Chiral Symmetry Breaking and Restoration and Consistent Meson Spectrum

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Content

- **Dynamically Generated Spontaneous Chiral Symmetry Breaking In Chiral Dynamical Model At Zero Temperature**

- **Chiral Thermodynamic Model of QCD and its Critical Behavior**

Dynamically Generated
Spontaneous Chiral Symmetry Breaking
In Chiral Dynamical Model
At Zero Temperature

QCD Lagrangian and Symmetry

Chiral limit: Taking vanishing quark masses $m_q \rightarrow 0$.

QCD Lagrangian

$$L_{QCD}^{(o)} = \bar{q}_L \gamma_\mu i D_\mu q_L + \bar{q}_R \gamma_\mu i D_\mu q_R - \frac{1}{4} G_{\mu\nu}^{\alpha} G^{\alpha\mu\nu}$$

$$D_\mu = \partial_\mu - g_s \lambda_\alpha / 2 G_\mu^\alpha$$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad q_{R,L} = \frac{1}{2}(1 \pm \gamma_5) q$$

has maximum global Chiral symmetry :

$$SU_L(3) \times SU_R(3) \times U_A(1) \times U_B(1)$$

The Problem

However, in the low-energy meson spectrum, we have not seen such symmetry patterns.

The natural question is how these (exact or approximate) symmetries are broken (explicitly or spontaneously)?

QCD Lagrangian and Symmetry

- QCD Lagrangian with massive light quarks

$$\mathcal{L}_{QCD} = \bar{q}\gamma^{\mu}(i\partial_{\mu} + g_s G^a_{\mu} T^a)q - \bar{q}Mq - \frac{1}{2}\mathrm{tr}G_{\mu\nu}G^{\mu\nu}$$

$$q = (u, d, s), \qquad M = \mathrm{diag.}(m_1, m_2, m_3) \equiv \mathrm{diag.}(m_u, m_d, m_s)$$

Approximate Global Chiral Symmetry

$$U(3)_L \times U(3)_R, \qquad m_i << \Lambda_{QCD}(i = 1, 2, 3)$$

Instanton Effects via t'Hooft Determination

$$\mathcal{L}^{inst} = \kappa_{inst} e^{i\theta_{inst}} \det(-\bar{q}_R q_L) + h.c., \qquad \kappa_{inst} \sim e^{-8\pi^2/g^2}$$

$$U(1)_L \times U(1)_R \to U(1)_V$$

Effective Lagrangian Based on Loop Regularization

Y.B. Dai and Y-L. Wu, Euro. Phys. J. C 39 s1 (2004)

Effective Lagrangian for Quarks and Bound States

Integrating over the gluon field and considering the bound state solution

$$\begin{aligned}\mathcal{L}_{\mathrm{eff}}(\mathrm{q},\bar{\mathrm{q}},\Phi) &= \bar{\mathrm{q}}\gamma^{\mu}\mathrm{i}\partial_{\mu}\mathrm{q} + \bar{\mathrm{q}}_{\mathrm{L}}\gamma_{\mu}\mathcal{A}_{\mathrm{L}}^{\mu}\mathrm{q}_{\mathrm{L}} + \bar{\mathrm{q}}_{\mathrm{R}}\gamma_{\mu}\mathcal{A}_{\mathrm{R}}^{\mu}\mathrm{q}_{\mathrm{R}} - [\,\bar{\mathrm{q}}_{\mathrm{L}}(\Phi - \mathrm{M})\mathrm{q}_{\mathrm{R}} + \mathrm{h.c.}\,] \\ &+ 2\mu_{\mathrm{f}}^{2}\mathrm{tr}\left(\Phi\mathrm{M}^{\dagger} + \mathrm{M}\Phi^{\dagger}\right) - \mu_{\mathrm{f}}^{2}\mathrm{tr}\Phi\Phi^{\dagger} + \mu_{\mathrm{inst}}\left(\det\Phi + \mathrm{h.c.}\right)\end{aligned}$$

Note that the field Φ has no kinetic terms, so it is an auxiliary field and can be integrated out.

Effective Four Quark Interactions-NJL at low energy

$$\mathcal{L}^{4\mathrm{q}} = \frac{1}{\mu_{\mathrm{f}}^{2}}(\bar{\mathrm{q}}_{\mathrm{Li}}\mathrm{q}_{\mathrm{Rj}})(\bar{\mathrm{q}}_{\mathrm{Rj}}\mathrm{q}_{\mathrm{Li}}) + \mathrm{h.c.}$$

Dynamically Generated Spontaneous Symmetry Breaking

Instead, if we integrate out the quark fields, we can obtain the following effective potential.

Dynamically Generated Effective Potential

$$
\begin{aligned}
V_{\rm eff}(\Phi) \;&=\; -{\rm tr}\hat{\mu}_{\rm m}^{2}\left(\Phi M^{\dagger} + M\Phi^{\dagger}\right) + \frac{1}{2}{\rm tr}\hat{\mu}_{\rm f}^{2}(\Phi\Phi^{\dagger} + \Phi^{\dagger}\Phi) \\
&+\; \frac{1}{2}{\rm tr}\lambda[\,(\hat{\Phi}\hat{\Phi}^{\dagger})^{2} + (\hat{\Phi}^{\dagger}\hat{\Phi})^{2}\,] - \mu_{\rm inst}\left(\det\Phi + {\rm h.c.}\right)
\end{aligned}
$$

with $\hat{\mu}_f^2$, $\hat{\mu}_m^2$ and λ the three diagonal matrices

$$
\begin{aligned}
\hat{\mu}_{\rm f}^{2} \;&=\; \mu_{\rm f}^{2} - \frac{N_c}{8\pi^{2}}\left(M_c^{2}T_2 + \bar{M}^{2}T_0\right) \\
\hat{\mu}_{\rm m}^{2} \;&=\; \mu_{\rm m}^{2} - \frac{N_c}{8\pi^{2}}\left(M_c^{2}T_2 + \bar{M}^{2}T_0\right), \qquad \lambda = \frac{N_c}{16\pi^{2}}T_0
\end{aligned}
$$

Dynamically Generated Spontaneous Symmetry Breaking

Recall that the pseudoscalar mesons, such as π, η , are much lighter than their scalar chiral partners. This indicates that we should choose our vacuum expectation values (VEVs) in the scalar fields:

Vacuum Expectation Values(VEVs)

$$\Phi(x) = \xi_L(x)\phi(x)\xi_R^\dagger(x), \quad \phi(x) = V + \varphi(x), \quad <\phi>= V = diag.(v_1, v_2, v_3)$$

$$U(x) \equiv \xi_L(x)\xi_R^\dagger(x) = \xi_L^2(x) = e^{i\frac{2\Pi(x)}{f}}$$

Dynamically Generated Spontaneous Symmetry Breaking

By differentiating the effective potential w.r.t. the VEVs, we obtain the following Minimal Conditions/Generalized Gap Equations:

Minimal Conditions/Generalized Gap Equations

$$-\left(\hat{\mu}_{\rm f}^2\right)_{\rm i} {\rm v_i} + \left(\hat{\mu}_{\rm m}^2\right)_{\rm i} {\rm m_i} - 2\lambda_{\rm i}\bar{\rm m}_{\rm i}^3 + \mu_{\rm inst}\bar{\rm v}^3/{\rm v_i} = 0, \quad {\rm i} = 1,2,3, \quad \bar{\rm v}^3 = {\rm v_1 v_2 v_3}$$

If we take the limit of vanishing instanton effects, we can recover the usual gap equation

Gap Equation without Instanton ($\rm v_{inst} = 0$)

$$\frac{\rm N_c}{8\pi^2\mu_{\rm f}^2}\,[\,{\rm M_c^2} - \mu_{\rm o}^2\left(\ln\frac{\rm M_c^2}{\mu_{\rm o}^2} - \gamma_{\rm w} + 1 + {\rm y_2}(\frac{\mu_{\rm o}^2}{\rm M_c^2})\right)\,] = 1$$

Scalars as Partner of Pseudoscalars

According to their quantum numbers, we can reorganize the mesons into a matrix form.

Scalar mesons:

$$\sqrt{\mathbf{2}}\varphi = \begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} f_8 + \sqrt{\frac{1}{3}} f_s & a_0^+ & \kappa_0^+ \\ a_0^- & -\frac{a_0^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} f_8 + \sqrt{\frac{1}{3}} f_s & \kappa_0^0 \\ \kappa_0^- & \bar{\kappa}_0^0 & -\frac{2}{\sqrt{6}} f_8 + \sqrt{\frac{1}{3}} f_s \end{pmatrix}$$

Pseudoscalar mesons :

$$\sqrt{\mathbf{2}}\mathbf{\Pi} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} \eta_8 + \sqrt{\frac{1}{3}} \eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} \eta_8 + \sqrt{\frac{1}{3}} \eta_0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta_8 + \sqrt{\frac{1}{3}} \eta_0 \end{pmatrix}$$

Mass Formula

Pseudoscalar mesons :

$$m^2_{\pi^\pm} \simeq \frac{2\mu_P^3}{f^2}(m_u + m_d)$$

$$m^2_{K^\pm} \simeq \frac{2\mu_P^3}{f^2}(m_u + m_s)$$

$$m^2_{K^0} \simeq \frac{2\mu_P^3}{f^2}(m_d + m_s)$$

$$m^2_{\eta_8} \simeq \frac{2\mu_P^3}{f^2}[\ \frac{1}{3}(m_u + m_d) + \frac{4}{3}m_s\ ] = \frac{1}{3}(4m_K^2 - m_\pi^2)$$

$$m^2_{\eta_8\eta_0} \simeq -\frac{2\mu_P^3}{f^2}\frac{\sqrt{2}}{3}[\ 2m_s - (m_u + m_d)\ ] = -\frac{2\sqrt{2}}{3}(m_K^2 - m_\pi^2)$$

$$m^2_{\eta_0} \simeq \frac{2\mu_P^3}{f^2}\frac{2}{3}(m_u + m_d + m_s) + \frac{12\bar{v}^3}{f^2}\mu_{inst} = \frac{1}{3}(2m_K^2 + m_\pi^2) + \frac{24\bar{v}^3}{f^2}\bar{\lambda}v_{inst}$$

$$\mu_P^3 = (\bar{\mu}_m^2 + 2\bar{\lambda}v_o^2)v_o \simeq 12\bar{\lambda}v_o^3 \simeq 3v_o f^2$$

Mixing Angles

$$\tan 2\theta_P = 2\sqrt{2}[1 - \frac{9v_{inst}v_3}{m_K^2 - m_{\pi^2}}]^{-1}$$

Instanton interactions only affect mass of SU(3) singlet

Mass Formula

Scalar Mesons - Lightest Composite Higgs Bosons

$$m_{a_0^\pm}^2 \simeq m_{a_0^0}^2 \simeq 2(2\bar{m}_u + \bar{m}_d)\bar{m}_u + 2v_{inst}v_3 \sim 8v_o^2$$

$$m_{k_0^\pm}^2 \simeq 2(2\bar{m}_u + \bar{m}_s)\bar{m}_u + 2v_{inst}v_2 \sim 8v_o^2$$

$$m_{k_0^0}^2 \simeq 2(2\bar{m}_d + \bar{m}_s)\bar{m}_d + 2v_{inst}v_1 \sim 8v_o^2$$

$$m_{f_8}^2 \simeq \bar{m}_u^2 + \bar{m}_d^2 + 4\bar{m}_s^2 + \frac{2}{3}v_{inst}(2v_1 + 2v_2 - v_3) \sim 8v_o^2$$

$$m_{f_s}^2 \simeq 2(\bar{m}_u^2 + \bar{m}_d^2 + \bar{m}_s^2) - \frac{4}{3}v_{inst}(v_1 + v_2 + v_3) \sim 2v_o^2$$

$$m_{f_s f_8}^2 \simeq \sqrt{2}(2\bar{m}_s^2 - \bar{m}_u^2 - \bar{m}_d^2) - \frac{\sqrt{2}}{3}v_{inst}(2v_3 - v_1 - v_2) \sim 0$$

Mixing Angles

$$\tan 2\theta_S = \frac{2m_{f_s f_8}^2}{m_{f_s}^2 - m_{f_8}^2}$$

Predictions for Mass Spectra & Mixings

Input Parameters

$f_\pi = 94\mathrm{MeV}$ $\quad v_o = 340\mathrm{MeV}$

$m_u \simeq 3.8\mathrm{MeV}$ $\quad m_d \simeq 5.7\mathrm{MeV}$ $\quad m_s/m_d \simeq 20.5$

Output Predictions

$\mu_f \simeq 144\mathrm{MeV},$ $\quad \mu_{inst} \simeq 8.0\mathrm{MeV}$

$M_c \simeq 922\mathrm{MeV},$ $\quad \mu_s \simeq 333\mathrm{MeV}$

$< \bar{u}u > \simeq < \bar{d}d > \simeq < \bar{s}s > = -(242\mathrm{MeV})^3$

$m_\pi \simeq 139\mathrm{MeV},$ $\quad m_\pi|_{exp} \simeq 139\mathrm{MeV}$

$m_{K^0} \simeq 500\mathrm{MeV},$ $\quad m_{K^0}|_{exp} \simeq 500\mathrm{MeV}$

$m_{K^\pm} \simeq 496\mathrm{MeV}$ $\quad m_{K^\pm}|_{exp} \simeq 496\mathrm{MeV}$

$m_\eta \simeq 503\mathrm{MeV},$ $\quad m_\eta|_{exp} \simeq 548\mathrm{MeV}$

$m_{\eta'} \simeq 986\mathrm{MeV},$ $\quad m_{\eta'}|_{exp} \simeq 958\mathrm{MeV}$

Predictions

$$\mathrm{m_{a_0}} \simeq 978 \text{ MeV}, \qquad \mathrm{m_{a_0}^{exp.}} = 984.8 \pm 1.4 \text{ MeV} \quad \mathrm{PDG}$$

$$\mathrm{m_{\kappa_0}} \simeq 970 \text{ MeV}, \qquad \mathrm{m_{\kappa_0}^{exp.}} = 797 \pm 19 \pm 43 \text{ MeV} \quad \mathrm{E7912}$$

$$\mathrm{m_{f_0}} \simeq 1126 \text{ MeV}, \qquad \mathrm{m_{f_0}^{epx.}} = 980 \pm 10 \text{ MeV} \quad \mathrm{PDG}$$

$$\mathrm{m_{\sigma}} \simeq 677 \text{ MeV}, \qquad \mathrm{m_{\sigma}^{exp.}} = (400 - 1200) \text{ MeV} \quad \mathrm{PDG}$$

$$\theta_{\mathrm{P}} \simeq -18^{\mathrm{o}}, \qquad \theta_{\mathrm{S}} \simeq -18^{\mathrm{o}}$$

$$\eta_8 = \cos\theta_{\mathrm{P}}\ \eta + \sin\theta_{\mathrm{P}}\ \eta'$$

$$\eta_0 = \cos\theta_{\mathrm{P}}\ \eta' - \sin\theta_{\mathrm{P}}\ \eta$$

$$\mathrm{f_8} = \cos\theta_{\mathrm{S}}\ \mathrm{f_0} + \sin\theta_{\mathrm{S}}\ \sigma$$

$$\mathrm{f_s} = \cos\theta_{\mathrm{S}}\ \sigma - \sin\theta_{\mathrm{S}}\ \mathrm{f_0}$$

Chiral Thermodynamic Model of QCD and its Critical Behavior

Motivation

Provided that chiral dynamical model works so well to show chiral symmetry breaking and to predict the meson spectrum, we want to see what happens for this model at finite temperature

-- Chiral symmetry will be restored at high enough temperature

Effective Lagrangian

by DH, Y-L Wu, 1110.4491 [hep-ph]

Effective Lagrangian for Quarks and Bound States

$$\mathcal{L}_{eff}(q,\bar{q},\Phi) = \bar{q}\gamma^{\mu} i\partial_{\mu} q + \bar{q}_L \gamma_{\mu} \mathcal{A}_L^{\mu} q_L + \bar{q}_R \gamma_{\mu} \mathcal{A}_R^{\mu} q_R - [\bar{q}_L(\Phi - M) q_R + h.c.]$$
$$+\mu_m^2 tr(\Phi M^{\dagger} + M\Phi^{\dagger}) - \mu_f^2 tr\Phi\Phi^{\dagger}$$

Two simplifications:

- Only consider two flavors
- Ignore the instanton effective action

Dynamically Generated Spontaneous Symmetry Breaking

By integrating out the quark fields with Closed Time Path (CTP) formalism, we obtain the following effective potential at finite temperature

Dynamically Generated Effective Potential

$$
\begin{aligned}
V_{eff}(\Phi) &= -tr_F \hat{\mu}_m^2(T)(\Phi M^\dagger + M\Phi^\dagger) + \frac{1}{2} tr_F \hat{\mu}_f^2(T)(\Phi\Phi^\dagger + \Phi^\dagger\Phi) \\
&+ \frac{1}{2} tr_F \lambda(T)[(\hat{\Phi}\hat{\Phi}^\dagger)^2 + (\hat{\Phi}^\dagger\hat{\Phi})^2]
\end{aligned}
$$

Dynamically Generated Spontaneous Symmetry Breaking

with $\hat{\mu}_f^2(T)$, $\hat{\mu}_m^2(T)$ and $\lambda(T)$ the three diagonal matrices

$$\begin{aligned}
\hat{\mu}_f^2(T) &\equiv \mu_f^2 - \frac{N_c}{8\pi^2}\left(M_c^2 L_2(T) + \bar{M}^2 L_0(T)\right) \\
\hat{\mu}_m^2(T) &\equiv \mu_m^2 - \frac{N_c}{8\pi^2}\left(M_c^2 L_2(T) + \bar{M}^2 L_0(T)\right) \\
\lambda(T) &\equiv \frac{N_c}{16\pi^2} L_0(T)
\end{aligned}$$

With the Two Regularized Diagonal Matrices $L_0(T)$ and $L_2(T)$

$$\begin{aligned}
L_0(T) &\equiv L_0\left(\frac{\mu^2}{M_c^2}\right) - \frac{1}{\pi}\int d^3k \, \frac{\beta\sqrt{\vec{k}^2+\bar{M}^2}\, e^{\beta\sqrt{\vec{k}^2+\bar{M}^2}} + e^{\beta\sqrt{\vec{k}^2+\bar{M}^2}} + 1}{(\vec{k}^2+\bar{M}^2)^{3/2}\left(e^{\beta\sqrt{\vec{k}^2+\bar{M}^2}}+1\right)^2} \\
L_2(T) &\equiv L_2\left(\frac{\mu^2}{M_c^2}\right) - \frac{4}{\pi M_c^2}\int d^3k \, \frac{1}{\sqrt{\vec{k}^2+\bar{M}^2}\left(e^{\beta\sqrt{\vec{k}^2+\bar{M}^2}}+1\right)}
\end{aligned}$$

Dynamically Generated Spontaneous Symmetry Breaking

Recall that the pseudoscalar mesons, such as π, η , are much lighter than their scalar chiral partners. This indicates that we should choose our vacuum expectation values (VEVs) in the scalar fields:

Vacuum Expectation Values(VEVs)

$$\Phi(x) = \xi_L(x)\phi(x)\xi_R^\dagger(x), \quad \phi(x) = V(T) + \varphi(x), \quad <\phi>= V(T) = diag.(v_1(T), v_2(T))$$

$$U(x) \equiv \xi_L(x)\xi_R^\dagger(x) = \xi_L^2(x) = e^{i\frac{2\Pi(x)}{f}}$$

Dynamically Generated Spontaneous Symmetry Breaking

By differentiating the effective potential with respect to the VEVs, we can obtain:

Minimal Conditions/Generalized Gap Equations

$$-\hat{\mu}_{fi}^2(T)v_i(T) + \hat{\mu}_{mi}^2(T)m_i - 2\lambda_i(T)\bar{m}_i^3(T) = 0, \quad i = 1,2$$

After we take the limit of zero current quark masses, the gap equation can be simplified to

Gap Equation with Vanishing Current Quark Masses

$$\mu_f^2 = \frac{N_c}{8\pi^2}[M_c^2 - \mu_o^2(T)(\ln\frac{M_c^2}{\mu_o^2(T)} - \gamma_\omega + 1 + y_2(\frac{\mu_o^2(T)}{M_c^2}))] - \frac{2N_c}{\pi^2}\int_0^\infty dk \frac{k^2}{\sqrt{k^2 + v_o^2(T)}(e^{\beta\sqrt{k^2+v_o^2(T)}} + 1)}$$

Critical Temperature For Chiral Symmetry Restoration

In order to determine the critical temperature for chiral symmetry restoration, we need to make the following assumption:

Further Assumption

$$\mu_f^2(T) = \gamma v_o^2(T)$$

Note that $\mu_f^{\ 2}(T)$ is the coupling of the four quark interaction in our model. In principle this interaction comes by integrating out the gluon fields. Thus, this assumption indicates that the low energy gluon dynamics have the same finite temperature dependence as the quark field.

Critical Temperature For Chiral Symmetry Restoration

With the above assumption, we can determine the critical temperature for chiral symmetry restoration in the chiral thermodynamic model (CTDM)

Critical Temperature for Chiral Symmetry Restoration (When $v_o(T)^2 \to 0$)

$$T_c = \sqrt{\frac{6}{8\pi^2}[M_c^2 - \mu_s^2(\ln\frac{M_c^2}{\mu_s^2} - \gamma_\omega + 1 + y_2(\frac{\mu_s^2}{M_c^2}))]}$$

Scalars as Partner of Pseudoscalars & Lightest Composite Higgs Bosons

Scalar mesons:

$$\sqrt{2}\varphi = \begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{\sigma}{\sqrt{2}} & a_0^+ \\ a_0^- & -\frac{a_0^0}{\sqrt{2}} + \frac{\sigma}{\sqrt{2}} \end{pmatrix}$$

Pseudoscalar mesons :

$$\sqrt{2}\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{2}} \end{pmatrix}$$

Mass Formula

Pseudoscalar mesons :

$$m^2_{\pi^{0,\pm}}(T) = m^2_\eta(T) \simeq \frac{2\mu_P^3(T)}{f_\pi^2(T)}(m_u + m_d) = \frac{4\mu_P^3(T)}{f_\pi^2(T)} m$$

$$\mu_P^3(T) = (\bar{\mu}_m^2(T) + 2\bar{\lambda}(T)v_o^2(T))v_o(T) = \mu_f^2(T)v_o(T) = \gamma v_o(T)^3$$

Scalar mesons (Lightest Composite Higgs Boson) :

$$m^2_{a_0^{0,\pm}}(T) = m^2_\sigma(T) \simeq 3(\bar{m}_u^2(T) + \bar{m}_d^2(T)) = 6\bar{m}^2(T)$$

Predictions for Mass Spectra

Input Parameters

$f_\pi \simeq 94\text{MeV}$, $m_{\pi^{0,\pm}} \simeq 139\text{MeV}$,
$m = 4.76\text{MeV}$,

Output Predictions

$$v_o \simeq 350MeV$$

$$M_c \simeq 881MeV, \quad \mu_s \simeq 312MeV$$

$$\mu_m^2 = 2\mu_f^2 = (226MeV)^2$$

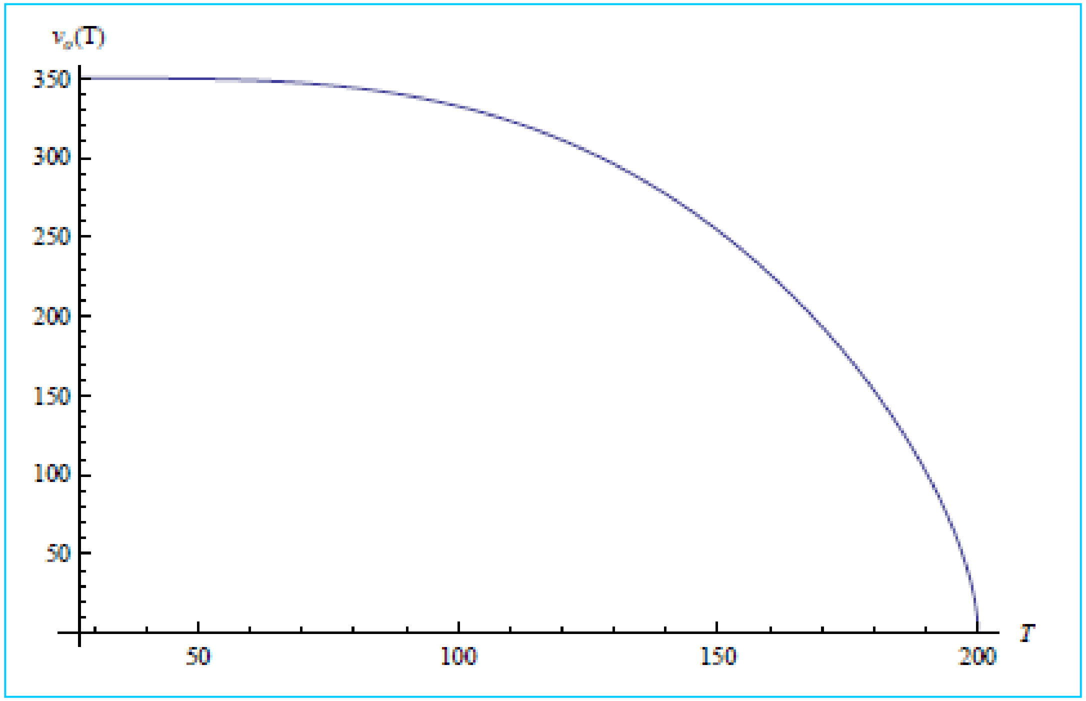
$$\beta_o = 2, \quad \gamma = \frac{\mu_f^2}{v_o^2} = 0.209$$

$$\langle \bar{q}q \rangle = -(262MeV)^3$$

$$T_c = 200MeV$$

Chiral Symmetry Restoration at Finite Temperature

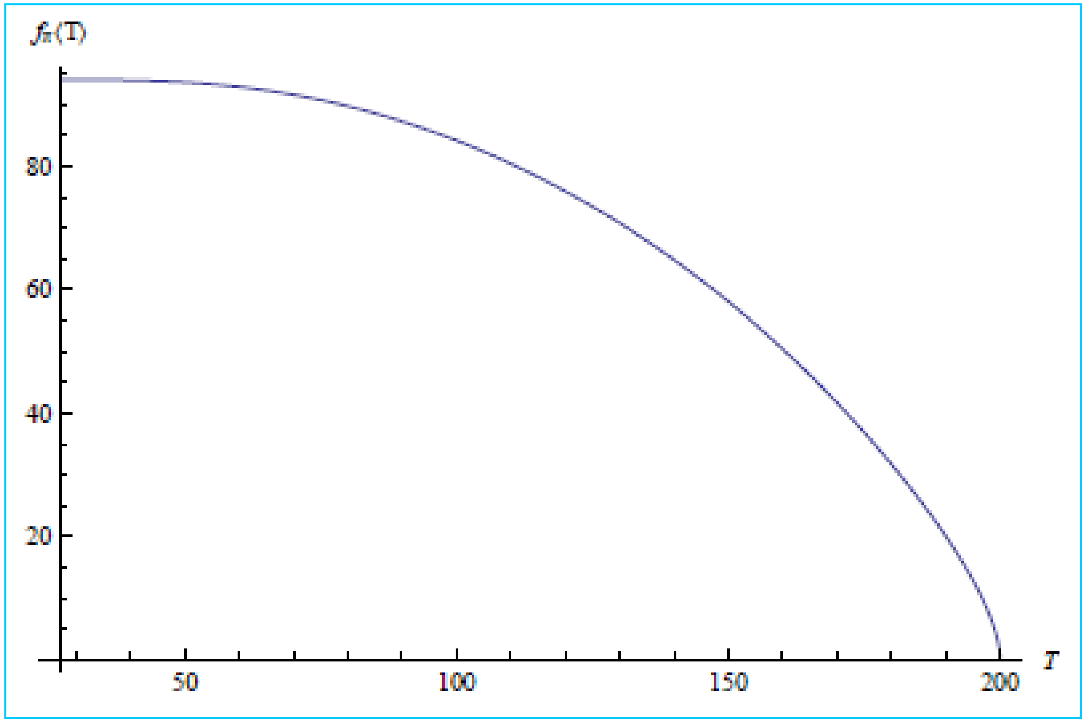
Vacuum Expectation Value (VEV)



Chiral Symmetry Restoration at Finite Temperature

Pion Decay Constant

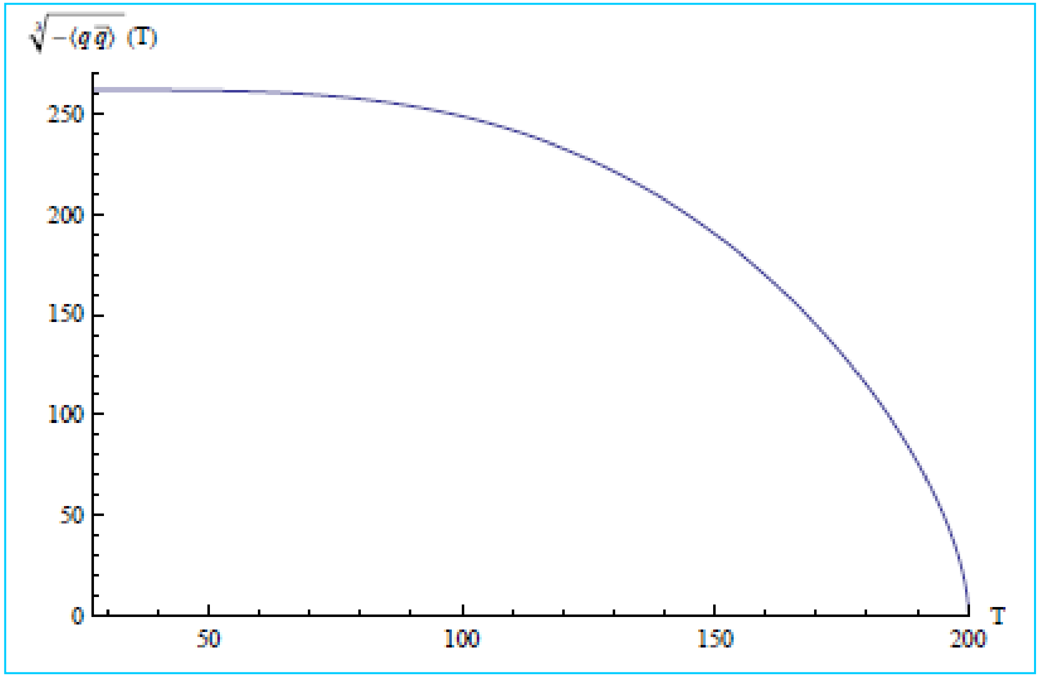
$$f_\pi(T) = \sqrt{4\bar{\lambda}(T) v_o(T)^2} = 2 v_o(T) \sqrt{\frac{N_c}{16\pi^2} \bar{L}_0(T)}$$



Chiral Symmetry Restoration at Finite Temperature

Quark Condensate

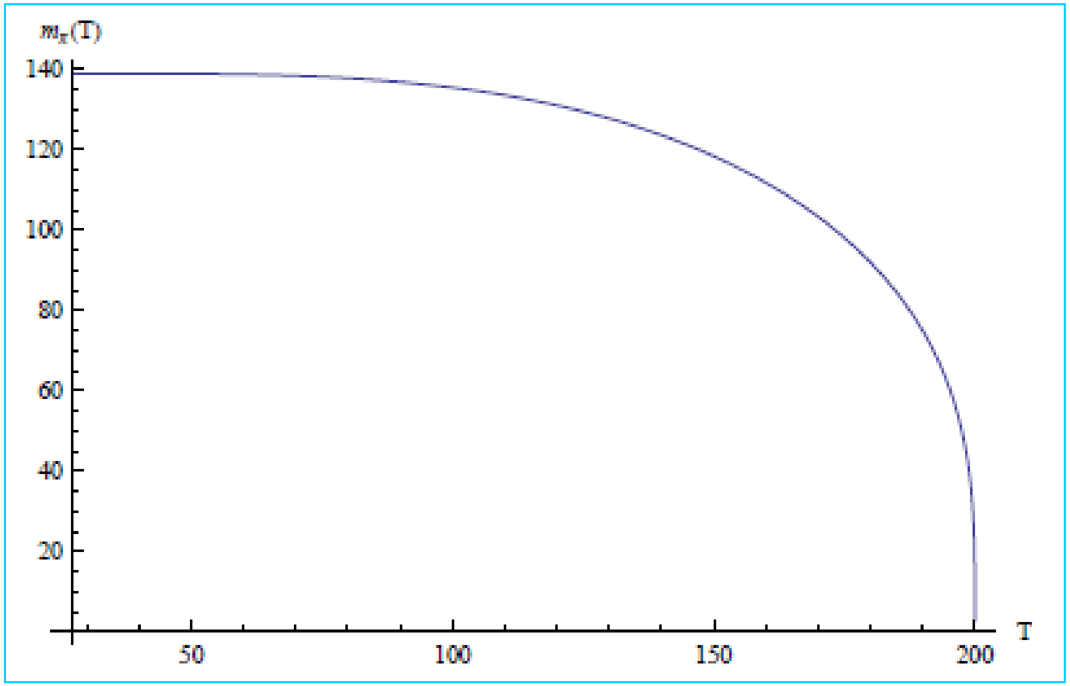
$$\langle \bar{q}q \rangle (T) = -2\mu_f^2(T) v_o(T) = -2\gamma v_o(T)^3$$



Chiral Symmetry Restoration at Finite Temperature

Mass for Pseudoscalar mesons

$$m^2_{\pi^{0,\pm}}(T) = \frac{4\mu_P^3(T)}{f_\pi^2(T)} m = \frac{\gamma m v_o(T)}{\bar{\lambda}(T)}$$



Chiral Symmetry Restoration at Finite Temperature

All of these quantities have the same scaling behavior near the critical temperature,

$$v_o(T \sim T_c) \propto |T - T_c|^\beta \quad \beta = 0.5$$

This feature can be traced back to the fact that all of these quantities are proportional to the VEV near the critical temperature

Summary and Outlook

- **In the simple chiral dynamical model, Prof. Dai and Prof. Wu derived a consistent spectrum of lowest lying meson states.**

- **Within this model, we discuss the chiral symmetry restoration at finite temperature by determining the critical temperature and critical behavior of some characteristic quantities.**

- **In the future work, we need to consider finite chemical potential effects. Also, we also want to extend the current work into three flavor case and consider instanton finite temperature effects.**

THANKS

谢谢！

Back up

Dynamically Generated Spontaneous Symmetry Breaking

Dynamically Generated Effective Lagrangian

$$\begin{aligned}\mathcal{L}_{eff}(\Phi) &= \frac{1}{2}\frac{N_c}{16\pi^2} tr_F L_0(T)[D_\mu \hat{\Phi}^\dagger D^\mu \hat{\Phi} + D_\mu \hat{\Phi} D^\mu \hat{\Phi}^\dagger - (\hat{\Phi}^\dagger\hat{\Phi} - \bar{M}^2)^2 - (\hat{\Phi}\hat{\Phi}^\dagger - \bar{M}^2)^2] \\ &+ \frac{N_c}{16\pi^2} M_c^2 tr_F L_2(T)[(\hat{\Phi}^\dagger\hat{\Phi} - \bar{M}^2) + (\hat{\Phi}\hat{\Phi}^\dagger - \bar{M}^2)] \\ &+ \mu_m^2 tr_F(\Phi M^\dagger + M\Phi^\dagger) - \mu_f^2 tr \Phi\Phi^\dagger\end{aligned}$$

With the Two Regularized Diagonal Matrices $L_0(T)$ and $L_2(T)$

$$L_0(T) \equiv L_0(\frac{\mu^2}{M_c^2}) - \frac{1}{\pi}\int d^3k \frac{\beta\sqrt{\vec{k}^2 + \bar{M}^2} e^{\beta\sqrt{\vec{k}^2+\bar{M}^2}} + e^{\beta\sqrt{\vec{k}^2+\bar{M}^2}} + 1}{(\vec{k}^2 + \bar{M}^2)^{3/2}(e^{\beta\sqrt{\vec{k}^2+\bar{M}^2}} + 1)^2}$$

$$L_2(T) \equiv L_2(\frac{\mu^2}{M_c^2}) - \frac{4}{\pi M_c^2}\int d^3k \frac{1}{\sqrt{\vec{k}^2 + \bar{M}^2}(e^{\beta\sqrt{\vec{k}^2+\bar{M}^2}} + 1)}$$