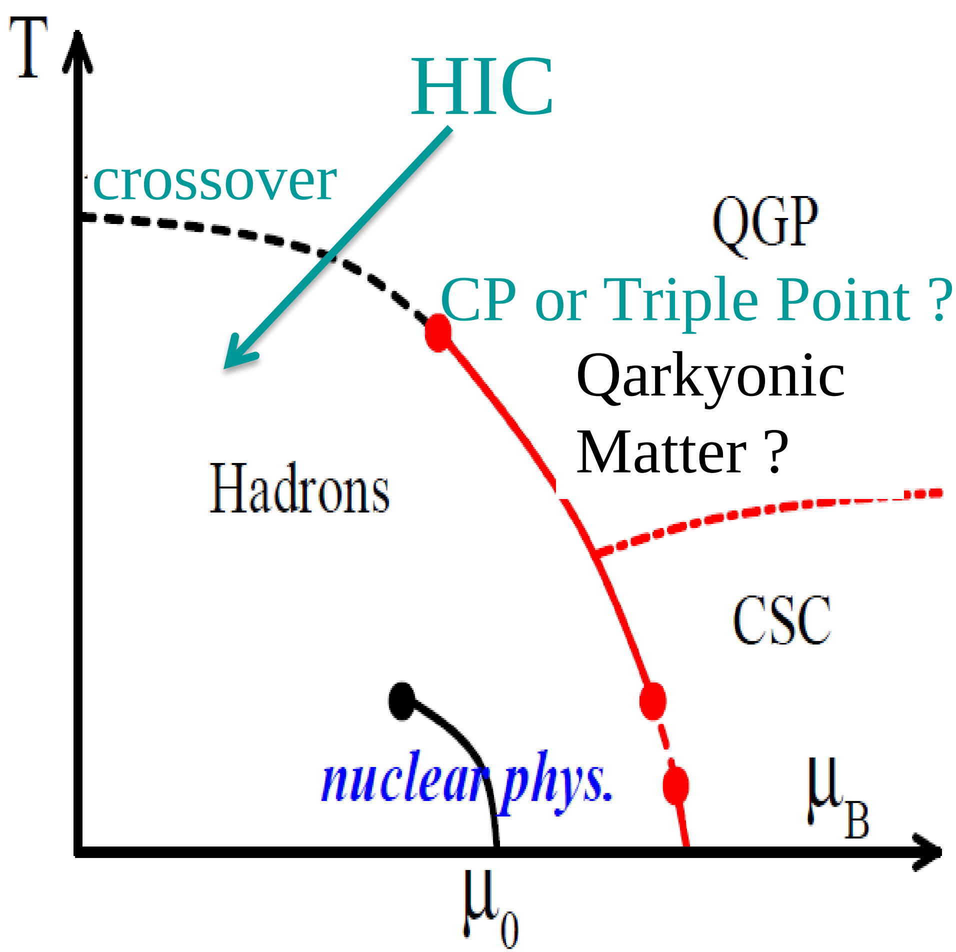
Probability distribution of conserved charges and the QCD phase transition

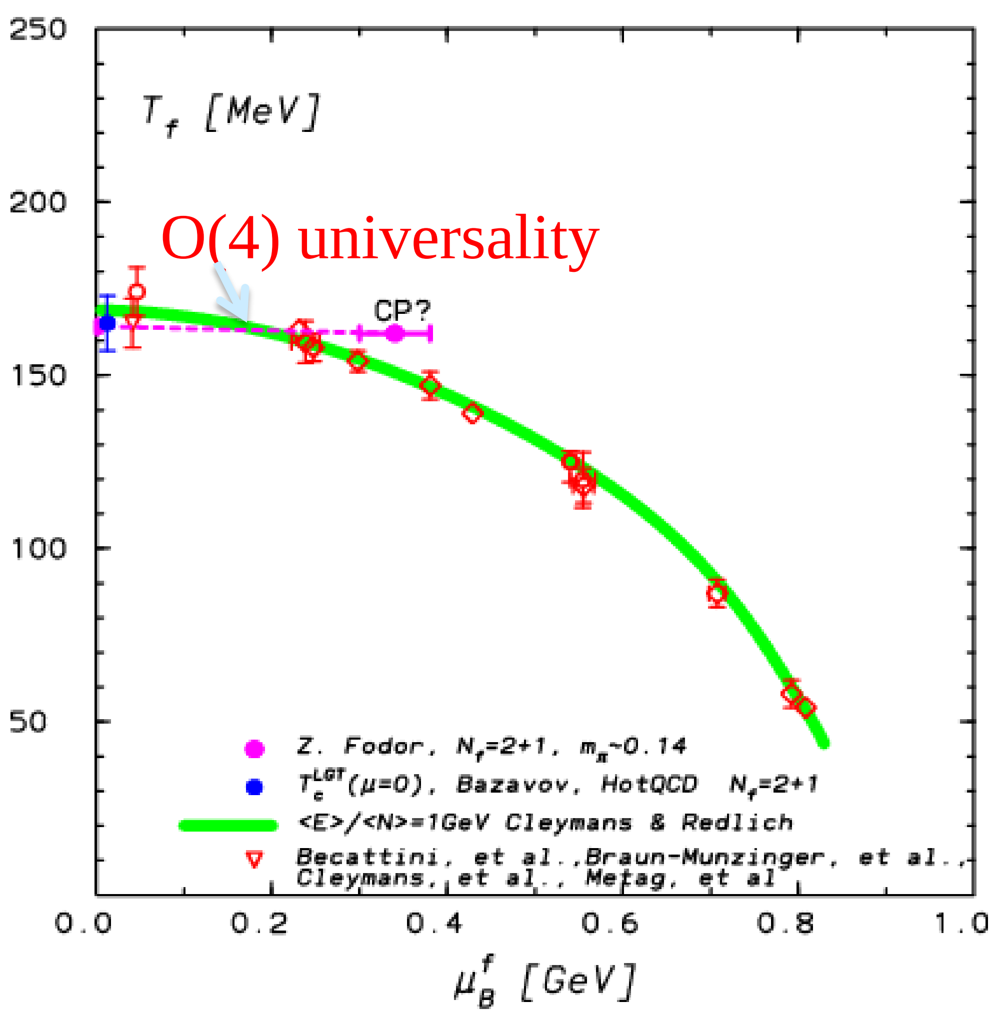
K. Redlich



- QCD phase boundary, its O(4) „scaling" & relation to freezeout in HIC
- Moments and probablility distributions of conserved charges as probes of the O(4) crossover criticality
- STAR data & expectiations

with: P. Braun-Munzinger, B. Friman
F. Karsch, V. Skokov

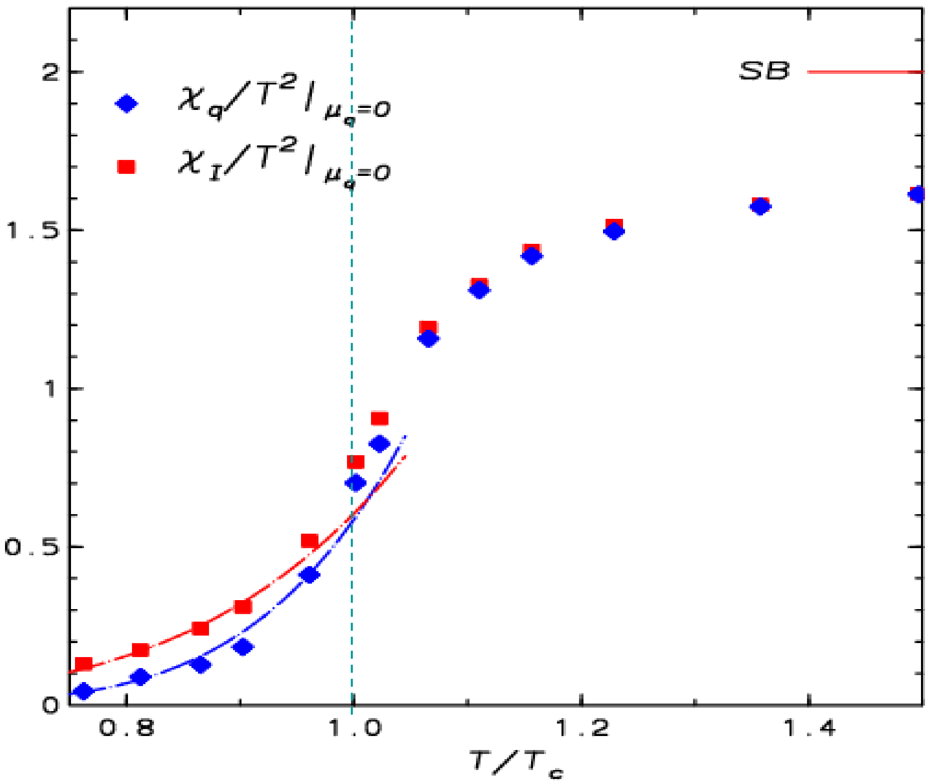
Chemical freezeout and QCD crtitical line



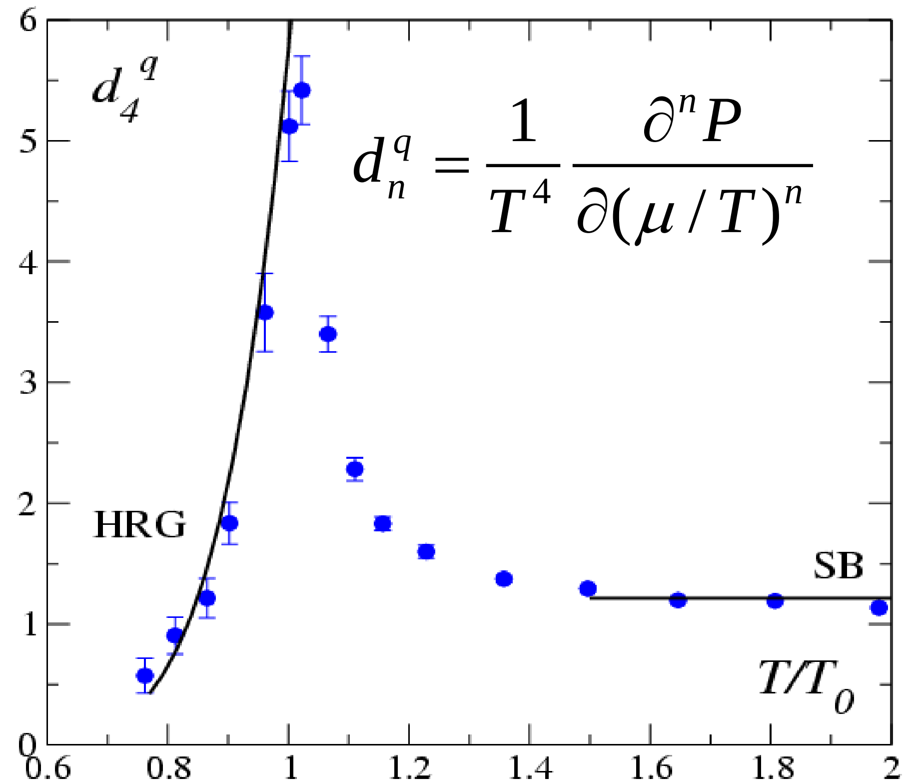
- Relating freezeout and QCD critical line at small baryon chemical potential
- Particle yields and their ratios as well as LGT thermodynamics at $T < T_c$ are well described by the HRG Partition Function
- QCD crossover line - remnant of the O(4) criticality

LGT and phenomenological HRG model

C. Allton et al.,



S. Ejiri, F. Karsch & K.R.



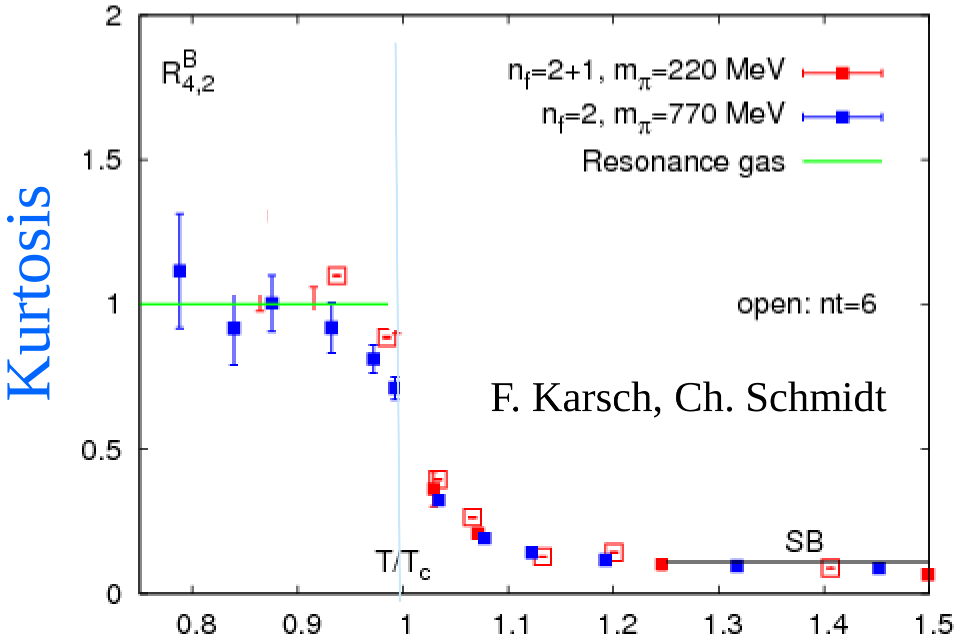
- The HRG partition function provides and excellent approximation of the QCD thermodynamics at $T < T_c$

See also: C. Ratti et al., P. Huovinen et al., B. Muller et al.,…….

Kurtosis as an excellent probe of deconfinement

S. Ejiri, F. Karsch & K.R.

$$R_{4,2}^{\;B} = \frac{1}{9}\frac{c_4}{c_2}$$



The $R_{4,2}^{\;B}$ measures the quark content of particles carrying baryon number

- HRG factorization of pressure:

$$P^B(T,\mu_q) = F(T)\cosh(3\mu_q/T)$$

consequently: $c_4/c_2 = 9$ in HRG

- In QGP, $SB = 6/\pi^2$
- Kurtosis=Ratio of cumulants

$$c_4^q/c_2^q = \frac{<(\delta N_q)^4>}{<(\delta N_q)^2>} - 3<(\delta N_q)^2>$$

excellent probe of deconfinement

O(4) scaling and critical behavior

- Near T_c critical properties obtained from the singular part of the free energy density

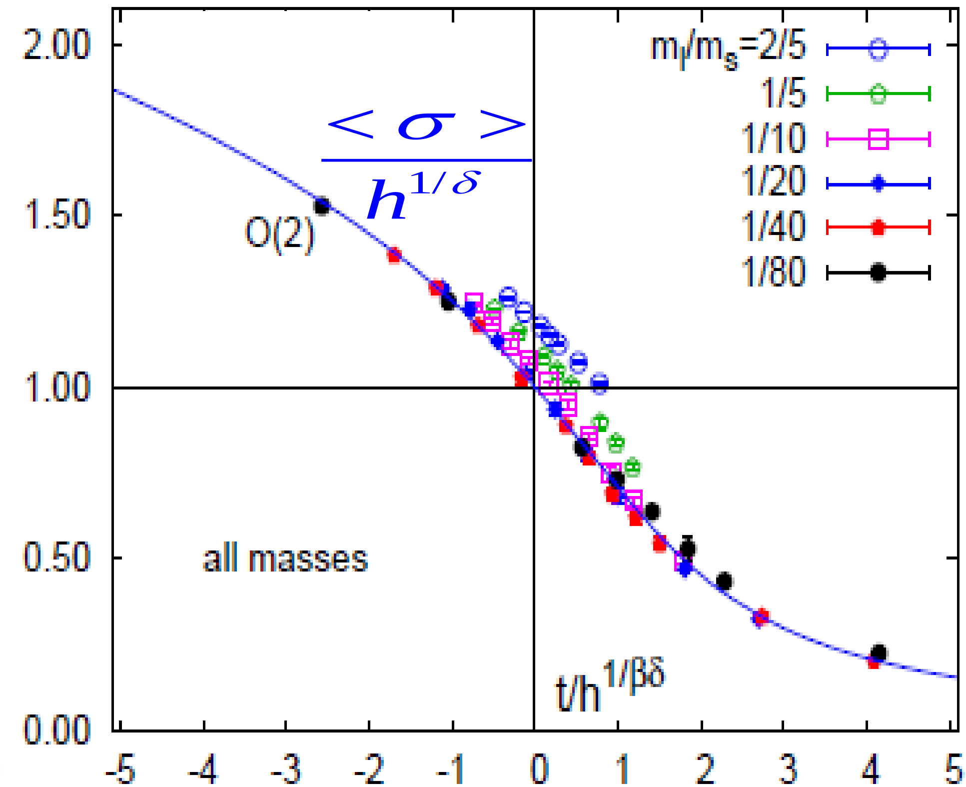
$$F = F_{reg} + F_S \qquad \text{with} \qquad F_S(t,h) = b^{-d} F(b^{1/\nu} t, b^{\beta\delta/\nu} h)$$

$$\text{with} \quad t = \frac{T - T_c}{T_c} + \kappa \left(\frac{\mu}{T_c} \right)^2$$

- Phase transition encoded in the "equation of state"

$$< \sigma > = -\frac{\partial F_s}{\partial h} \Rightarrow \text{pseudo-critical line}$$

$$\frac{< \sigma >}{h^{1/\delta}} = F_h(z) \ , \quad z = t h^{-1/\beta\delta}$$



O(4) scaling of net-baryon number fluctuations

- The fluctuations are quantified by susceptibilities

$$\chi_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} := c_n \qquad \text{with} \qquad P = P_{reg} + P_{Singular}$$

- From free energy and scaling function one gets

$$\chi_B^{(n)} \approx \chi_r^{(n)} + c\; h^{2-\alpha-n/2} f_{\pm}^{(n/2)}(z) \quad for \quad \mu = 0 \quad and \quad n \quad even$$

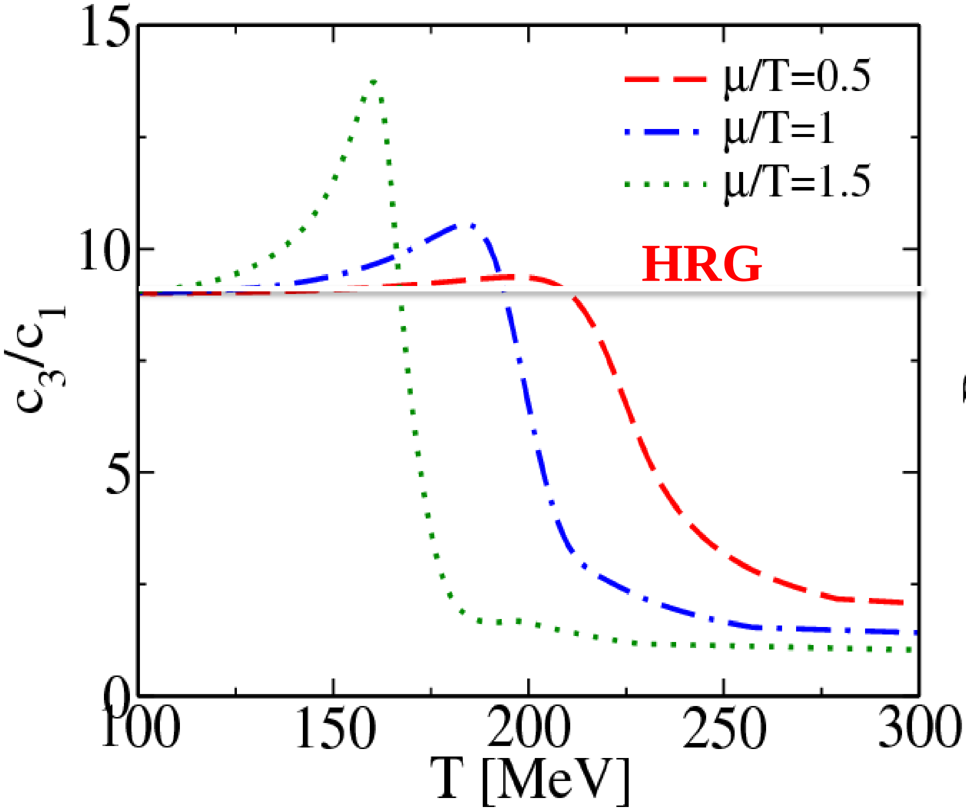
$$\chi_B^{(n)} \approx \chi_r^{(n)} + c_\mu\; h^{2-\alpha-n} f_{\pm}^{(n)}(z) \quad for \quad \mu \neq 0$$

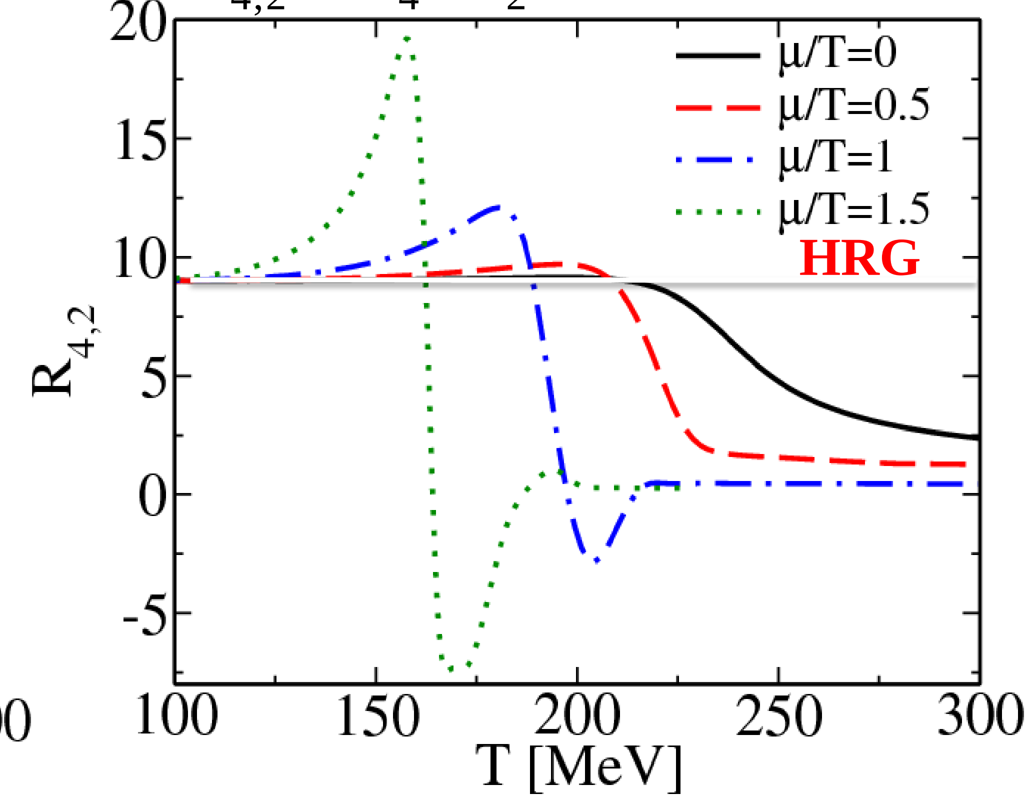
- Resulting in singular structures in n-th order moments which appear for $n \geq 6 \quad at \quad \mu = 0$ and for $n \geq 3 \quad at \quad \mu \neq 0$ since $\alpha \approx -0.2$ in O(4) univ. class

Ratio of cumulants at finite density PQM -RG

See talk of Vladimir Skokov

$R_{4,2} = c_4 / c_2$



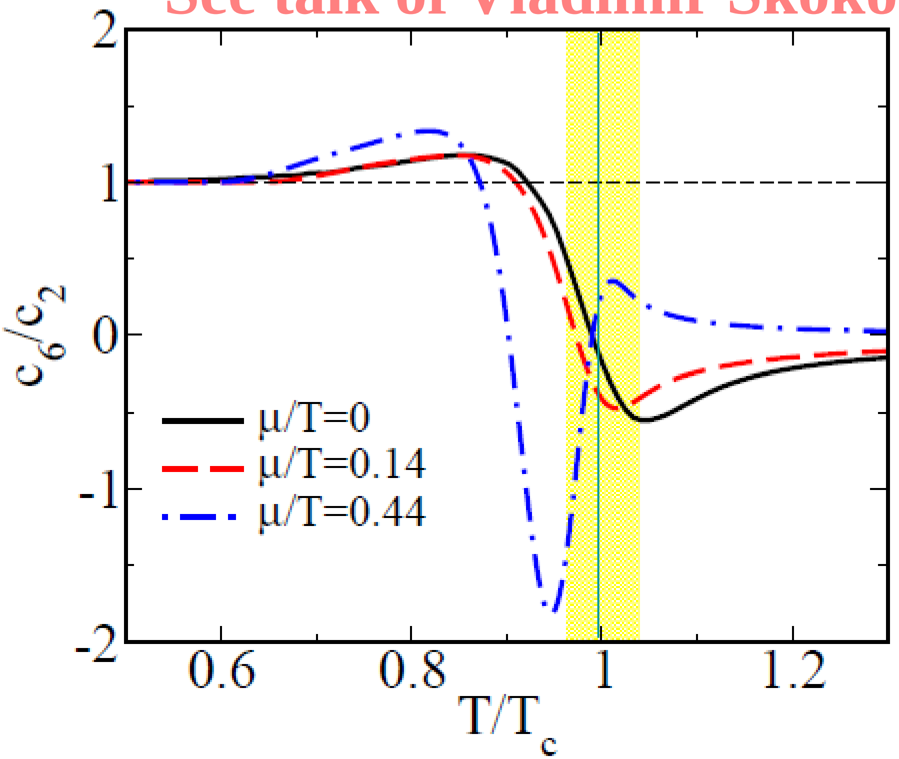


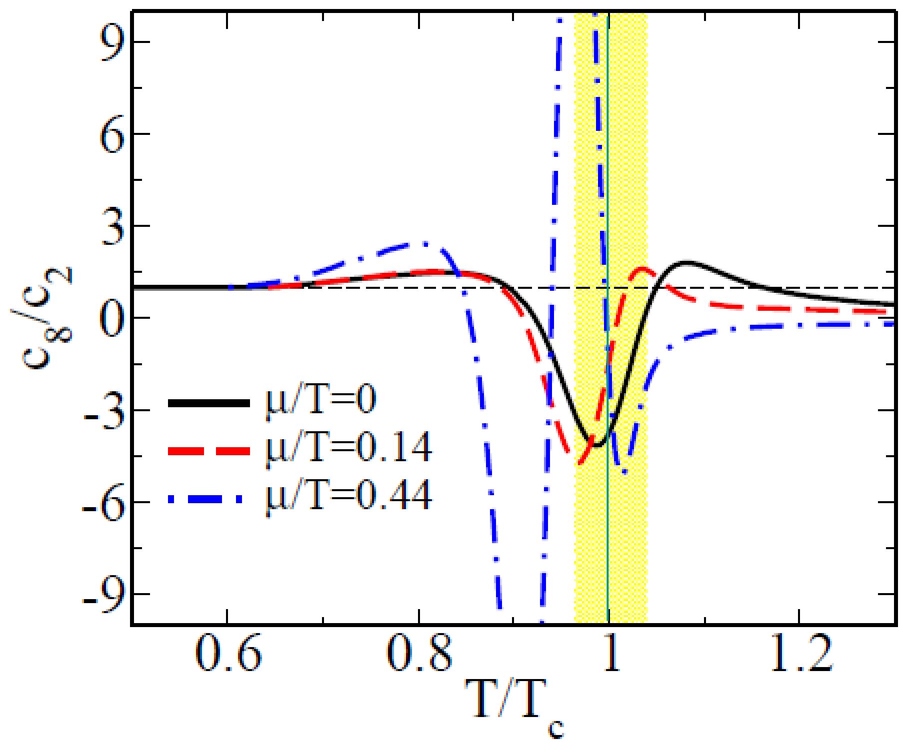
Deviations of the ratios of odd and even order cumulants from their asymptotic, low T-value, $c_4 / c_2 = c_3 / c_1 = 9$ are increasing with μ / T and the cumulant order

Properties essential in HIC to discriminate the phase change by measuring baryon number fluctuations !

Ratio of higher order cumulants at finite density

See talk of Vladimir Skokov





Deviations of the ratios from their asymptotic, low T-value, are increasing with the order of the cumulant order and with increasing chemical potential

Properties essential in HIC to discriminate the phase change by measuring baryon number fluctuations !

Properties of fluctuations in HRG

Calculate generalized susceptibilities:

$$\chi_B^{(n)} = \frac{\partial^n (P(T,\mu_B)/T^4)}{\partial(\mu_B/T)^n}$$

from Hadron Resonance Gas (HRG) partition function:

$$P^{HRG} = P_{mesons} + P_{baryons} \quad \text{and} \quad P_{baryons} \approx T^4 F(m/T) \cosh(\mu_B/T)$$

$$\frac{P_{baryons}}{T^4} = \frac{1}{\pi^2} \sum_i d_i (m_i/T)^2 K_2(m_i/T) \cosh[(B_i\mu_B + S_i\mu_S + Q_i\mu_Q)/T]$$

$$\frac{\chi_B^{(4)}}{\chi_B^{(2)}} = 1 \qquad \frac{\chi_B^{(3)}}{\chi_B^{(1)}} = 1 \qquad \frac{\chi_B^{(2)}}{\chi_B^{(1)}} \approx \coth(\mu_B/T) \quad \text{and} \quad \frac{\chi_B^{(3)}}{\chi_B^{(2)}} \approx \tanh(\mu_B/T)$$

resulting in:

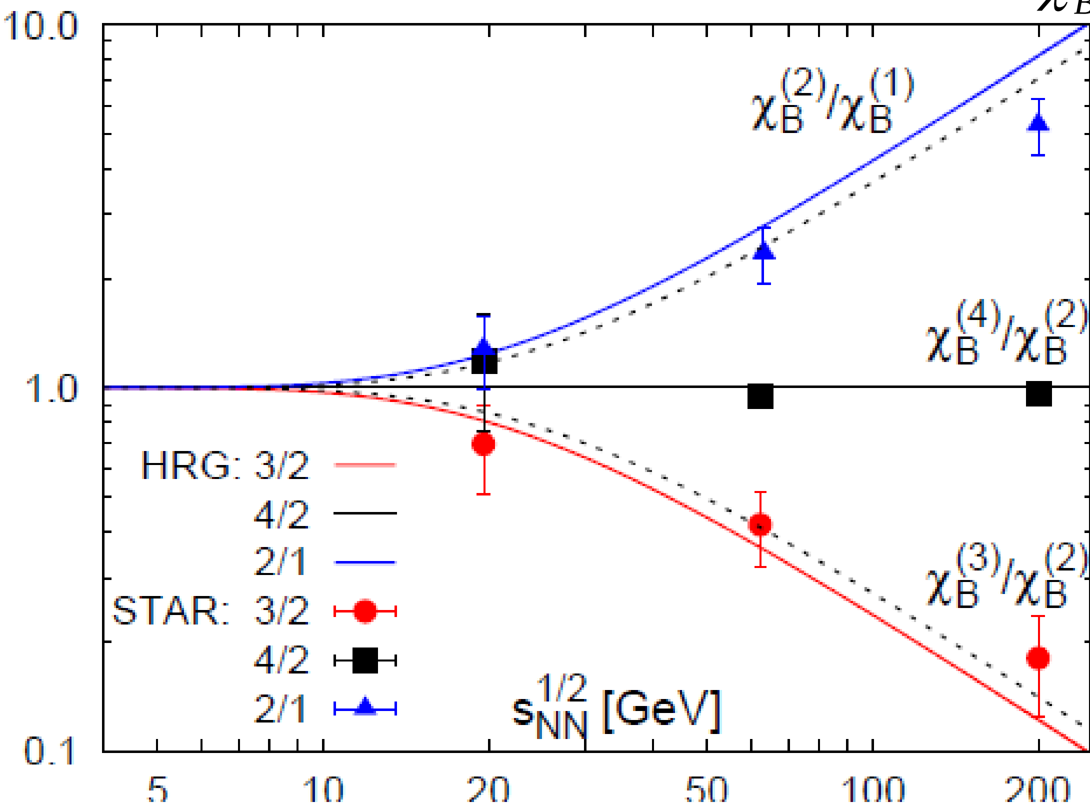
$$\kappa_B \sigma_B^2 = \frac{\chi_B^{(4)}}{\chi_B^{(2)}} \qquad \frac{\sigma_B^2}{M_B} = \frac{\chi_B^{(2)}}{\chi_B^{(1)}} \qquad S_B \sigma_B{}^2 = \frac{\chi_B^{(3)}}{\chi_B^{(2)}}$$

Compare these HRG model predictions with STAR data at RHIC:

Coparison of the Hadron Resonance Gas Model with STAR data

- Frithjof Karsch & K.R.

$$\frac{\chi_B^{(2)}}{\chi_B^{(1)}} \approx \coth(\mu_B / T)$$



$$\frac{\chi_B^{(4)}}{\chi_B^{(2)}} = 1$$

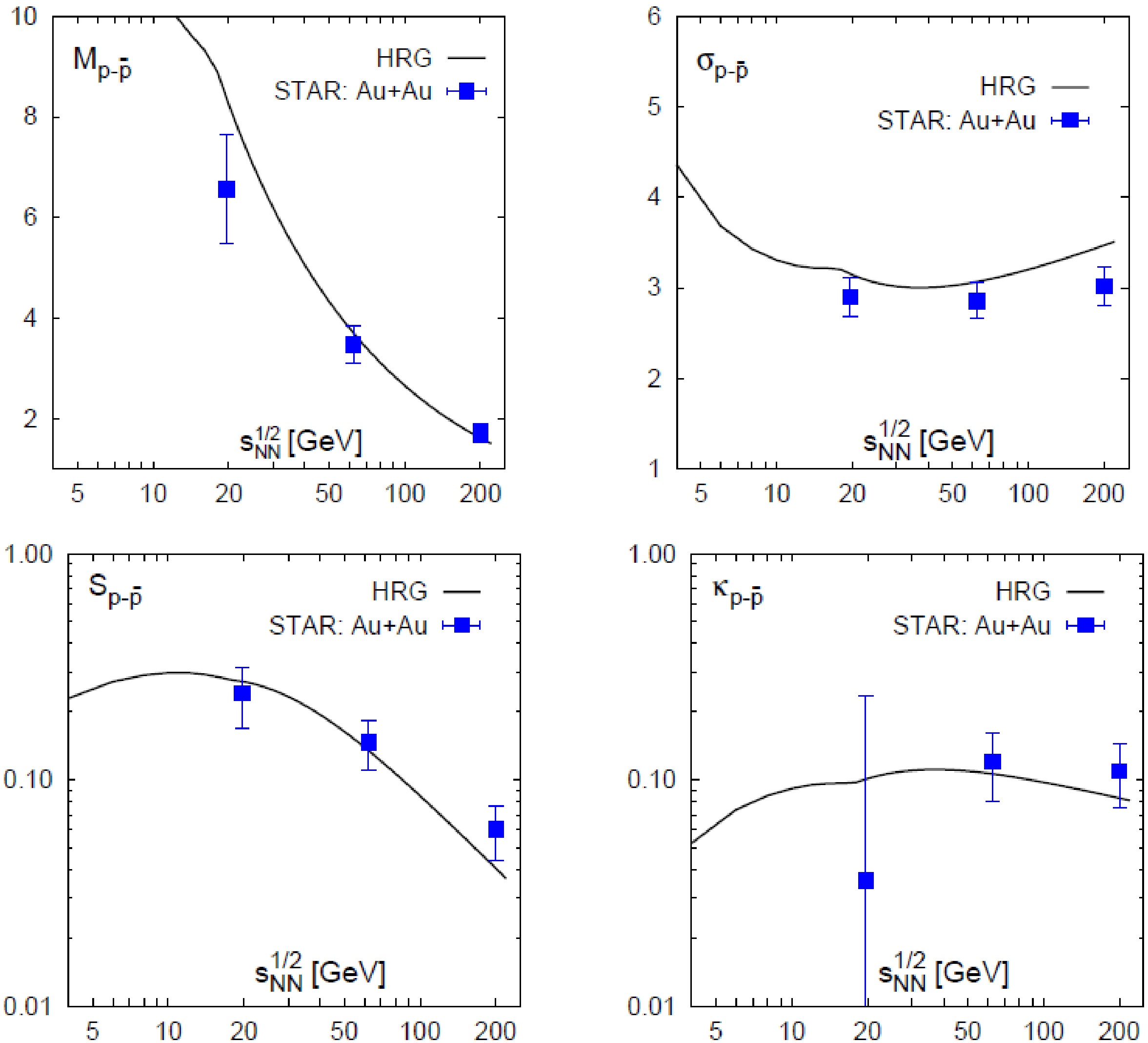
$$\frac{\chi_B^{(3)}}{\chi_B^{(2)}} \approx \tanh(\mu_B / T)$$

- RHIC data follow generic properties expected within HRG model for different ratios of the first four moments of baryon number fluctuations

deviations between HRG model and data for the variance ($\chi_B^{(2)}$)?

Error Estimation for Moments Analysis: Xiaofeng Luo arXiv:1109.0593v

Mean, variance, skewness and kurtosis obtained by STAR and rescaled HRG



- STAR Au-Au $\sqrt{s} = 200$

  $M_{p-\bar{p}} \approx 8.5$

- STAR Au-Au $\sqrt{s} = 200$

  $M_{p-\bar{p}} \approx 1.8$ these data, due to transverse momentum cut.

Account effectively for the above in the HRG model by rescaling the volume parameter by the factor 1.8/8.5

Moments obtained from probability distributions

- Moments obtained form the probability distribution

$$< N^k >= \sum_N N^k P(N)$$

- Probability quantified by all moments

$$P(N) = \frac{1}{2\pi} \int_0^{2\pi} dy \exp[iyN - \chi(iy)]$$

Moments generating function: $\chi(y) = \beta V[p(T, y+\mu) - p(T,\mu)] = \sum_k \chi_k y^k$

- In statistical physics

$$P(N) = \frac{Z_C(N)}{Z_{GC}} e^{\frac{\mu N}{T}}$$

$$e^{i\phi S} H e^{i\phi S} = H \leftrightarrow \quad [S, H] = 0$$

conservation on the average

exact conservation

$$Z^{GC}(T, \mu_S, V) = Tr\ [e^{-\beta(H - \mu_S S)}]$$

$$Z_S^C(T, V) = Tr_S[e^{-\beta H}]$$

$$Z^{GC} = \sum_{S=-\infty}^{S=+\infty} e^{S\mu_S/T} Z_S^C$$

$$Z_S(T,V) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi e^{-iS\varphi} Z^{GC}(T, \frac{\mu_S}{T} \to i\varphi)$$

$$P(S) = \left(\frac{\bar{S}_1}{\bar{S}_{\bar{1}}}\right)^{\frac{S}{2}} \exp\left[\sum_{n=1}^{3}(\bar{S}_n + \bar{S}_{\bar{n}})\right] \sum_{i=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\left(\frac{\bar{S}_3}{\bar{S}_{\bar{3}}}\right)^{k/2} I_k(2\sqrt{\bar{S}_3\bar{S}_{\bar{3}}}) \left(\frac{\bar{S}_2}{\bar{S}_{\bar{2}}}\right)^{i/2} I_i(2\sqrt{\bar{S}_2\bar{S}_{\bar{2}}}) \left(\frac{\bar{S}_1}{\bar{S}_{\bar{1}}}\right)^{-i-3k/2} I_{2i+3k-S}(2\sqrt{\bar{S}_1\bar{S}_{\bar{1}}})$$

- Probability quantified by $S_n, \overline{S}_n$: mean numbers of charged 1,2 and 3 particles & their antiparticles

P(N) in the Hadron Resonance Gas for net-proton number

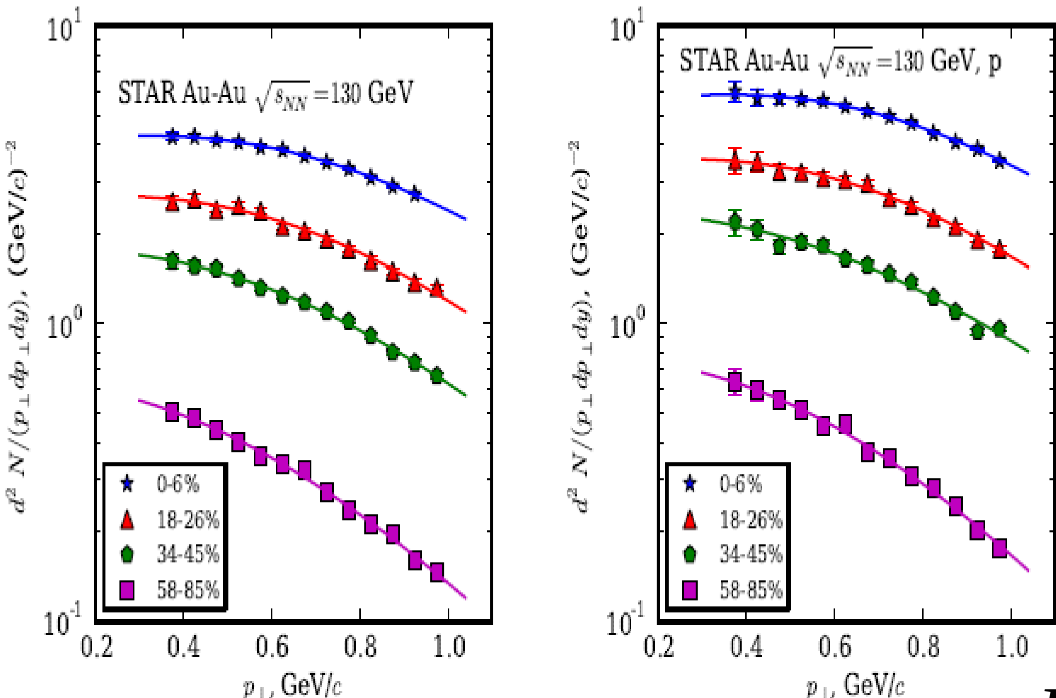
The probability distribution for net baryon number N is governed in HRG by the Skellam distribution

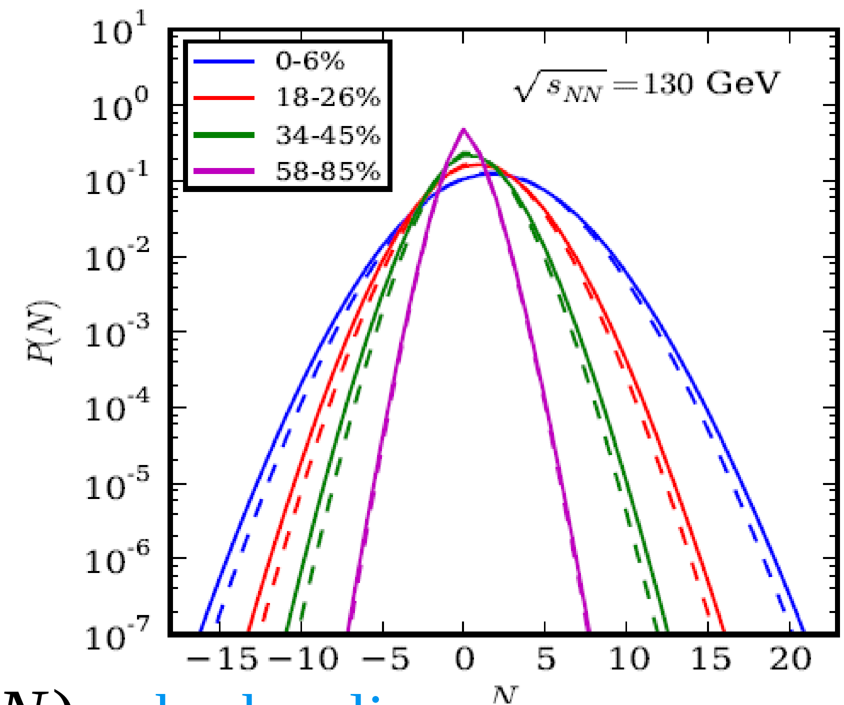
$$P(N) = \left(\frac{b}{\bar{b}}\right)^{N/2} I_N(2\sqrt{b\bar{b}}\,)\,\exp[-(b+\bar{b})]$$

The probability distribution for net proton number N is entirely given in terms of (measurable) mean number of protons b and anti-protons $\bar{b}$

Probability distribution obtained form measured particle spectra: $P(N) = f(< p >_{\Delta p_t}, < \bar{p} >_{\Delta p_t})$

STAR data



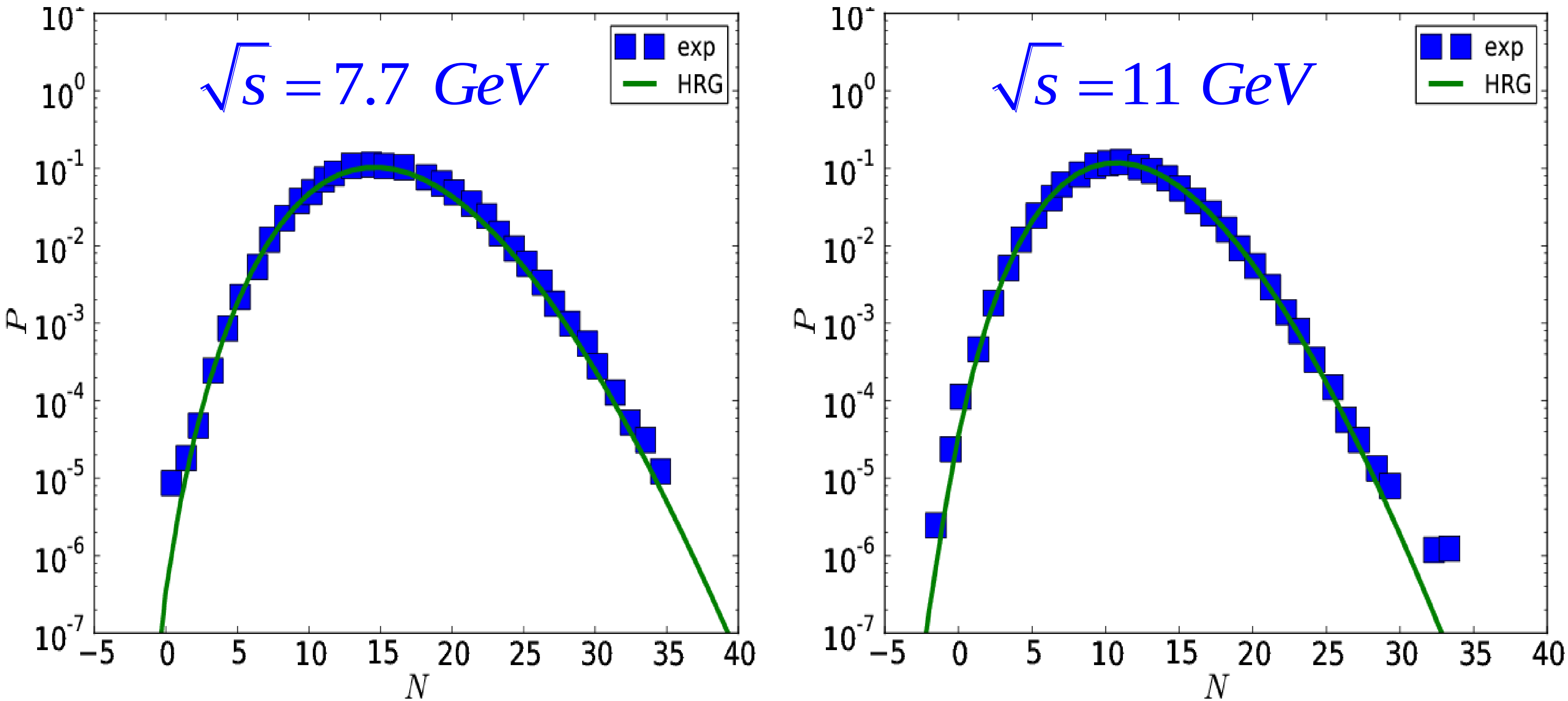


$P(N) -$ broken lines

- Mean proton and anti-proton calculated in the HRG with (T, μ_B) taken at freezeut curve and Volume obtained form the net-mean proton number

Comparing HRG Model with Preliminary STAR data: efficiency uncorrected

Data presented at QM'11, H.G. Ritter, GSI Colloquium

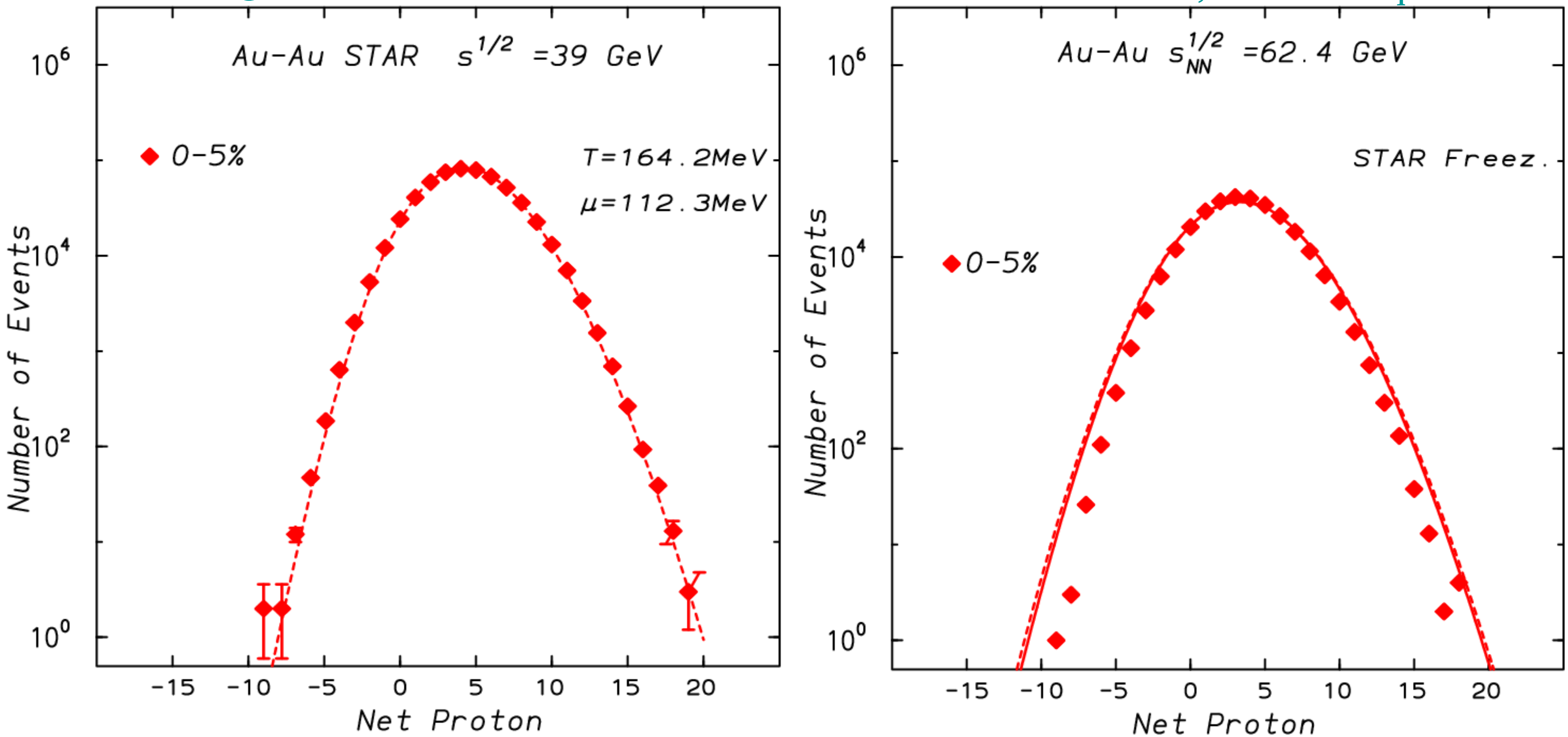


Data consistent with Skellam distribution: No sign for criticality

HRG energy dependence versus STAR data

arXiv:1106.2926
Xiaofeng Luo for the STAR Collaboration

Data presented at QM'11,
H.G. Ritter, GSI Colloquium



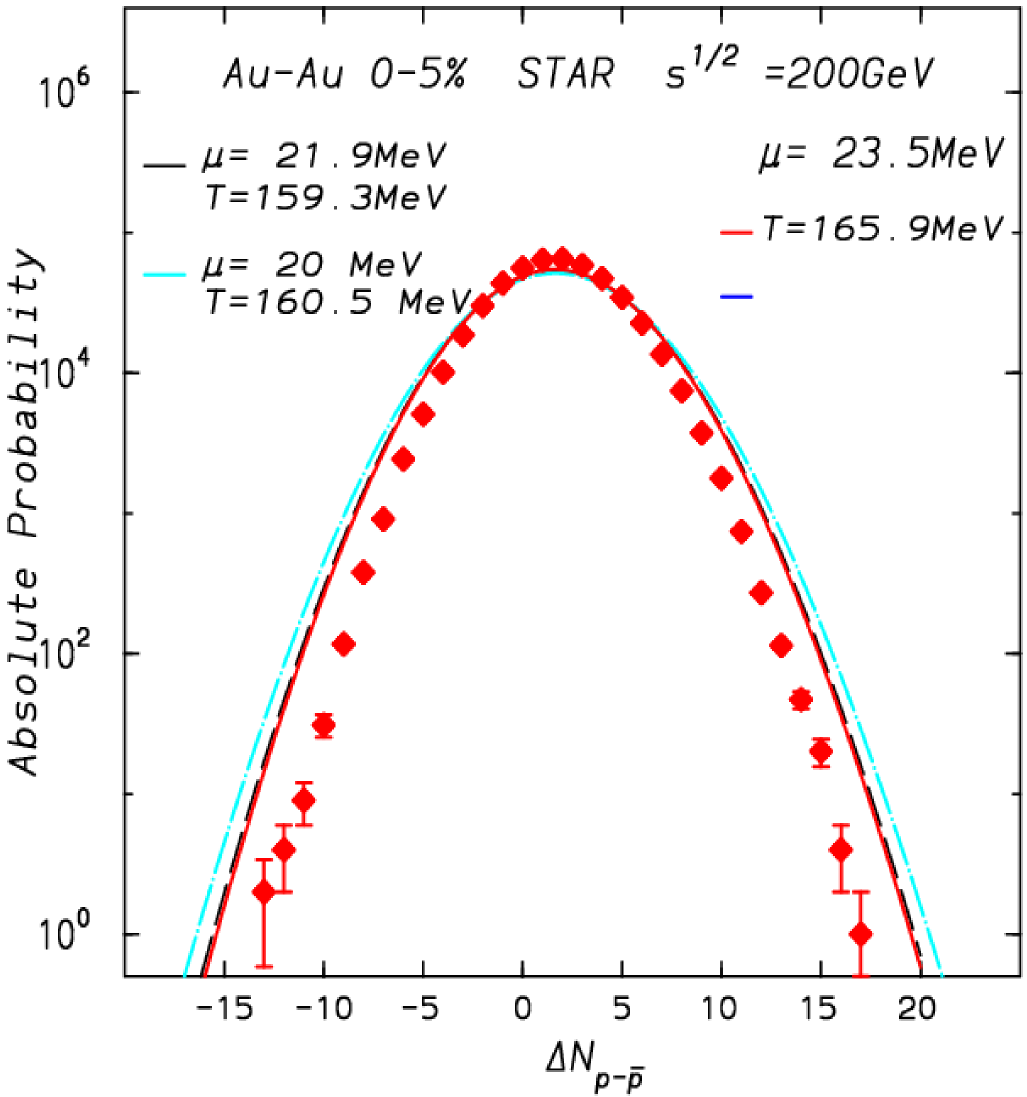
Data consitent with Skellam distribution

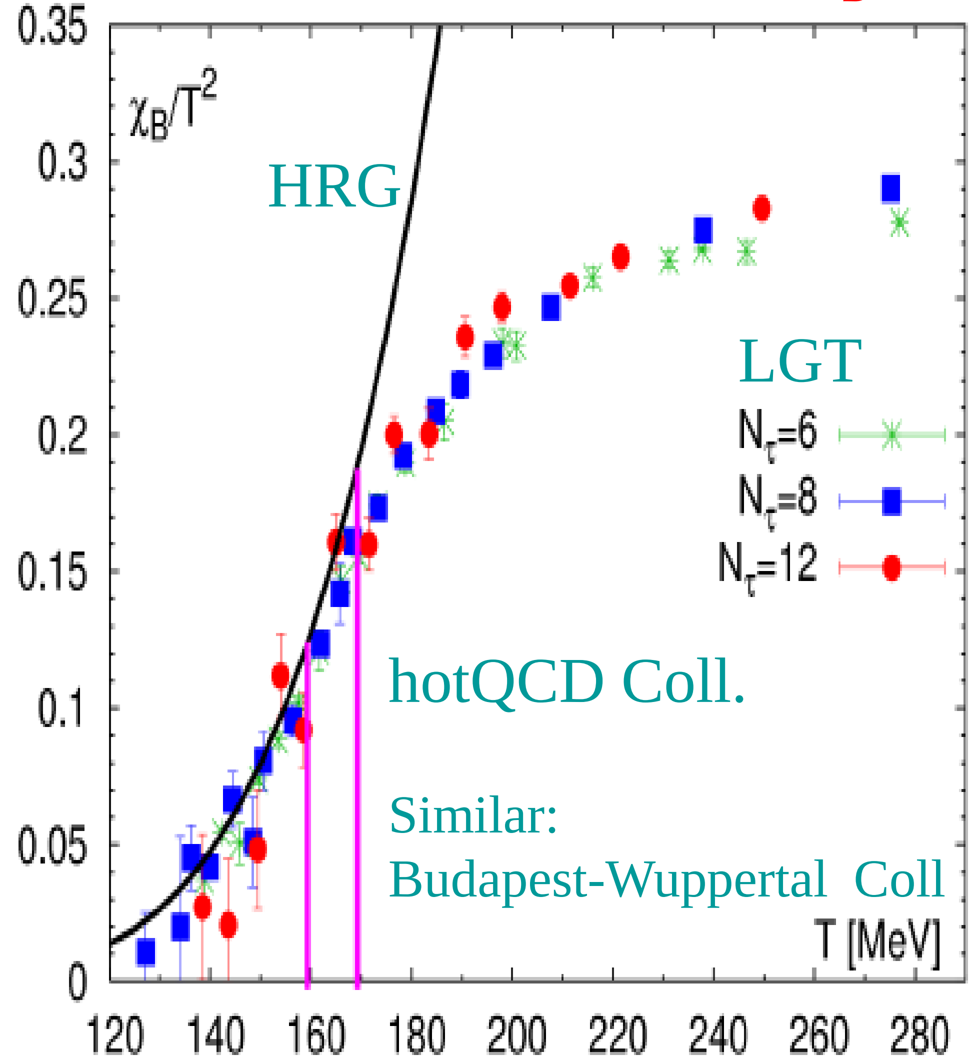
HRG shows broader distribution

Observed schrinking of the probability distribution already expected due to deconfinement properties of QCD

In the first approximation the χ_B quantifies the width of P(N)

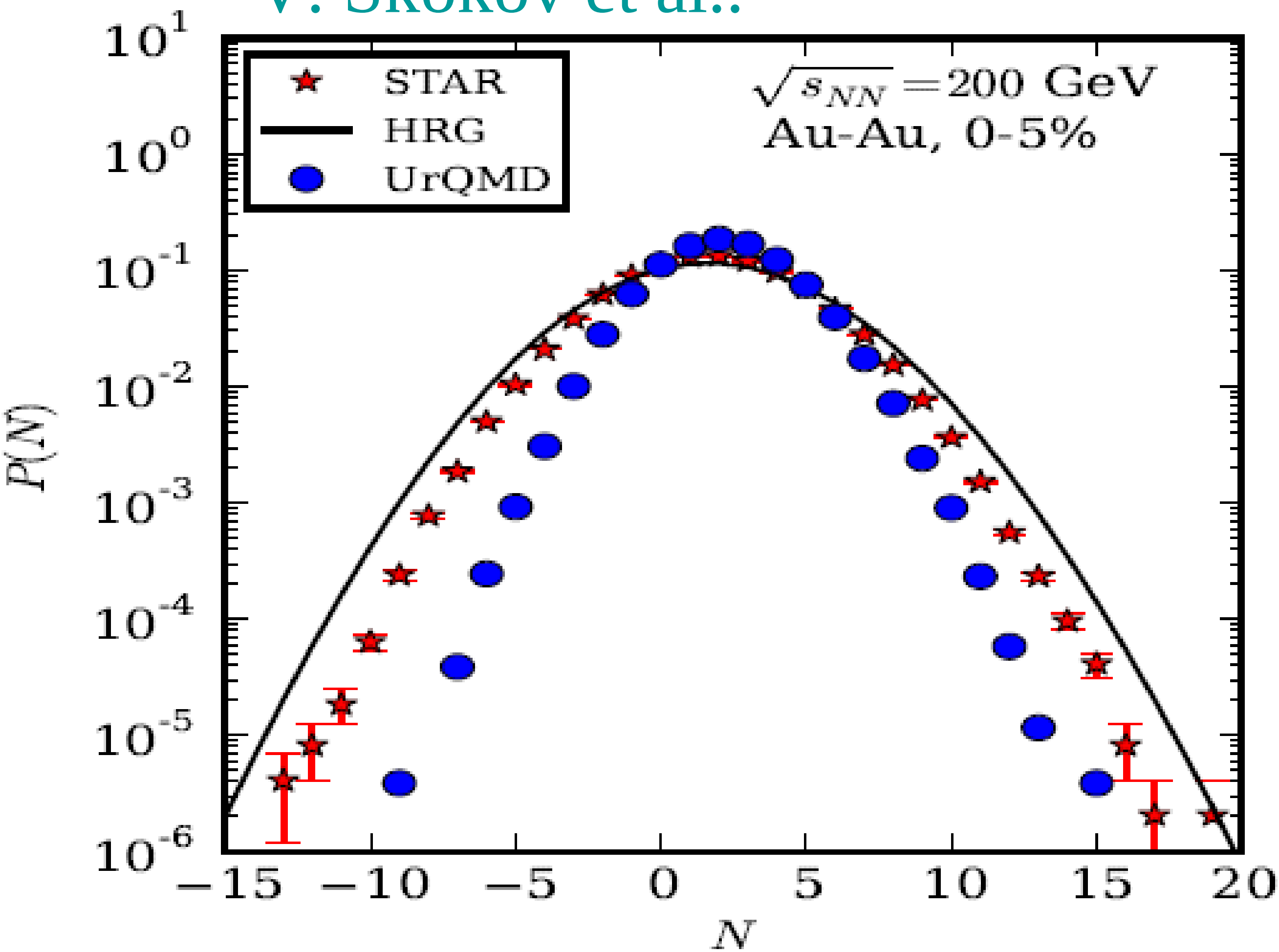
$$P(N) \approx \exp(-N^2 / 2VT^3 \chi_B)$$





Data versus UrQMD

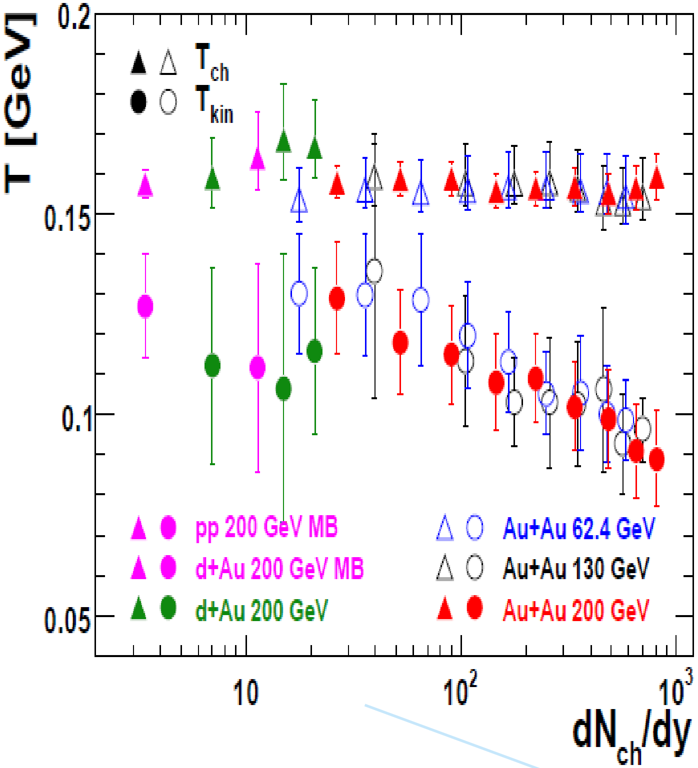
V. Skokov et al..



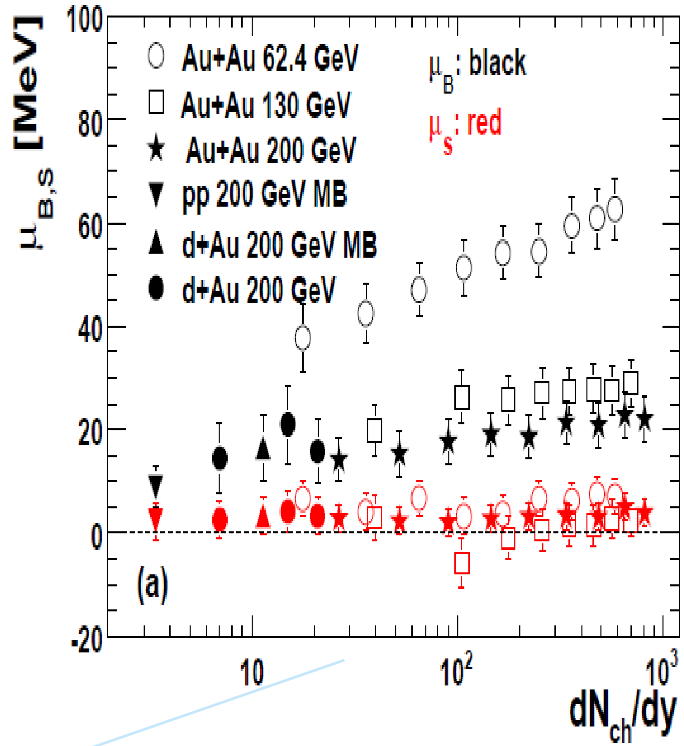
- UrQMD provide much too narower probability distribution of net proton number

Centrality dependence of O(4) criticality?

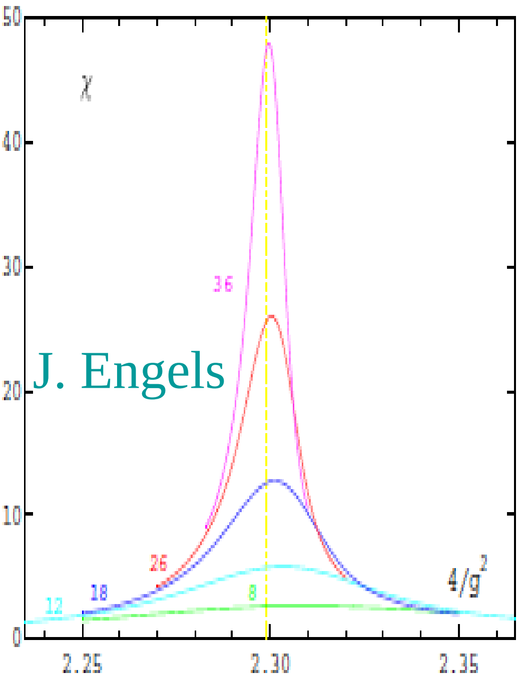
Weak dependence of chemcial Freezeout on centrality



Decrease of chemical potentil



In the chiral limit decrease of volume reduce criticlity



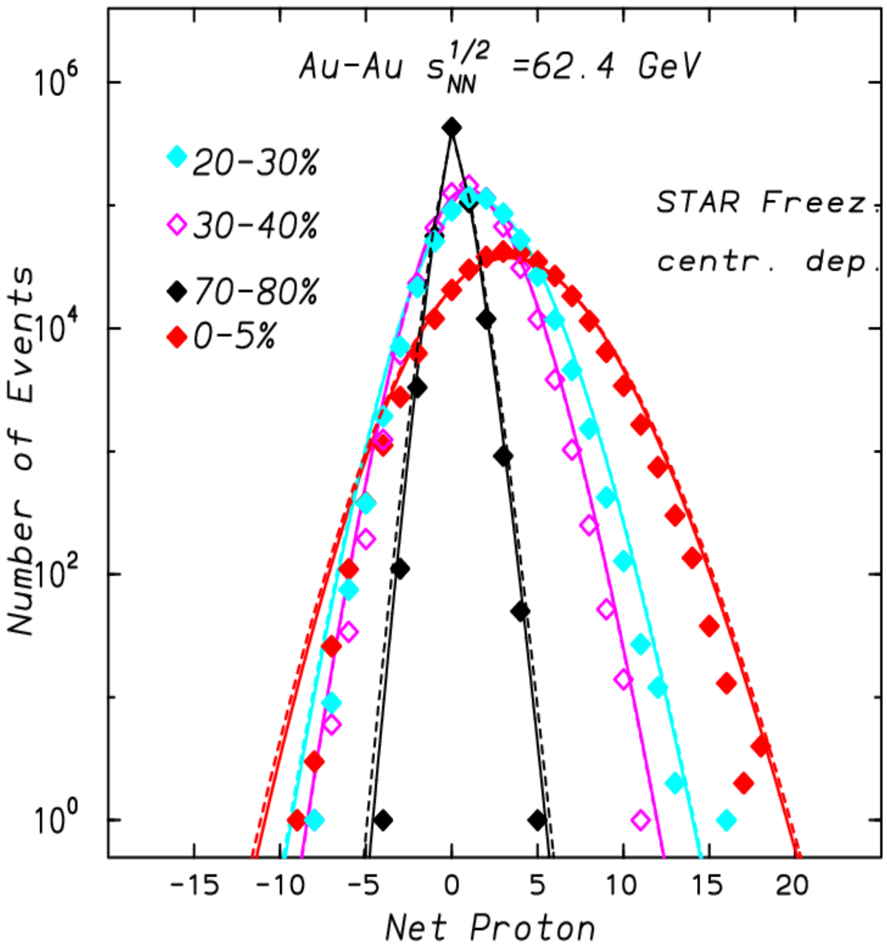
J. Engels

Decrease of centrality shifts the chemical line out of chiral O(4) cross over

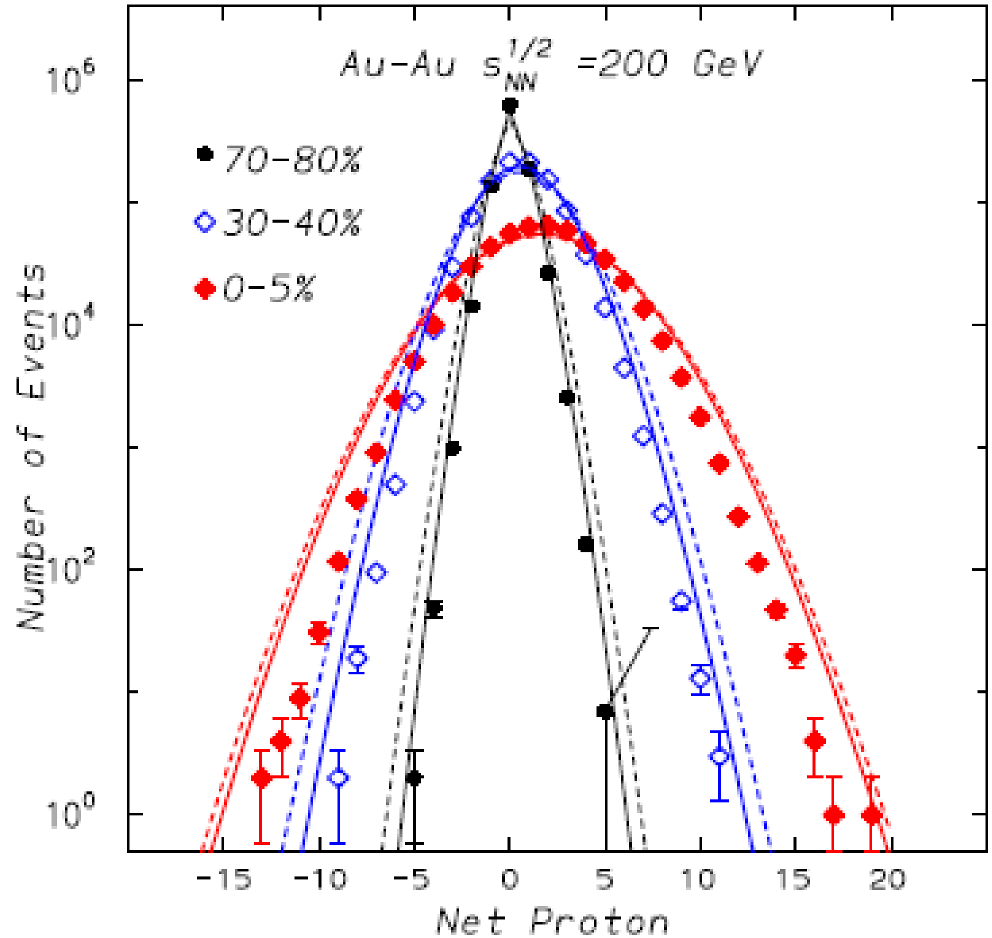
Influence of volume on criticality requires detaile knowlege of Finite Size Scaling Function for free energy in O(4) universality class

Centrality dependence at $\sqrt{s_{NN}} = 62$ and 200 GeV

Preliminary STAR data
H.G. Ritter, GSI Colloquium



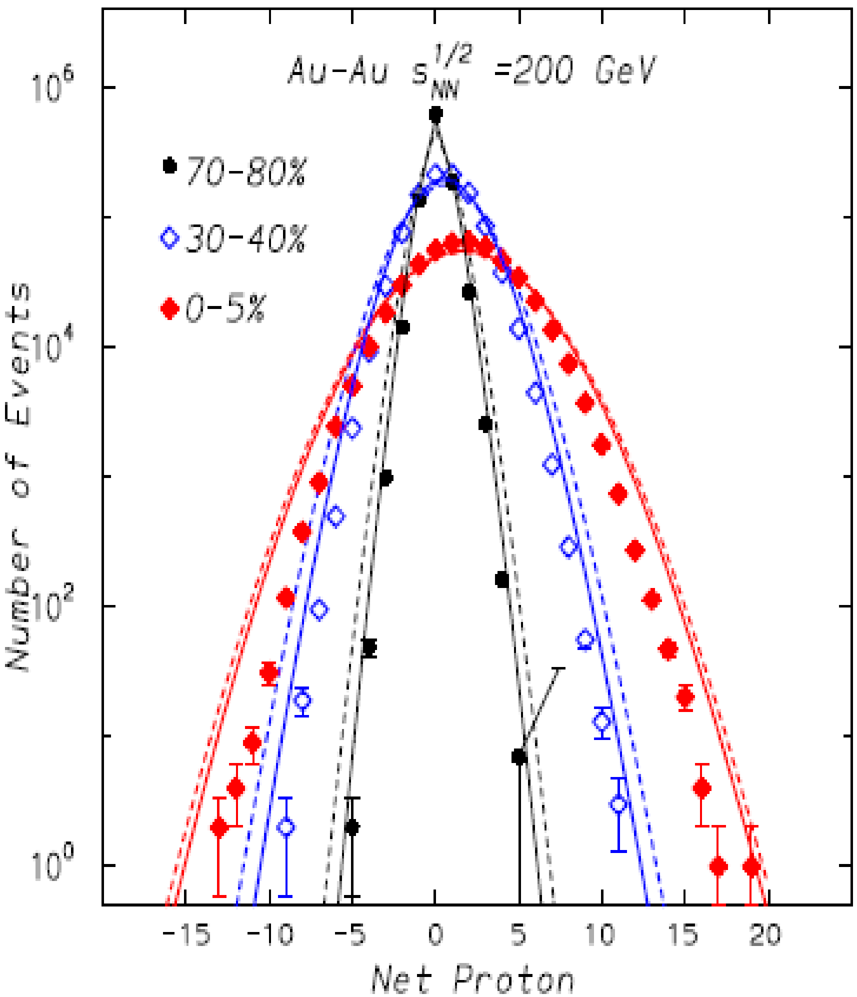
Published STAR data-efficiency uncorected

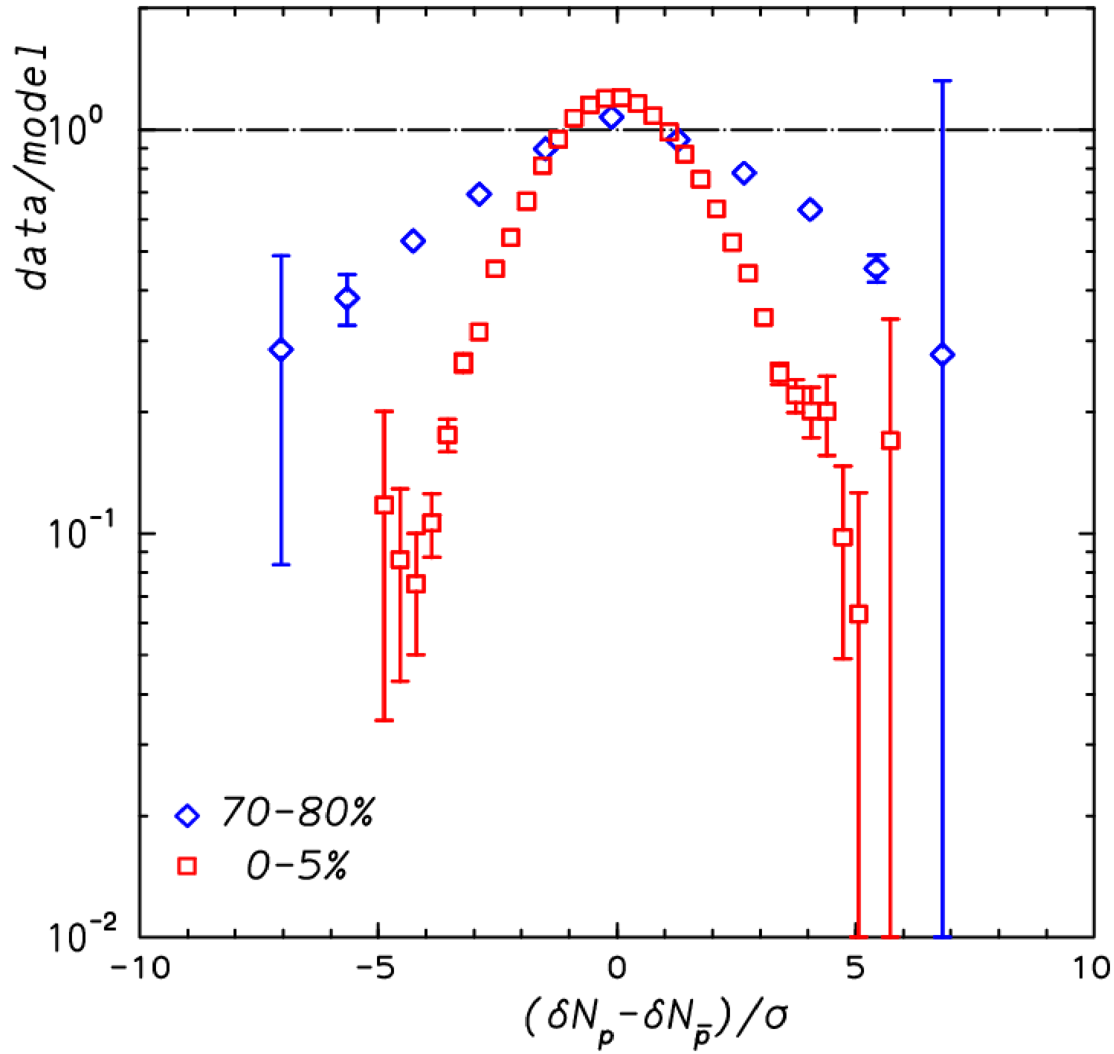


HRG shows increasing broadening of distributions with centrality

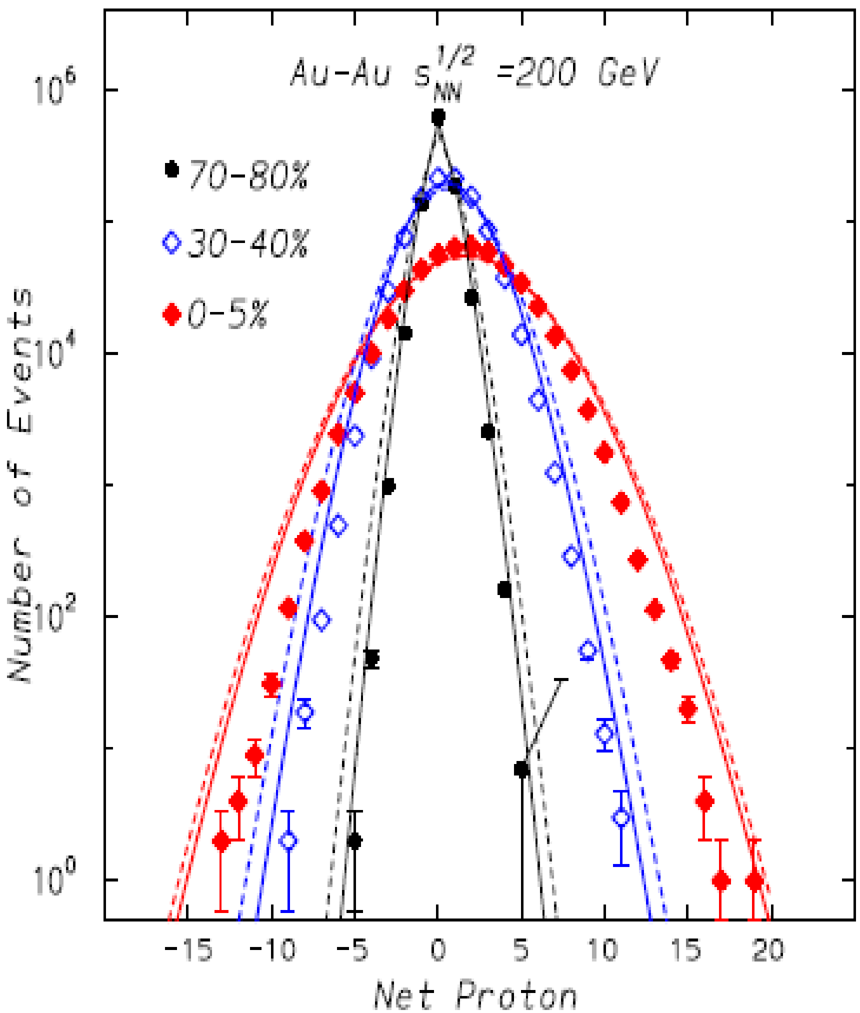
Centrality dependence of HRG model deviations from data:

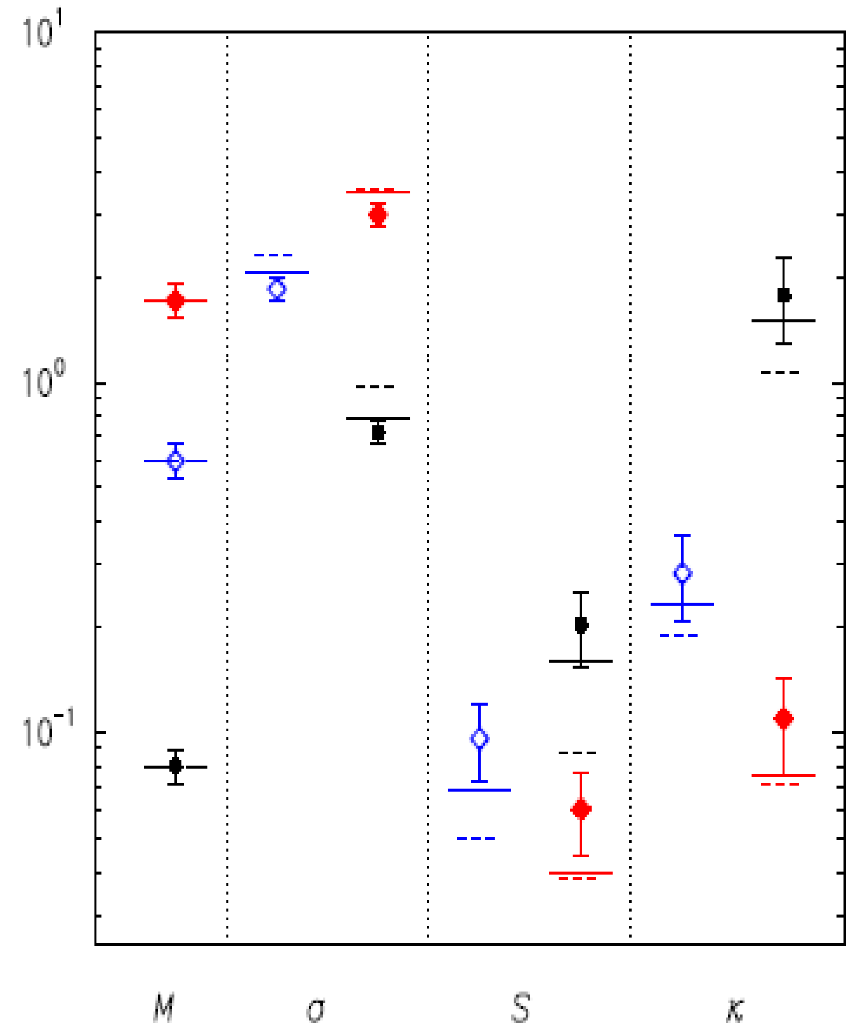
Thanks to Xu Nu



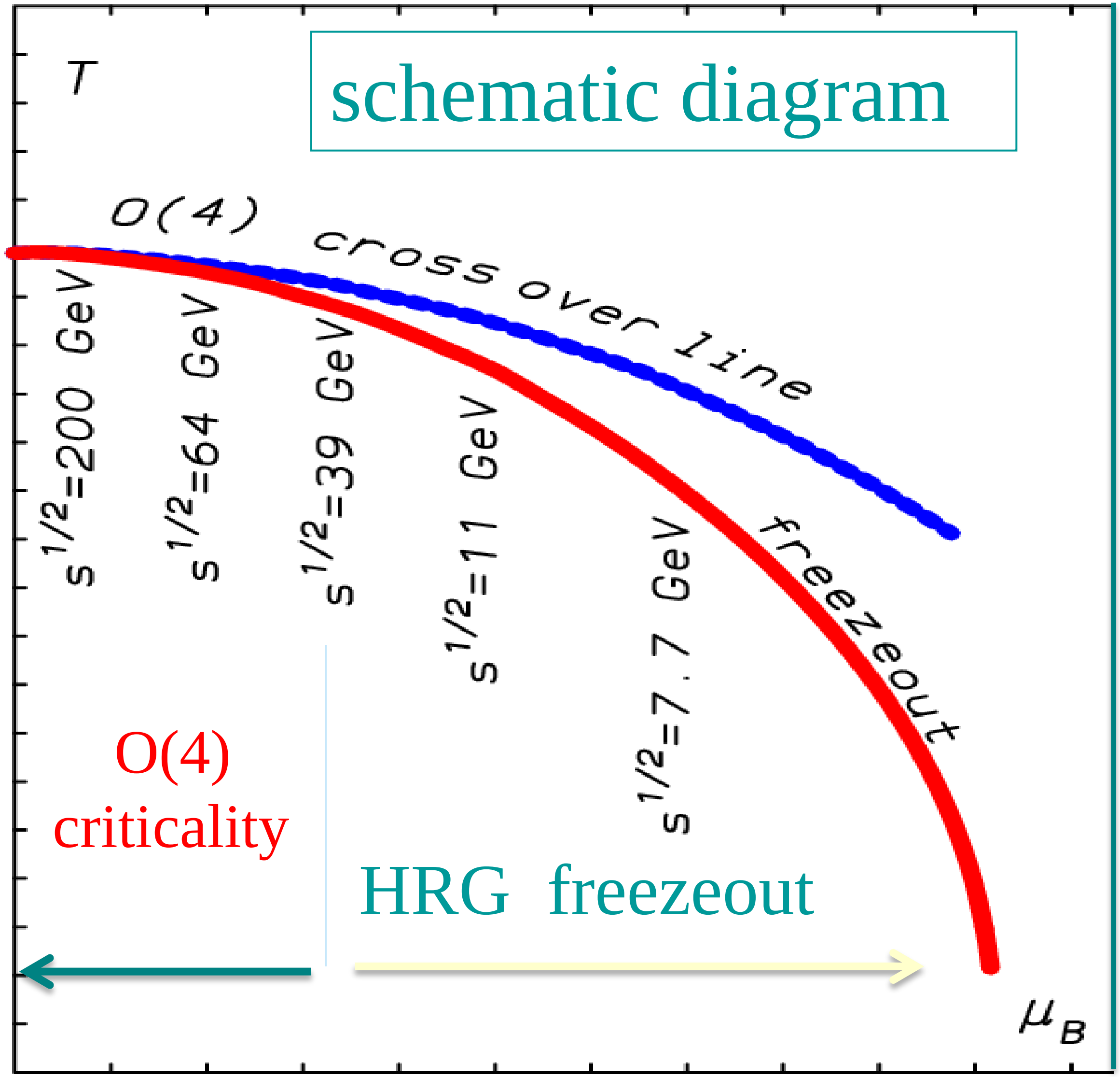


Criticality increases with centrality: seen in the probability distributions and moments compared to data





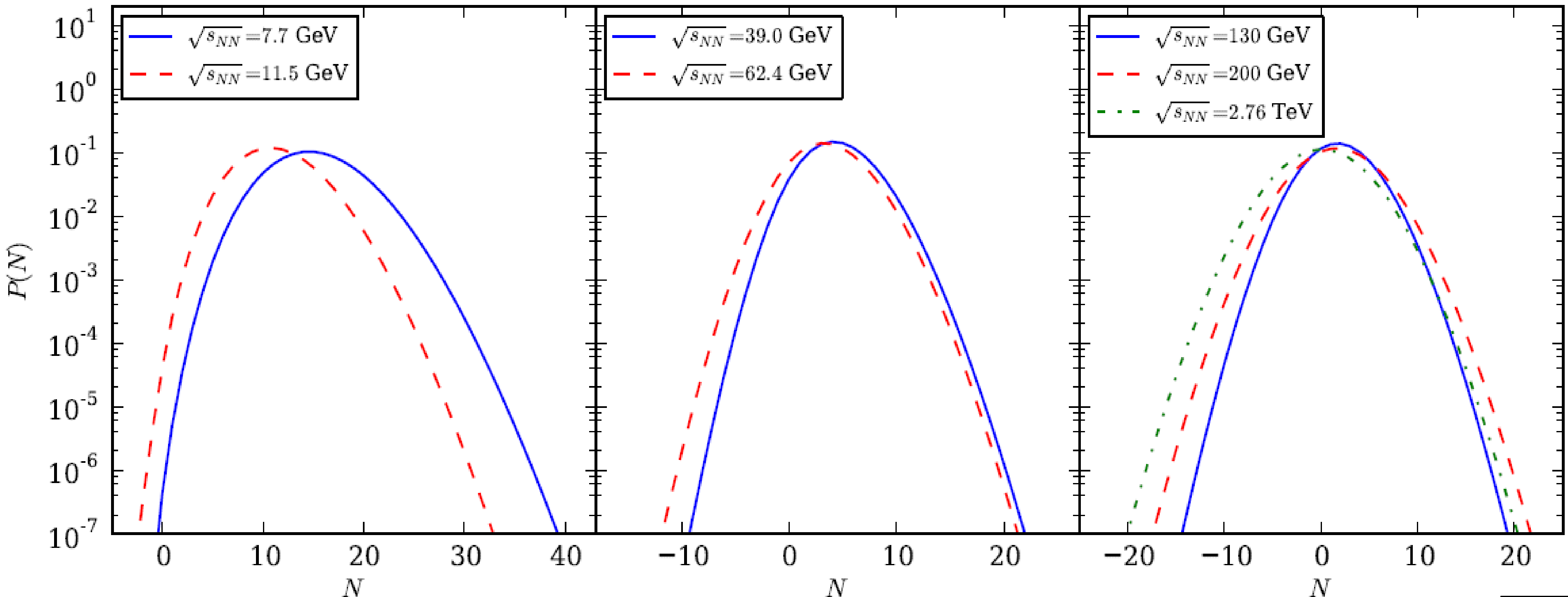
O(4) criticality ⟺ Hadronic freezout



Based on the relation between HRG model and presently aveliable data we could conclude that:

- The separation between chemical freezeout line and the O(4) chiral line might appear in HIC at collision energies larger than 39 GeV??

Probability distributions and their energy dependence for $0.4\ GeV < p_t < 0.8\ GeV$



- The maxima of $P(N)$ have very similar values for all $\sqrt{s_{NN}}$

$$P_{max} \approx \frac{1}{\sqrt{2\pi(\bar{N}_p + \bar{N}_{\bar{p}})}}$$

thus $N_p + N_{\bar{p}} \approx$ const. for all

$$7\ GeV < \sqrt{s_{NN}} < 2.8\ TeV$$

Conclusions:

- Probability distributions and higher order cumulants are excellent probes of O(4) criticality in HIC
- Hadron resonance gas provides reference for O(4) critical behavior in HIC and LGT results
- In the HRG the P(N) is uniqually determined by

  the measure multiplicities of charged particles. This avoids ambiguity in determining freezeout parameters and volume of the system
- Deviation of P(N) form HRG sets in at $\sqrt{s} > 39GeV$ and increases with centrality:

  This might indicated remnants of O(4) criticality

  in STAR data on net proton probability distribution

  in central Au-Au collisions at $\sqrt{s} > 39GeV$