QCD phase diagram with functional methods

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C.F., J. A. Mueller, PRD 84 (2011) 054013.
C.F., J. Luecker, J. A. Mueller, PLB 702 (2011) 438-441.
C.F., A. Maas and J. A. Mueller, EPJC 68 (2010) 165-181.

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HIC for FAIR

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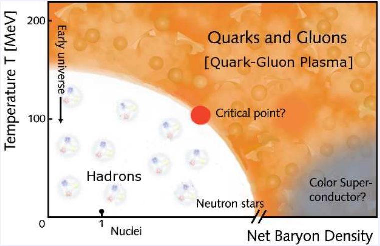
QCD phase transitions I

Open questions:

- Gluons and quarks in QGP
- Existence and location of CEP



Phase transitions:

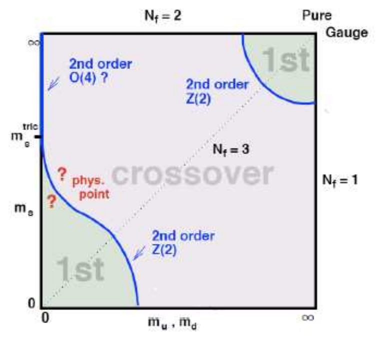
- Chiral limit ($M_{weak} \to 0$): order parameter chiral condensate

$$\langle\bar{\psi}\psi\rangle = Z_2 N_c Tr_D \int \frac{d^4p}{(2\pi)^4} S(p)$$

- Static quarks ($M_{weak} \to \infty$): order parameter Polyakov-loop

$$\Phi \sim e^{-F_q/T}$$

Quark mass dependence:



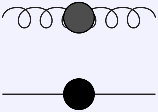
- Deconfinement transition at large masses
- Chiral transition at small masses

in this talk: pure gauge and $N_f = 2$

QCD in covariant gauge

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \; \exp\left\{ -\int_0^{1/T} dt \int d^3x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\,(F^a_{\mu\nu})^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Landau gauge ($\xi = 0$) propagators in momentum space, $q = (\vec{q}, \omega_q)$:



$$D^{\text{Gluon}}_{\mu\nu}(q) = \frac{Z_T(q)}{q^2} P^T_{\mu\nu}(q) + \frac{Z_L(q)}{q^2} P^L_{\mu\nu}(q)$$

$$S^{\text{Quark}}(q) = \frac{1}{-i\,\vec{\gamma}\vec{q}\;A(q) - i\,\gamma_4\omega_n\;C(q) + B(q)}$$

The Goal:

Gauge invariant information from gauge fixed functional approach

Lattice QCD vs. DSE/FRG: Complementary!

- Lattice simulations
  - Ab initio
  - Gauge invariant

- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  - Analytic solutions at small momenta
  - Space-Time-Continuum
  - Chiral symmetry: light quarks and mesons
  - Multi-scale problems feasible: e.g. $(g\text{-}2)_\mu$

    T. Goecke, C.F., R. Williams, PLB 704 (2011); PRD 83 (2011)
  - Chemical potential: no sign problem

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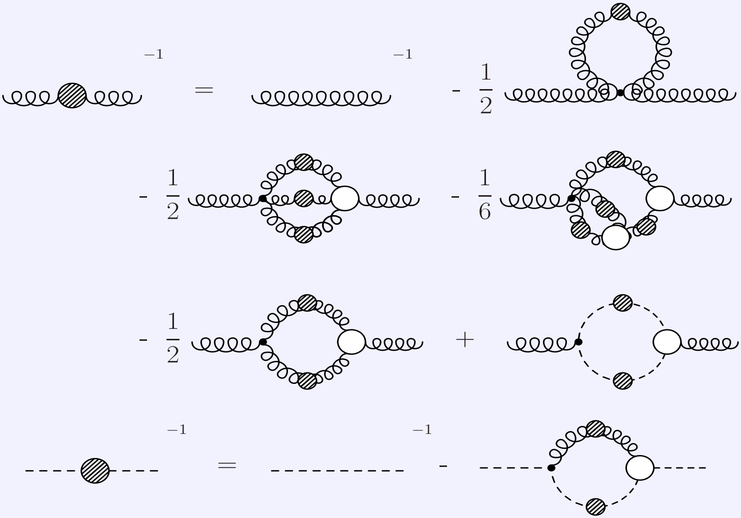
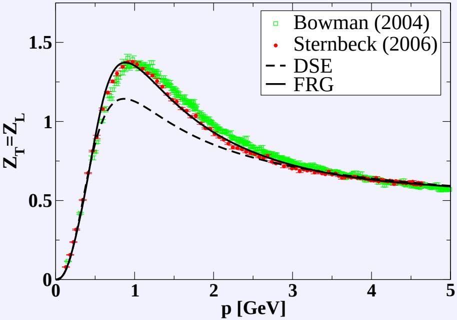
Dyson-Schwinger equations (DSEs)



DSEs vs Lattice ($T = 0$)



C.F., A. Maas and J. M. Pawlowski, Annals Phys. **324** (2009) 2408-2437.

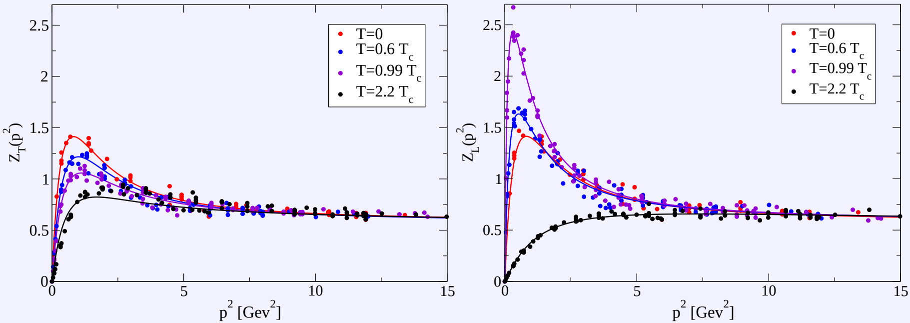
- Gluon mass generation in agreement with lattice results

  Cucchieri, Mendes, PoS **LAT2007** (2007) 297.

- Schwinger function/analytic structure: positivity violations
  $\rightarrow$ gluon screening

Glue at finite temperature $T \neq 0$

T-dependent gluon propagator from lattice simulations:



- Difference between electric and magnetic gluon
- Maximum of electric gluon around T_c

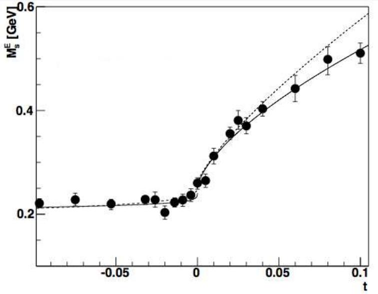
Cucchieri, Maas, Mendes, PRD 75 (2007)

C.F., Maas and Mueller, EPJC 68 (2010)

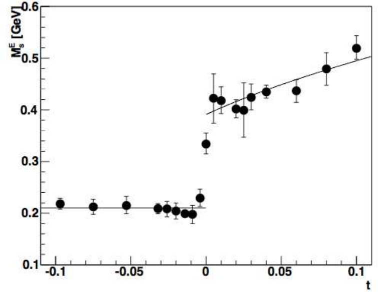
Cucchieri, Mendes, PoS FACESQCD (2010) 007.

Aouane, Bornyakov, Ilgenfritz, Mitrjushkin, Muller-Preussker, Sternbeck, [arXiv:1108.1735 [hep-lat]].

SU(2)



SU(3)



$t = (T - T_c)/T_c$

Maas, Pawlowski, Smekal, Spielmann, arXiv:1110.6340.

C.F., Maas and Mueller, EPJC 68 (2010)

- phase transition of second and first order
  now clearly visible in electric screening mass

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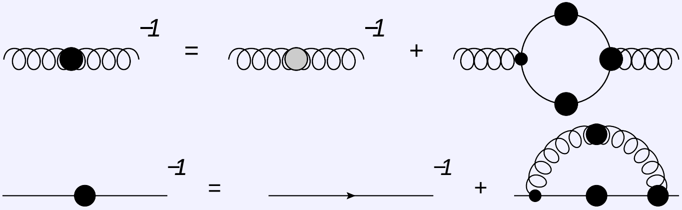
The ordinary chiral condensate



- quenched lattice gluon propagator + DSE-quark-loop ($N_f = 2$)
- $T = 0$: quark-gluon vertex studied via DSEs

  Alkofer, C.F., Llanes-Estrada, Schwenzer, Annals Phys.324:106-172,2009.

  C.F, R. Williams, PRL **103** (2009) 122001

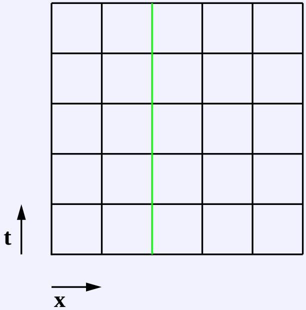
  $T \neq 0$: ansatz, T, μ and mass dependent (STI)
- Order parameter for chiral symmetry breaking:

$$\langle \bar{\psi}\psi \rangle = Z_2 N_c T \sum_{n_p} \int \frac{d^3p}{(2\pi)^3} \, Tr_D \, S(\vec{p}, \omega_p)$$

The Polyakov Loop

$$\Phi = \left\langle \frac{1}{N_c} Tr_D \mathcal{P} \exp\left\{ i \int_0^{1/T} A_4\, dt \right\} \right\rangle \sim e^{-F_q/T}$$

Lattice:



Order parameter for center symmetry breaking:

$\Phi = 0$: confined

$\Phi \neq 0$: deconfined

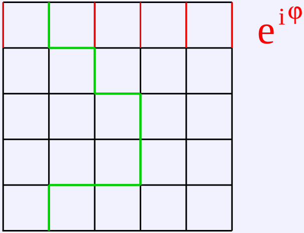
The dual condensate I

Consider general $U(1)$-valued boundary conditions in temporal direction for quark fields ψ:

$$\psi(\vec{x}, 1/T) = e^{i\varphi}\psi(\vec{x}, 0)$$

Matsubara frequencies: $\omega_p(n_t) = (2\pi T)(n_t + \varphi/2\pi)$

Lattice:



$$\langle\overline{\psi}\psi\rangle_\varphi \sim \sum \frac{exp[i\,\varphi\,n]}{(am)^l} \text{ Closed Loops}$$

E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).

The dual condensate II

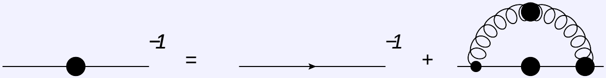
Then define dual condensate Σ_n:

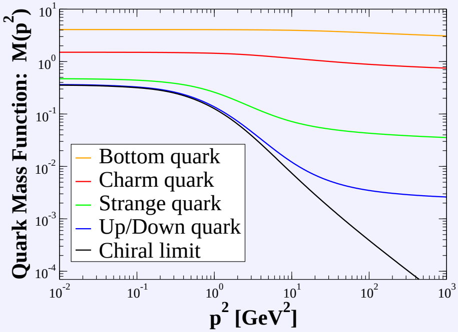
$$\Sigma_n = -\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi n} \langle\overline{\psi}\psi\rangle_\varphi$$

- $n = 1$ projects out loops with $n(l) = 1$: dressed Polyakov loop
- transforms under center transformation exactly like ordinary Polyakov loop: order parameter for center symmetry breaking
- Σ_1 is accessible with functional methods

  C.F., PRL **103** (2009) 052003

C. Gattringer, PRL **97**, 032003 (2006)
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** 094007 (2008).
F. Synatschke, A. Wipf and K. Langfeld, PRD **77**, 114018 (2008).
J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski, PRL 106 (2011)

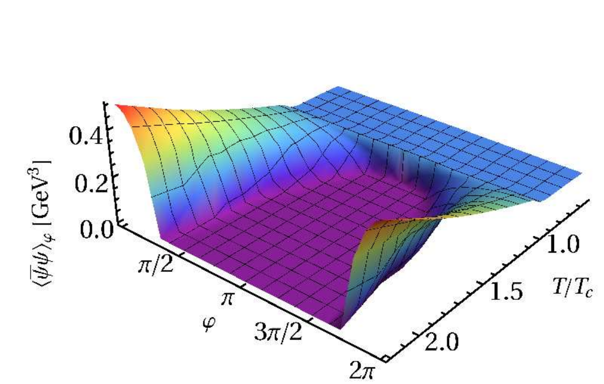




C.F. J.Phys.G G32 (2006) R253-R291

- $M(p^2) = B(p^2)/A(p^2)$: momentum dependent!
- Dynamical masses $M_{strong}(0) \approx 350$ MeV
- Flavour dependence because of M_{weak}
- $\langle \bar{\psi}\psi \rangle \approx (250\text{MeV})^3$

Condensate: angular dependence in chiral limit



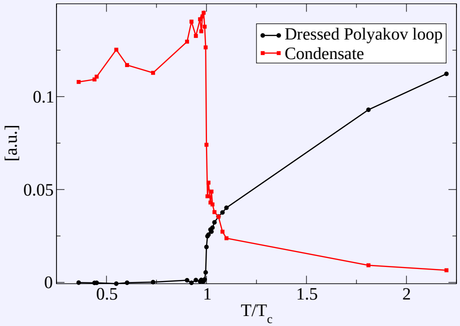
$$\Sigma_1 \;=\; -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi}\, \langle\bar\psi\psi\rangle_\varphi$$

- Width of plateau is T-dependent, $\langle\bar\psi\psi\rangle_\varphi(\varphi = 0) \sim T^2$

C.F. and Jens Mueller, PRD **80** (2009) 074029.

Transition temperatures, quenched

quenched, DSE



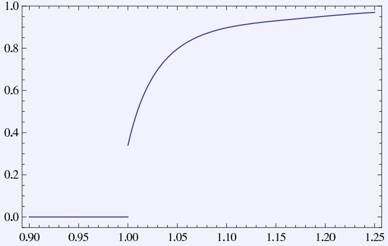
Luecker, C.F., arXiv:1111.0180

C.F., Maas, Mueller, EPJC 68 (2010).

- SU(2): $T_c \approx 305$ MeV
  SU(3): $T_c \approx 270$ MeV
- increasing condensate due to electric part of gluon

cf. Buividovich, Luschevskaya, Polikarpov, PRD 78 (2008) 074505.

quenched, FRG

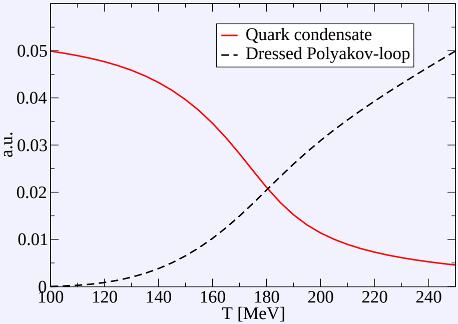


Braun, Gies, Pawlowski, PLB 684 (2010) 262-267.

- Polyakov loop potential via gluon/ghost propagators

$N_f = 2$: Transition temperatures at $\mu = 0$

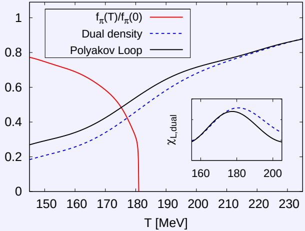
DSE



C.F., J. Luecker, J. A. Mueller, PL **B702** (2011) 438-441.

- $T_\chi \approx 185$ MeV
- $T_{conf} \approx 195$ MeV
- condensate in qualitative agreement with χPT

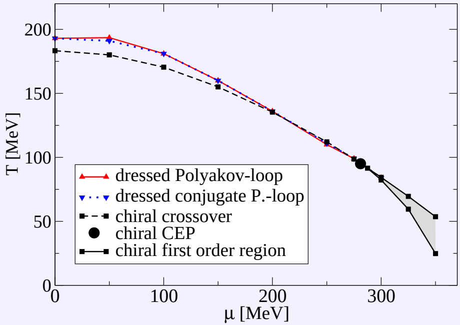
FRG

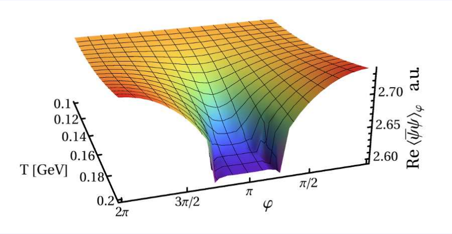


J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski, PRL 106 (2011) 022002

- chiral limit
- $T_\chi \simeq T_{conf} \simeq 180$ MeV

$N_f = 2$: QCD phase diagram





C.F., J. Luecker, J. A. Mueller, PLB 702 (2011) 438-441.

- chiral CEP

  Qin, Chang, Chen, Liu, Roberts, PRL 106 (2011) 172301.
- crucial: backreaction of quark onto gluon
- qualitative agreement with RG-improved PQM model

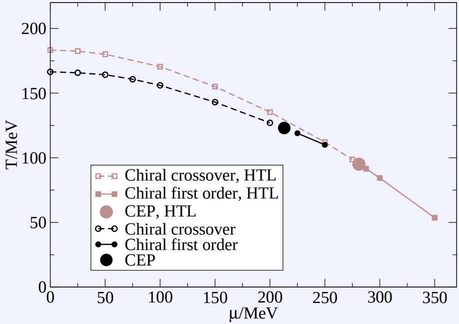
  Herbst, Pawlowski, Schaefer, PLB **696** (2011)
- no CEP at $\mu_c / T_c < 1$ in agreement with lattice

  de Forcrand, Philipsen, JHEP **0811** (2008) 012; Nucl. Phys. **B642** (2002) 290-306.

  Endrodi, Fodor, Katz, Szabo, JHEP **1104** (2011) 001.

$N_f = 2$: QCD phase diagram (preliminary)

Full backreaction included:



J. Luecker, C.F. in preparation

- CEP: $\mu_c/T_c \simeq 3 \longrightarrow \mu_c/T_c \simeq 1.7$
- no quarkyonic region
- To do: $N_f = 2+1$, curvature...

Summary: QCD transitions

Summary:

- Temperature dependent gluon propagator:
  - characteristic behavior of electric screening mass at T_c
  - 'melting' of magnetic gluon with temperature
- Deconfinement T_c from dressed Polyakov-loop calculated from DSEs
- $N_f = 2$-QCD with finite chemical potential (beyond mean field)
  - backreaction of quarks onto gluon crucial
  - $N_f = 2$: CEP at $\mu_c/T_c > 1$

Thank you for your attention!

Helmholtz Young Investigator Group ”Nonperturbative Phenomena in QCD”

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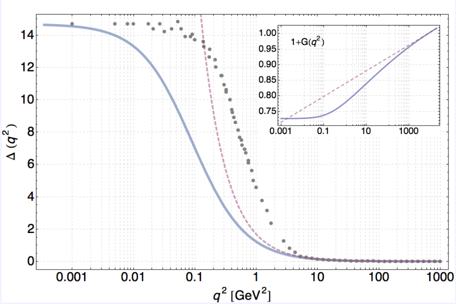
LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

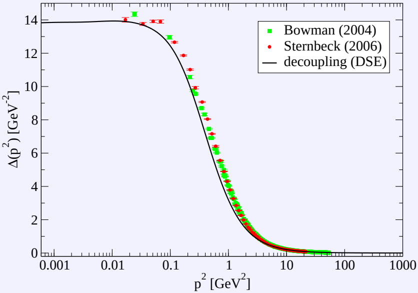
Gluon and Quark-Gluon-Vertex

Ansatz for Quark-Gluon-Vertex:

$$
\begin{aligned}
\Gamma_\nu(q,k,p) &= \widetilde{Z}_3 \left(\delta_{4\nu}\gamma_4 \frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j \frac{A(k)+A(p)}{2} \right) \times \\
&\quad \times \left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2+1]}{4\pi} \right)^{2\delta} \right) .
\end{aligned}
$$

Gluon propagator ($T = 0$)





Aguilar, Binosi, Papavassiliou, PRD **78**, 025010 (2008).

C.F., Maas and Pawlowski, Annals Phys. **324** (2009) 2408.

Cornwall, PRD **26** (1982) 1453.

$$\Delta(p^2) = \frac{Z(p^2)}{p^2}$$

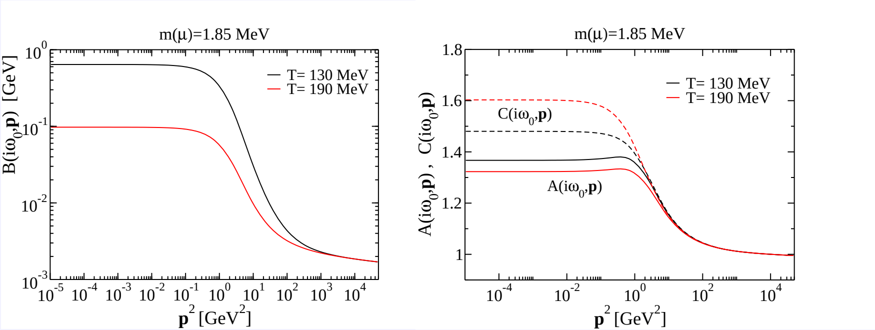
- Gluon mass generation in agreement with lattice results

  Cucchieri, Mendes, PoS **LAT2007** (2007) 297.

- see however
  Sternbeck, L. von Smekal, Eur. Phys. J. C68 (2010) 487;
  Cucchieri, Mendes, Phys. Rev. **D81** (2010) 016005.
  Maas, Phys. Lett. **B689** (2010) 107-111.

$T \neq 0$: Chiral symmetry restauration

$$S^{\text{Quark}}(q) = \frac{1}{-i\,\vec{\gamma}\vec{q}\,A(q) - i\,\gamma_4\omega_n\,C(q) + B(q)}$$



- dynamical effects below T_c $\leftrightarrow$ 'HTL-ish' above T_c