Dynamic fluctuations near the chiral phase transition

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SUBATECH, Nantes & FIAS, Frankfurt

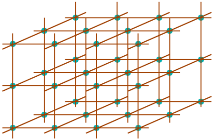
$7^t h$ International Workshop on Critical Point and Onset of Deconfinement, Wuhan, 2011

How to study the QCD phase diagram...

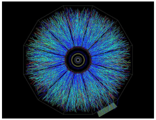
... be brave and solve

$$Z(T,\mu_B) = \int \mathcal{D}(A,q,q^\dagger)\mathrm{e}^{-S^E_{\mathrm{QCD}}}$$

ab initio and nonperturbatively,



... be strong and collide heavy ions at ultrarelativistic energies,



... be creative and study effective models of QCD.

$$\mathcal{L}_{\mathrm{eff}}$$

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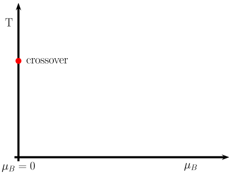

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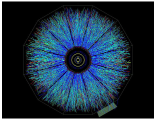

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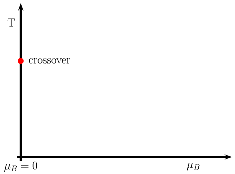

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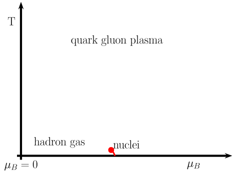

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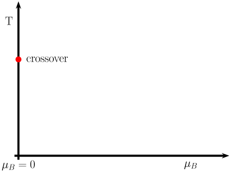

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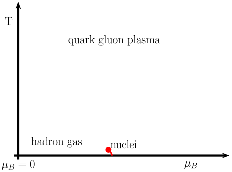

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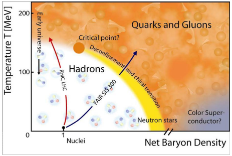
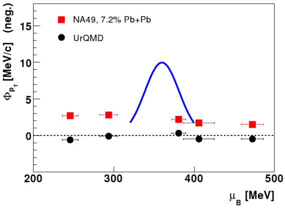


Fluctuations at the critical point

non-monotonic fluctuations (ebe) in pion and proton multiplicities

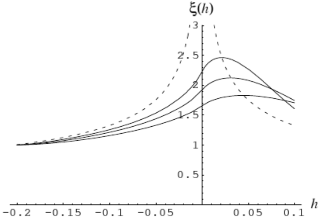
$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}$$

(M. A. Stephanov, K. Rajagopal and E. V. Shuryak, PRD **60** (1999))



(NA49 collaboration J. Phys. G **35** (2008))

BUT: critical slowing down



(B. Berdnikov and K. Rajagopal, PRD **61** (2000))

Fluctuation measures based on the second moments are not conclusive about the critical behavior.

Definition of the kurtosis

susceptibilities of conserved charges (N: net-baryon, net-charge number) or the experimentally feasible net-proton number

$$\chi_n(T,\mu_N) = \frac{1}{VT^3}\frac{\partial^n \ln Z(V,T,\mu_N)}{\partial(\mu_N/T)^n}\bigg|_T$$

effective kurtosis:

$$K^{\text{eff}} = \frac{\chi_4}{\chi_2} = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3\langle \delta N^2 \rangle \equiv \kappa\sigma^2 \, .$$

Higher moments of the distribution of conserved quantities are more sensitive to critical phenomena.

$$\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2 \propto \xi^7$$

(M. A. Stephanov, PRL **102**, 032301 (2009))

The kurtosis is negative on the crossover side of the critical point!

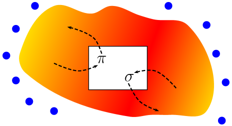
(M. A. Stephanov, PRL **107**, 052301, (2011))

1. Dynamic fluctuations of the order parameter of chiral symmetry in chiral fluid dynamics.
2. Net-baryon and net-proton kurtosis in UrQMD.

Motivation

- Fluctuations have so far been investigated in static systems.
- However, systems created in a heavy-ion collisions are finite in size and time and inhomogeneous.
- Necessary to propagate fluctuations explicitly!

- Nonequilibrium chiral fluid dynamics:
  - phase transition model +
  - dissipation and noise +
  - fluid dynamics

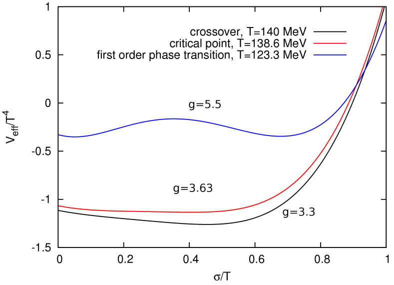


The linear sigma model with constituent quarks

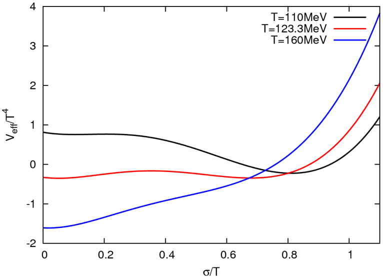
$$\mathcal{L} = \overline{q}\left[i\gamma^{\mu}\partial_{\mu} - g\left(\sigma + i\gamma_5\tau\vec{\pi}\right)\right]q + 1/2\left(\partial_{\mu}\sigma\right)^2 + 1/2\left(\partial_{\mu}\vec{\pi}\right)^2 - U\left(\sigma,\vec{\pi}\right)$$

effective potential at $\mu_B = 0$

$$V_{\text{eff}} = -\frac{T}{V}\ln Z = -d_q T\int\frac{\mathrm{d}^3p}{(2\pi)^3}\ln\left(1+\exp\left(-\frac{E}{T}\right)\right) + U\left(\sigma,\vec{\pi}\right)$$



Tune the strength of the phase transition via the coupling g.



dynamic symmetry breaking
first order phase transition

Nonequilibrium chiral fluid dynamics

- Langevin equation for the sigma field: damping and noise from the interaction with the quarks

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

- Fluid dynamic expansion of the quark fluid = heat bath

$$T_{\rm q}^{\mu\nu} = (e+p)u^\mu u^\nu - pg^{\mu\nu}$$

- Energy and momentum exchange

$$\partial_\mu T_{\rm q}^{\mu\nu} = S^\nu = -\partial_\mu T_\sigma^{\mu\nu}$$

$\Longrightarrow$ Selfconsistent approach within the two-particle irreducible effective action!

(MN, S. Leupold, C. Herold, M. Bleicher, PRC **84** (2011))

The two-particle irreducible (2PI) effective action

for the σ mean field and the full quark propagators S^{ab}

$$\Gamma[\sigma, S] = S_{\rm cl}[\sigma] - i{\rm Tr}\ln S^{-1} - i{\rm Tr} S_0^{-1} S + \Gamma_2[\sigma, S]\,,$$

equation of motion for σ and S^{ab}

$$\frac{\delta\Gamma[\sigma, S]}{\delta\sigma^a} = 0 \quad \text{and} \quad \frac{\delta\Gamma[\sigma, S]}{\delta S^{ab}} = 0$$

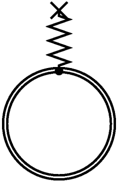
give conserving transport equations if the self-energy is given by

$$-i\Sigma^{ab}(x,y) = -\frac{\delta\Gamma_2[\sigma, S]}{\delta S^{ab}(x,y)}\,.$$

(J. M. Luttinger, J. C. Ward, Phys. Rev. **118** (1960); G. Baym, L. P. Kadanoff, Phys. Rev. **124** (1961); G. Baym, Phys. Rev. **127** (1962))

The 2PI effective action

$$\Gamma_2[\sigma, S] = g \int_{\mathcal{C}} \mathrm{d}^4 x \mathrm{tr}(S^{++}(x,x)\sigma^+(x) + S^{--}(x,x)\sigma^-(x))$$

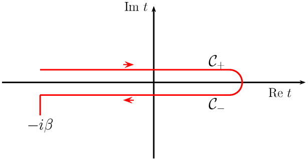


equation of motion for the σ mean field

$$-\frac{\delta S_{\mathrm{cl}}[\sigma]}{\delta \sigma^a} = \frac{\delta \Gamma_2[\sigma, S]}{\delta \sigma^a} = g \mathrm{tr} S^{aa}(x,x)$$

the effective action along the contour

$$\begin{aligned}\Gamma[\sigma, S] =& g \mathrm{tr} S_{\mathrm{th}}^{++}(x,x)\Delta\sigma(x) - \frac{T}{V} \ln Z_{\mathrm{th}} \\ &+ \int \mathrm{d}^4 x D[\bar{\sigma}](x)\Delta\sigma(x) \\ &+ \frac{i}{2} \int \mathrm{d}^4 x \int \mathrm{d}^4 y \Delta\sigma(x) \mathcal{I}[\bar{\sigma}](x,y)\Delta\sigma(y)\end{aligned}$$



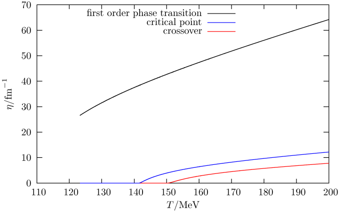
with $\Delta\sigma = \sigma^+ - \sigma^-$ and $\bar{\sigma} = 1/2(\sigma^+ + \sigma^-)$ on the contour.

Semiclassical equation of motion for the sigma field

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

damping term η and noise ξ for $\mathbf{k} = 0$

$$\eta = g^2 \frac{d_q}{\pi} \left(1 - 2n_{\mathrm{F}}\left(\frac{m_\sigma}{2}\right)\right) \frac{\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{\frac{3}{2}}}{m_\sigma^2}$$



$$\langle \xi(t) \xi(t') \rangle = \frac{1}{V} \delta(t - t') m_\sigma \eta \coth\left(\frac{m_\sigma}{2T}\right)$$

below T_c damping by the interaction with the hard pion modes, apply $\eta = 2.2/\mathrm{fm}$ from (T. S. Biro and C. Greiner, PRL **79** (1997))

Fluid dynamics with energy-momentum exchange

Energy-momentum tensor of the coupled system is conserved for the full propagator:

$$
\begin{aligned}
\partial_\mu T_{\mathrm{q}}^{\mu\nu} &= g \mathrm{tr} S^{++}(x,x) \\
\partial_\mu T_{\sigma}^{\mu\nu} &= -g \mathrm{tr} S^{++}(x,x)
\end{aligned}
$$

then $\partial_\mu (T_{\mathrm{q}}^{\mu\nu} + T_{\sigma}^{\mu\nu}) = 0$
HERE, approximation of an ideal fluid and the source term

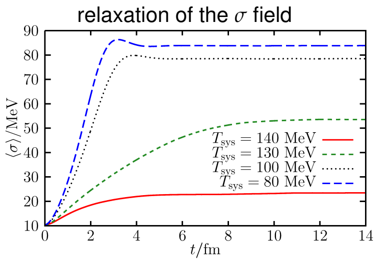
$$
\begin{aligned}
\partial_\mu T_{\mathrm{q}}^{\mu\nu} =& g \mathrm{tr} S_{\mathrm{th}}^{++}(x,x) \\
&= 4 d_q \int \frac{\mathrm{d}^4 p}{(2\pi)^4} p^\mu p^\nu f_{\mathrm{FD}}(E_p) \\
&= -\partial_\mu T_{\sigma}^{\mu\nu} = S^\nu = (g\rho_s + \eta \partial_t \sigma)\partial^\nu \sigma
\end{aligned}
$$

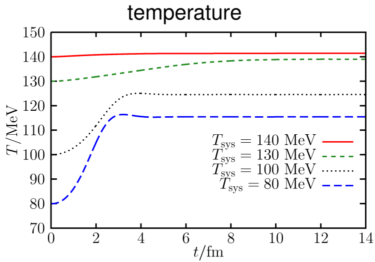
Evolution in a box

- nonexpanding, finite heat bath
- initialize the sigma field in equilibrium at $T > T_c$
- initialize the energy density at a $T_{\mathrm{sys}} < T_c$
- update sigma field on the grid according to the Langevin equation

Equilibration for a heat bath with reheating

Critical point





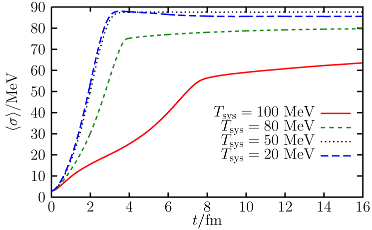
(MN, S. Leupold, M. Bleicher, arXiv:1105.1396)

- During relaxation of the σ-field the temperature of the heat bath increases.
- Coupled dynamics equilibrate at a given T_{eq} and σ_{eq}.
- Green curves correspond to scenarios with T_{eq} near T_c.
  $\Rightarrow$ Critical slowing down!

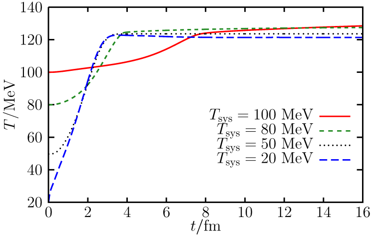
Equilibration for a heat bath with reheating

First order phase transition

relaxation of the σ field



temperature



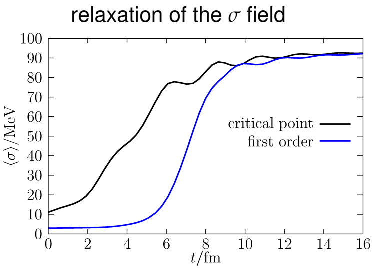
- Strong reheating during relaxation of the σ-field.
- Long (initial) relaxation times for T_{sys} close to the phase transition.
- Except for the scenario with $T_{\mathrm{sys}} = 20$ MeV the heat bath reheats to $T > T_c$.
- System gets trapped in metastable states.

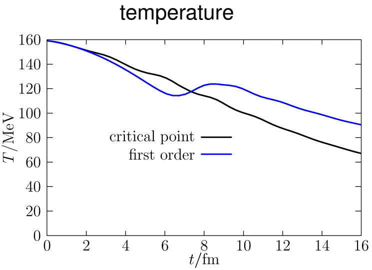
(MN, S. Leupold, M. Bleicher, arXiv:1105.1396)

Fluid dynamic expansion of the heat bath

- very simple initial conditions: almond-shaped initial temperature distribution, sigma field and energy density in equilibrium at $T(x)$
- 3+1d fluid dynamic expansion
- update sigma field on the grid according to the Langevin equation
- very good energy conservation

Reheating and supercooling





- oscillations at the critical point
- supercooling of the system at the first order phase transition
- reheating effect visible at the first order phase transition

MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962

Intensity of sigma fluctuations

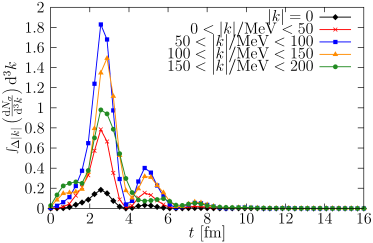
in one event

$$\frac{\mathrm{d}N_\sigma}{\mathrm{d}^3k} = \frac{(\omega_k^2|\sigma_k|^2 + |\partial_t\sigma_k|^2)}{(2\pi)^3 2\omega_k}$$

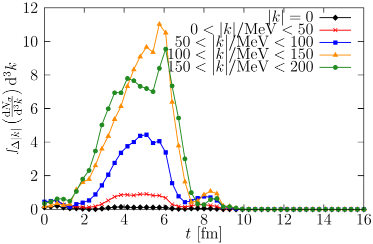
$$\omega_k = \sqrt{|k|^2 + m_\sigma^2}$$

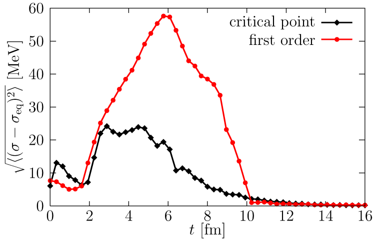
$$m_\sigma = \sqrt{\partial^2 V_{\mathrm{eff}}/\partial\sigma^2|_{\sigma=\sigma_{\mathrm{eq}}}}$$

critical point



first order phase transition

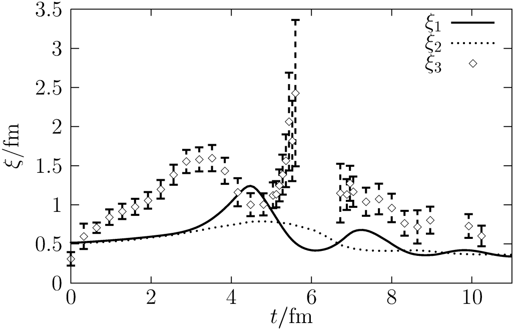




deviation from equilibrium

MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962

Correlation length at the critical point



ξ_1: averaged correlation length from $\xi^{-1} = \sqrt{\frac{\partial^2 \Omega}{\partial \sigma^2}\Big|_{\sigma=\sigma(x)}}$

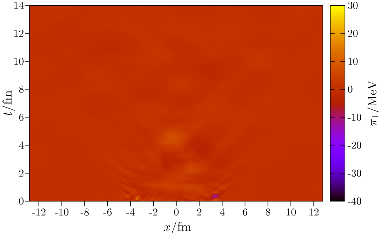
ξ_3: averaged correlation length from $\xi^{-1} = \sqrt{\frac{\partial^2 \Omega}{\partial \sigma^2}\Big|_{\sigma=\sigma_{\mathrm{eq}}}}$

ξ_2: correlation length obtained from fits to $G(r) = \sigma_{\mathrm{eq}}^2 + \frac{1}{r}\exp(-\frac{r}{\xi})$

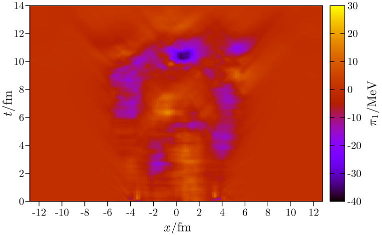
Pion fluctuations

So far: pion fluctuations were not considered and $\vec{\pi} = \langle \vec{\pi} \rangle = 0$.
Now: extend the model to explicitly propagate pion fluctuations, too.

critical point



first order phase transition



Larger isospin fluctuations in a scenario with a first order phase transition!

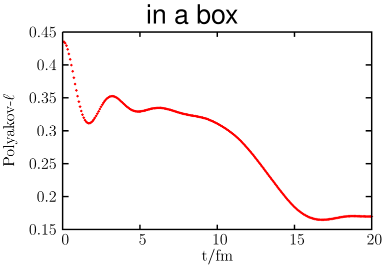
Polyakov-loop extended chiral fluid dynamics

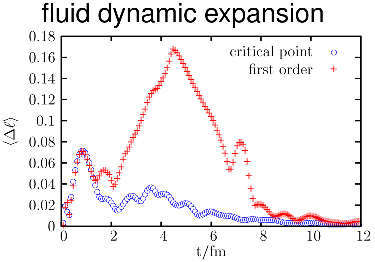
$$\mathcal{L} = \overline{q}\left[\mathrm{i}\left(\gamma^\mu\partial_\mu - \mathrm{i}g_{QCD}\gamma^0 A_0\right) - g\sigma\right]q + 1/2\left(\partial_\mu\sigma\right)^2 - U(\sigma) - \mathcal{U}(\ell,\bar{\ell})$$

(C. Ratti, M. A. Thaler, W. Weise, Phys. Rev. D **73** (2006), B.-J. Schaefer, J. M. Pawlowski and J. Wambach, Phys. Rev. D **76** (2007))

dynamics of the Polyakov loop (A. Dumitru and R. D. Pisarski, Nucl. Phys. A **698** (2002))

$$\frac{2N_c}{g_{QCD}^2}\partial_\mu\partial^\mu\ell T^2 + \eta_\ell\partial_t\ell + \frac{\partial V_{eff}}{\partial\ell} = \xi_\ell$$





(C.Herold, MN, I.Mishustin, M.Bleicher, in preparation

See talk by C.Herold on Friday 14.00, Parallel 5!

Relativistic Transport Approach

cover more effects in realistic simulations of heavy-ion collisions, here: UrQMD (www.urqmd.org)

issues:

- eventwise baryon number and charge conservation instead of grandcanonical ensembles
- centrality selection and centrality bin width effects

Analytic toy model

Baryon number conservation limits fluctuations of net-baryon number.

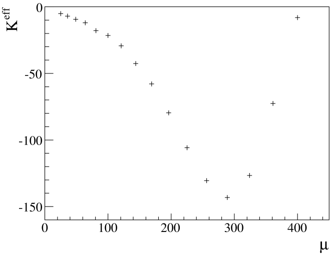
$$P_\mu(N, C) = \mathcal{N}(\mu, C) e^{-\mu} \frac{\mu^N}{N!} \quad \text{on} \quad [\mu - C, \mu + C]$$

μ: the expectation value of the original Poisson distribution, $\mathcal{N}(\mu, C)$: normalization factor, $C > 0$: cut parameter

$$C = \alpha \sqrt{\mu} \left(1 - \left(\frac{\mu}{N_{\text{tot}}} \right)^2 \right) .$$

$\alpha = 3$, $N_{\text{tot}} = 416$.

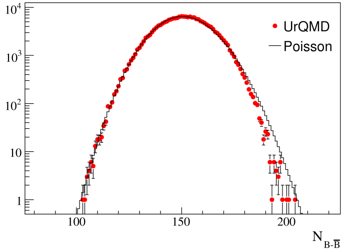
- An increase of the average net-baryon number does not lead to stronger fluctuations.
- At the upper limit of $N_{\text{tot}} = 416$ the distribution changes to a δ-function ($K_\delta^{\text{eff}} = 0$).

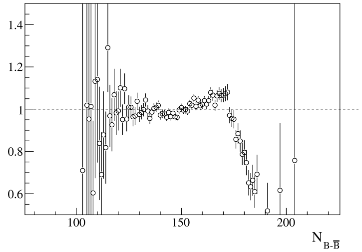


Net-baryon number distribution in UrQMD

- central Pb+Pb collisions at $E_{\mathrm{lab}} = 20A\mathrm{GeV}$
- fit to a Poisson distribution
- shoulders are enhanced
- tails are cut

$\Longrightarrow$ decrease from $K^{\mathrm{eff}}_{\mathrm{Poisson}} = 1$ to $K^{\mathrm{eff}}_{\mathrm{UrQMD}} = -22.2$



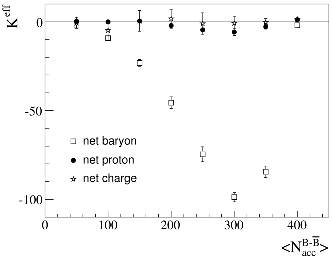


ratio of UrQMD to Poisson distribution

Rapidity window dependence of the effective kurtosis

- Same qualitative behavior of the net-baryon kurtosis as expected from the analytic toy model.

- $E_{\mathrm{lab}} = 158A\mathrm{GeV}$
- The net-proton kurtosis slightly follows this trend.
- The net-charge kurtosis is not influenced, but error bars are larger.

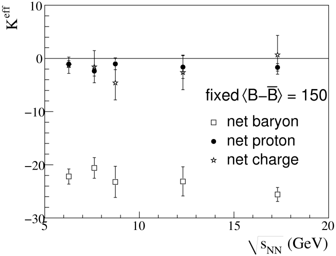


- For small net-baryon numbers in the acceptance, the values of net-baryon, net-proton and net-charge kurtosis are compatible with values of $0 - 1$.

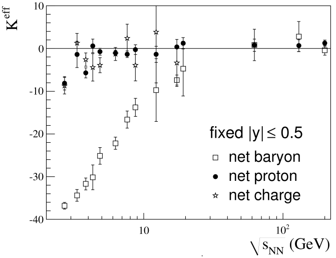
T. Schuster, MN, M. Mitrovski, R. Stock, M. Bleicher, [arXiv:0903.2911 [hep-ph]]

Energy dependence of the effective kurtosis

- adapting the rapidity window to fix the mean net-baryon number
- net-baryon effective kurtosis does not show an energy dependence



- fixed rapidity cut
- the net-baryon number varies with $\sqrt{s}$
- for lower $\sqrt{s}$ $K^{\rm eff}$ becomes increasingly negative
- at $E_{\rm lab} = 2A$GeV: $\langle N_{B-\bar{B}} \rangle \simeq 240$



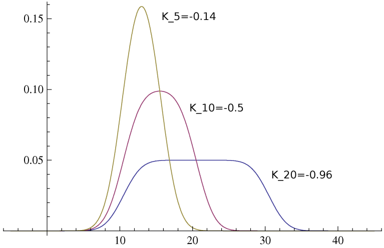
T. Schuster, MN, M. Mitrovski, R. Stock, M. Bleicher, [arXiv:0903.2911 [hep-ph]]

Centrality selection, e.g. by impact parameter

We investigate central collisions with $b \leq 2.75$ fm.
The superposition of two Gauss distributions (with mean $\mu_{1,2}$ and variance $\sigma_{1,2}$) has a negative kurtosis

$$K_2 = \frac{1/8\Delta\mu^4 + 3\Sigma^2\Delta\mu^2 + 6\Sigma^4}{1/8\Delta\mu^4 + \Sigma^2\Delta\mu^2 + 2\Sigma^4} - 3 < 0$$

with $\Delta\mu = |\mu_2 - \mu_1|$ and $\Sigma^2 = \sigma_1^2 + \sigma_2^2$.

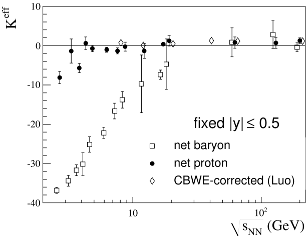


The distribution approaches a box-distribution with a $K_{\mathrm{box}} = -1.2$.

Effects of centrality selection

Suggestion by STAR to reduce centrality bin width effects:

- calculate moments for each fixed N_{charge} in one wider centrality bin
- take the weighted average



(MN et al., QM 2011 proceedings)

problems:

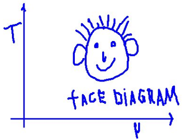
- (anti-) protons constitute a larger fraction of all charged particles with decreasing energy
- fixing N_{charge} puts a bias on the fluctuations
- baryon conservation and bias on fluctuation by centrality selection lead to negative values of the kurtosis

Summary

- supercooling and reheating effects in nonequilibrium chiral fluid dynamics
- enhanced sigma and pion fluctuations at a first order phase transition
- dynamic correlation length at the critical point
- baryon number conservation and bias on fluctuations by centrality selection lead to negative kurtosis

Outlook:

- extend chiral fluid dynamics to finite μ_B and study baryon density fluctuations
- use realistic initial conditions and study event-by-event fluctuations in chiral fluid dynamics



Fluid dynamics

Equation of state

EoS depends on the actual value of σ

pressure:

$$p(\sigma, T) = -V_{\text{eff}}(\sigma, T) + U(\sigma)$$

energy density:

$$e(\sigma, T) = T\frac{\partial p(\sigma, T)}{\partial T} - p(\sigma, T)$$

This relation is obtained from thermodynamic consistency, which is guaranteed by the 2PI effective action!

Fluid dynamics

Initial conditions

temperature profile, $T_{\rm ini} = 160$ MeV:

$$T(\vec{x}, t=0) = \frac{T_{\rm ini}}{(1+\exp((\tilde{r}-\tilde{R})/\tilde{a}))(1+\exp((|z|-l_z)/\tilde{a}))}$$

sigma field:

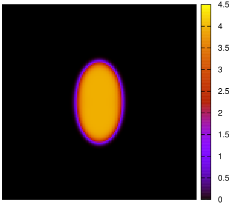
$$\sigma(\vec{x}, t=0) = \sigma_{\rm eq} + \delta\sigma(\vec{x})\,.$$

with

$$\langle \delta\sigma^2 \rangle = \frac{T}{V}\frac{1}{m_\sigma^2}\,.$$

energy density in units of e_0

$$e(\vec{x}, t=0) = e_{\rm eq}(T,\sigma)$$



Energy-momentum conservation

for the full propagator:

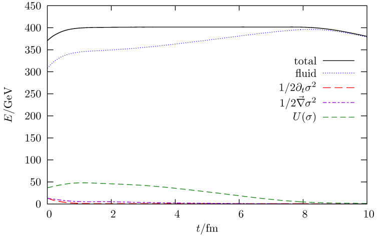
$$\partial_\mu (T_{\mathrm{q}}^{\mu\nu} + T_\sigma^{\mu\nu}) = 0$$

HERE, approximation of an ideal fluid and the source term

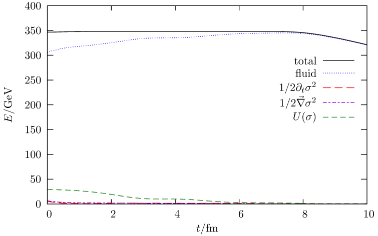
$$\partial_\mu T_{\mathrm{q}}^{\mu\nu} = g \,\mathrm{tr} S_{\mathrm{th}}^{++}(x,x) = -\partial_\mu T_\sigma^{\mu\nu} = S^\nu$$

(MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962)

first order phase transition



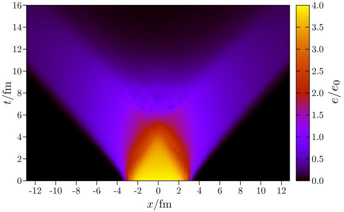
critical point



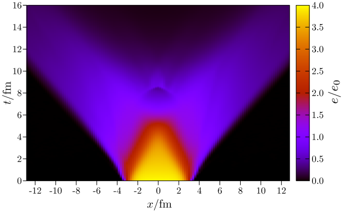
Time evolution

energy density

critical point

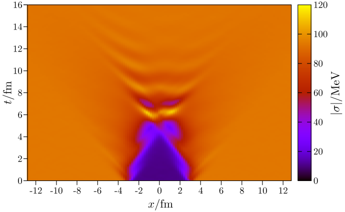


first order phase transition

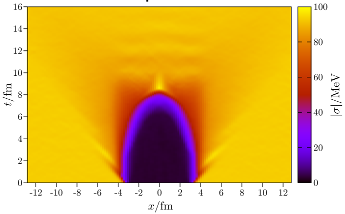


sigma field

critical point



first order phase transition

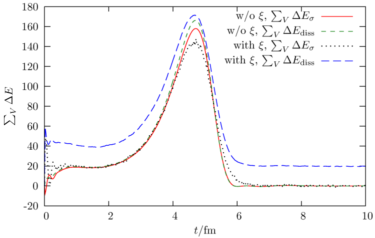


Energy transfer between the field and the heat bath

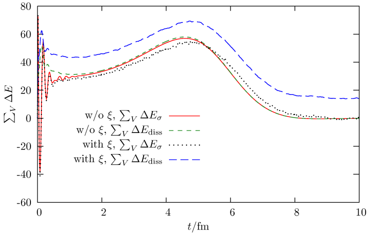
$$\Delta E_{\text{diss}} \simeq -\partial_\mu T_\sigma^{\mu 0} \Delta t = (g\rho_s + \eta \partial_t \sigma)\partial_t \sigma \Delta t$$

The total energy of the σ field

$$E_\sigma = 1/2 \partial_t \sigma^2 + 1/2 \vec{\nabla} \sigma^2 + U(\sigma)$$



first order phase transition
quench from $T = 160$ MeV to $T = 100$ MeV



critical point
quench from $T = 160$ MeV to $T = 130$ MeV