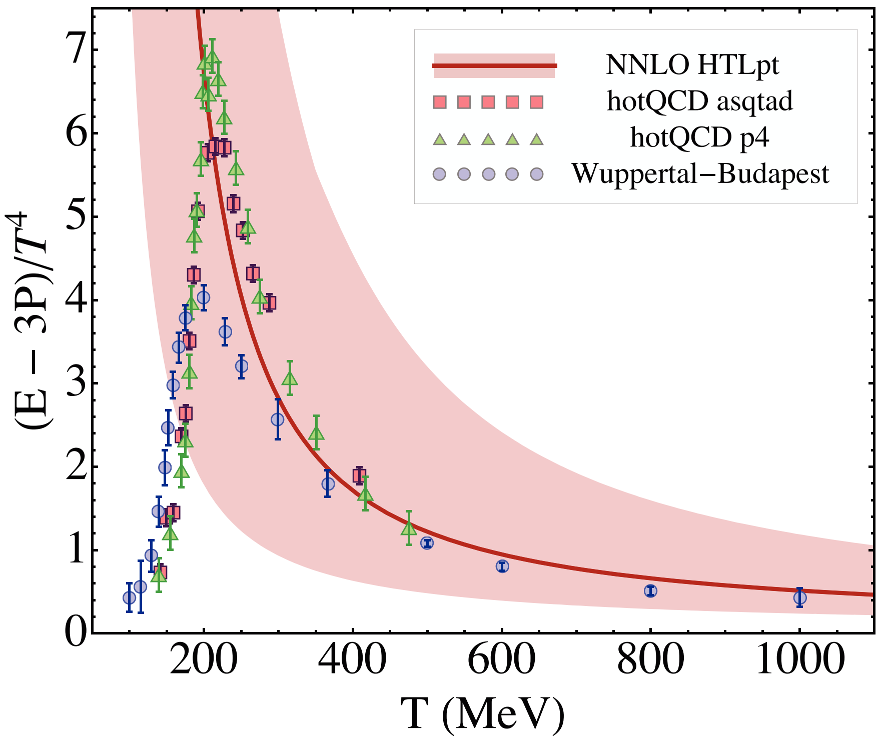
HTLpt: NNLO trace anomaly scaled by T^4

Pure-glue QCD



Andersen, Strickland and Su, JHEP 08, 113 (2010)

QCD with $N_f = 3$

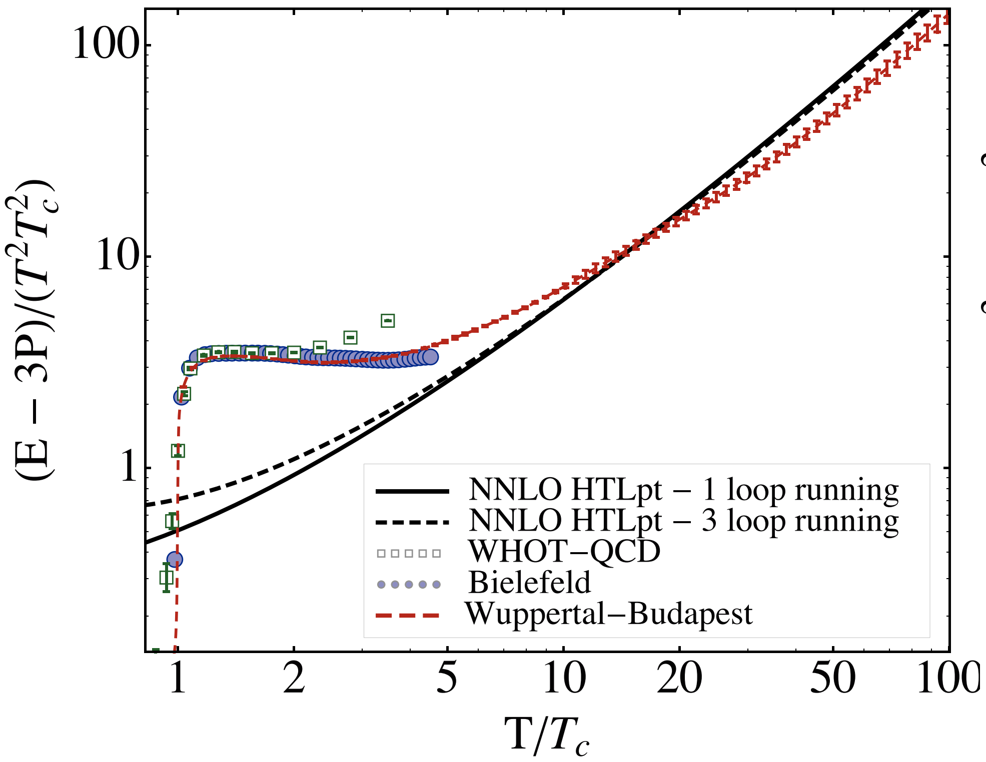


Andersen, Leganger, Strickland and Su,

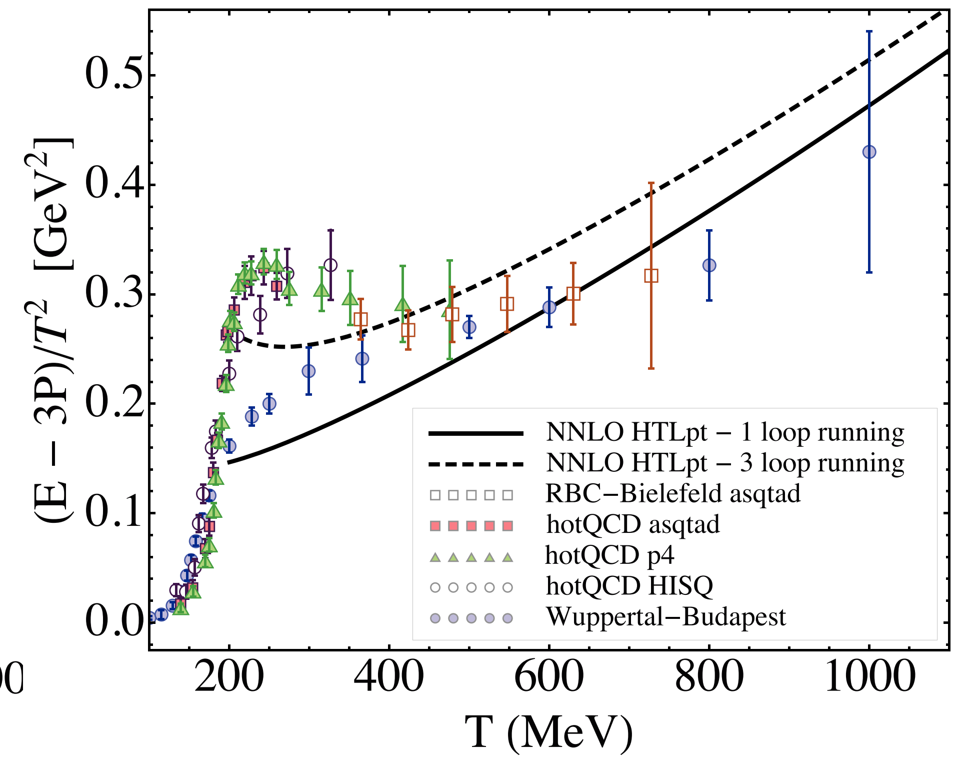
PLB 696, 468 (2011) & JHEP 08 ,053 (2011)

HTLpt: NNLO trace anomaly scaled by T^2

Pure-glue QCD



QCD with $N_f = 3$



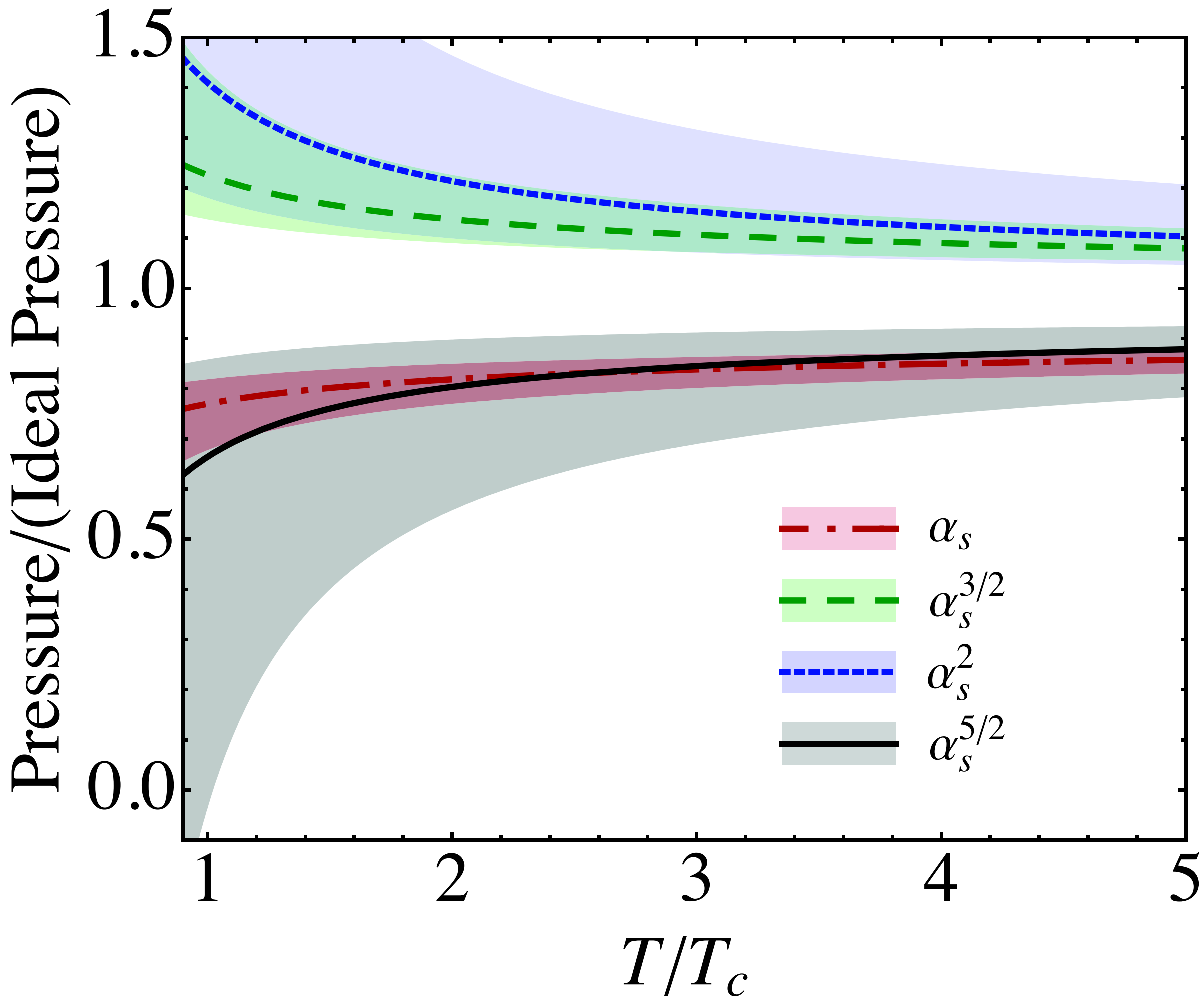
Andersen, Leganger, Strickland and Su, PRD 84, 087703 (2011)

Conclusions and Outlook

- HTLpt can improve the convergence of perturbative calculations in a gauge-invariant manner.
- The NNLO HTLpt results for pure-glue QCD look very good for $T \gtrsim 2 - 3\,T_c$, and the full QCD ones are even better! Especially considering that there are no free parameters to fit.
- Since HTLpt is formulated in Minkowski space, it provides a general systematic calculation scheme for both thermodynamics and real-time dynamics.
- The NNLO QCD thermodynamics calculation sets the stage of generalizing HTLpt to dynamic quantities, such as jet energy loss, momentum diffusion, viscosities, et al. for LHC temperatures.
- Explore the applications to other systems, e.g. cold atoms, compact stars...

Back-up

Weak-coupling expansion of pure-glue pressure

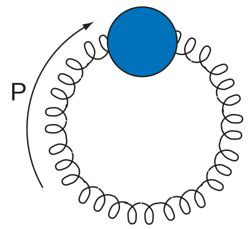


Kastening and Zhai, 95 & Braaten and Nieto, 96

HTLpt: LO free energy for pure-glue QCD

- Separation into hard and soft contributions ($d = 3 - 2\epsilon$)

$$\mathcal{F}_g = -\frac{1}{2}\sum\!\!\!\!\!\!\!\int_P \left\{(d-1)\ln[-\Delta_T^{\rm HTL}(P)] + \ln[\Delta_L^{\rm HTL}(P)]\right\}$$



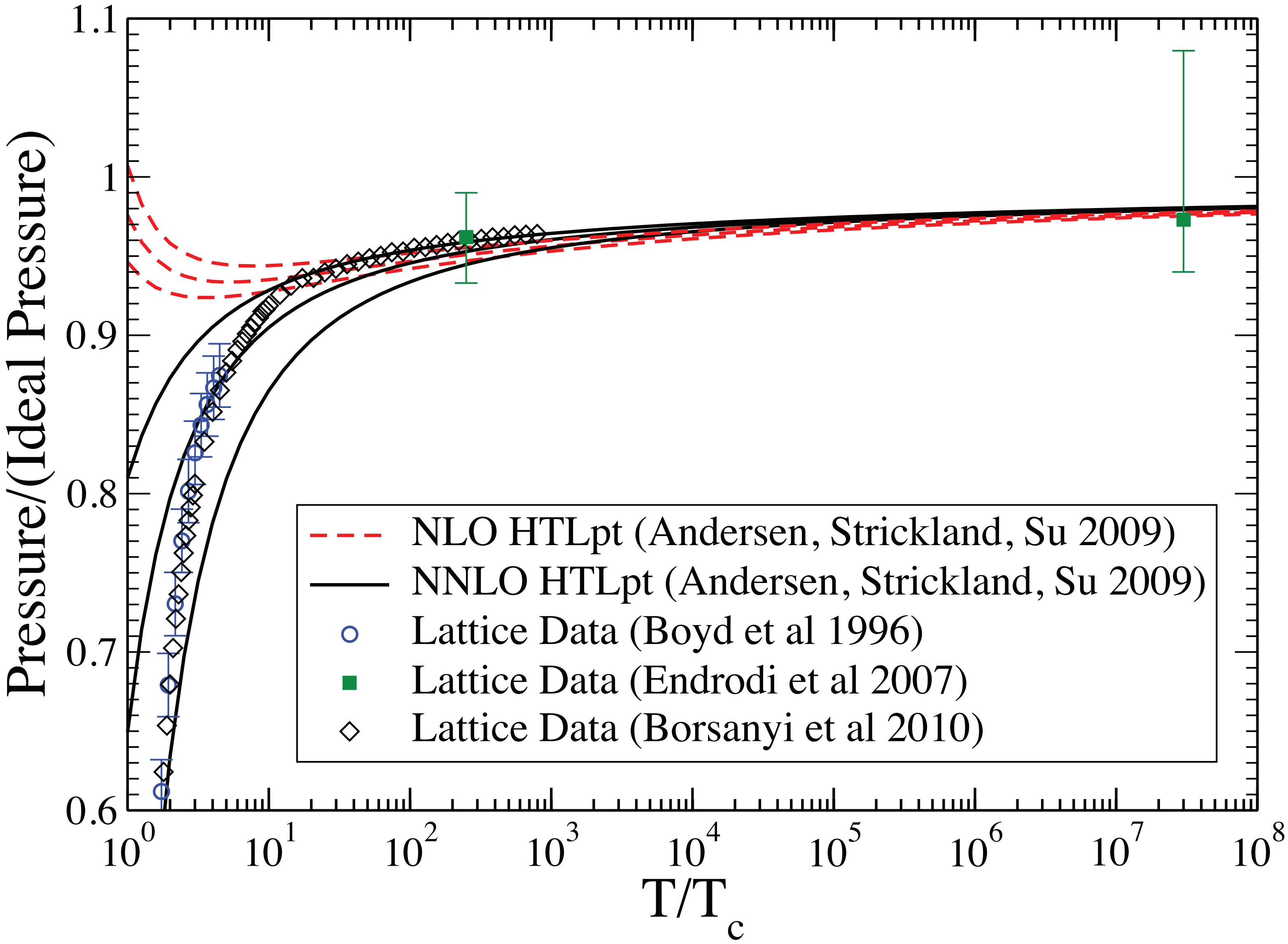
  - Hard momenta $(\omega, \mathbf{p} \sim T)$

$$\mathcal{F}_g^{(h)} = \frac{d-1}{2}\sum\!\!\!\!\!\!\!\int_P \ln(P^2) + \frac{1}{2}m_D^2 \sum\!\!\!\!\!\!\!\int_P \frac{1}{P^2} - \frac{1}{4(d-1)}m_D^4 \sum\!\!\!\!\!\!\!\int_P \left[\frac{1}{(P^2)^2} - 2\,\frac{1}{p^2 P^2} - 2d\,\frac{1}{p^4}\mathcal{T}_P + 2\,\frac{1}{p^2 P^2}\mathcal{T}_P + d\,\frac{1}{p^4}(\mathcal{T}_P)^2\right] + \mathcal{O}(m_D^6)$$

  - Soft momenta $(\omega, \mathbf{p} \sim gT)$

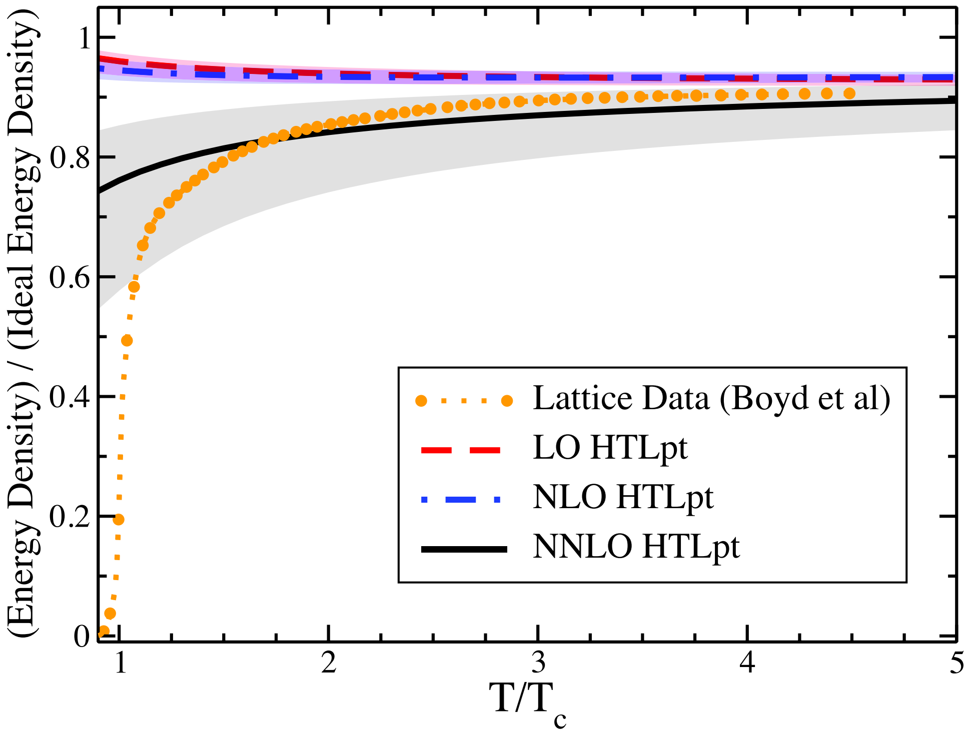
$$\mathcal{F}_g^{(s)} = \frac{1}{2}T\int_{\mathbf{p}} \ln(p^2 + m_D^2)$$

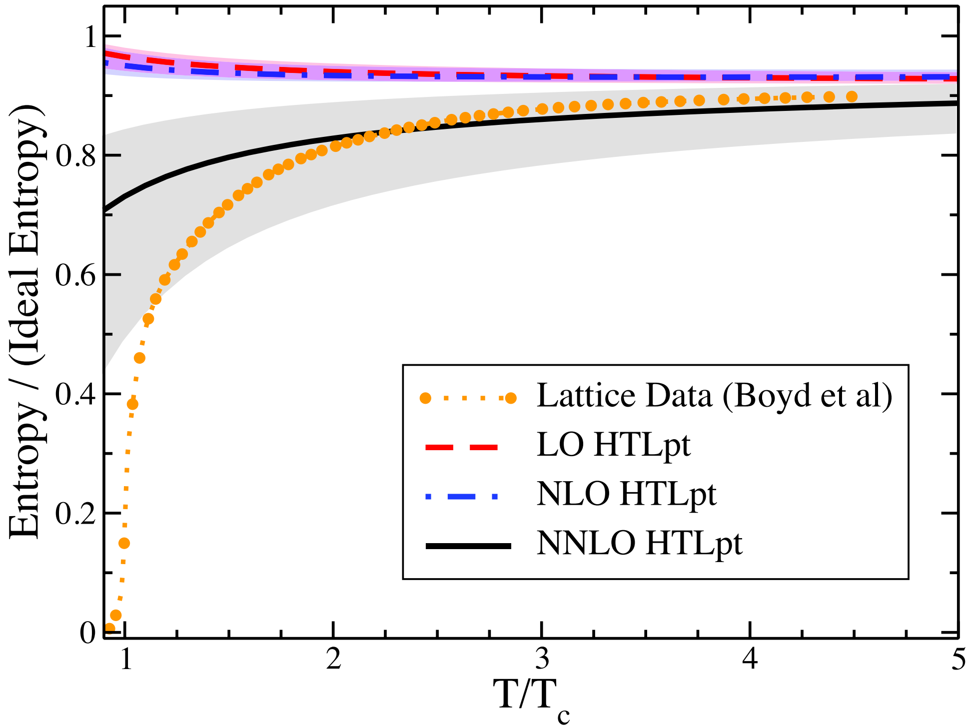
HTLpt: Pure-glue QCD high T pressure



HTLpt: Pure-glue QCD energy and entropy

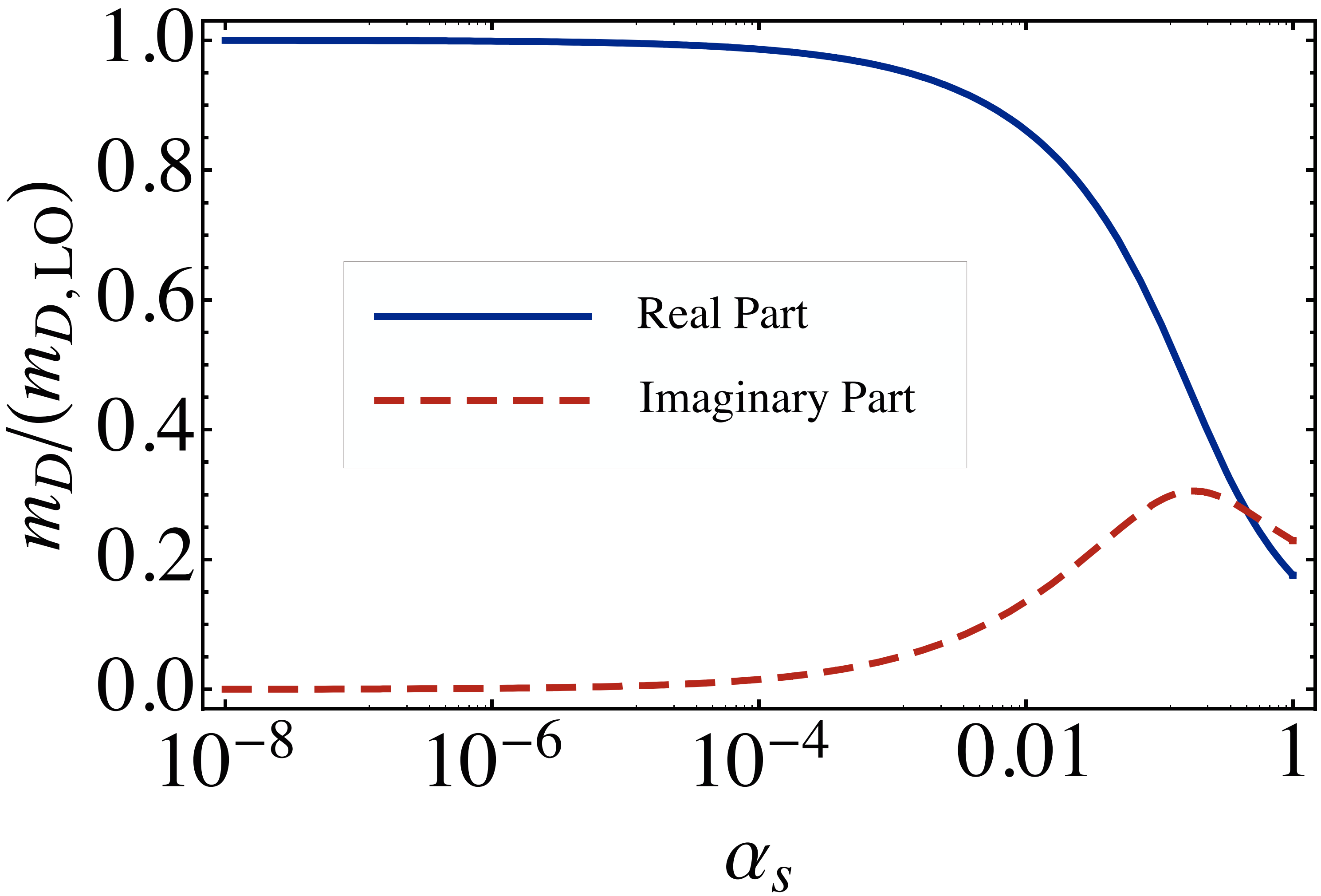
From the free energy we can evaluate other thermodynamic variables using standard relations: $\mathcal{P} = -\mathcal{F}, \mathcal{E} = \mathcal{F} - T\frac{d\mathcal{F}}{dT}, \mathcal{S} = -\frac{d\mathcal{F}}{dT}$.





Andersen, Strickland and Su, PRL, 104, 122003 (2010) & JHEP 08, 113 (2010)

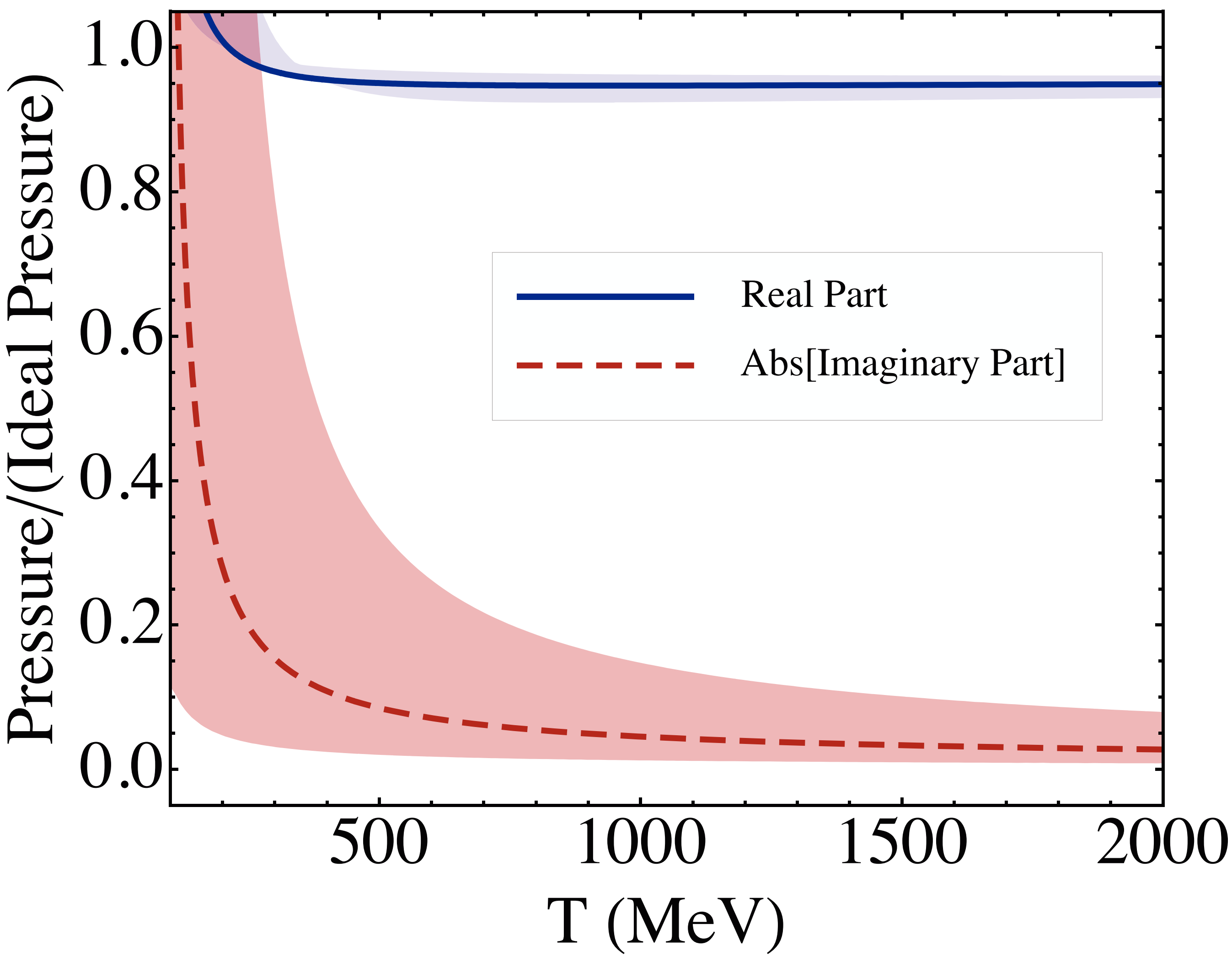
HTLpt: NNLO variational Debye mass



Comparison of the real and imaginary parts of the NNLO variational Debye mass, with $N_c = 3$ & $N_f = 3$

Andersen, Leganger, Strickland and Su, JHEP 08 ,053 (2011)

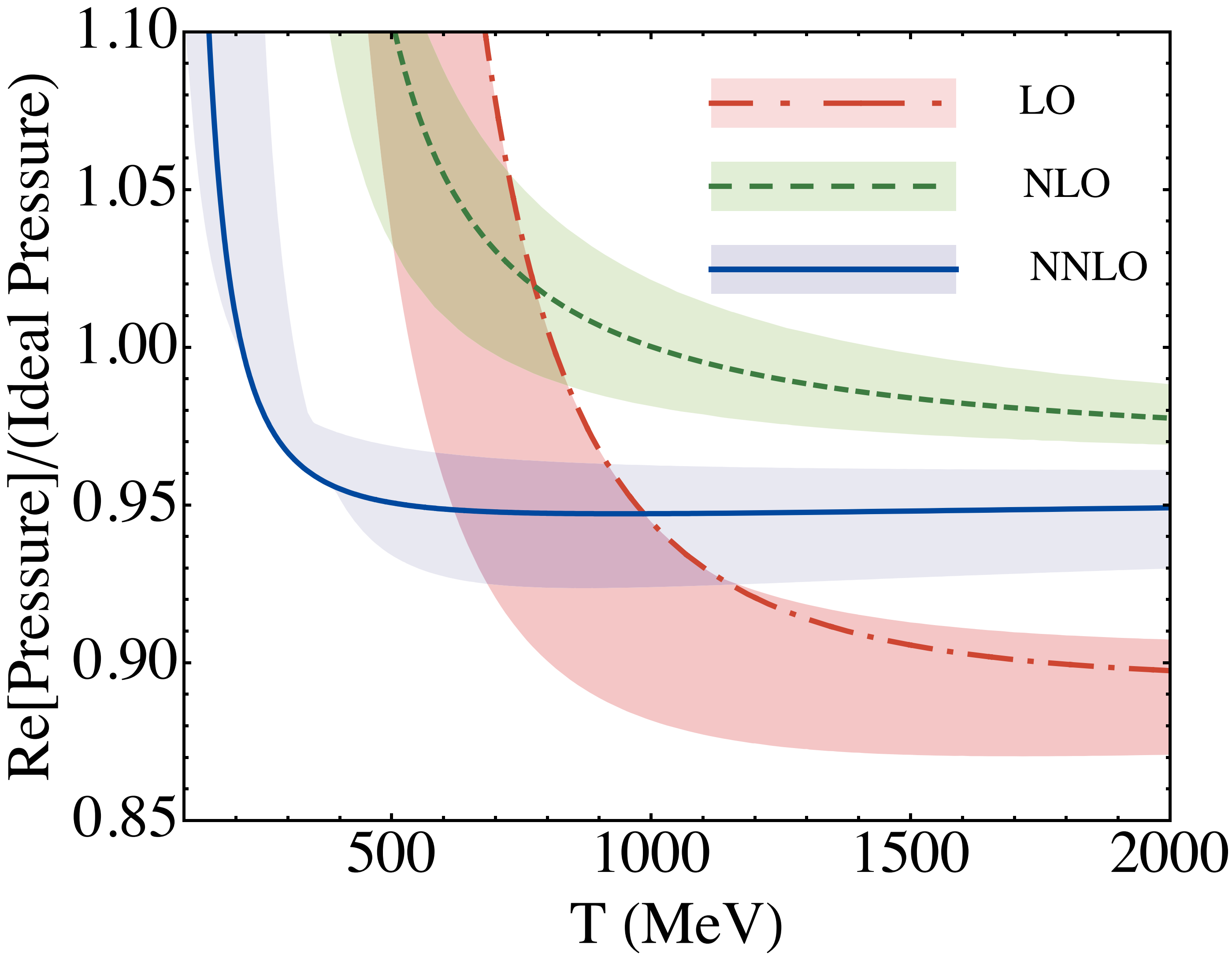
HTLpt: NNLO variational pressure



Comparison of the real and imaginary parts of the NNLO variational pressure, with $N_c = 3$ & $N_f = 3$

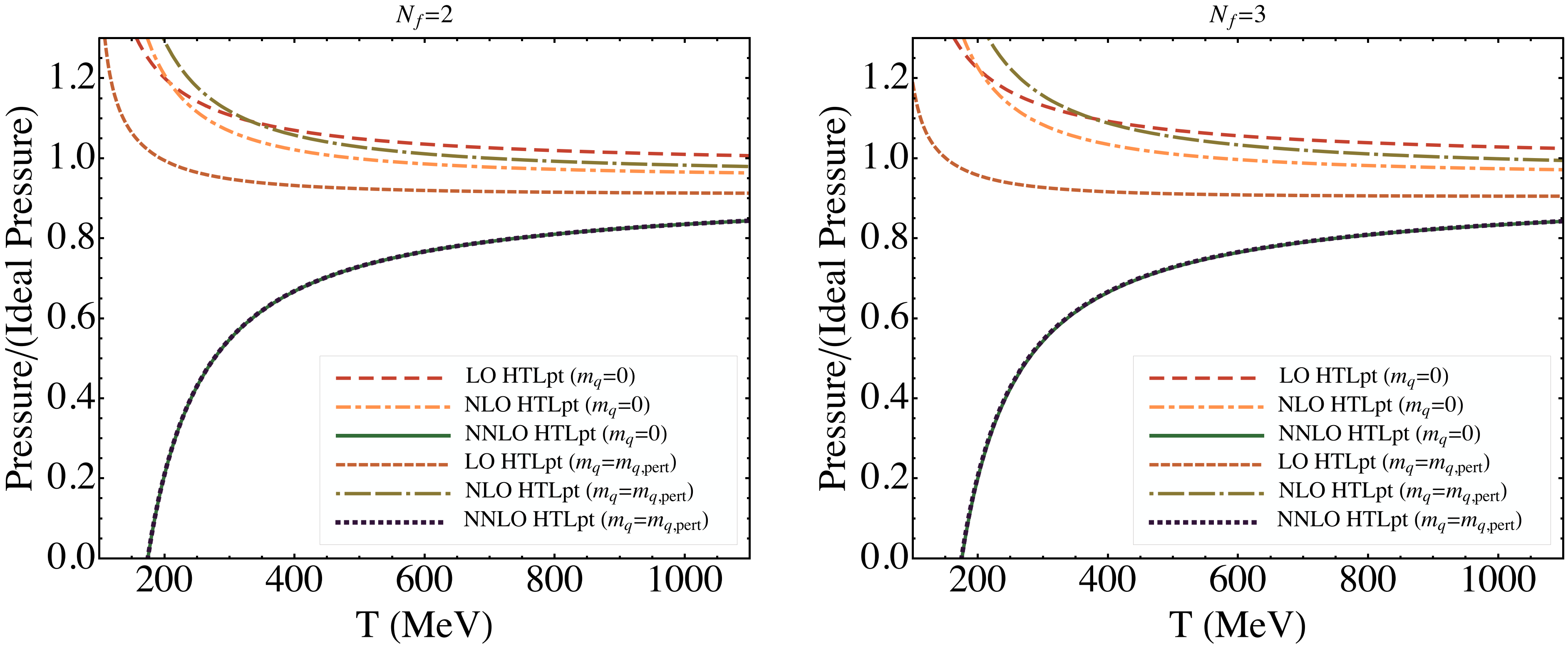
Andersen, Leganger, Strickland and Su, JHEP 08 ,053 (2011)

HTLpt: $N_c = 3$ & $N_f = 3$ variational pressure



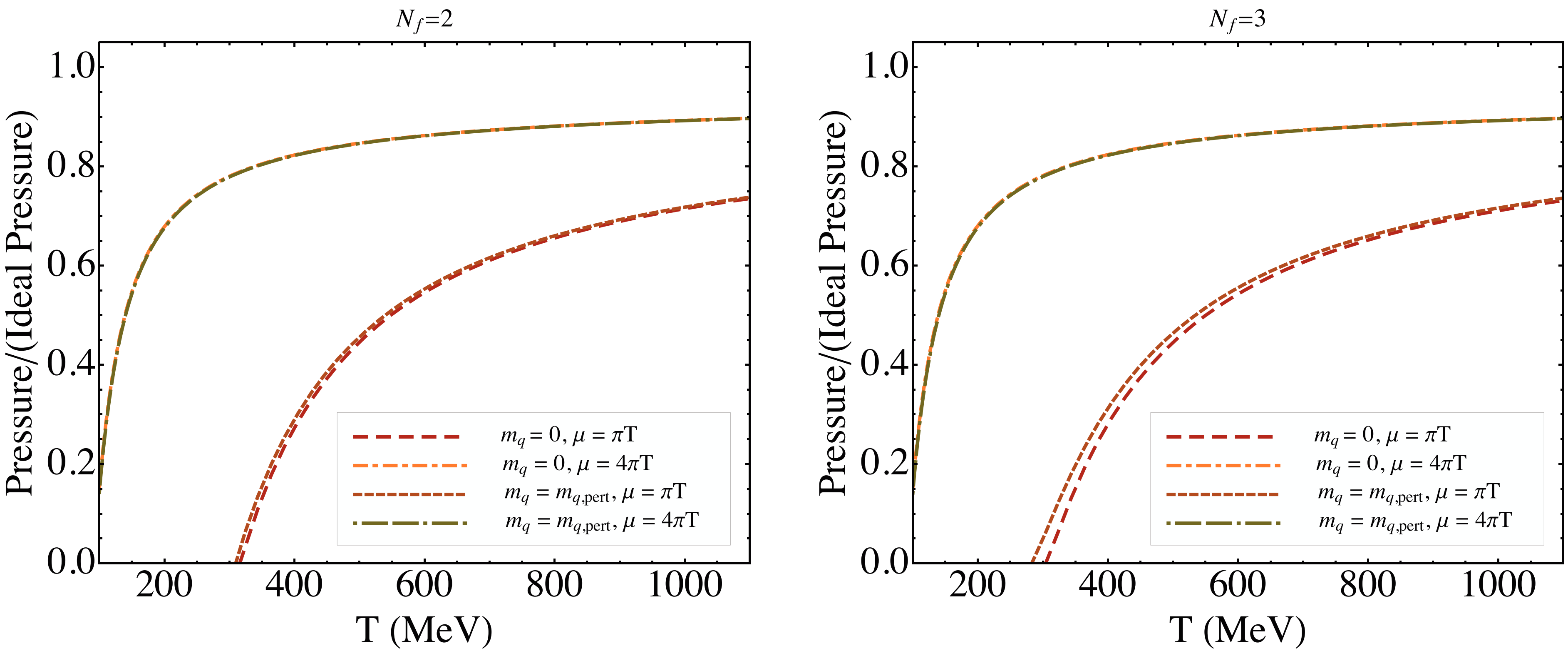
Andersen, Leganger, Strickland and Su, JHEP 08 ,053 (2011)

HTLpt: QCD pressure with different m_f



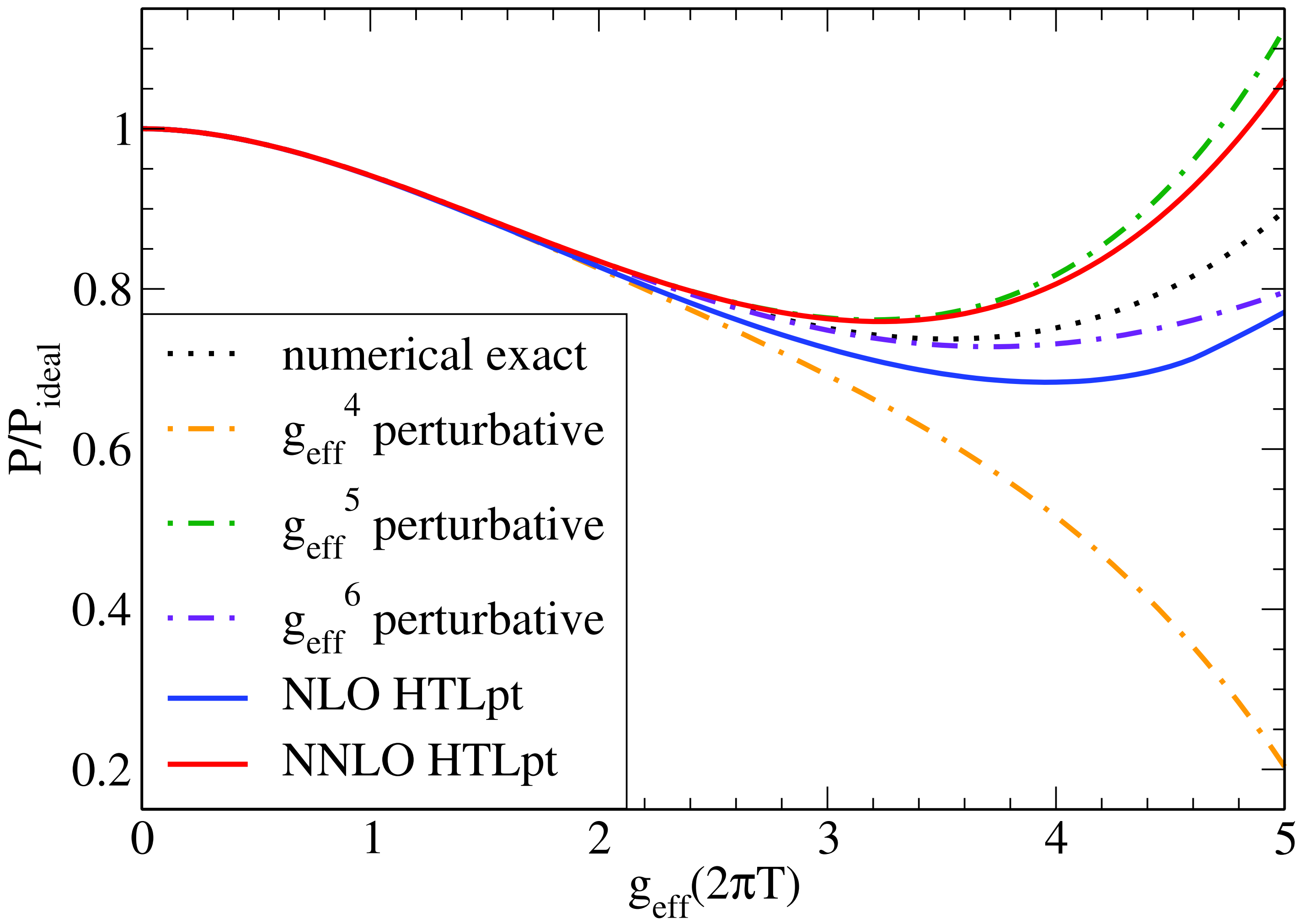
Andersen, Leganger, Strickland and Su, JHEP 08 ,053 (2011)

HTLpt: NNLO scale variation with different m_f



Andersen, Leganger, Strickland and Su, JHEP 08 ,053 (2011)

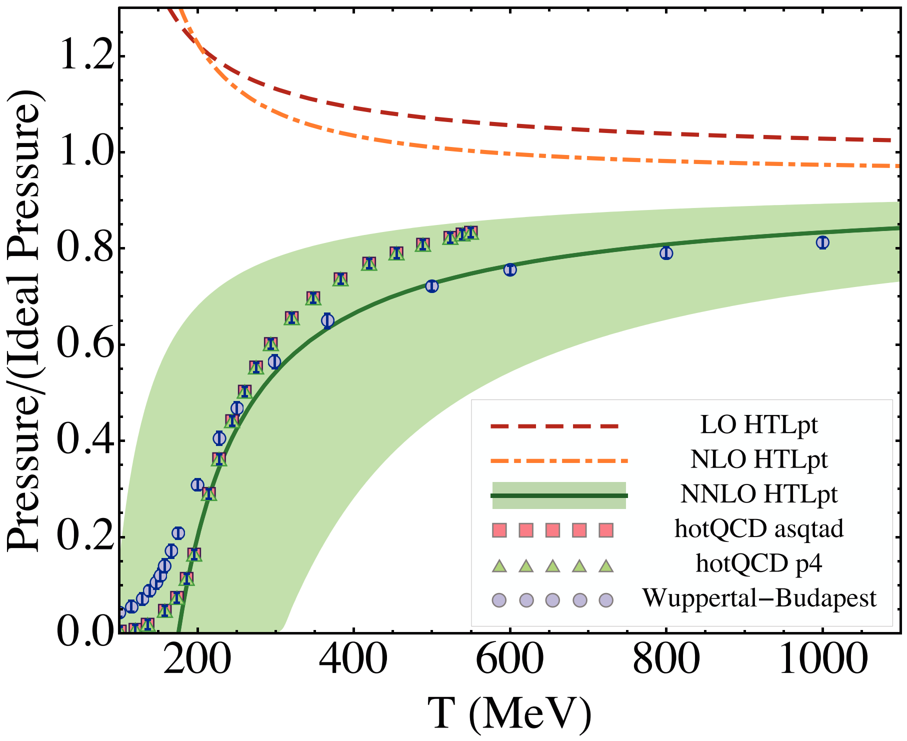
HTLpt: QCD pressure at large N_f



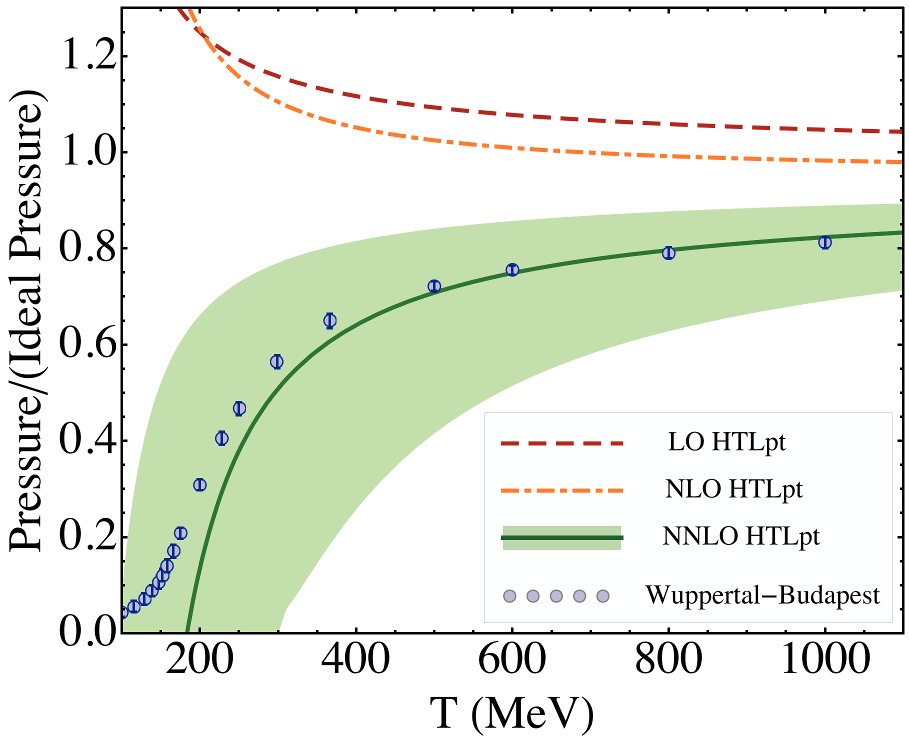
Andersen, Leganger, Strickland and Su, JHEP 08 ,053 (2011)

HTLpt: QCD pressure with different N_f

$N_f = 3$

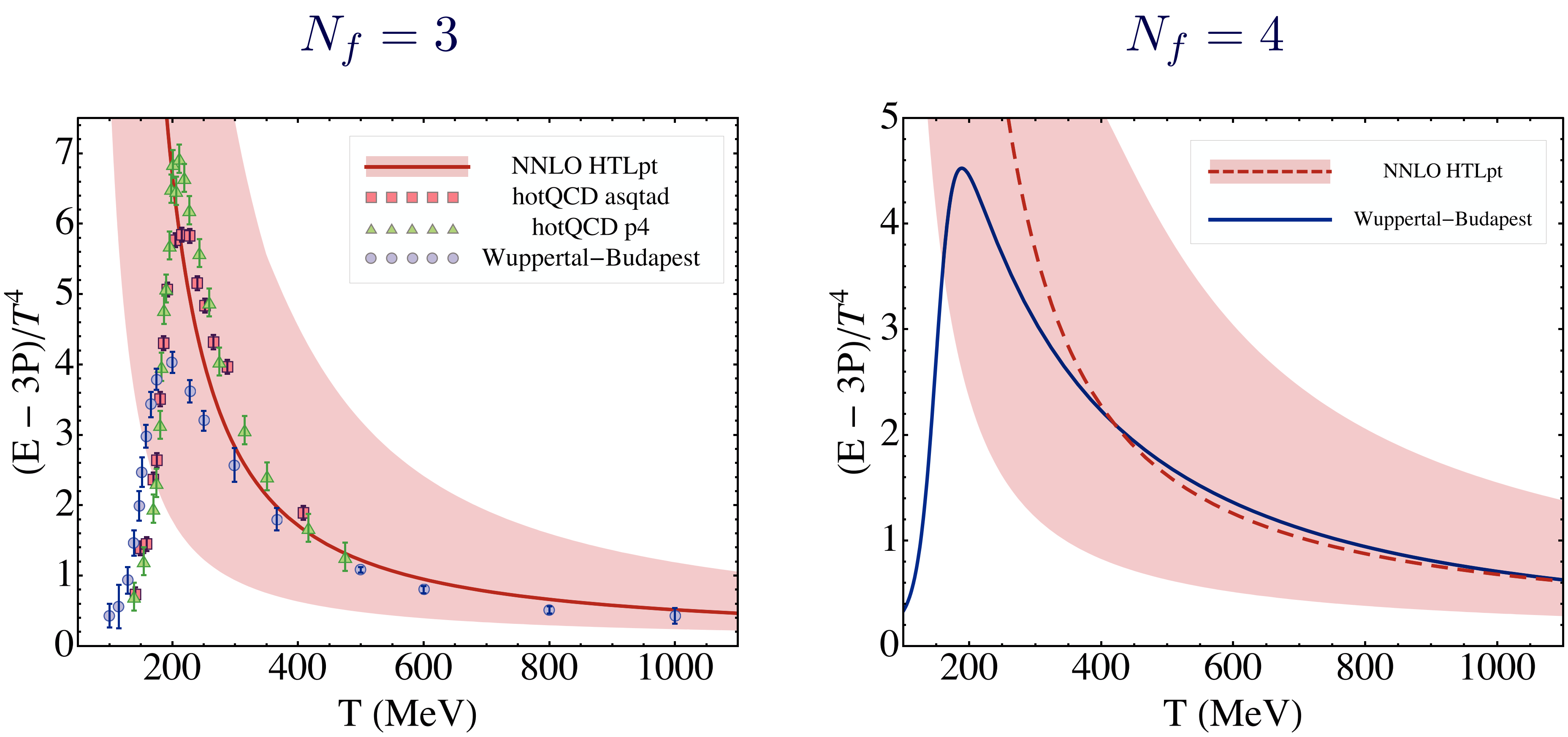


$N_f = 4$



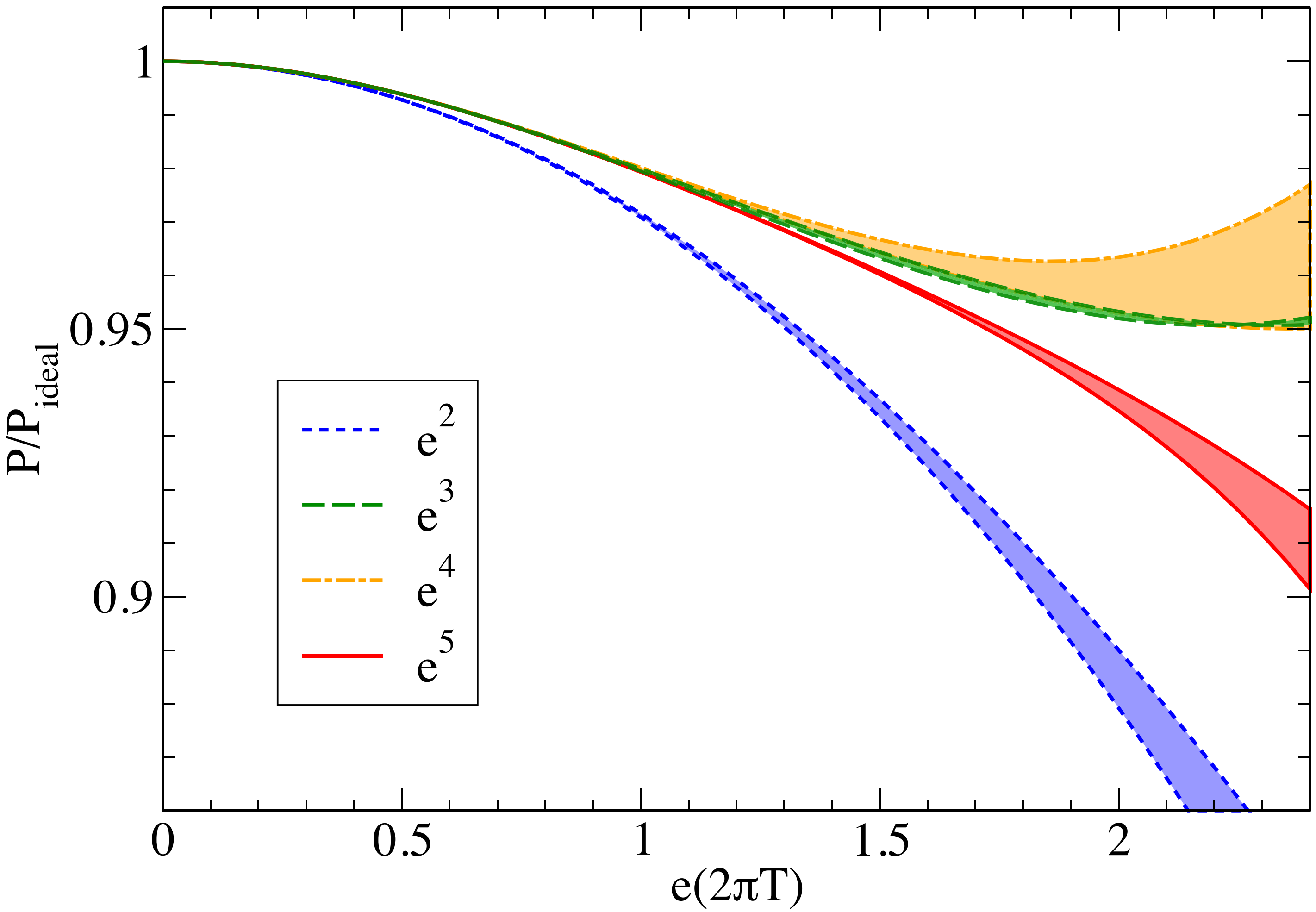
Andersen, Leganger, Strickland and Su, PLB 696, 468 (2011) & JHEP 08 ,053 (2011)

HTLpt: QCD trace anomaly with different N_f



Andersen, Leganger, Strickland and Su, PLB 696, 468 (2011) & JHEP 08 ,053 (2011)

Weak-coupling expansion of QED pressure

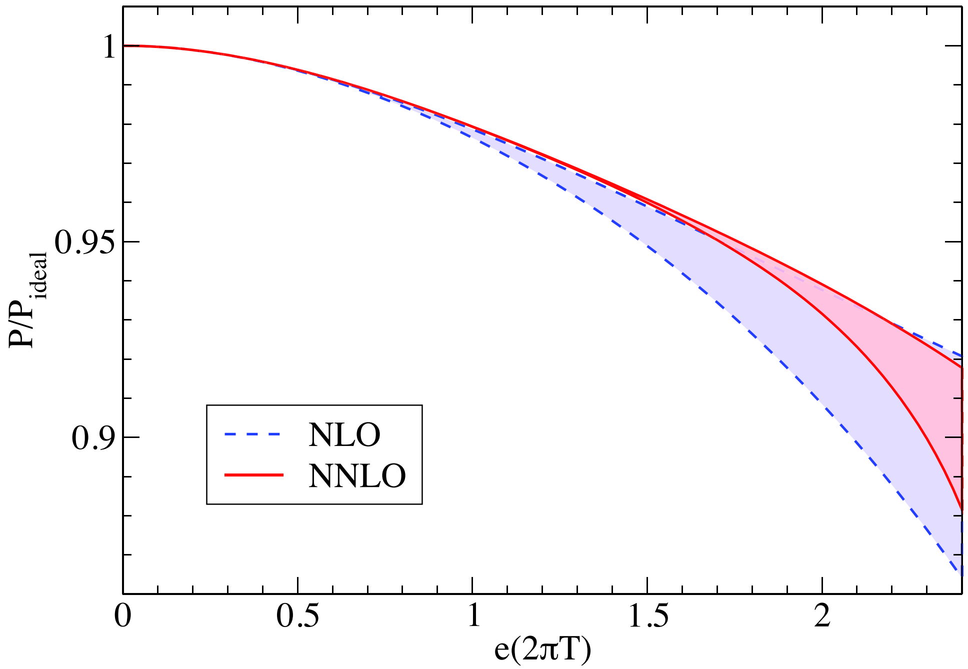


Same nonconvergence pattern as the QCD case

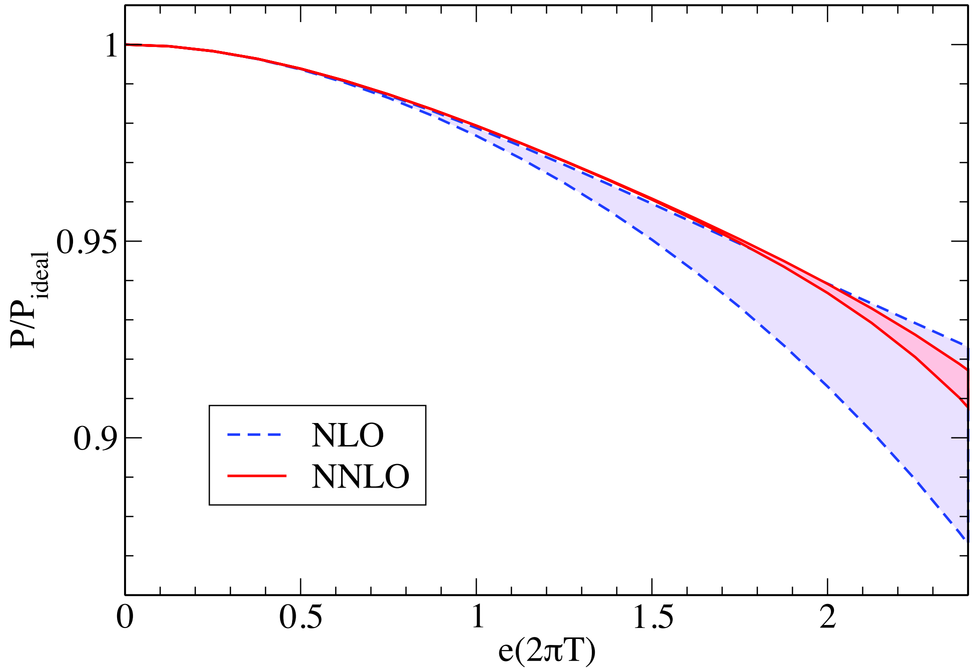
Parwani, 94; Parwani and Coriano, 95; Zhai and Kastening, 95; Andersen, 96.

HTLpt: QED pressure with different mass prescriptions

Variational masses

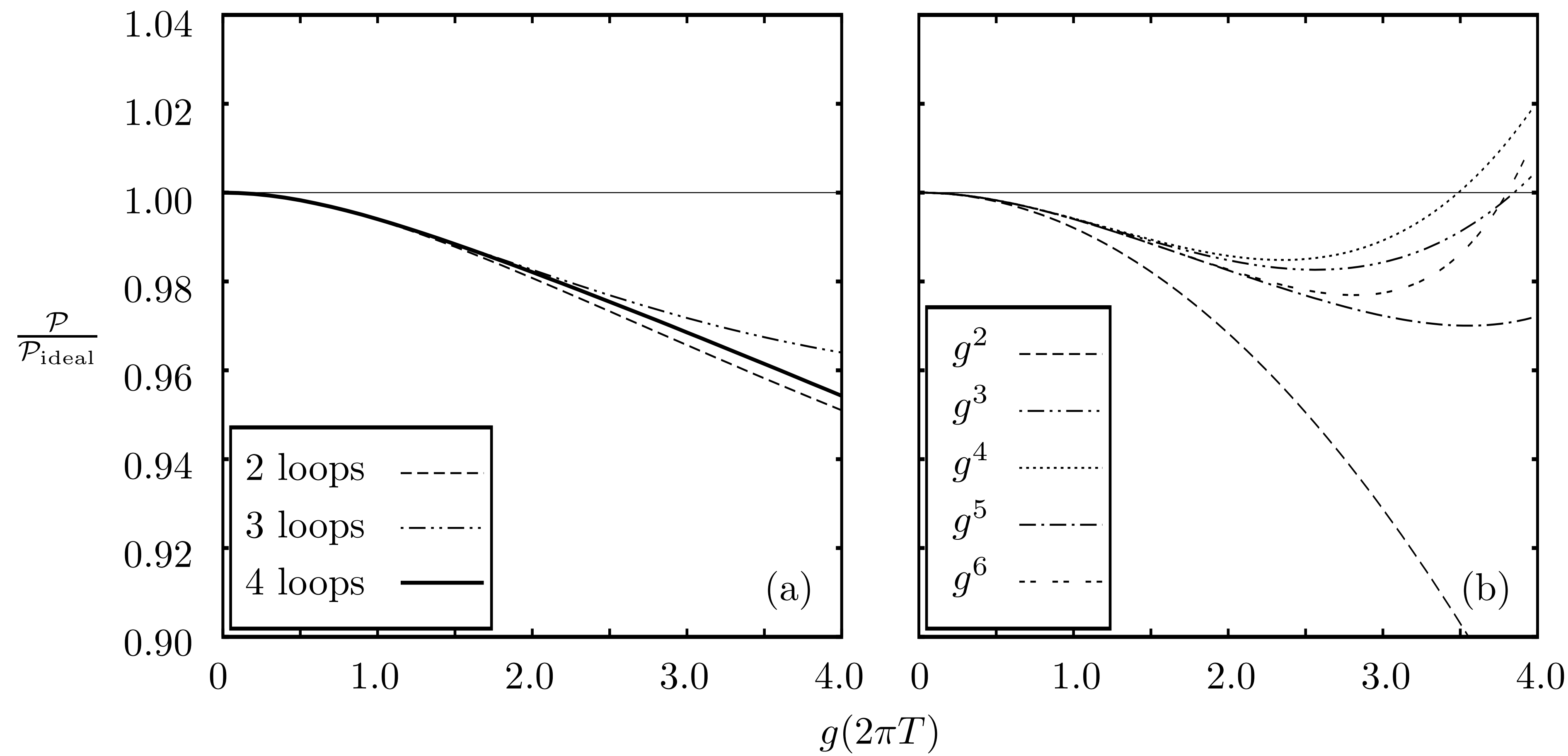


Perturbative masses



Andersen, Strickland and Su, PRD 80, 085015 (2009)

Screened Perturbation Theory



4-loop SPT pressure vs weak-couping pressure

Andersen, Braaten and Strickland, 00. Andersen and Strickland, 01.
Andersen and Kyllingstad, 08.