Isospin Matter

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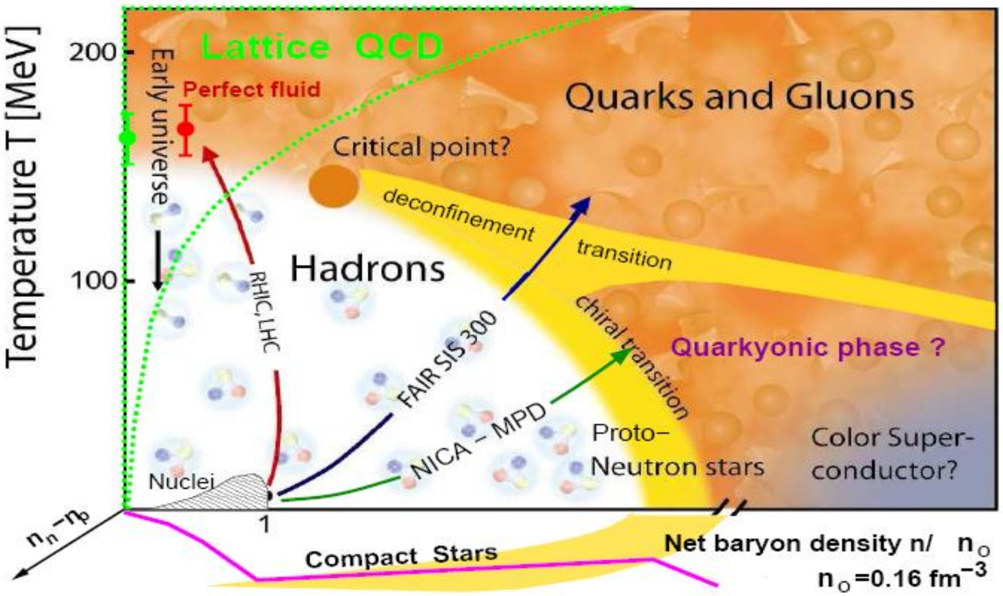
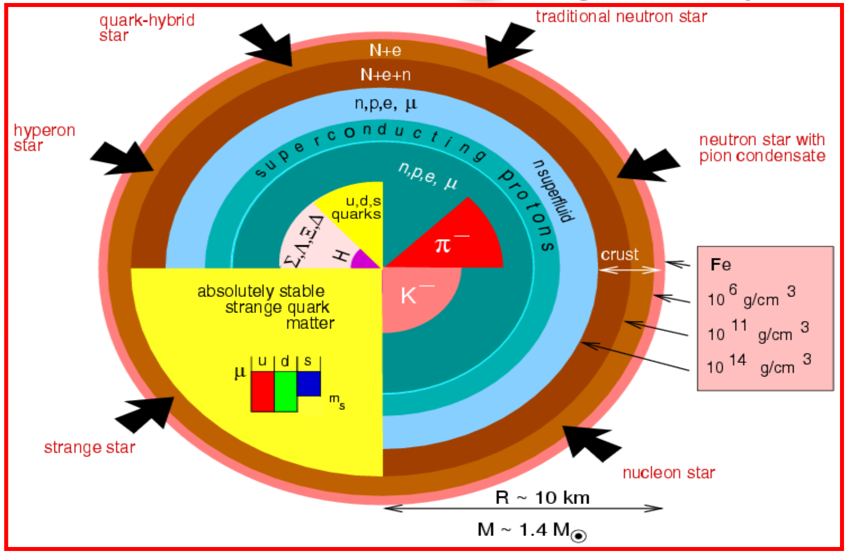
- Phase Diagram at finite μ_I
- BCS-BEC Crossover in pion superfluid

QCD Phase Diagram

Tsinghua University



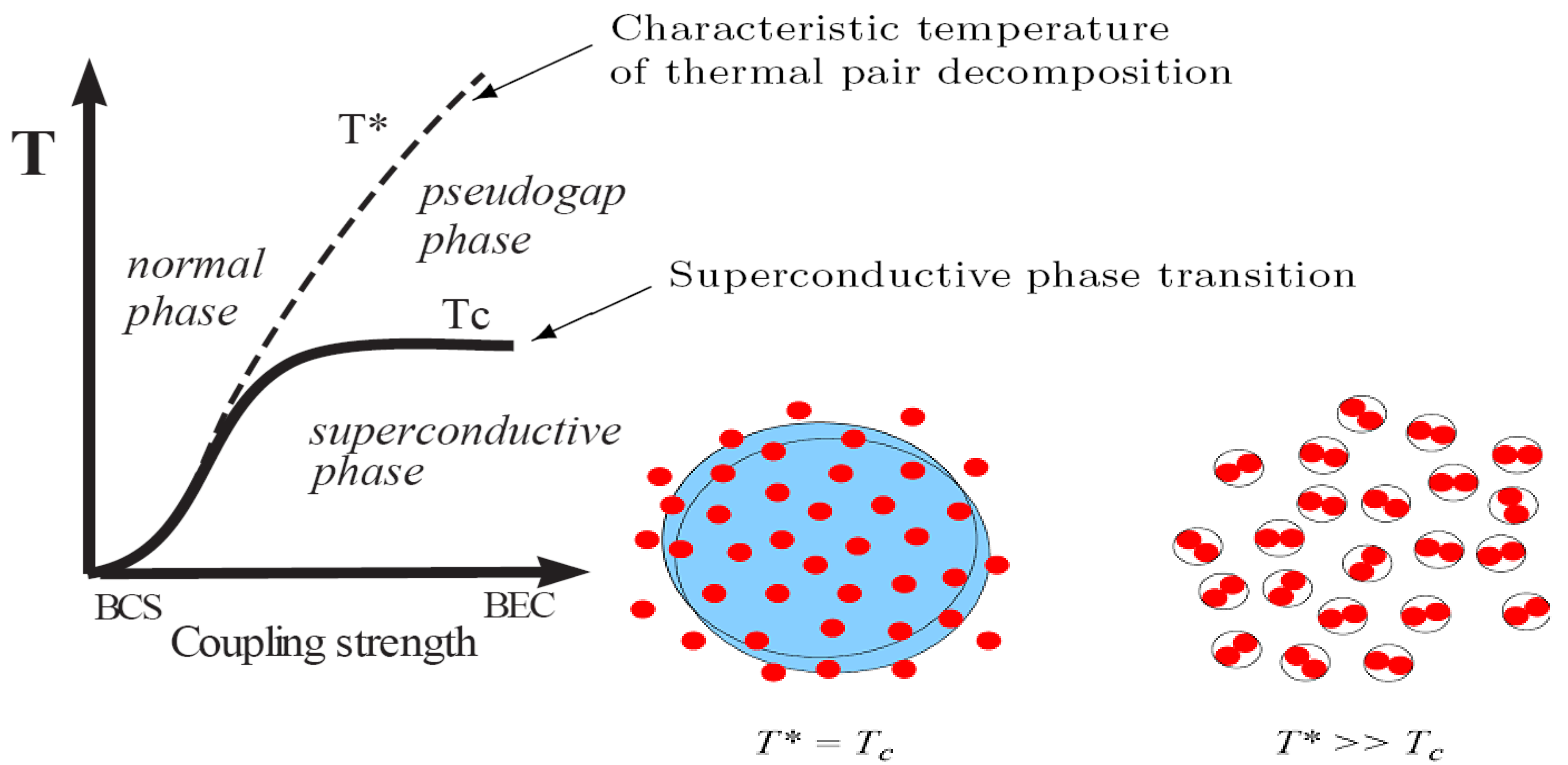


Questions: 1) What is the phase of quark matter at finite μ_I ?

2) Is μ_I effect similar to μ_B effect?

D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 86, 592(2001)
J. B. Kogut, D. K. Sinclair, Phys. Rev. D 66, 034505(2002)
K. Splittorff, D. T. Son, and M. A. Stephanov, Phys. Rev. D64, 016003 (2001)
M. Loewe and C. Villavicencio, Phys. Rev. D 67, 074034(2003)
Michael C. Birse, Thomas D. Cohen, and Judith A.McGovern, Phys. Lett. B 516, 27 (2001)
D. Toublan and J. B. Kogut, Phys. Lett. B 564, 212 (2003)
A.Barducci, R. Casalbuoni, G. Pettini, and L. Ravagli, Phys. Rev. D 69, 096004 (2004)
M. Frank, M. Buballa and M. Oertel, Phys. Lett. B 562, 221 (2003)
L. He and P. Zhuang, Phys. Lett. B 615, 93 (2005)

BCS and BEC Pairing



in BCS, T_c is determined by thermal excitation of fermions,
in BEC, T_c is controlled by thermal excitation of collective modes.

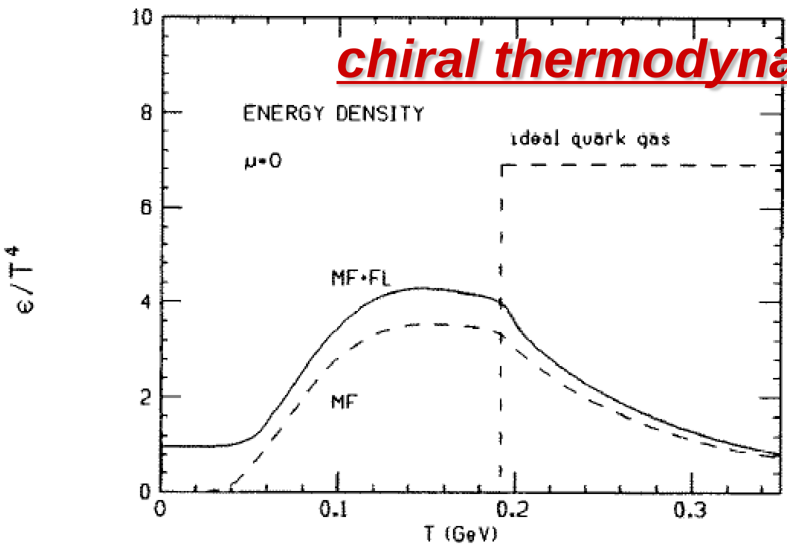
Is there a similar BCS-BEC structure for QCD condensed matter ?

NJL Model

- there is reliable lattice QCD result at finite T, but not yet precise lattice simulation at finite μ, we have to consider effective models.
- the physics is vacuum excitation at finite T but vacuum condensate at finite μ.
- the BCS inspired Nambu-Jona-Lasinio (NJL) model successfully describes the chiral condensate and color condensate.

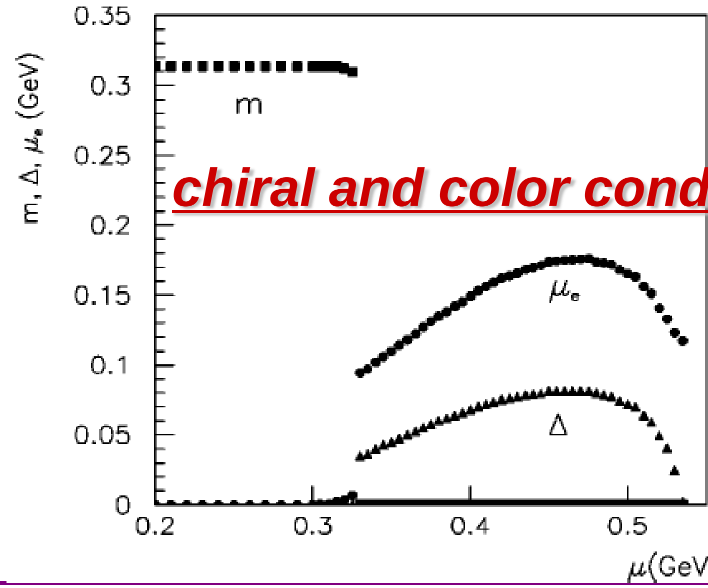
P. Zhuang et al. / Nuclear Physics A 576 (1994) 525–552

chiral thermodynamics



HUANG, ZHUANG, AND CHAO
PHYSICAL REVIEW D 67, 065015 (2003)

chiral and color condensate



NJL at Finite μ_I

He, Jin and PZ, PRD71, (2005)116001

NJL with isospin symmetry breaking

$$L_{NJL}=\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu}-m_0+\mu\gamma_0\right)\psi+G\left(\left(\bar{\psi}\psi\right)^2+\left(\bar{\psi}i\tau_i\gamma_5\psi\right)^2\right)$$

quark chemical potentials

$$\mu=\begin{pmatrix}\mu_u & 0\\ 0 & \mu_d\end{pmatrix}=\begin{pmatrix}\mu_B/3+\mu_I/2 & 0\\ 0 & \mu_B/3-\mu_I/2\end{pmatrix}$$

chiral and pion condensates with finite pair momentum

$$\sigma=\left\langle\bar{\psi}\psi\right\rangle=\sigma_u+\sigma_d,\quad \sigma_u=\left\langle\bar{u}u\right\rangle,\quad \sigma_d=\left\langle\bar{d}d\right\rangle$$

$$\pi_+=\sqrt{2}\left\langle\bar{u}i\gamma_5 d\right\rangle=\frac{\pi}{\sqrt{2}}e^{2i\vec{q}\cdot\vec{x}}\ \ (for\ \mu_I>0),\qquad \pi_-=\sqrt{2}\left\langle\bar{d}i\gamma_5 u\right\rangle=\frac{\pi}{\sqrt{2}}e^{-2i\vec{q}\cdot\vec{x}}\ \ (for\ \mu_I<0)$$

quark propagator in MF

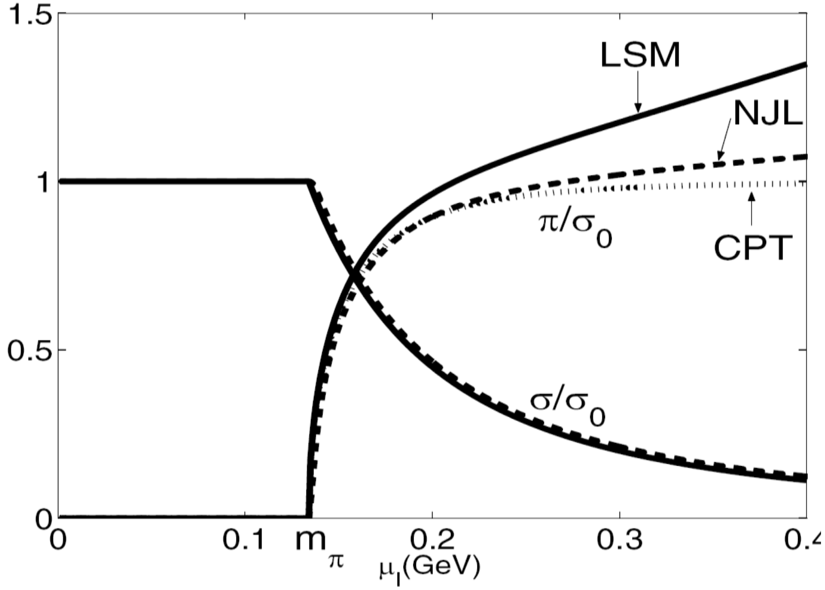
$$S^{-1}(p,\vec{q})=\begin{pmatrix}\gamma^{\mu}p_{\mu}-\vec{\gamma}\cdot\vec{q}+\mu_u\gamma_0-m & 2iG\pi\gamma_5\\ 2iG\pi\gamma_5 & \gamma^{\mu}p_{\mu}+\vec{\gamma}\cdot\vec{q}+\mu_d\gamma_0-m\end{pmatrix}\qquad m=m_0-2G\sigma$$

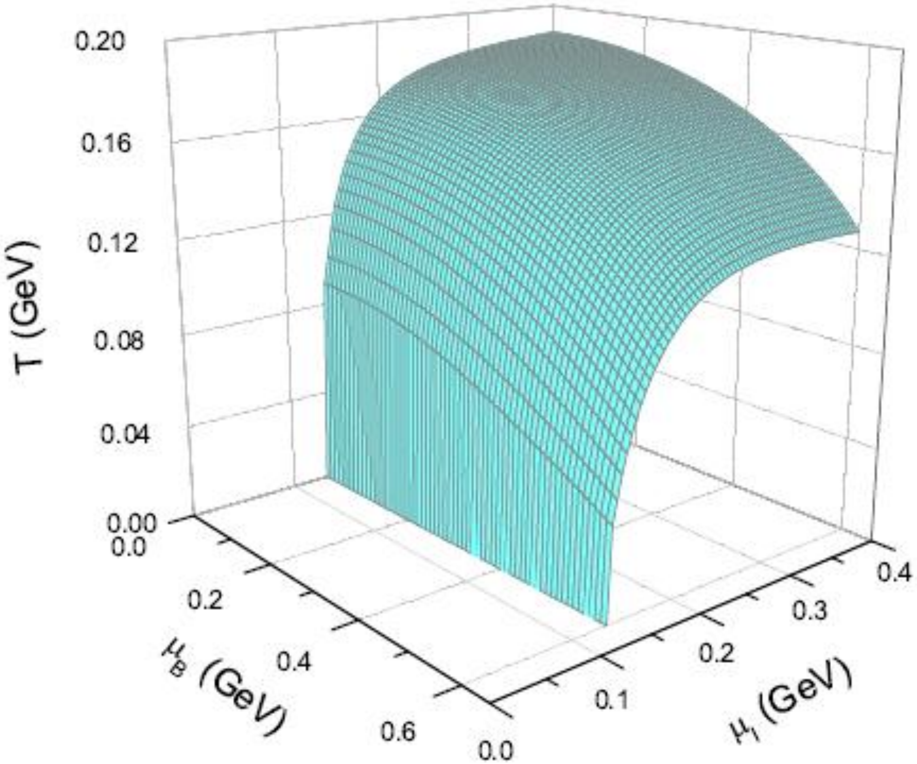
gap equations:

$$\Omega=G(\sigma^2+\pi^2)-\frac{T}{V}\mathrm{Tr\,Ln}\,S^{-1}$$

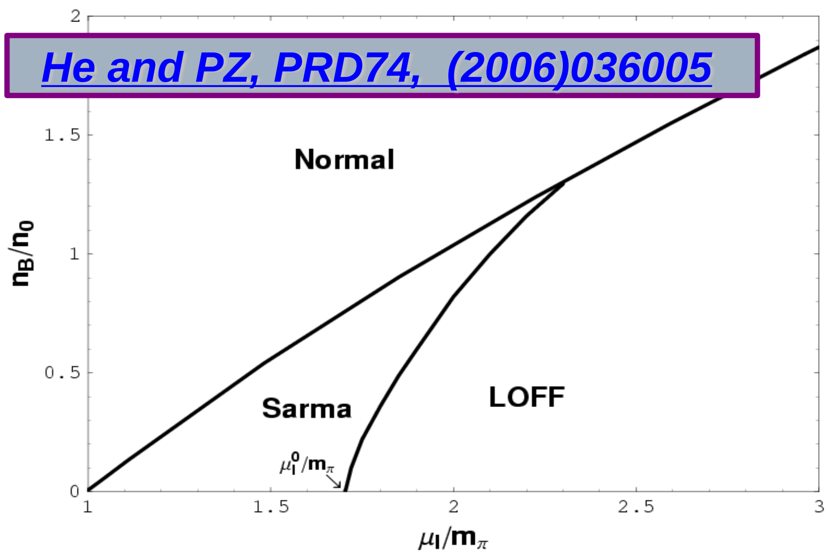
$$\frac{\partial\Omega}{\partial\sigma_u}=0,\ \frac{\partial^2\Omega}{\partial\sigma_u^2}\ge 0,\qquad \frac{\partial\Omega}{\partial\sigma_d}=0,\ \frac{\partial^2\Omega}{\partial\sigma_d^2}\ge 0,\qquad \frac{\partial\Omega}{\partial\pi}=0,\ \frac{\partial^2\Omega}{\partial\pi^2}\ge 0,\qquad \frac{\partial\Omega}{\partial q}=0,\ \frac{\partial^2\Omega}{\partial q^2}\ge 0$$

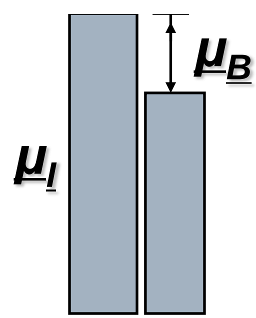
Phase Structure of Pion Superfluid





He and PZ, PRD74, (2006)036005





Jiang and PZ, arXiv:1104.0094

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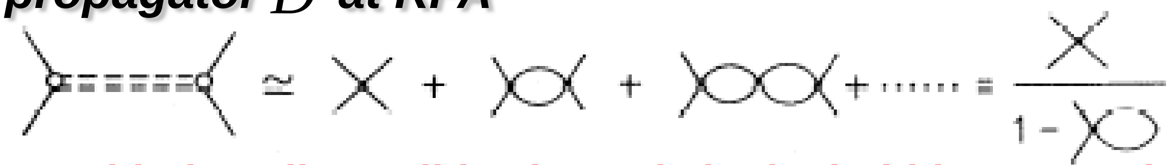
He, Jin, and PZ, PRD71, (2005)116001

meson propagator D at RPA



considering all possible channels in the bubble summation

meson polarization functions

$$\Pi_{mn}(k) = i\int \frac{d^4 p}{(2\pi)^4} \mathrm{Tr}\left(\Gamma_m^* S(p+k)\Gamma_n S(p)\right) \qquad \Gamma_m = \begin{cases} 1, & m=\sigma \\ i\tau_+\gamma_5, & m=\pi_+ \\ i\tau_-\gamma_5, & m=\pi_- \\ i\tau_3\gamma_5, & m=\pi_0 \end{cases}$$

mixing among normal σ, π_+, π_- in pion superfluid phase

pole of the propagator determines meson masses M_m

$$\det\begin{pmatrix} 1-2G\Pi_{\sigma\sigma}(k) & -2G\Pi_{\sigma\pi_+}(k) & -2G\Pi_{\sigma\pi_-}(k) & -2G\Pi_{\sigma\pi_0}(k) \\ -2G\Pi_{\pi_+\sigma}(k) & 1-2G\Pi_{\pi_+\pi_+}(k) & -2G\Pi_{\pi_+\pi_-}(k) & -2G\Pi_{\pi_+\pi_0}(k) \\ -2G\Pi_{\pi_-\sigma}(k) & -2G\Pi_{\pi_-\pi_+}(k) & 1-2G\Pi_{\pi_-\pi_-}(k) & -2G\Pi_{\pi_-\pi_0}(k) \\ -2G\Pi_{\pi_0\sigma}(k) & -2G\Pi_{\pi_0\pi_+}(k) & -2G\Pi_{\pi_0\pi_-}(k) & 1-2G\Pi_{\pi_0\pi_0}(k) \end{pmatrix}_{k_0=M_m,\vec{k}=0} = 0$$

the new eigen modes $\bar{\sigma}, \bar{\pi}_+, \bar{\pi}_-$ are linear combinations of σ, π_+, π_-

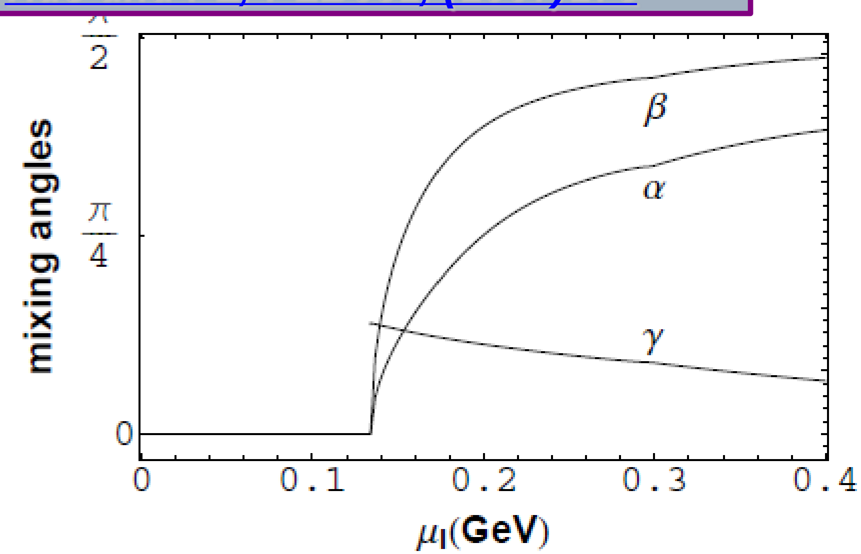
Meson Mixing and Goldstone Mode

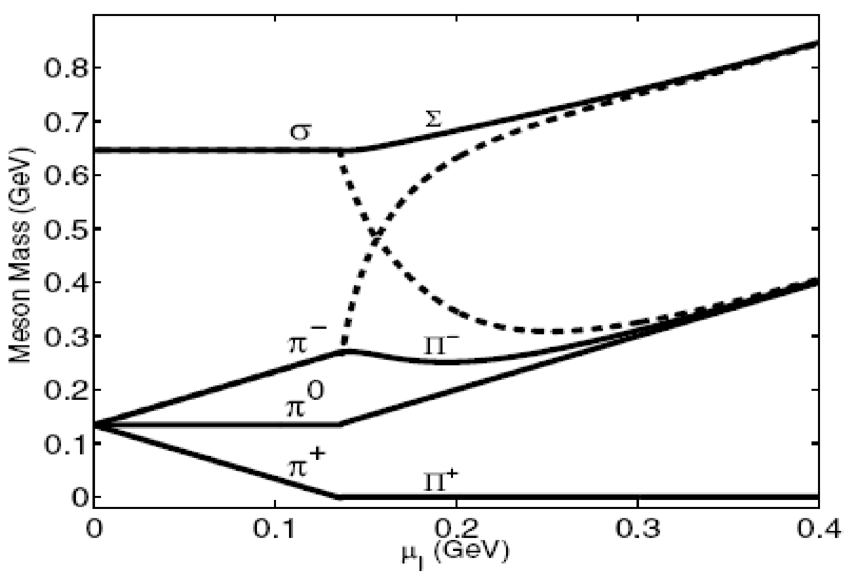
Hao and PZ, PLB652, (2007)275

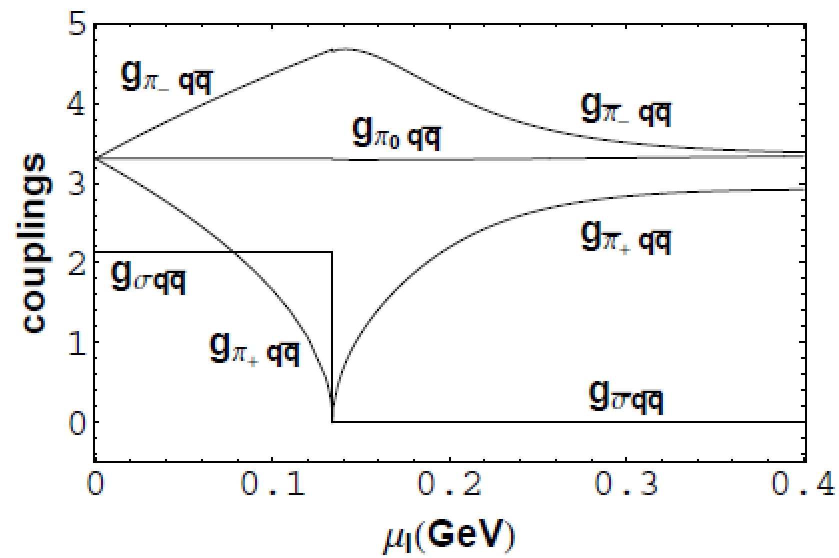
mixing angles α between σ and π_+ ,

β between σ and π_- ,

and γ between π_+ and π_-





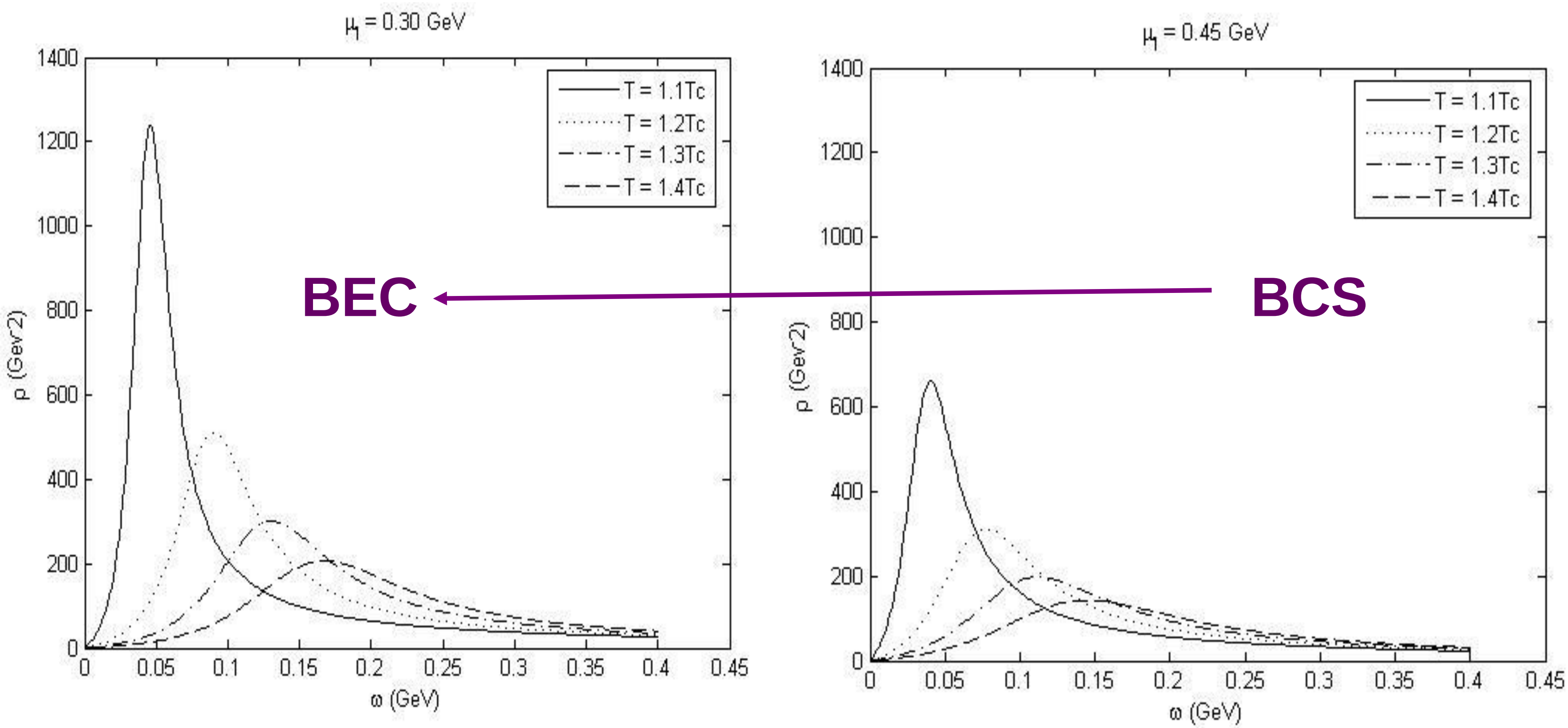


Meson Spectral Functions

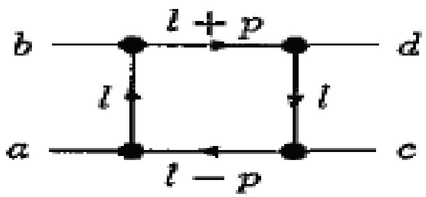
Sun, He and PZ, PRD75, (2007)096004

meson spectra function between the temperatures T_c and T^*

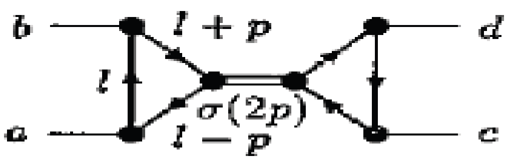
$$\rho(\omega,\vec{k}) = -2\,\mathrm{Im}\,D_R(\omega,\vec{k})$$



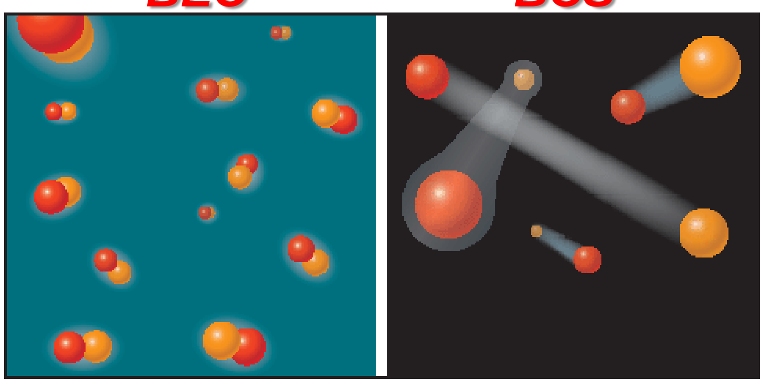
$\pi - \pi$ scattering and BCS-BEC Crossover

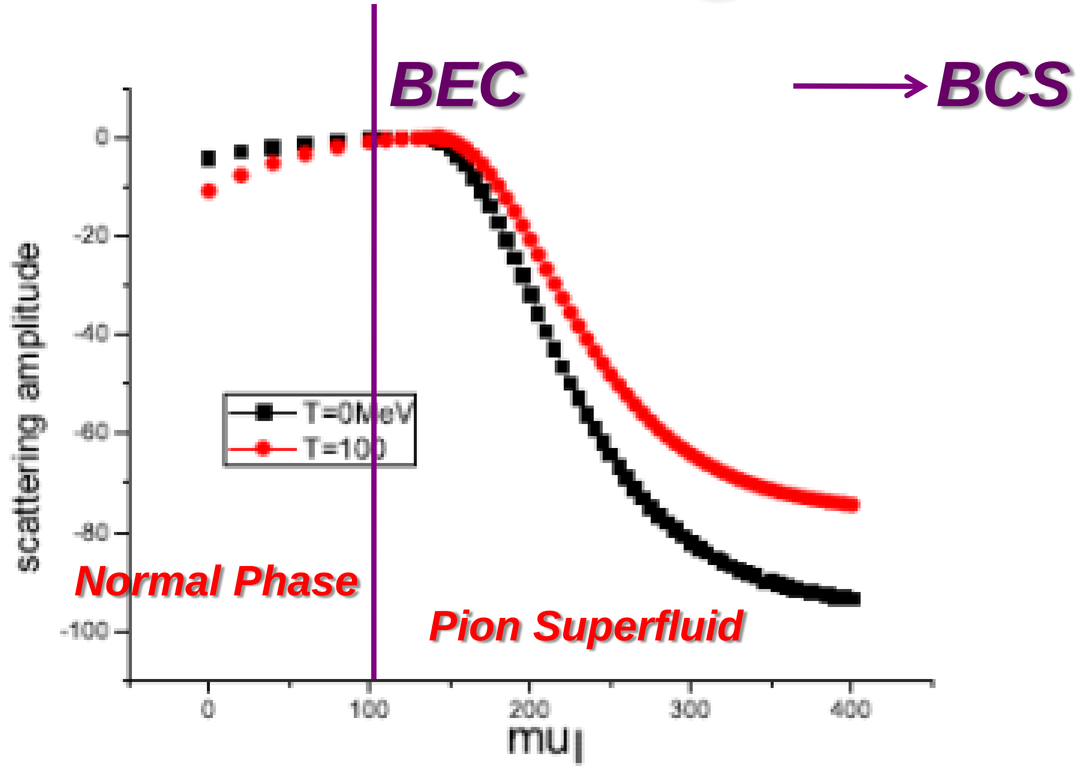


(a)



BEC BCS





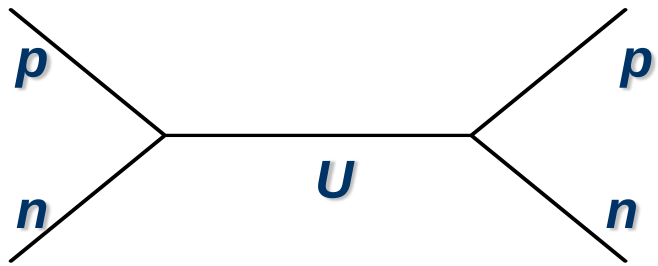
talk by Shijun Mao, November 10

BCS: overlapped molecules, large $\pi - \pi$ cross section
BEC: identified molecules, ideal Boson gas limit

Yukawa Potential in Quark Matter

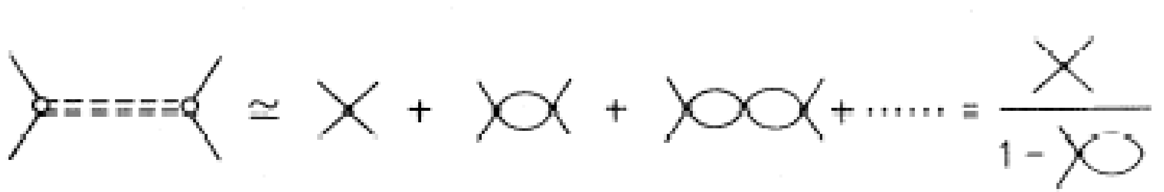
Mu and PZ, Eur. Phys. J C58, (2008)271

meson exchange in nucleon scattering (1934-1935)



$$V(r) \sim \frac{e^{-Mr}}{r}$$

meson exchange in NJL at quark level



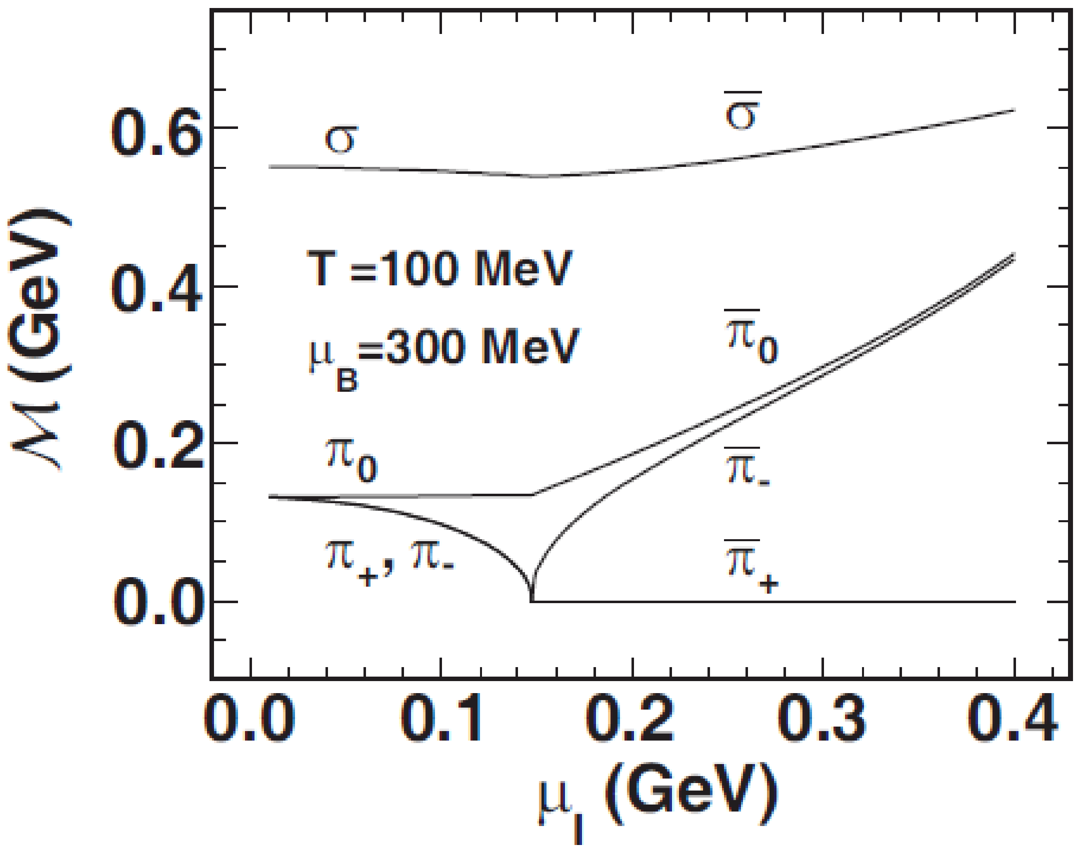
stable quark potential

$$V(r) = \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} TrD(0,\vec{k}^2)$$

screen mass

$$1 - 2G\Pi_m(0,(iM_m + \Gamma_m)^2) = 0,$$

Meson Screening Mass



Jiang and PZ, arXiv:1104.0094

corresponding to the global isospin symmetry breaking, there is a Goldstone mode in the pion superfluid, which leads to a long range force between two quarks !

Quark Potential in Pion Superfluid

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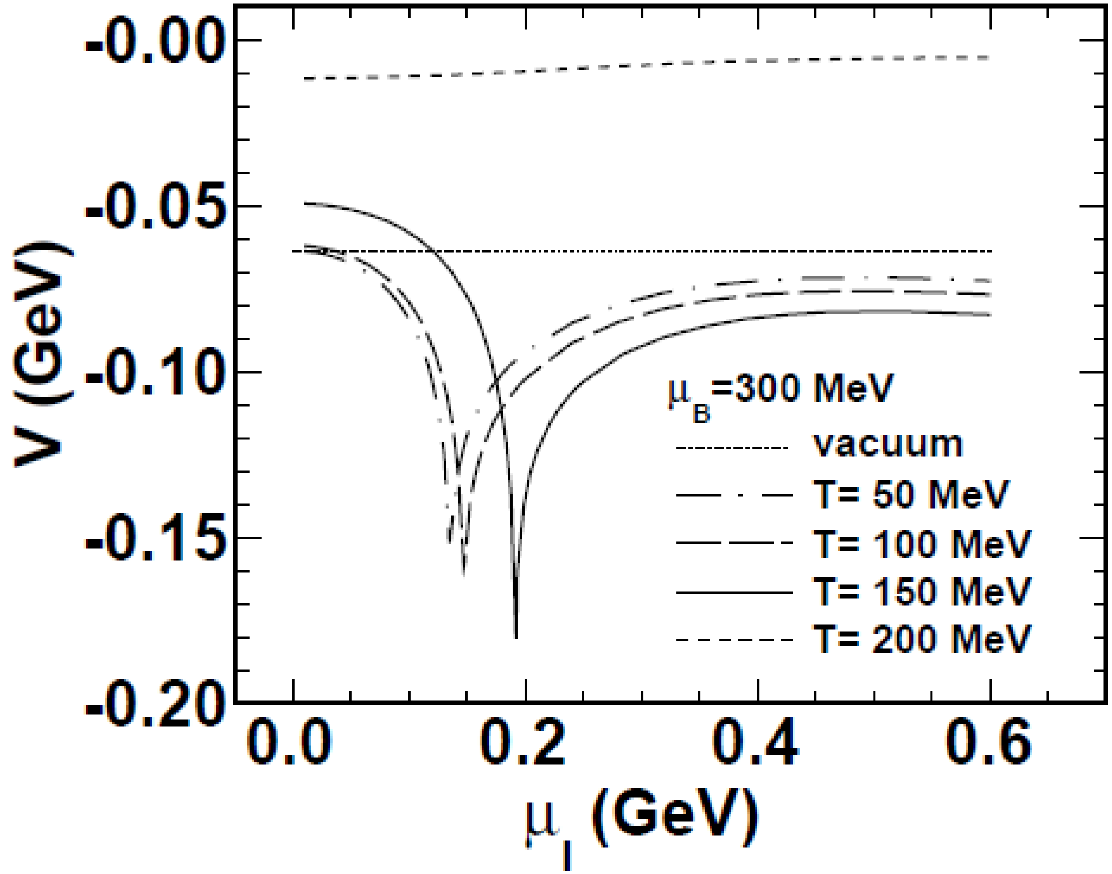

talk by Yin Jiang, November 11

1) the maximum potential is located at the phase boundary .

2) the potential in pion superfluid is non-zero at extremely high isospin density, totally different from the temperature and baryon density effects !

Conclusion and Outlook

1)There exists a pion superfluid at high isospin density, and the Goldstone mode controls the thermodynamics of the system.

2) There exists a BCS-BEC crossover in the pion superfluid.

3) The maximum coupling is located at the phase transition boundary, similar to the temperature and baryon density effects.

4) The coupling is non-zero even at extremely high isospin density, totally different from the temperature and baryon density effects.

Applications in neutron stars and intermediate energy nuclear collisions, like mass-radius relation, pion superfluid in curved space, and pion_-/pion_+ ratio.