What Favors and Disfavors the Critical Point of QCD?

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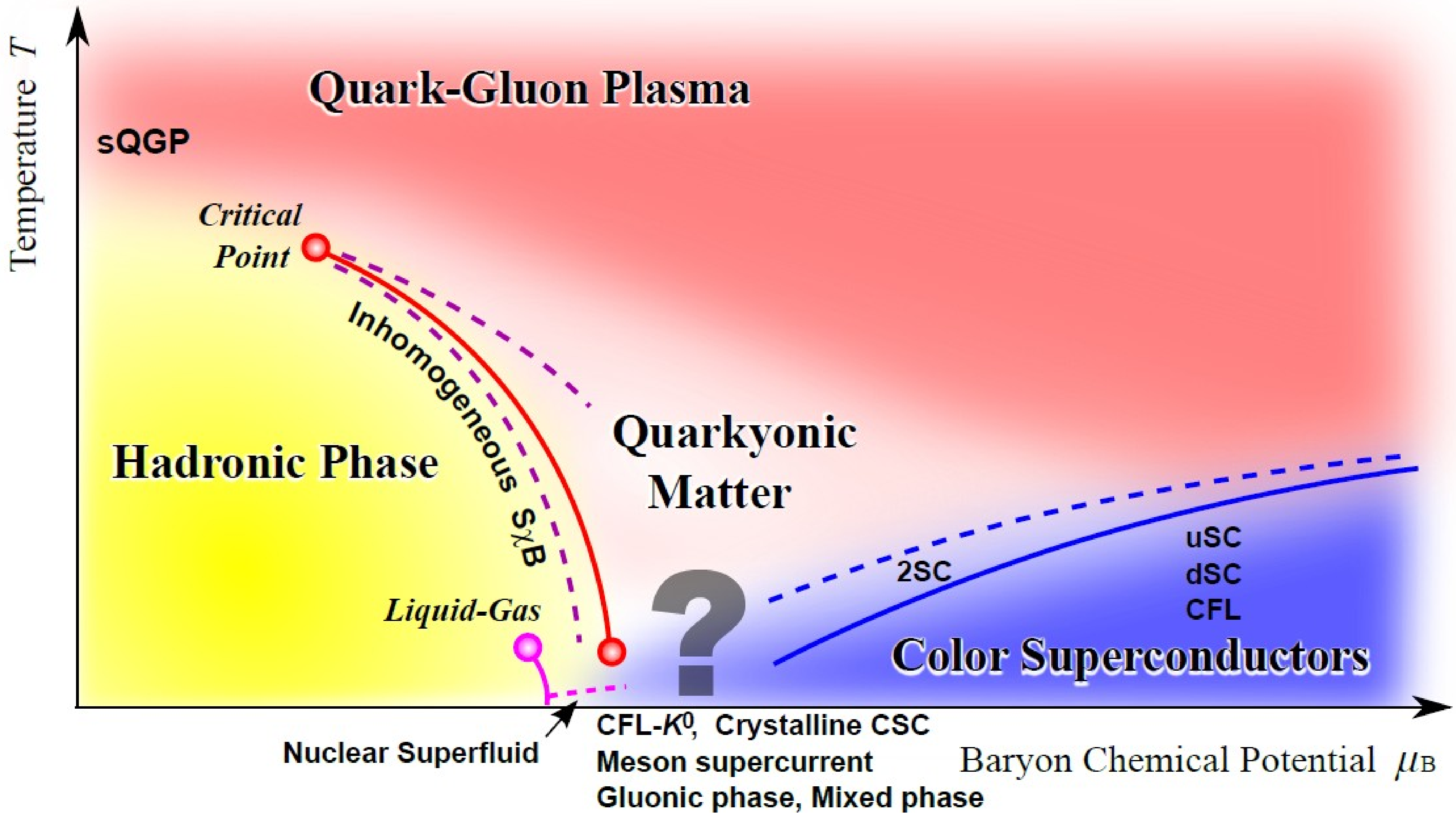
Talk Plan

- **Why many chiral models predict a first-order phase transition and the QCD critical point?**
- **Why those predictions are easily changed even qualitatively?**
- **Liquid-gas phase transition of nuclear matter**
  "Established" critical point of QCD
- **Understanding in analogy to nuclear matter**
- **Summary**

“Guessed” Phase Diagram of QCD

Fukushima-Hatsuda (2010)



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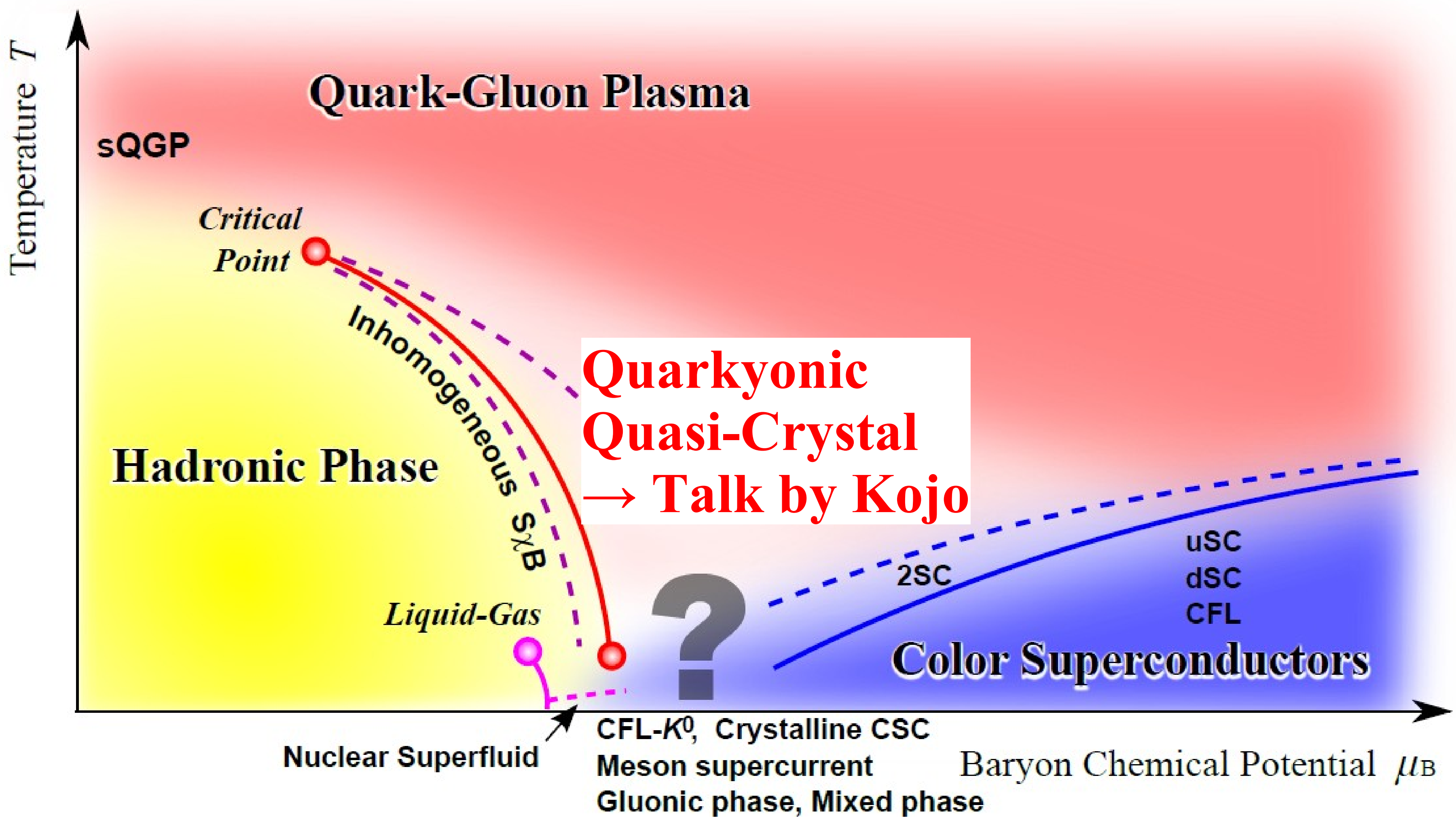

"Guessed" Phase Diagram of QCD

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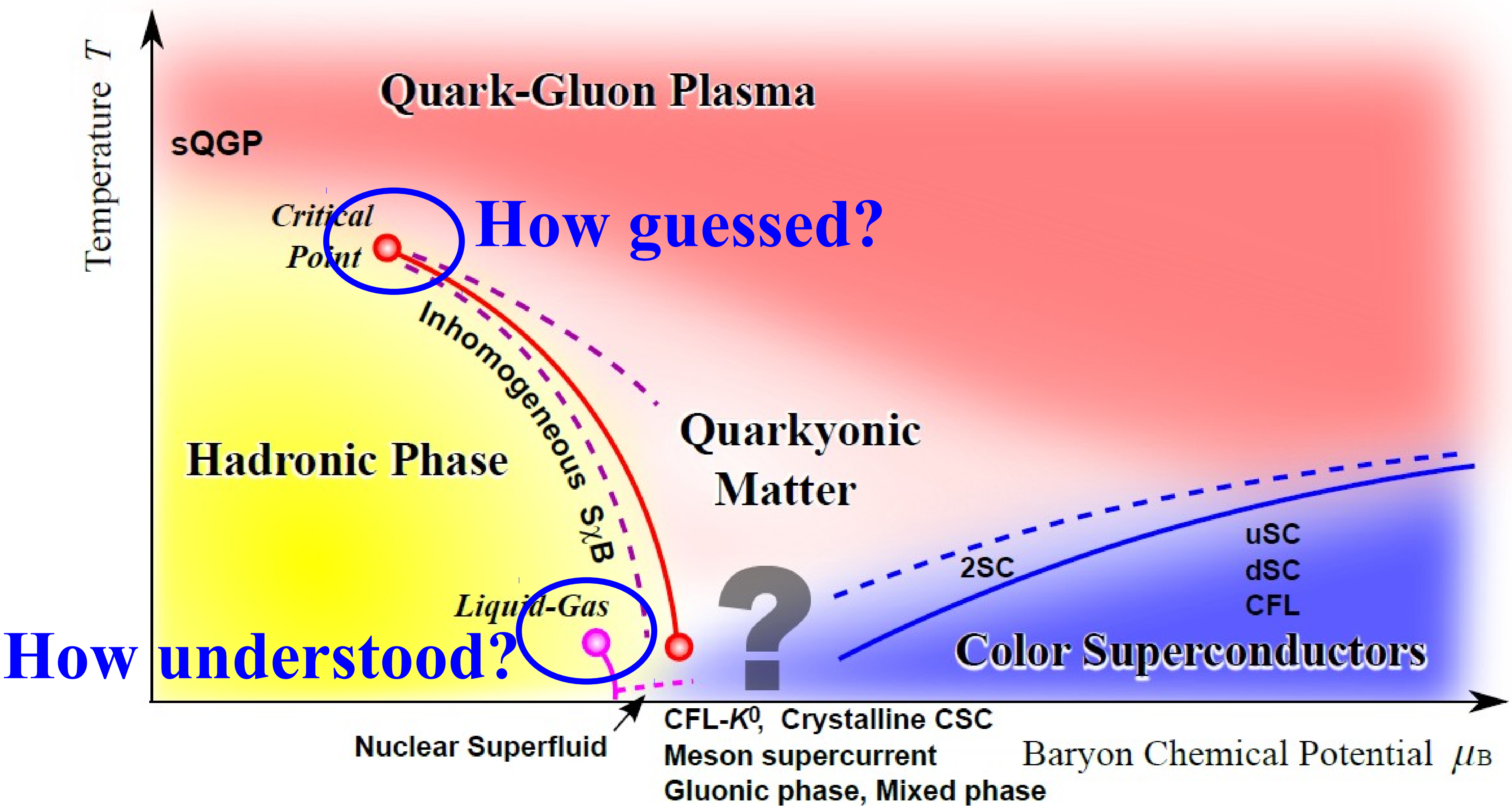
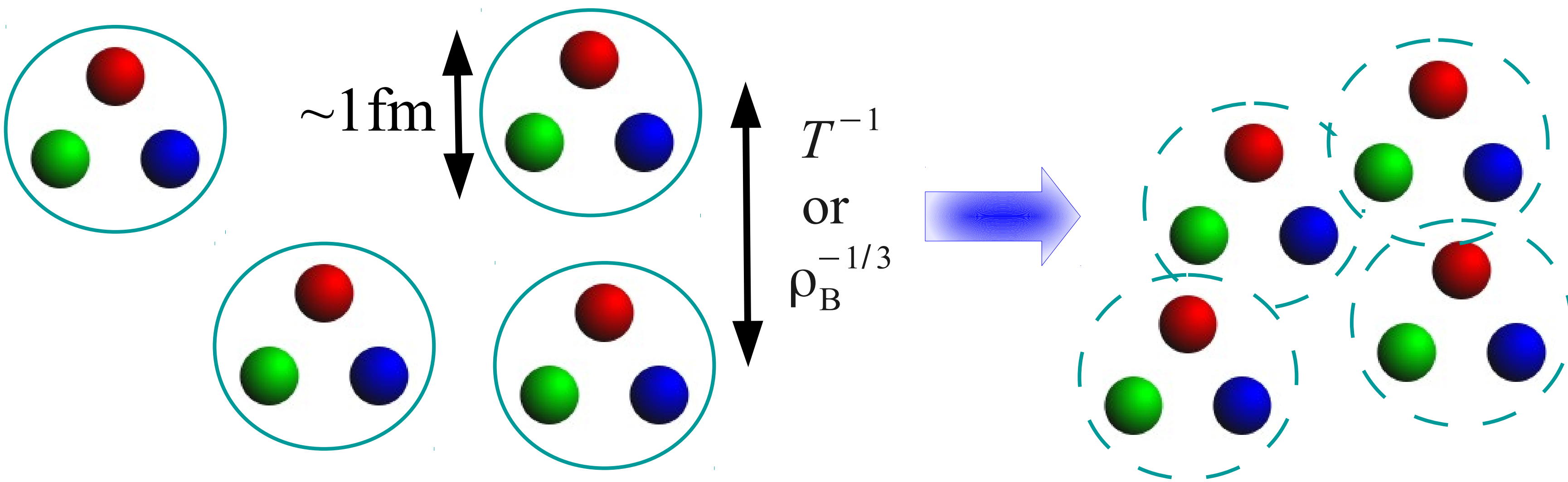


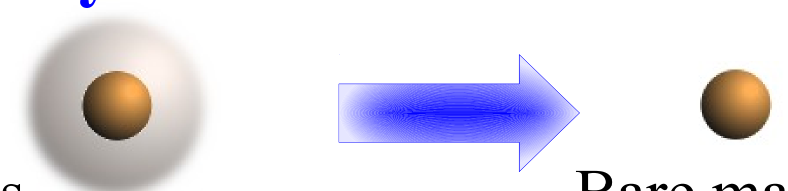
QCD Phase Transitions

Relativistic Heavy-Ion Collisions aim to see:

Color (or Quark) Deconfinement → Talk by Huang



Chiral Symmetry Restoration



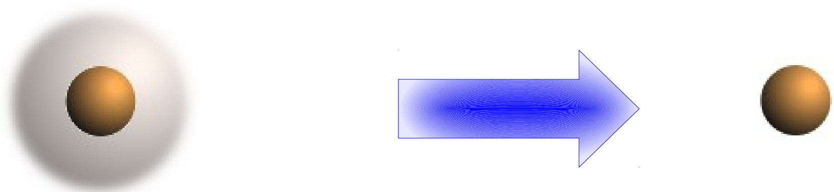
Constituent mass
$M \sim$ a third of M_B (~ 340MeV)

Bare mass
$m \sim$ a few MeV (< 5MeV)

QCD Phase Transitions

Relativistic Heavy-Ion Collisions aim to see:

Chiral Symmetry Restoration



QCD Critical Point (formerly called **C**ritical **E**nd-**P**oint)

If this is found, it would be the first clear indication for the chiral phase transition in the heavy-ion experiment.

(Dilepton measurement may give a signature, but indirect.)

Deconfinement is, on the other hand, already evident...

(Quark number scaling for example)

Coherent Tendency

Many chiral models coherently predict a 1st-order phase transition at high baryon density.

Nambu—Jona-Lasinio (NJL) Model **Asakawa-Yazaki (1989)**

Quark-Meson (QM) Model (~ Linear-σ Model)

Polyakov-loop Coupled NJL (PNJL) Model

Polyakov-loop Coupled QM (PQM) Model

Chiral Random Matrix Model (~ NJL Model)

Strong-coupling Expansion (~ NJL Model)

Look like various models, but they are relatives...

"Model-independent" Consideration

Pressure in a "Quasi-Quark" Description

$$P = 2N_c N_f \int^{\Lambda} \frac{d^3 p}{(2\pi)^3} \sqrt{p^2 + M^2}$$ ← Zero-point Energy

$$+ \; 2N_c N_f T \int \frac{d^3 p}{(2\pi)^3} \left(\ln\left[1 + e^{-(\sqrt{p^2+M^2}-\mu)/T}\right] + \ln\left[1 + e^{-(\sqrt{p^2+M^2}+\mu)/T}\right] \right)$$ ← Medium Effects

$$- \frac{M^2}{4G}$$ ← Interaction Energy

Zero-point Energy favors larger *M*
Interaction Energy favors smaller *M* ⇒ **Optimal *M* chosen**

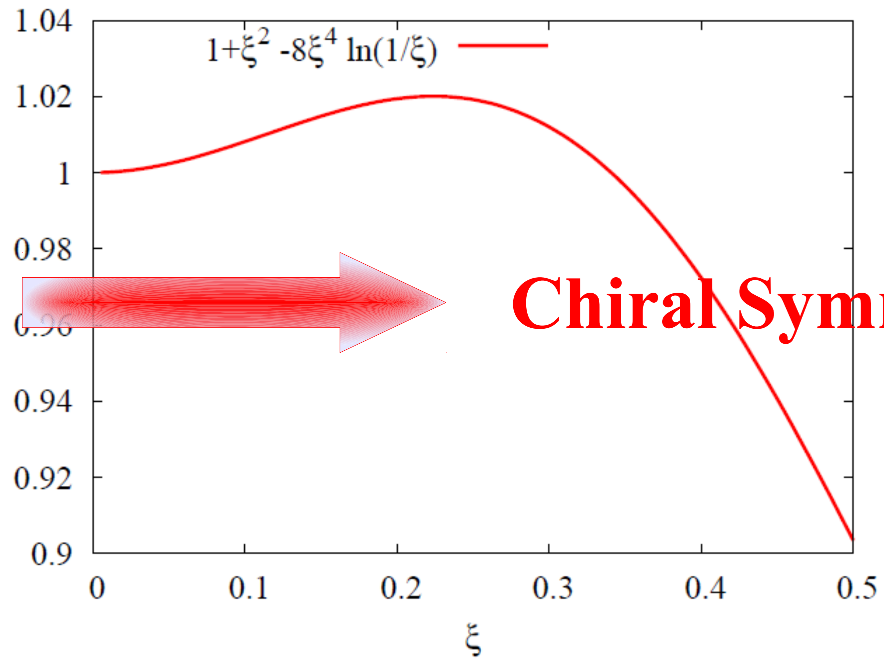
Medium Effects favor smaller *M* ⇒ **Chiral Phase Transition**

Zero-Point Energy

$$2N_c N_f \int^\Lambda \frac{d^3p}{(2\pi)^3}\sqrt{p^2+M^2}$$

$$=\frac{N_c N_f \Lambda^4}{\pi^2}\int_0^1 dx\ \ x^2\sqrt{x^2+\xi^2} \qquad (\xi = M/\Lambda)$$

$$\simeq \frac{N_c N_f \Lambda^4}{4\pi^2}\left(1+\xi^2-\frac{1}{2}\xi^4\ln(1/\xi)\right) \qquad (\text{for small } \xi)$$



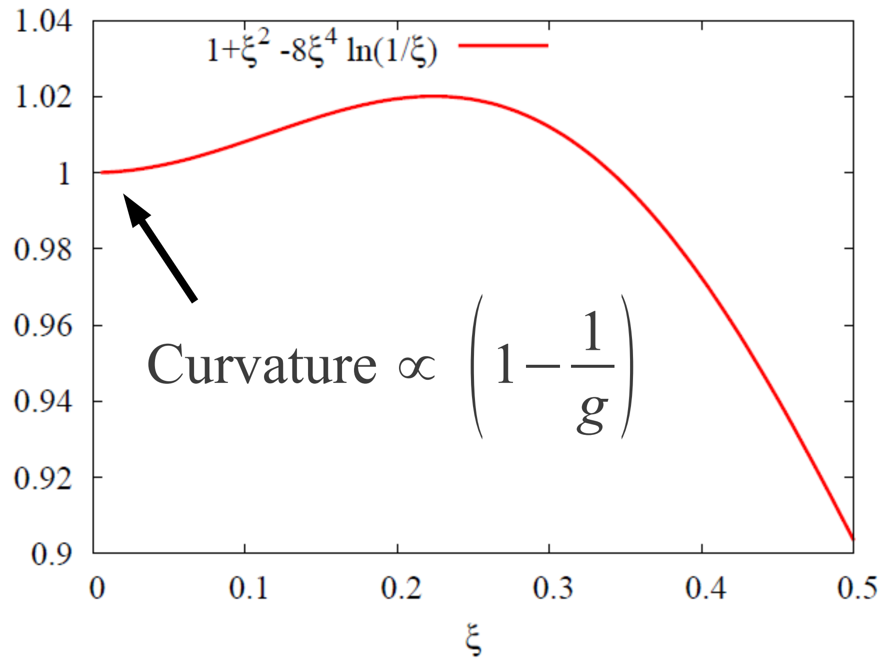
Chiral Symmetry Breaking

Interaction Energy

$$-\frac{M^2}{4G} = -\frac{N_c N_f \Lambda^4}{4\pi^2 g}\xi^2 \qquad \left(G = \frac{\pi^2 g}{N_c N_f \Lambda^2}\right)$$

Vacuum Energy = Zero-point Energy + Interaction Energy

$$P_\chi \simeq \frac{N_c N_f \Lambda^4}{4\pi^2}\left(1 + \left(1 - \frac{1}{g}\right)\xi^2 - \frac{1}{2}\xi^4 \ln\frac{1}{\xi}\right)$$



Medium Effects

$$P_\mu = 2 N_c N_f T \int \frac{d^3 p}{(2\pi)^3} \left(\ln\left[1 + e^{-(\sqrt{p^2+M^2}-\mu)/T}\right] + \ln\left[1 + e^{-(\sqrt{p^2+M^2}+\mu)/T}\right] \right)$$

$$= 2 N_c N_f \int_0^\mu d\mu' \int \frac{d^3 p}{(2\pi)^3} \left(\frac{1}{e^{(\sqrt{p^2+M^2}-\mu')/T}+1} - \frac{1}{e^{(\sqrt{p^2+M^2}+\mu')/T}+1} \right)$$

$$4 N_c N_f T \int \frac{d^3 p}{(2\pi)^3} \ln\left[1 + e^{-\sqrt{p^2+M^2}/T}\right]$$

Density

Temperature

$$\rightarrow 2 N_c N_f \int_0^\mu d\mu' \int \frac{d^3 p}{(2\pi)^3} \theta\left(\mu' - \sqrt{p^2+M^2}\right) \qquad (T \to 0)$$

$$= \frac{N_c N_f}{3\pi^2} \int_M^\mu d\mu' (\mu'^2 - M^2)^{3/2} \; \theta(\mu - M)$$

$$= \frac{N_c N_f}{12\pi^2} \left(p_F \mu^3 - \frac{5}{2} M^2 p_F \mu + \frac{3}{4} M^4 \ln\left(\frac{\mu + p_F}{\mu - p_F} \right) \right) \theta(\mu - M)$$

$$\simeq \frac{N_c N_f \mu^4}{12\pi^2} \left(1 - 3 \left(\frac{M}{\mu} \right)^2 \right) \theta(\mu - M)$$

Double-Peak Structure

Fukushima (2008)

Pressure as a function of *M*

$$P_\chi \simeq -a\left(M_0^2 - M^2\right)^2$$

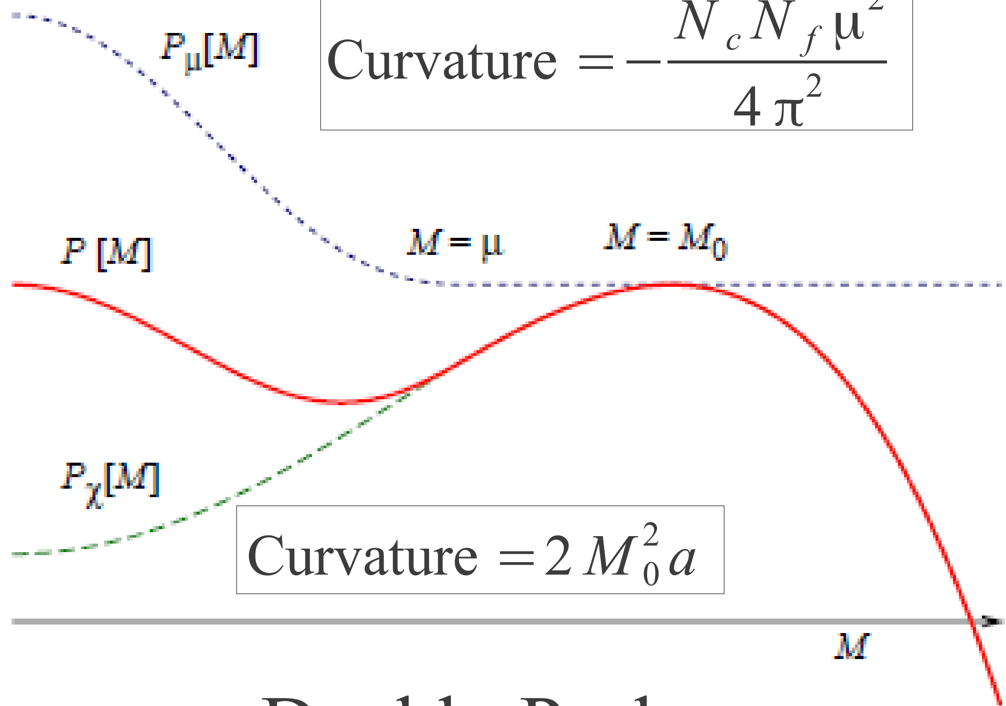
$$a = \frac{1}{2M_0^2}\left(\frac{N_c N_f \Lambda^2}{4\pi^2} - \frac{1}{4G}\right) \sim 0.067 \quad \text{(in NJL)}$$

$$a = \frac{m_\sigma^2 f_\pi^2}{8M_0^4} \sim 0.02 \sim 0.04 \quad \text{(in LSM)}$$

with $M_0 = 340\,\text{MeV}$ $f_\pi = 93\,\text{MeV}$
$m_\sigma = 500 \sim 1000\,\text{MeV}$

→ Talk by Schaefer

$$\text{Curvature} = -\frac{N_c N_f \mu^2}{4\pi^2}$$



$$\text{Curvature} = 2M_0^2 a$$

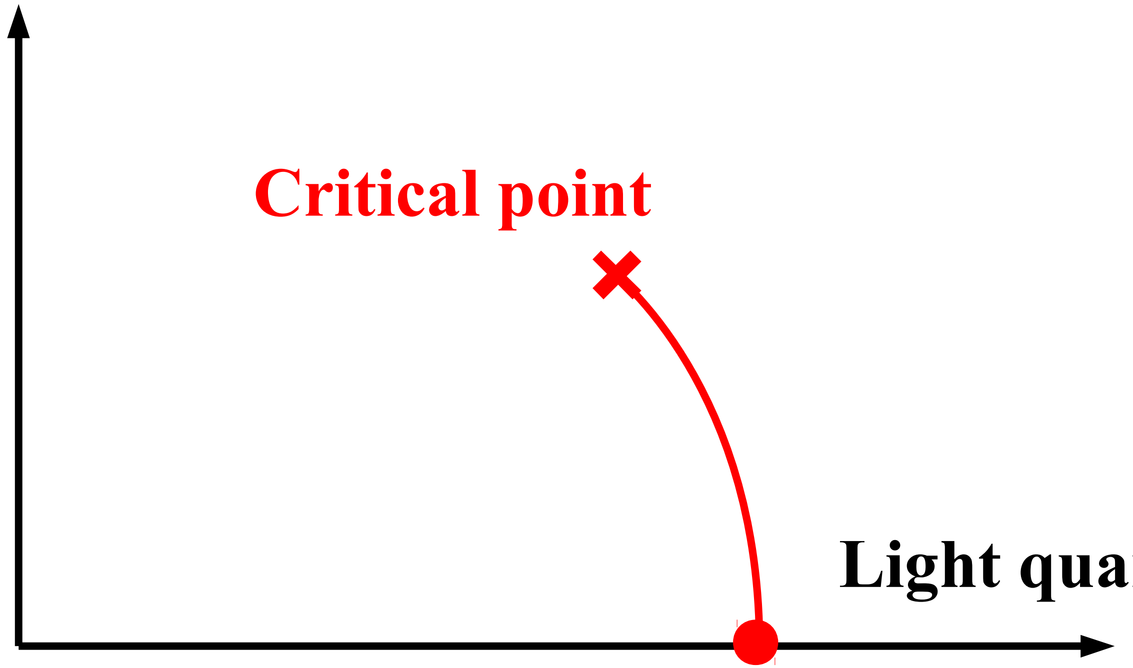
Double-Peak

$$a < \frac{N_c N_f}{8\pi^2} \sim 0.076$$

a at *M* =0 is not physical information
→ Model dependence

If a is small enough...

Chiral phase transition in T =0 quark matter



This simple analysis tells us...

- 1st-order phase transition depends on the tachyonic mass (negative curvature) at $M = 0$.

  Not constrained by observables around the physical vacuum at $M = M_0$.

- Curvature is model-dependent.

  NJL → Weak 1st-order (CP at lower T)

  LSM → Strong 1st-order (CP at higher T)

  c.f. Stephanov diagram (scattering plot)

- Roughly speaking...

  Weaker χSB (smaller bag const.) → CP favored

  Stronger χSB (larger bag const.) → CP disfavored

Source of Ambiguity

Is the Interaction Energy really so simple?

- **Simplest Choice** $-\frac{M^2}{4G}$

- **Higher-order Interaction *M*-Terms**

$$-\frac{M^2}{4G}+\eta M^3$$

U(1)-axial anomaly with $N_f = 3$

- **Different-type of Interaction *n*-Terms**

$$-\frac{M^2}{4G}-G_V n^2$$

P as a function of *n* as well as *M*
Not modify the vacuum properties

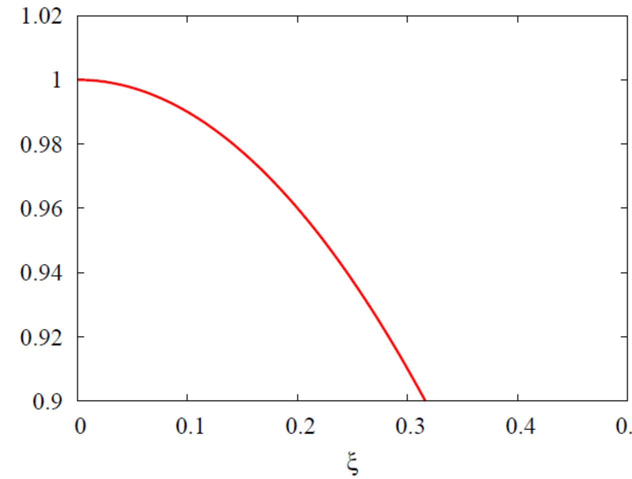
- **Etc, etc,...**

Cubic Term

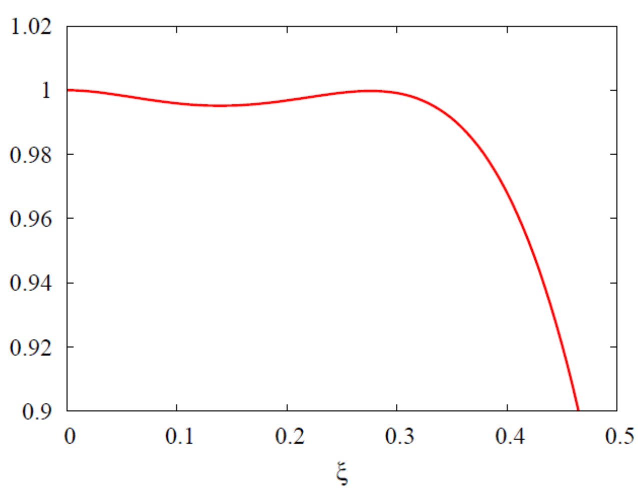
Without cubic term

If a is not small enough

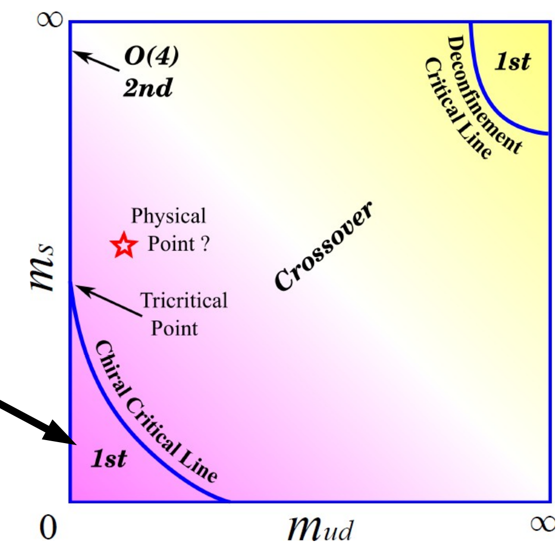
→ 2nd-order phase transition



With cubic term



1st-order on the Columbia plot



→ 1st-order phase transition (weakened by quark masses)

Density Terms

Vector Interaction (for example)

Talk by Sasaki

$$L_V = -G_V (\bar{\psi}\gamma_\mu \psi)(\bar{\psi}\gamma^\mu \psi)$$

Not affect chiral symmetry
Non-zero in general

$$\delta P_\mu = -G_V n^2 = -\frac{G_V N_c^2 N_f^2}{9\pi^4}(\mu^2 - M^2)^3 \theta(\mu - M)$$

$$P_\mu \simeq \frac{N_c N_f \mu^4}{12\pi^2}\left(1 - 3\left(\frac{M}{\mu}\right)^2\right) \theta(\mu - M)$$

$$P_\mu + \delta P_\mu \simeq \frac{N_c N_f \mu^4}{12\pi^2}\left(1 - \frac{4 G_V N_c N_f \mu^2}{3\pi^2}\right)\left(1 - 3\left(\frac{M}{\mu}\right)^2\right) \theta(\mu - M)$$

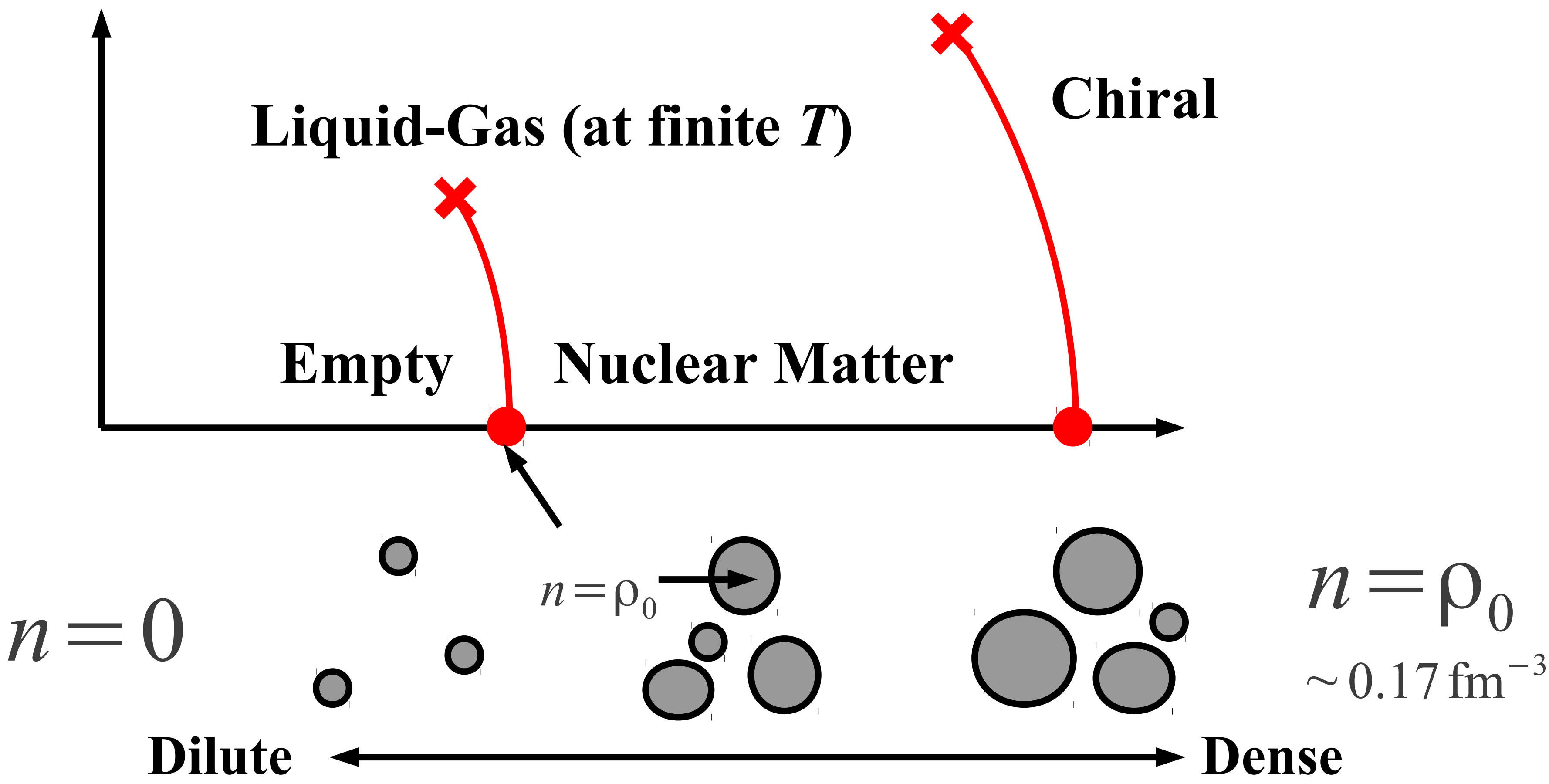
Condition for 1st-order

$$\underbrace{a}_{0.067} < \underbrace{\frac{N_c N_f}{8\pi^2}}_{0.076}\left(1 - \frac{4 G_V N_c N_f \mu^2}{3\pi^2}\right)$$

Not satisfied in NJL
for $G_V > 0.25\ G$
CP disappears!

Liquid-Gas Transition

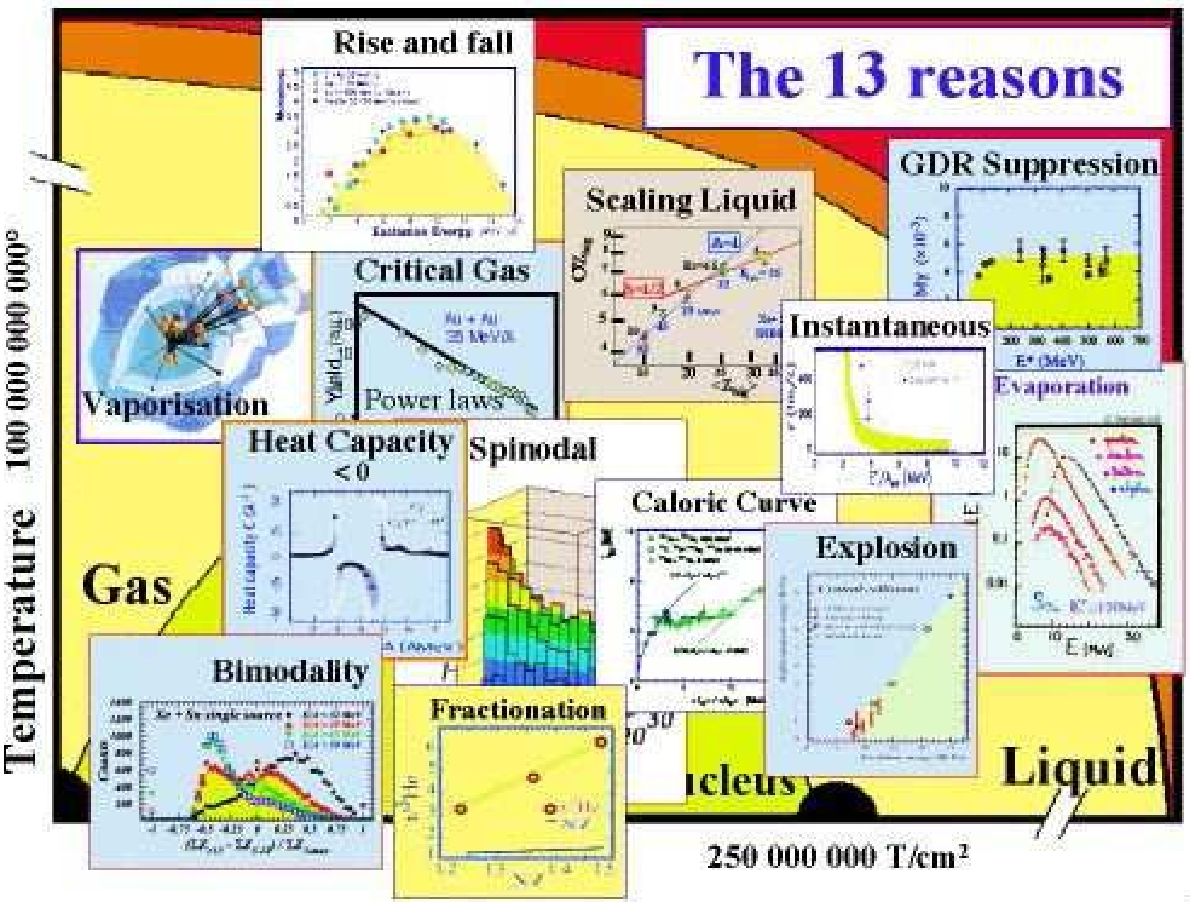
1st-order phase transition at μ_B=(939-16)MeV



Our world is in a mixed phase → Evidence for 1st-order

Experimental Evidences

(At least) 13 evidences (Chomaz: nucl-ex/0410024)



Scaling-law in the size distribution of the fragments

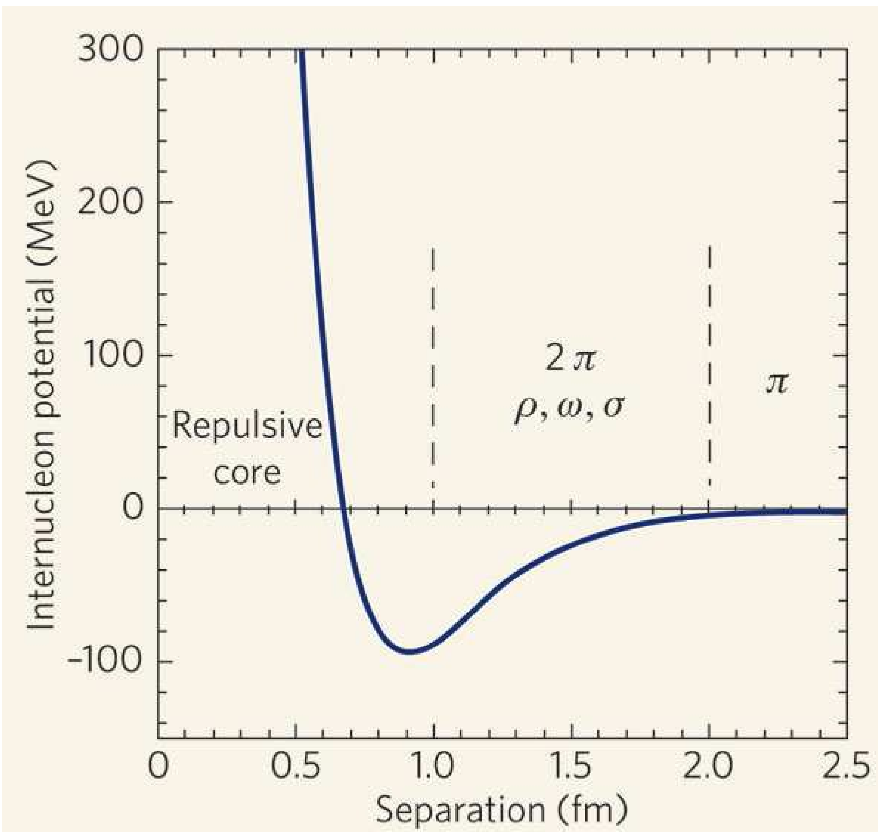
Fragment size fluctuations with predicted powers

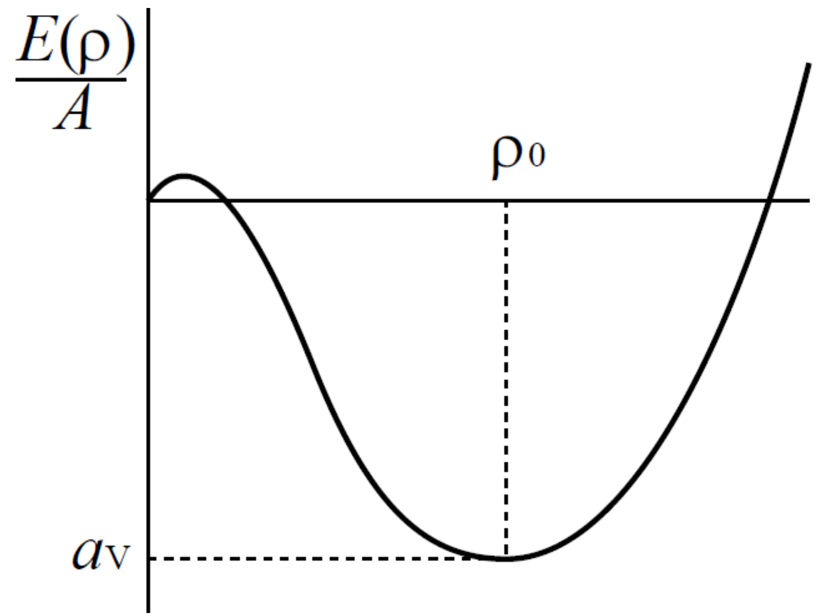
Temperature not changed by diff. *E* (Caloric Curve)

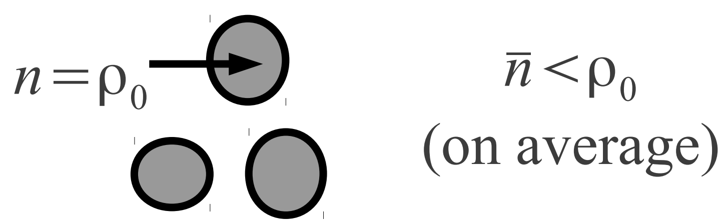
Saturation of Nuclear Matter

1st-order phase transition is a natural consequence from the *saturation property* and that *n is conserved*

N-*N* potential → *G*-matrix →







QCD Critical Point in Terms of n

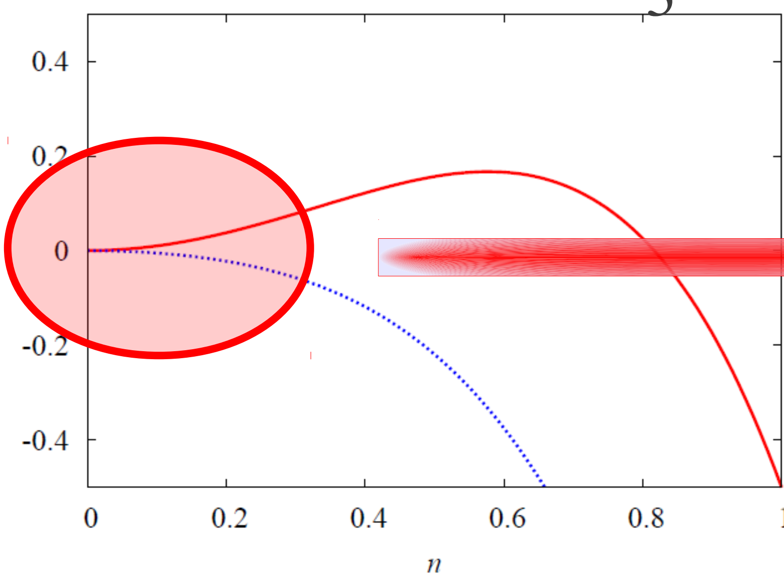
Medium Effects

$$P_\mu \simeq \frac{N_c N_f \mu^4}{12\pi^2}\left(1-3\left(\frac{M}{\mu}\right)^2\right)\theta(\mu - M) \simeq \frac{3\pi^2 n^2}{4 N_c N_f \mu^2}$$

Vacuum Energy

$$P_\chi = -a\left(M_0^2 - M^2\right)^2 \simeq -a M_0^4 + 2 a M_0^2 M^2 + \cdots$$

$$\simeq -a M_0^4 + \frac{2}{3} a M_0^2 \mu^2 - \frac{6\pi^4}{N_c^2 N_f^2} a M_0^2 \mu^2 n^2 + \cdots$$



Saturation appears when

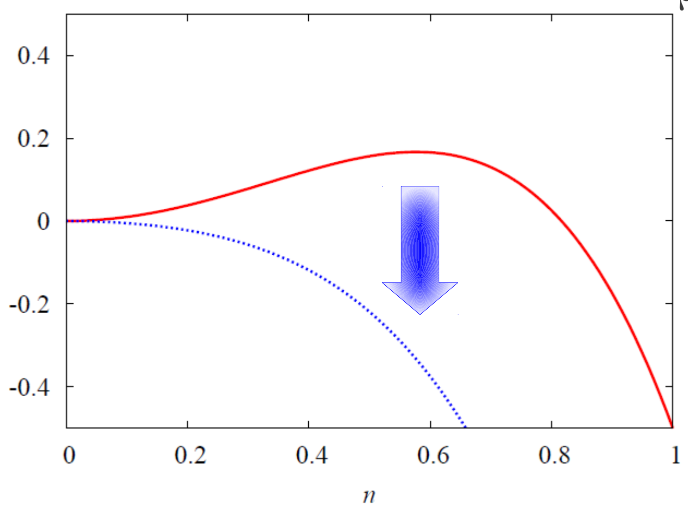
$$a < \frac{N_c N_f}{8\pi^2}$$

Same Condition

Density Terms Revisited

Vector-interaction generate a term like

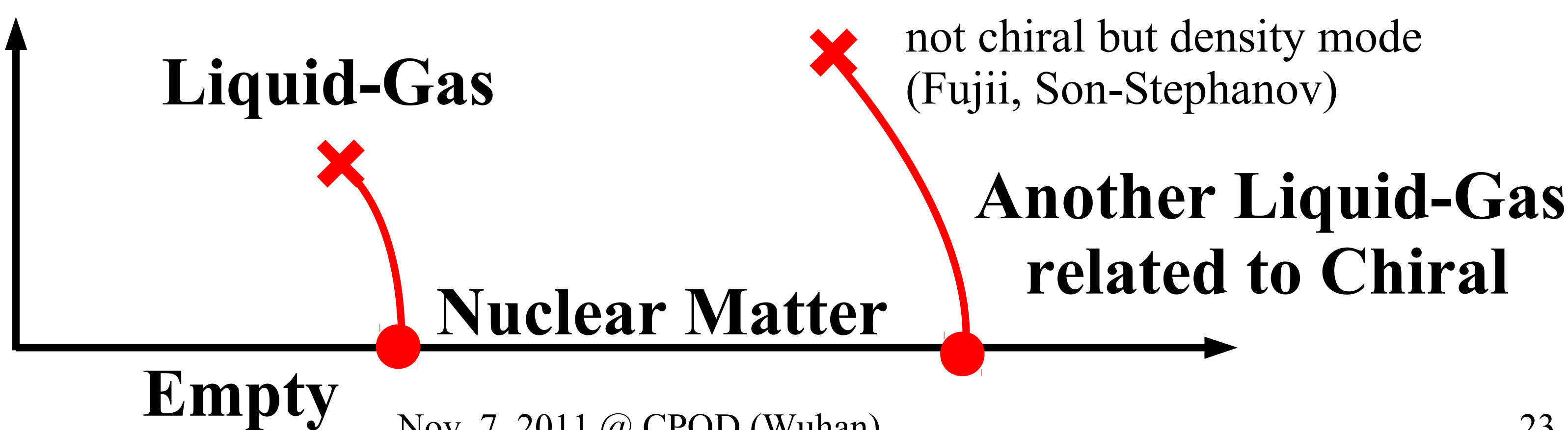
$$\delta P_\mu = -G_V n^2$$



If G_V is too strong, the pressure is pushed down, and there appears no saturation point → no CP

Effect of the vector-interaction is trivially understandable from the point of view of liquid-gas picture.

c.f. Slow-mode at CP is not chiral but density mode (Fujii, Son-Stephanov)



Summary

- Many chiral models predict a 1st-order phase transition at high baryon density because the density-induced pressure is the largest at $M = 0$.

- Strength of the 1-st order transition depend on unphysical curvature at $M = 0 \rightarrow$ Model dependent!

- Higher-order M-terms and additional n-terms would change the nature of the phase transition.

- More natural understanding is of a liquid-gap phase transition in terms of n just like in nuclear matter.