Higher order Fluctuations and Conservation Laws

- Introduction
- Details
- Results
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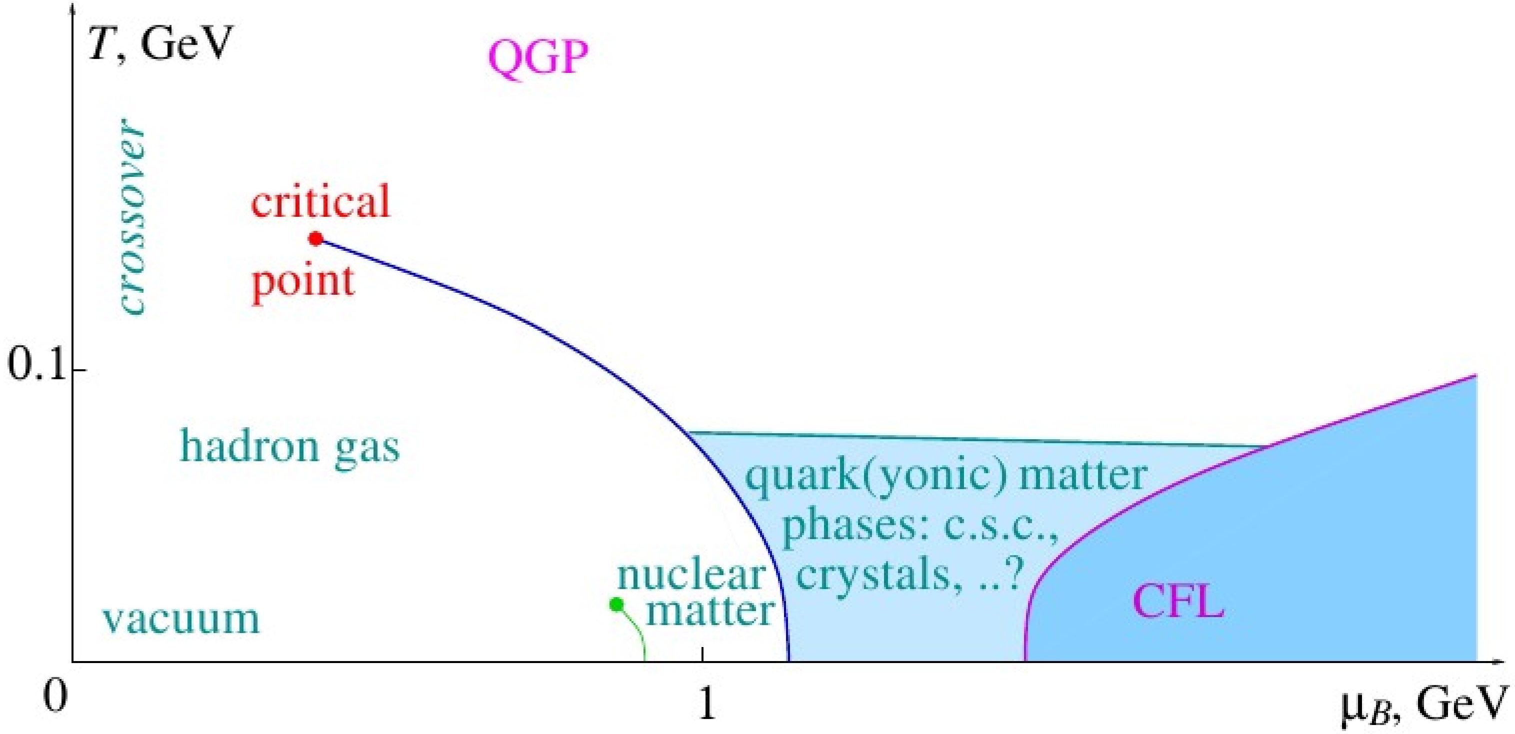
The quest



T, GeV
QGP
crossover
critical
point
0.1
hadron gas
quark(yonic) matter
phases: c.s.c.,
crystals, ..?
nuclear
matter
vacuum
CFL
0
1
μB, GeV

Higher moments (cumulants) and ξ

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},$$

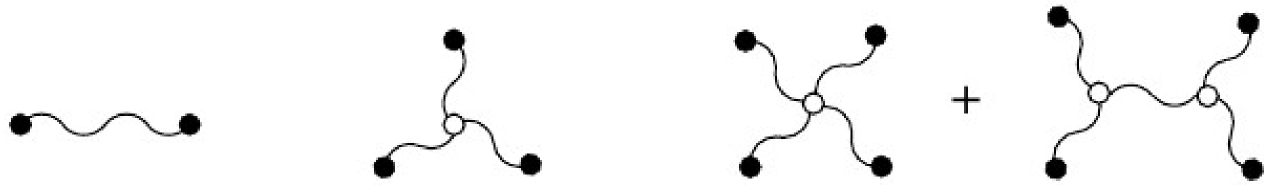
$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \dots\right]. \qquad \Rightarrow \qquad \xi = m_\sigma^{-1}$$

- Moments (connected) of $\boldsymbol{q} = 0$ mode $\sigma_V \equiv \int d^3x\,\sigma(x)$:

$$\kappa_2 = \langle\sigma_V^2\rangle = VT\,\xi^2\,; \qquad \kappa_3 = \langle\sigma_V^3\rangle = 2VT^2\,\lambda_3\,\xi^6\,;$$

$$\kappa_4 = \langle\sigma_V^4\rangle_c \equiv \langle\sigma_V^4\rangle - 3\langle\sigma_V^2\rangle^2 = 6VT^3\,[\,2(\lambda_3\xi)^2 - \lambda_4\,]\,\xi^8\,.$$

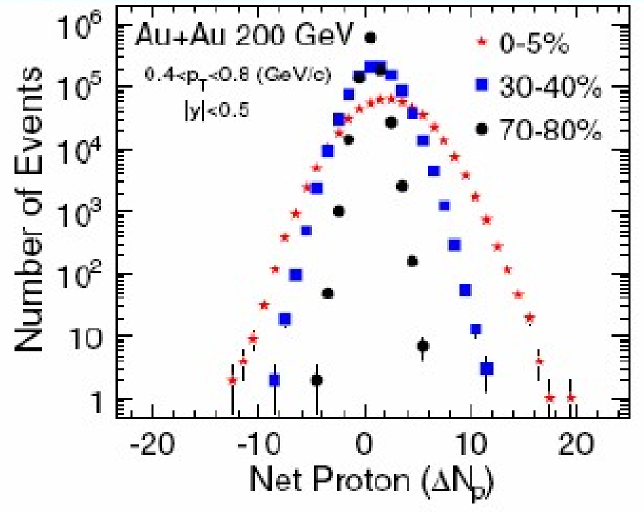
- Tree graphs. Each propagator gives ξ^2.

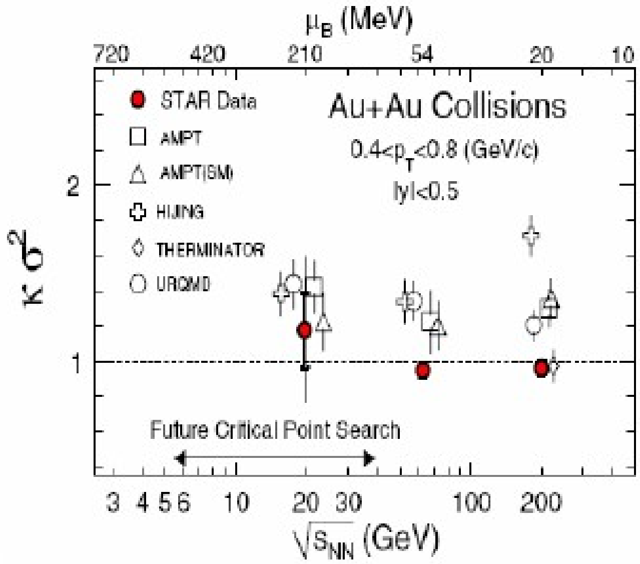


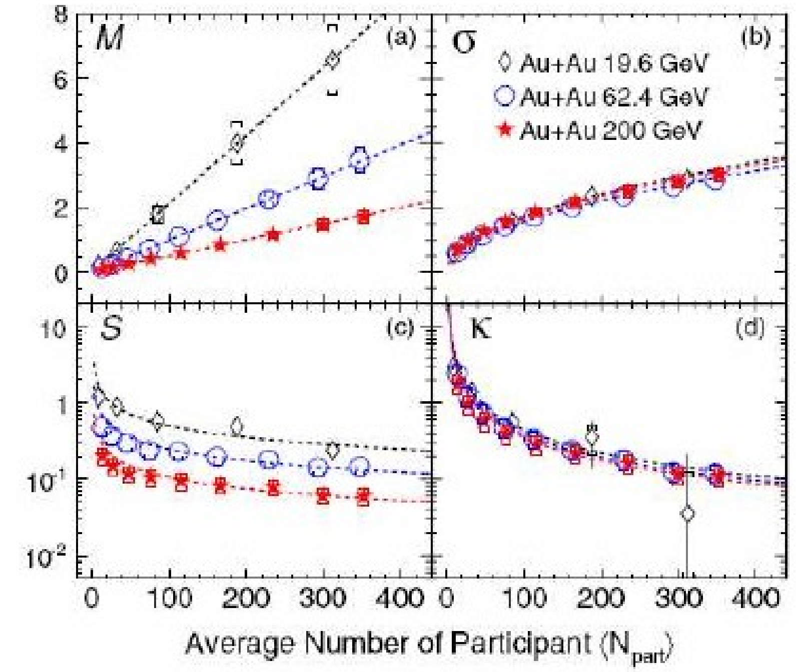
- Scaling requires “running”: $\lambda_3 = \tilde\lambda_3 T (T\xi)^{-3/2}$ and $\lambda_4 = \tilde\lambda_4 (T\xi)^{-1}$, i.e.,

$$\kappa_3 = \langle\sigma_V^3\rangle = 2VT^{3/2}\,\tilde\lambda_3\,\xi^{4.5}\,; \quad \kappa_4 = 6VT^2\,[\,2(\tilde\lambda_3)^2 - \tilde\lambda_4\,]\,\xi^7\,.$$

STAR First result on higher moments of net-proton



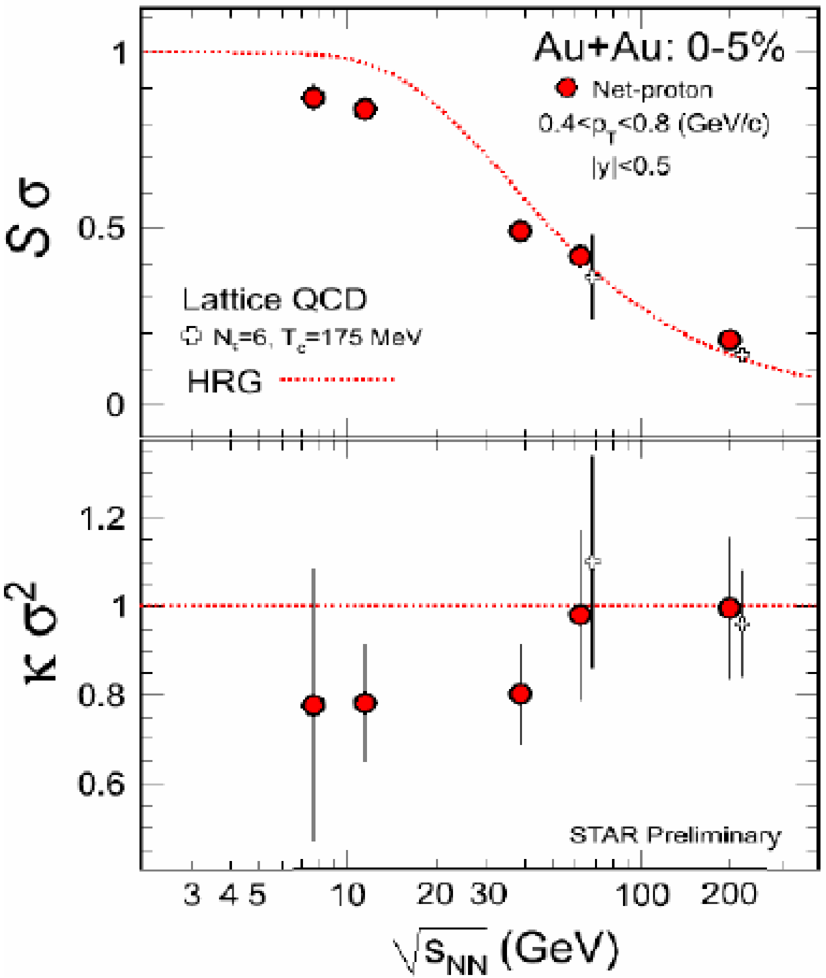




STAR Collaboration, PRL 105 (2010) 022302.

- STAR first results on higher moments analysis are up to fourth order.
- Using ratios used to establish base line measurements for the QCD critical point search.
- **This talk:** C_6 / C_2 **and** C_4 / C_2.

Results (I): Energy dependence of product moments

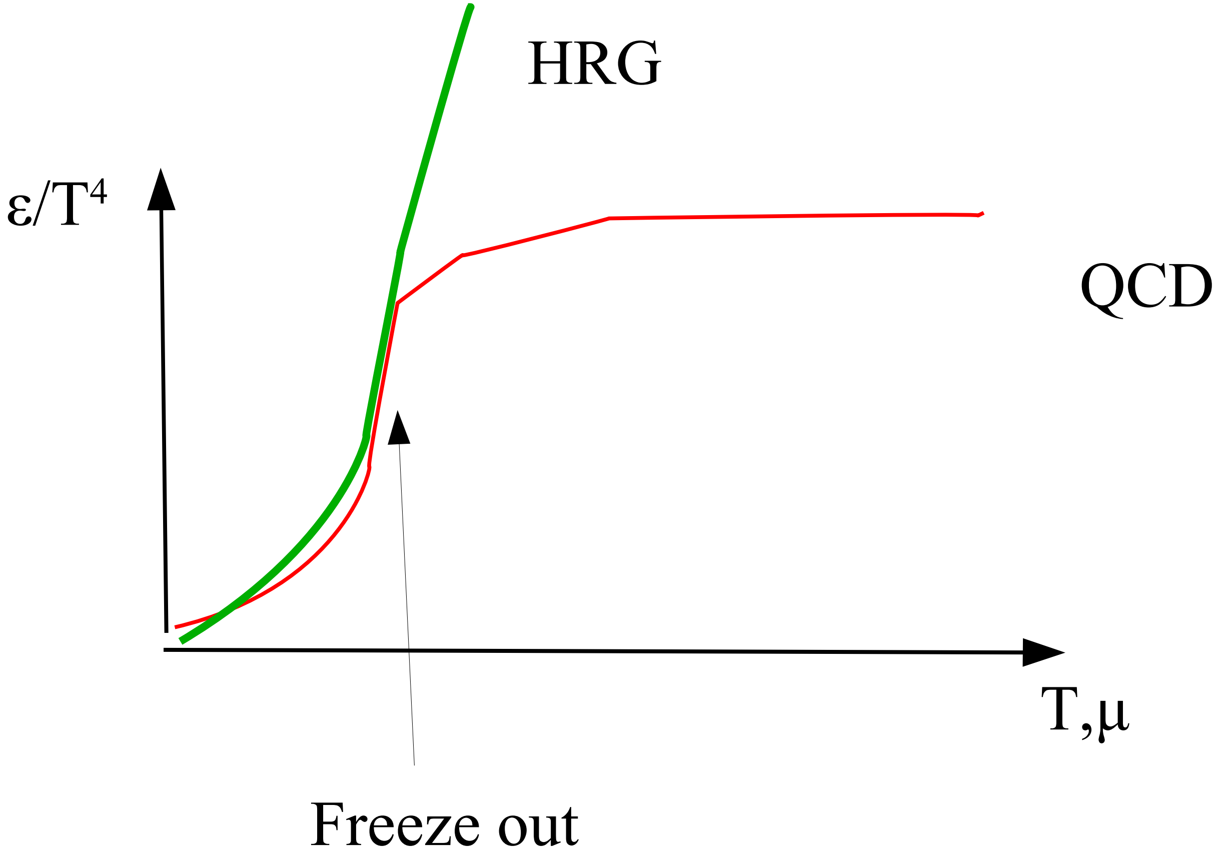


X. Luo, SQM 2011

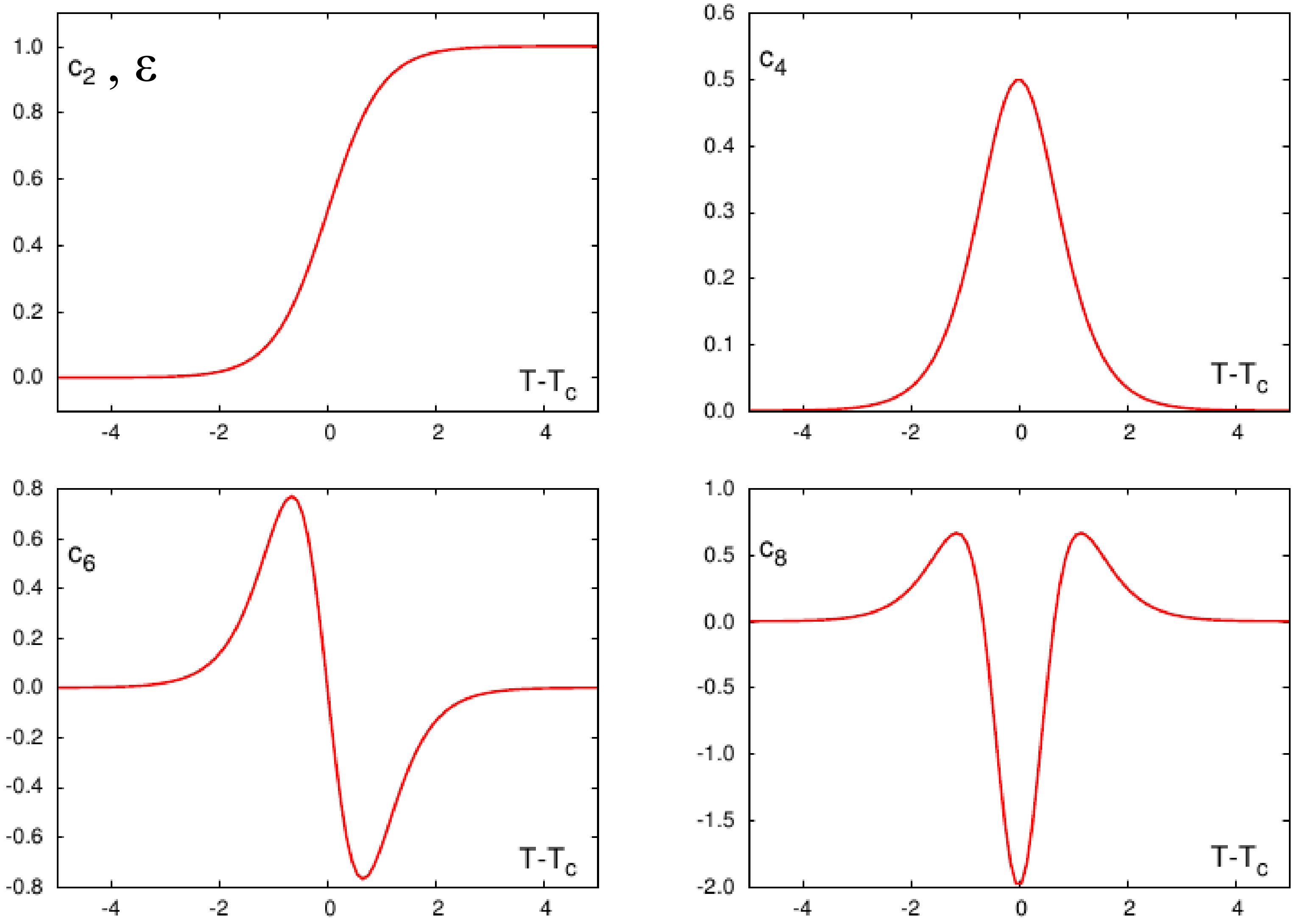
- Data are compared with HRG and Lattice QCD.

Another use

- Study higher moments at large sqrt(s)
- Establish that EOS bends over



Generic expansion coefficients



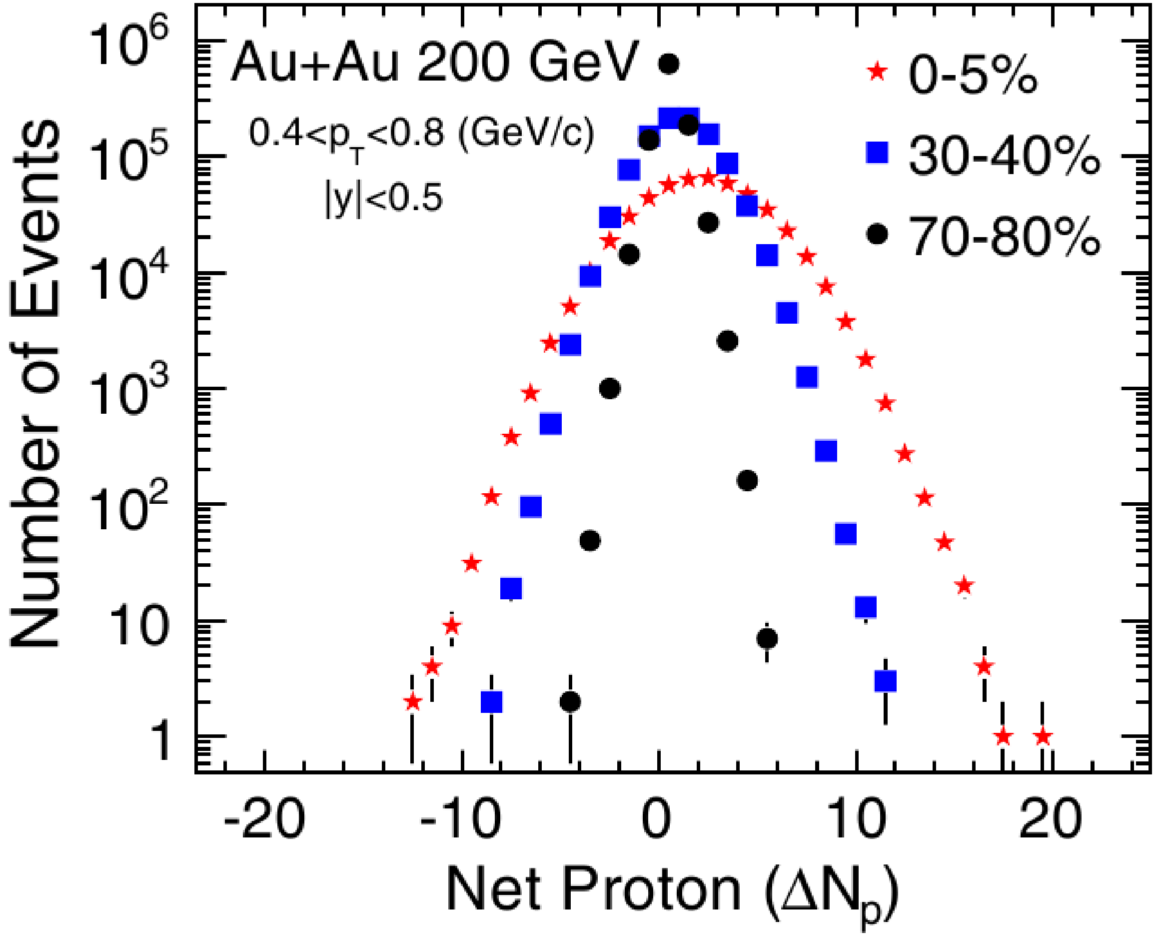
similar in PNJL model: S. Roessner et al, PR D75 (2007) 034007

Higher cumulants can tell us a lot!

Details

D-

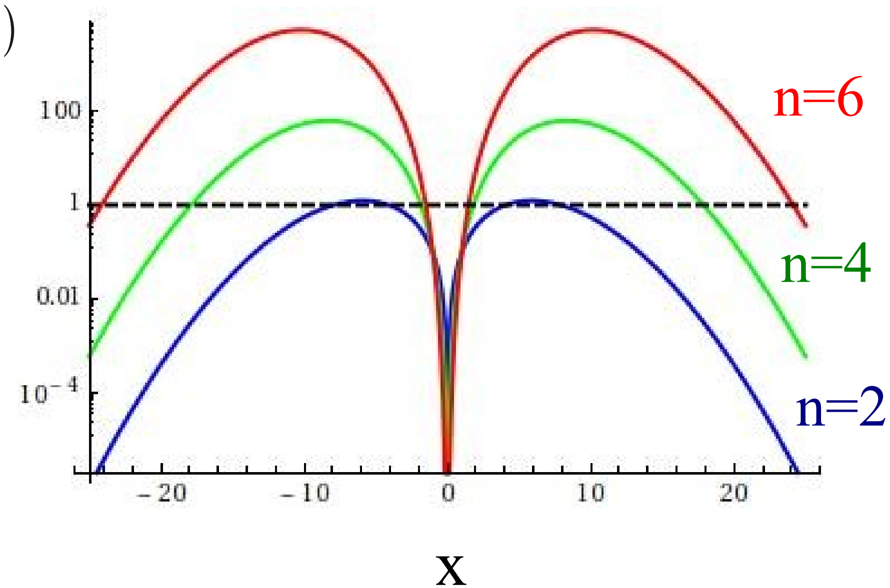
The tails matter



$$20^4 = 1.6\,10^5$$
$$20^6 = 6.4\,10^7$$

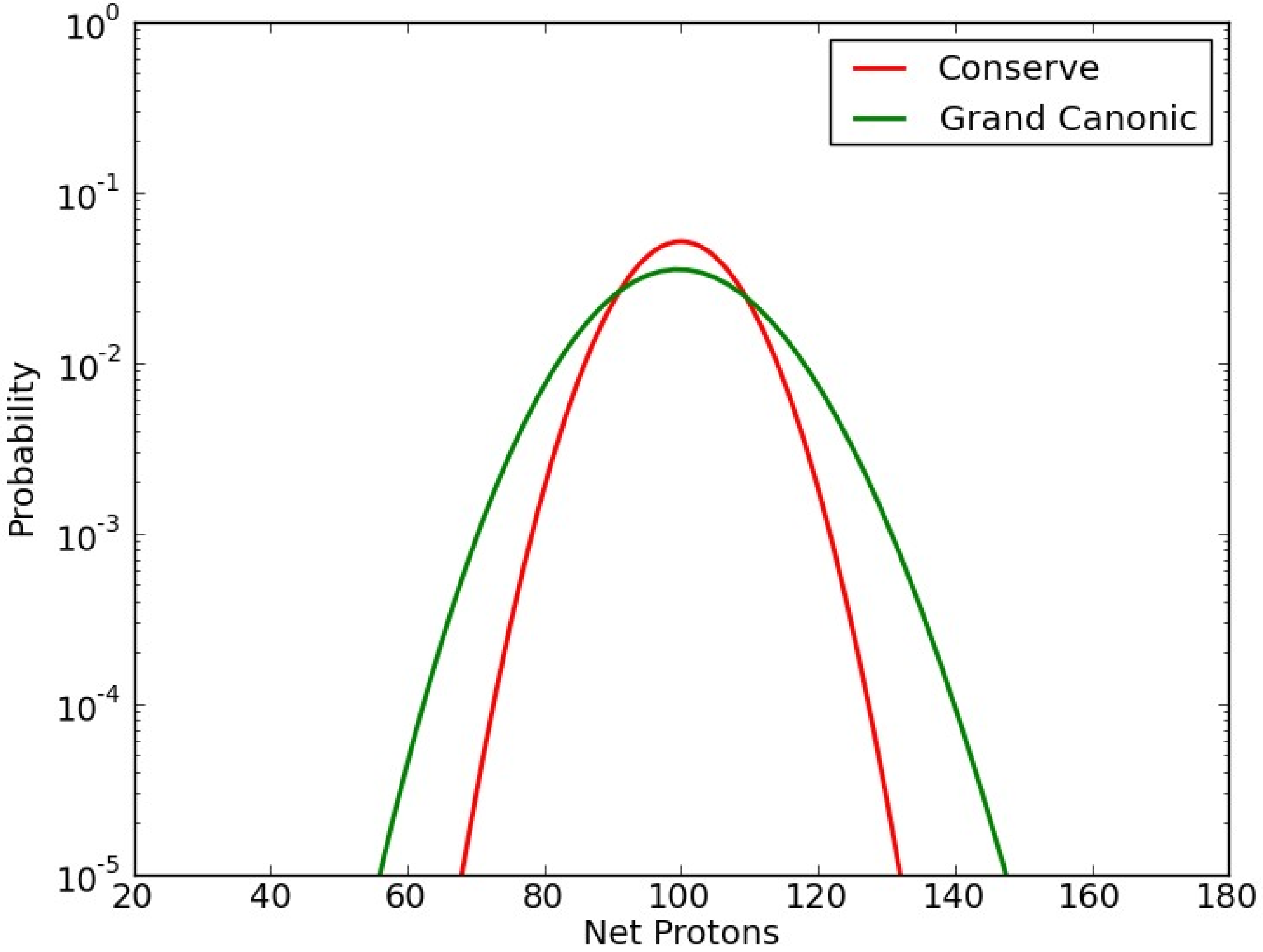
Higher Moments probe the tails

$$P_n(x) = x^n\, Prob(x)$$



Gaussian with width = 4.2,
corresponds roughly to STAR net proton distribution at 200 GeV

Conservation laws affect the tails



Baryon number conservation

- Affects all susceptibilities: Variance, Kurtosis....
- Proton Fluctuations are also affected
  - Distinguish from Isospin fluctuations

- Still LARGE baryon DENSITY fluctuations

Protons vs Baryons

All protons are baryons but not all baryons are protons

See also talks by Asakawa and Nahrgang

Partition Function

Consider only neutrons, protons, and charged "pions"

Conserved quantities: electric Charge Q and baryon number B

Still fluctuations: Protons ↔ neutrons and $\pi^+ \leftrightarrow \pi^-$

$$Z = \sum_{p,\bar{p},n,\bar{n},\pi_+,\pi_-} \frac{N_{nucl}^{p+\bar{p}+n+\bar{n}}}{p!\,\bar{p}!\,n!\,\bar{n}!} \frac{N_{pions}^{\pi^+ + \pi^-}}{\pi^+!\,\pi^-!} \delta\left(B - (p + n - \bar{p} - \bar{n})\right) \delta\left(Q - (p + \pi^+ - \bar{p} - \pi^-)\right)$$

$$N_{nucl} = Z_1(Nucleon)$$

$$N_{pions} = Z_1(Pion)$$

Single particle partition function

Ignore strangeness, energy, and momentum conservation

Partition Function

$$Z=\sum_{p,\bar{p},n,\bar{n},\pi_+,\pi_-}\frac{N_{nucl}^{p+\bar{p}+n+\bar{n}}}{p!\,\bar{p}!\,n!\,\bar{n}!}\frac{N_{pions}^{\pi^++\pi^-}}{\pi^+!\,\pi^-!}\delta\big(B-(p+n-\bar{p}-\bar{n})\big)\delta\big(Q-(p+\pi^+-\bar{p}-\pi^-)\big)$$

$$\Pi=p-\bar{p}$$
$$\nu=n-\bar{n}$$
$$q=\pi^+-\pi^-$$

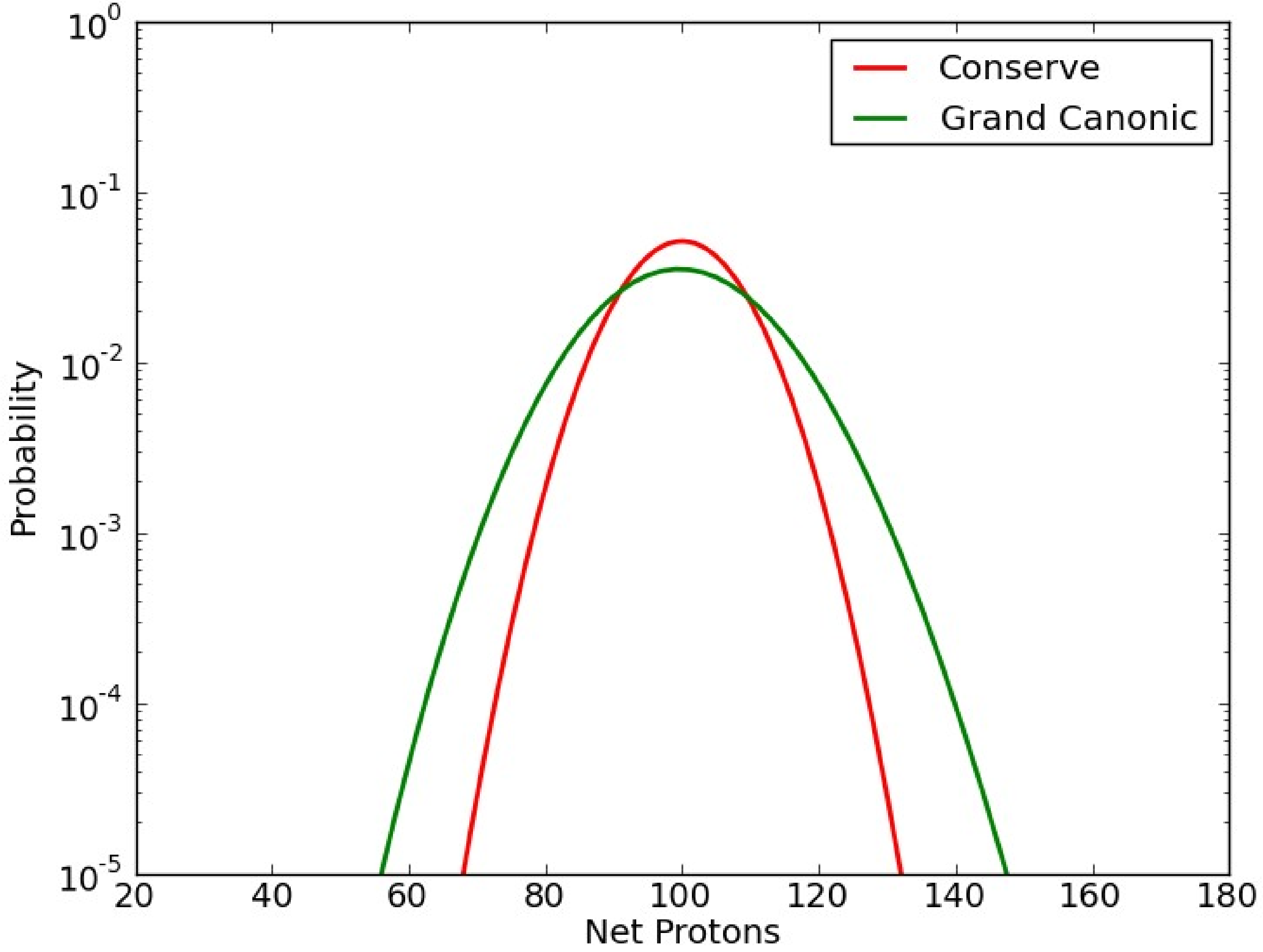
$$Z=\sum_{\Pi,\nu,q} I_\Pi(2N_{nucl})\,I_\nu(2N_{nucl})\,I_q(2N_{pions})\,\delta(B-\Pi-\nu)\,\delta(Q-\Pi-q)$$

$$Z=\sum_\Pi I_\Pi(2N_{nucl})\,I_{B-\Pi}(2N_{nucl})\,I_{q-\Pi}(2N_{pions})$$

Probability distribution

$$P(\Pi)=\frac{I_\Pi(2N_{nucl})\,I_{B-\Pi}(2N_{nucl})\,I_{q-\Pi}(2N_{pions})}{Z}$$

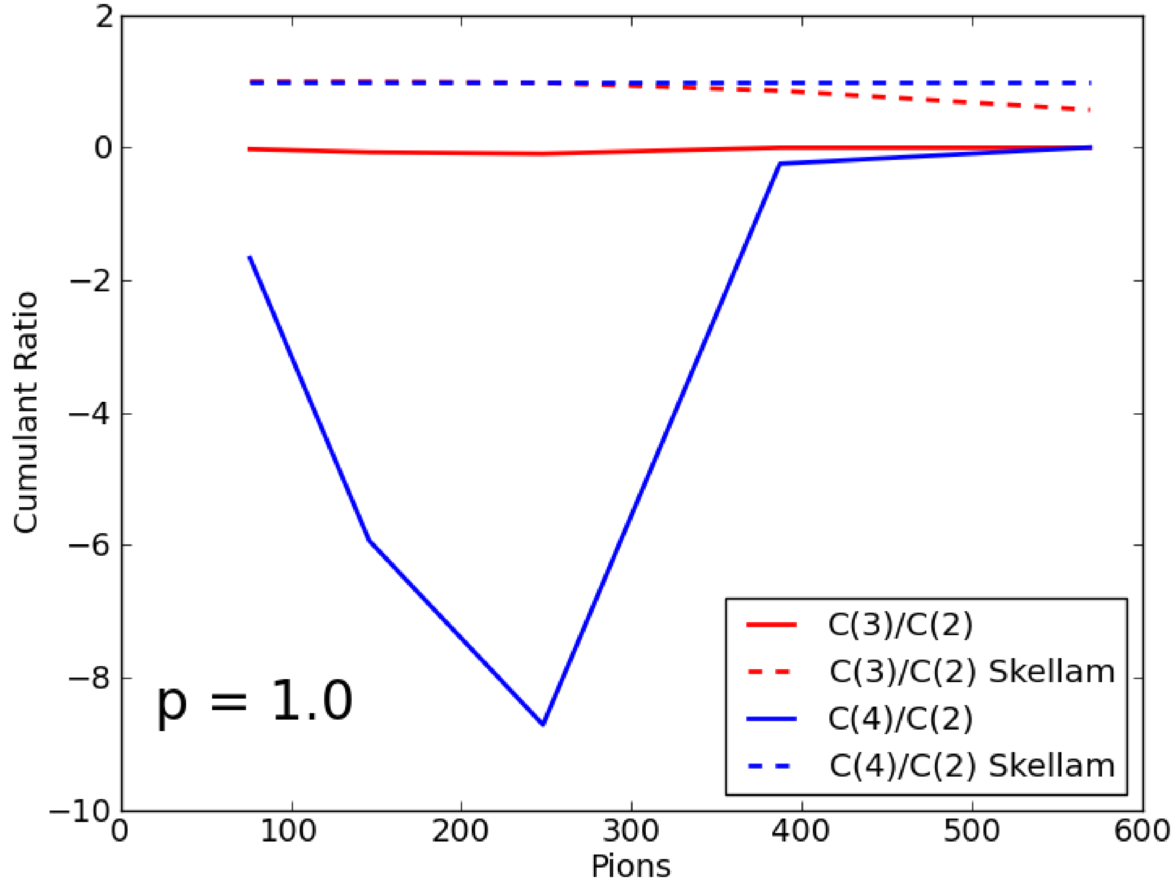
Probability distribution



B=200, Q=100, Z_1(nucleons)=40, Z_1(pion)=500

“Temperature Dependence”

Temperature (beam energy) dependence enters through the single particle Partition functions Z_1(pion) and Z_1(nucleon)

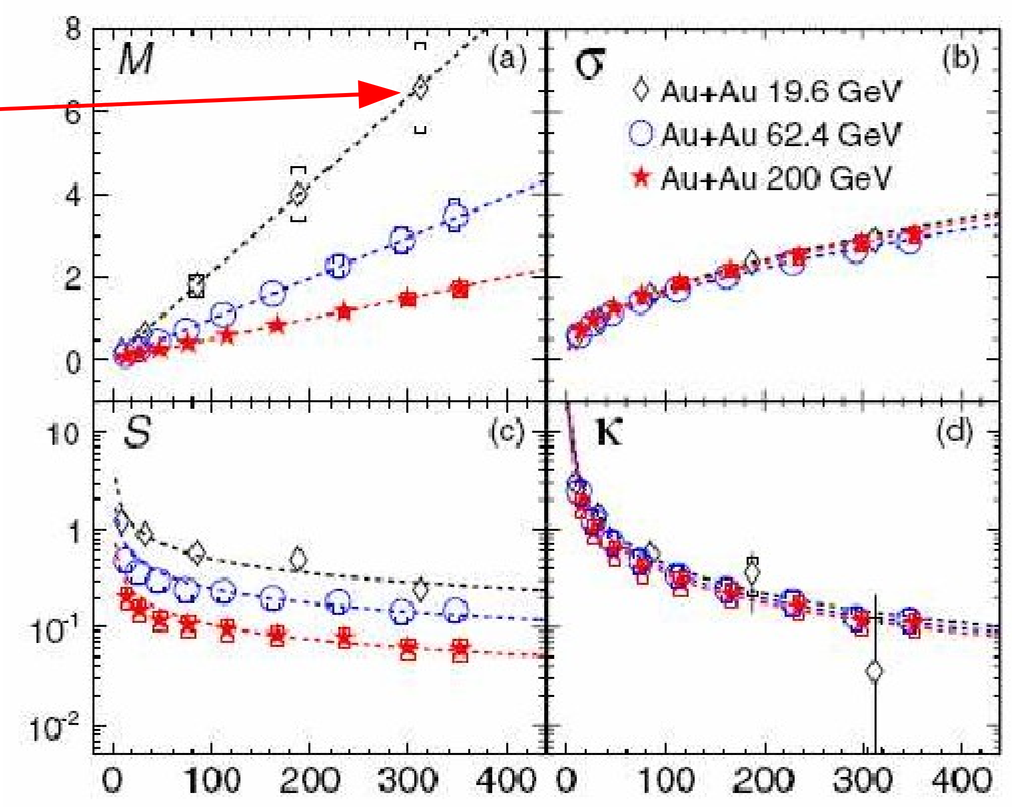


T=100 MeV

T=180 MeV

Finite acceptance

8 protons!!!



$$\sigma^2 \approx N_p + N_{\bar{p}}$$
$$\Rightarrow N_p + N_{\bar{p}} \approx const$$

Finite Acceptance

Effect of conservation laws get reduced with finite acceptance:
"Equilibration via ignorance"

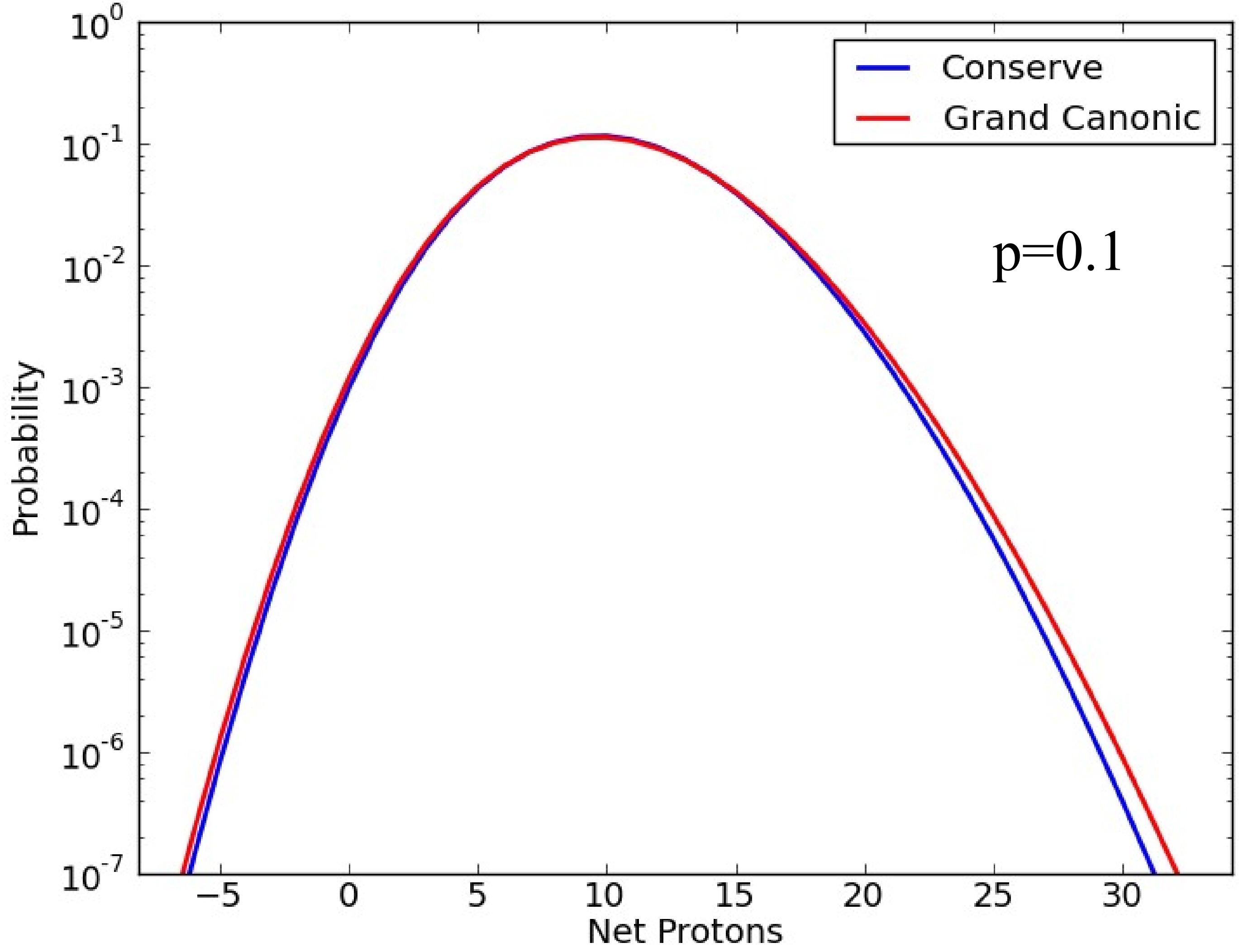
Model with biomial distribiution: p = probability to see particle

$$P(n;N)=\frac{N!}{n!(N-n)!}p^n(1-p)^{N-n}$$

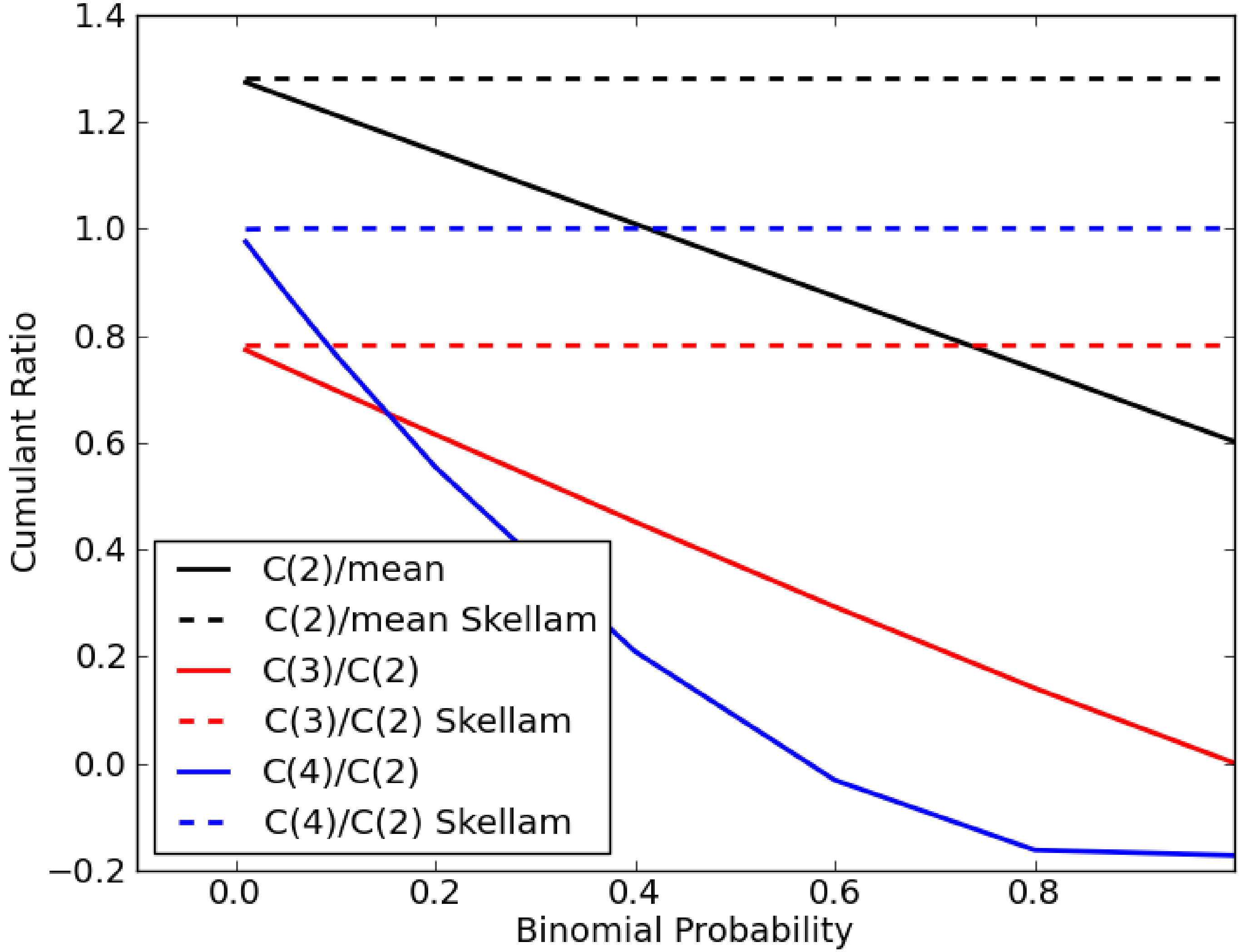
$$\begin{aligned}
C_{2,bin} &= p\mu + p^2(-\mu + k_2) \\
C_{3,bin} &= p\mu + p^2(-3\mu + 3k_2) + \underline{p^3}(2\mu - 3k_2 + \underline{k_3}) \\
C_{4,bin} &= p\mu + p^2(-7\mu + 7k_2) + p^3(12\mu - 18k_2 + 6k_3) \\
&\quad + \underline{p^4}(-6\mu + 11k_2 - 6k_3 + \underline{k_4})
\end{aligned}$$

μ, k_i = original cumulants

Probability Distribution



Cumulants



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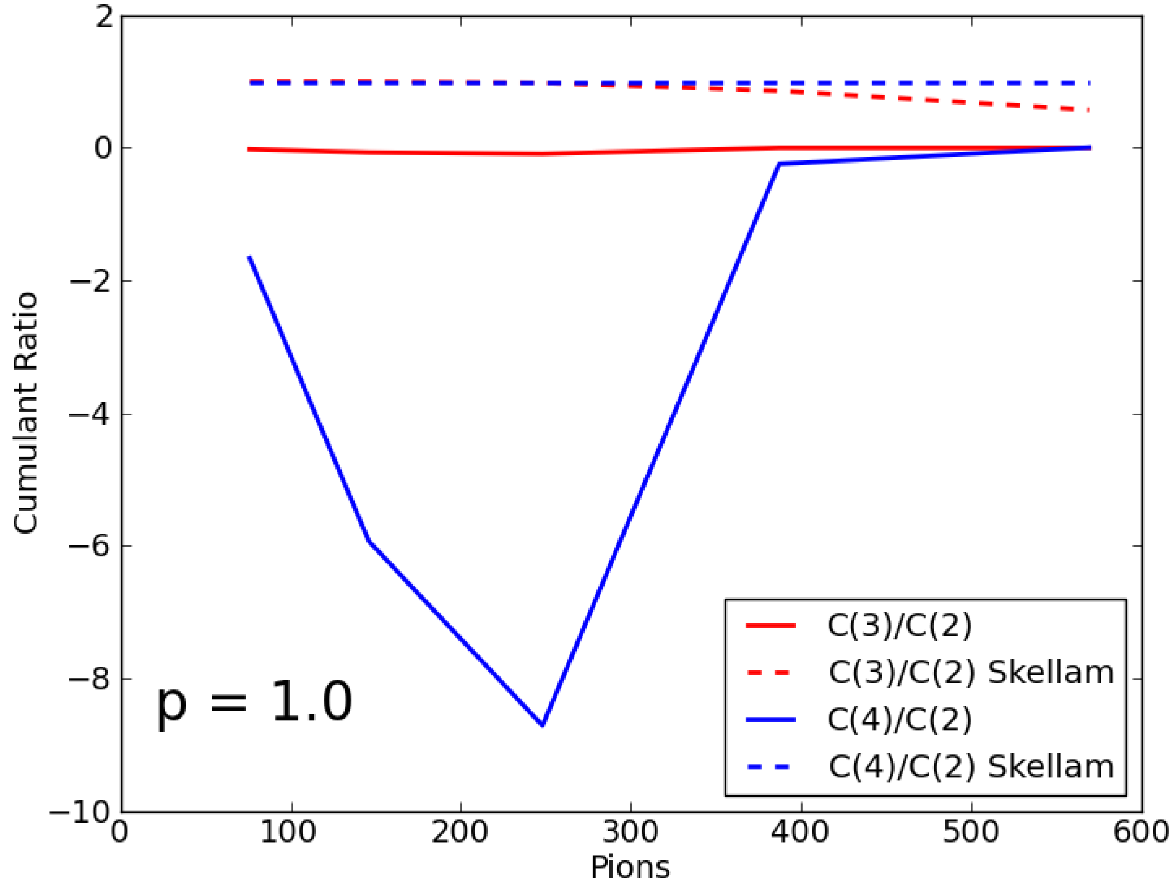

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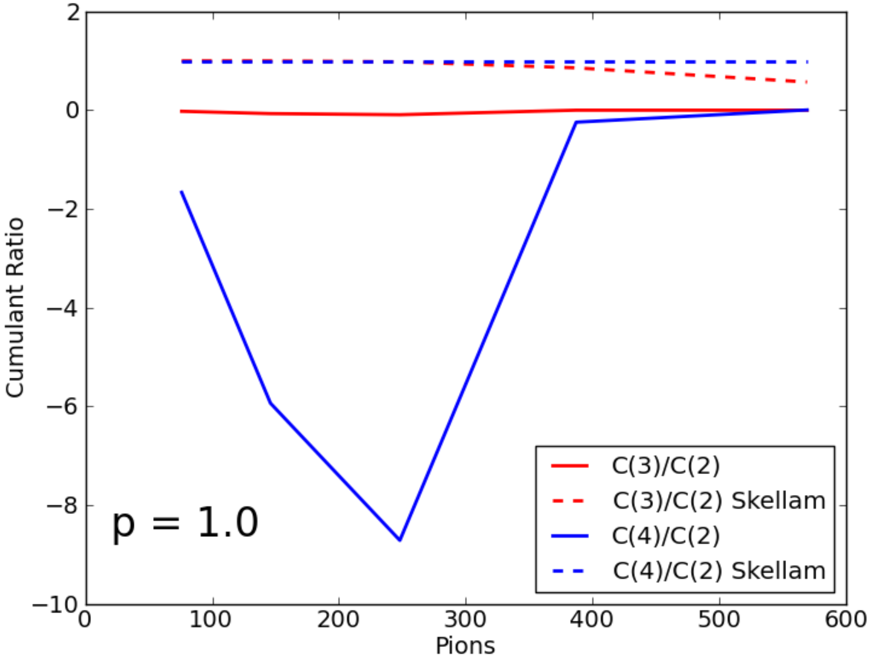
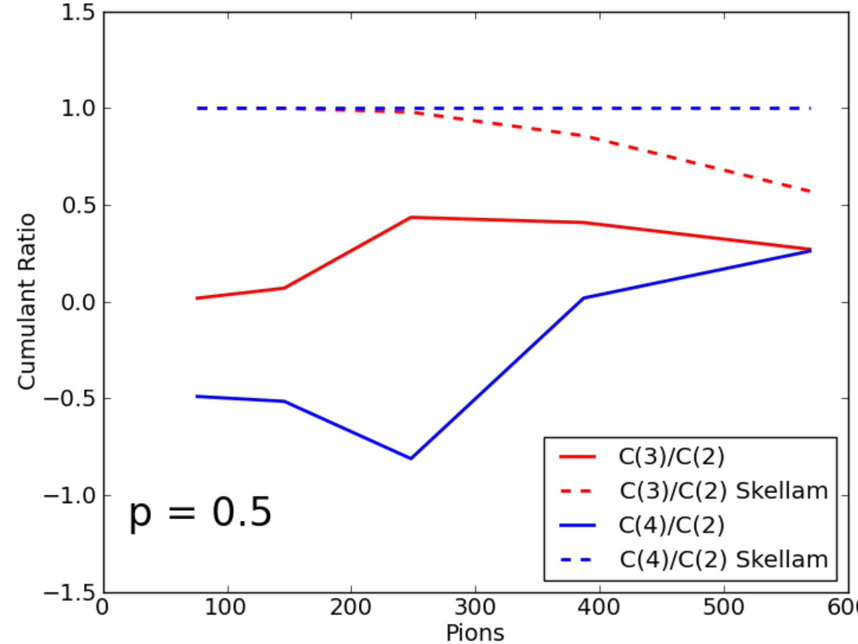
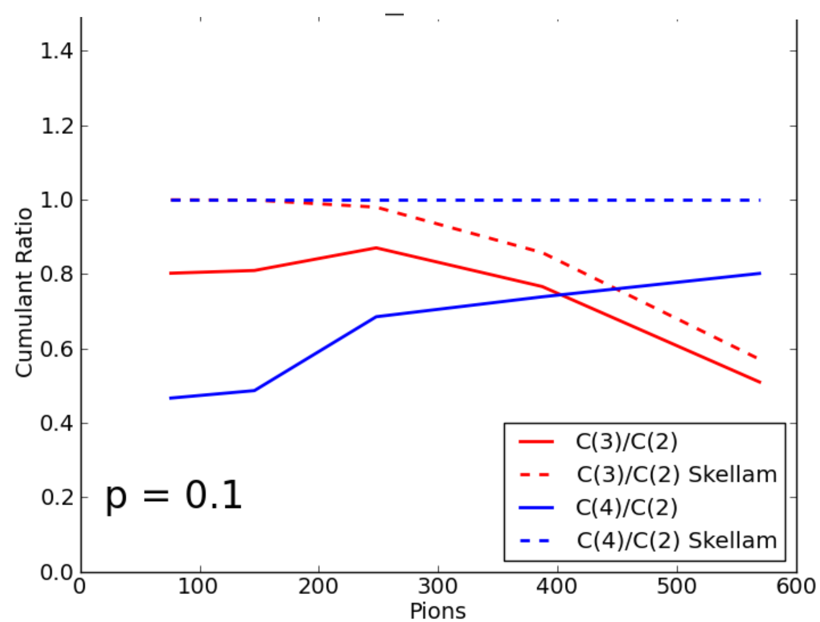






Furthermore

- Impact parameter (volume) fluctuations
  - C(4)/C(2) is independent of volume but NOT of volume fluctuations
- "Glauber" fluctuations
  - At high energy large fraction of baryons not part of the system
  - Can be addressed with transport (in principle)
- Can we unfold the effects of charge conservation?
  - Maybe.... at least at high energies.

Summary

- Higher cumulants are very sensitive to “charge” conservation
- Finite acceptance suppresses higher cumulants more strongly
- Not all baryons are protons
- Need to study dependence of e.g. size of rapidity window
- Sixth order cumulants for central Au+Au needs LOTS of statistics

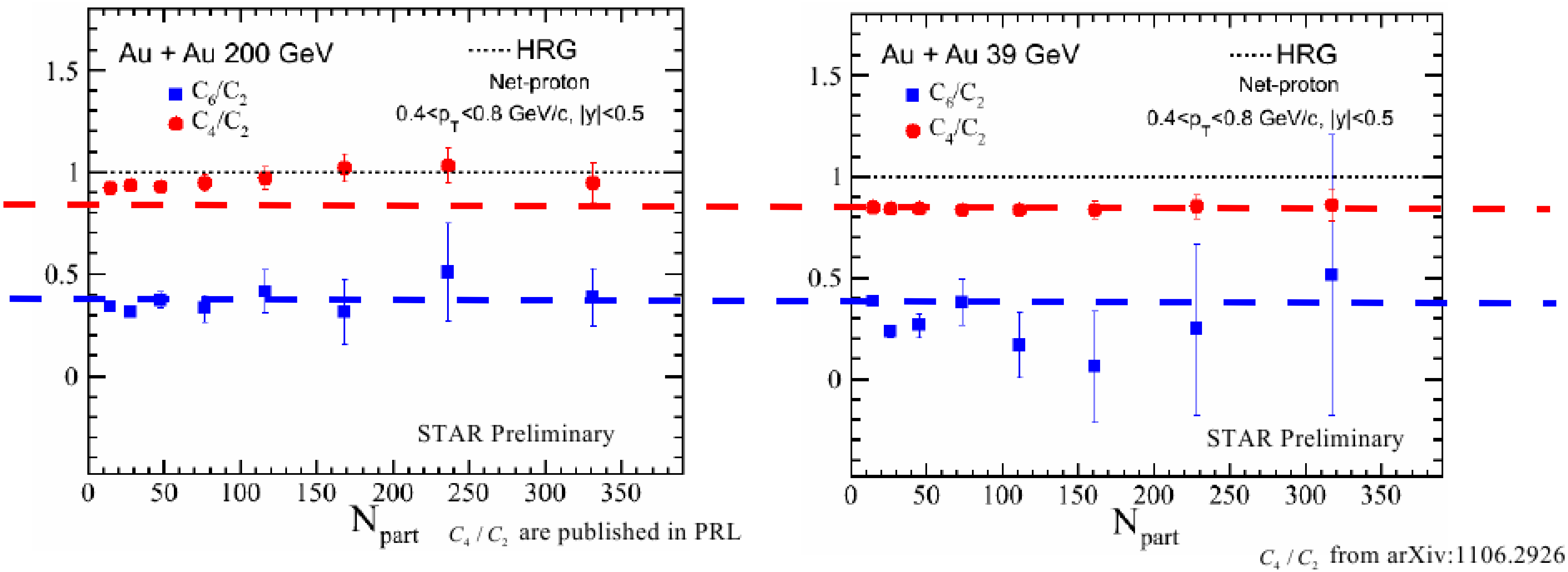
Volume Fluctuations

$$N = \rho V$$
$$\delta N = V \delta\rho + \rho \delta V$$

$$\langle(\delta N)^2\rangle = \langle V\rangle^2 \langle(\delta\rho)^2\rangle + \langle\rho\rangle^2 \langle(\delta V)^2\rangle$$
$$\langle(\delta N)^4\rangle = \langle V\rangle^4 \langle(\delta\rho)^4\rangle + 6\langle V\rangle^2 \langle\rho\rangle^2 \langle(\delta V)^2\rangle \langle(\delta\rho)^2\rangle + \langle\rho\rangle^4 \langle(\delta V)^4\rangle$$

$K\sigma^2 = \frac{(\delta N)^4}{(\delta N)^2} - 3(\delta N)^2$ is independent of the volume but not independent of volume fluctuations

The End



Centrality dependence ???