BEC-BCS Crossover and Phase Diagram of Dense QCD --- A NJL Model Study

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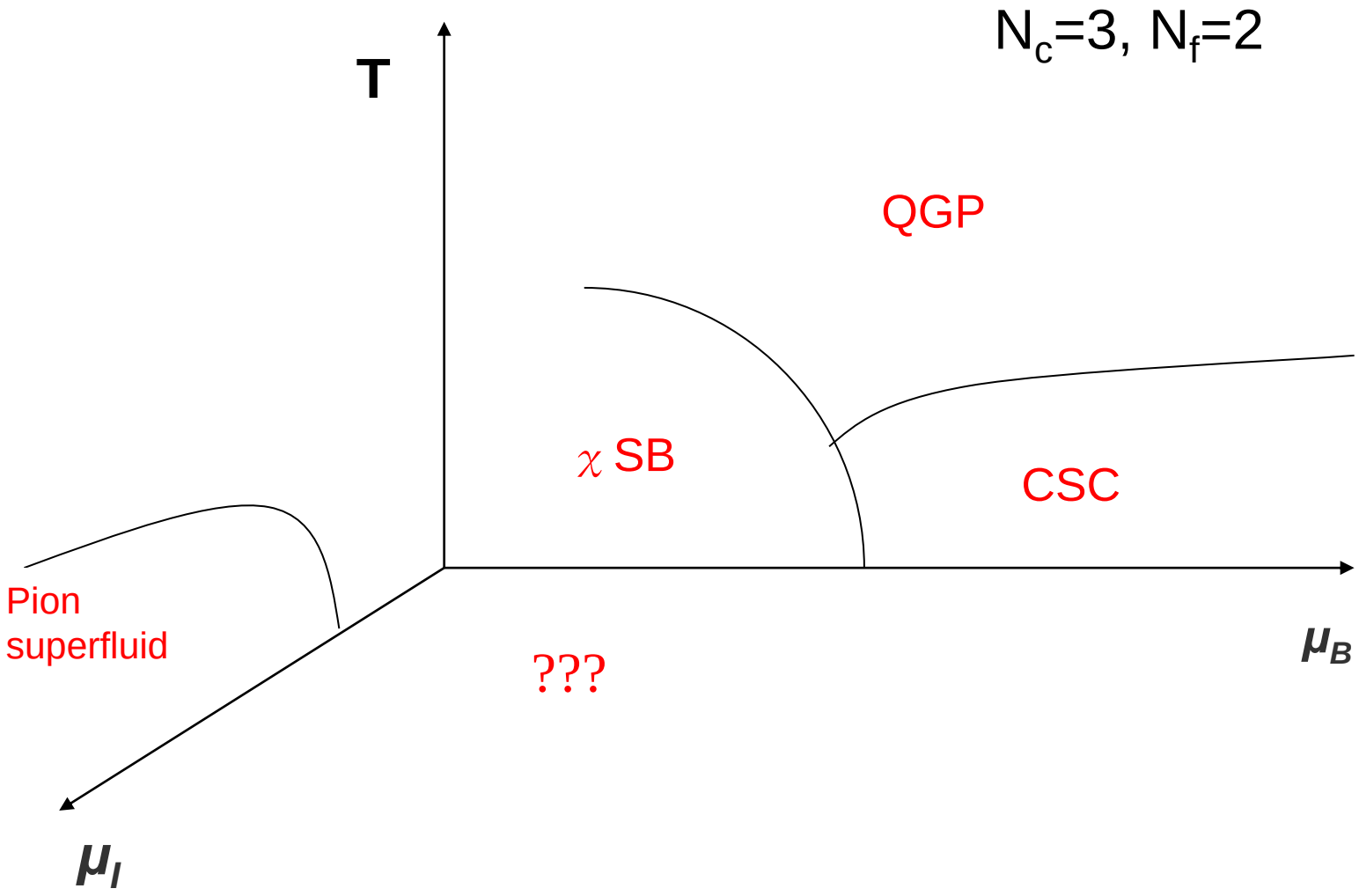
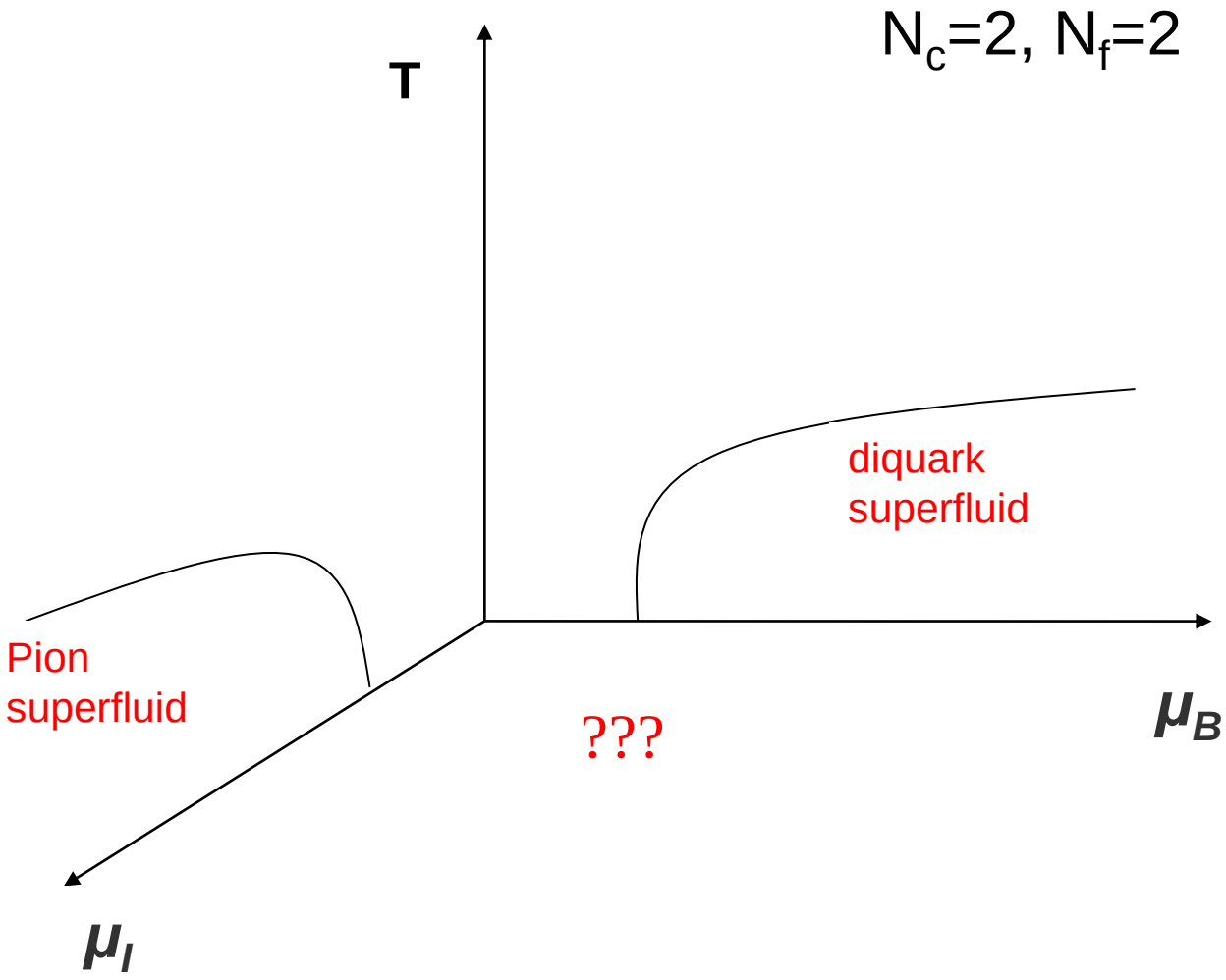
Outline

- BEC-BCS Crossover: from Cold Atoms to QCD
- Stressed Pairing: Quantum Phase Diagram in the BEC-BCS Crossover
- Quantum ($\mu_B - \mu_I$) Phase Diagram of Dense QCD from NJL Model
- Summary

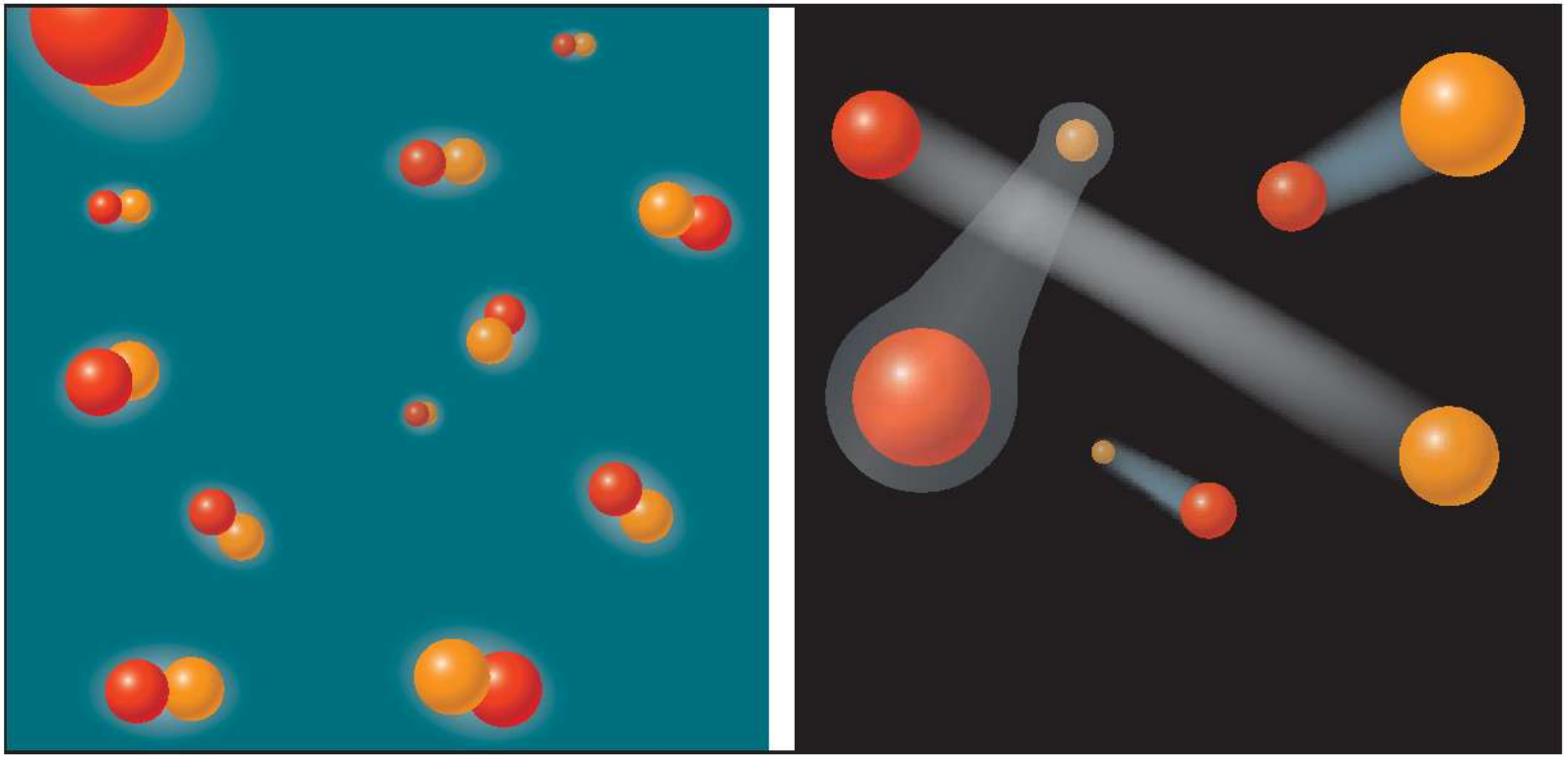
QCD Phase Diagram



QCD Phase Diagram



BEC-BCS Crossover



Tango or twist? In a magnetic field, atoms in different spin states can form molecules (*left*). Vary the field, and they might also form loose-knit Cooper pairs.

BEC-BCS Crossover in QCD?

- QCD-like Theories: QCD at finite isospin chemical potential or two-color QCD at finite chemical potentials, etc.
- Free from sign problem, lattice simulation OK
- Low density: BEC of pions or bosonic baryons (diquarks) --- chiral perturbation theory OK
- High density: weakly coupled BCS superfluid

  --- perturbative QCD OK

D. T. Son & M. A. Stephanov, PRL, 2001

BEC-BCS Crossover in QCD?

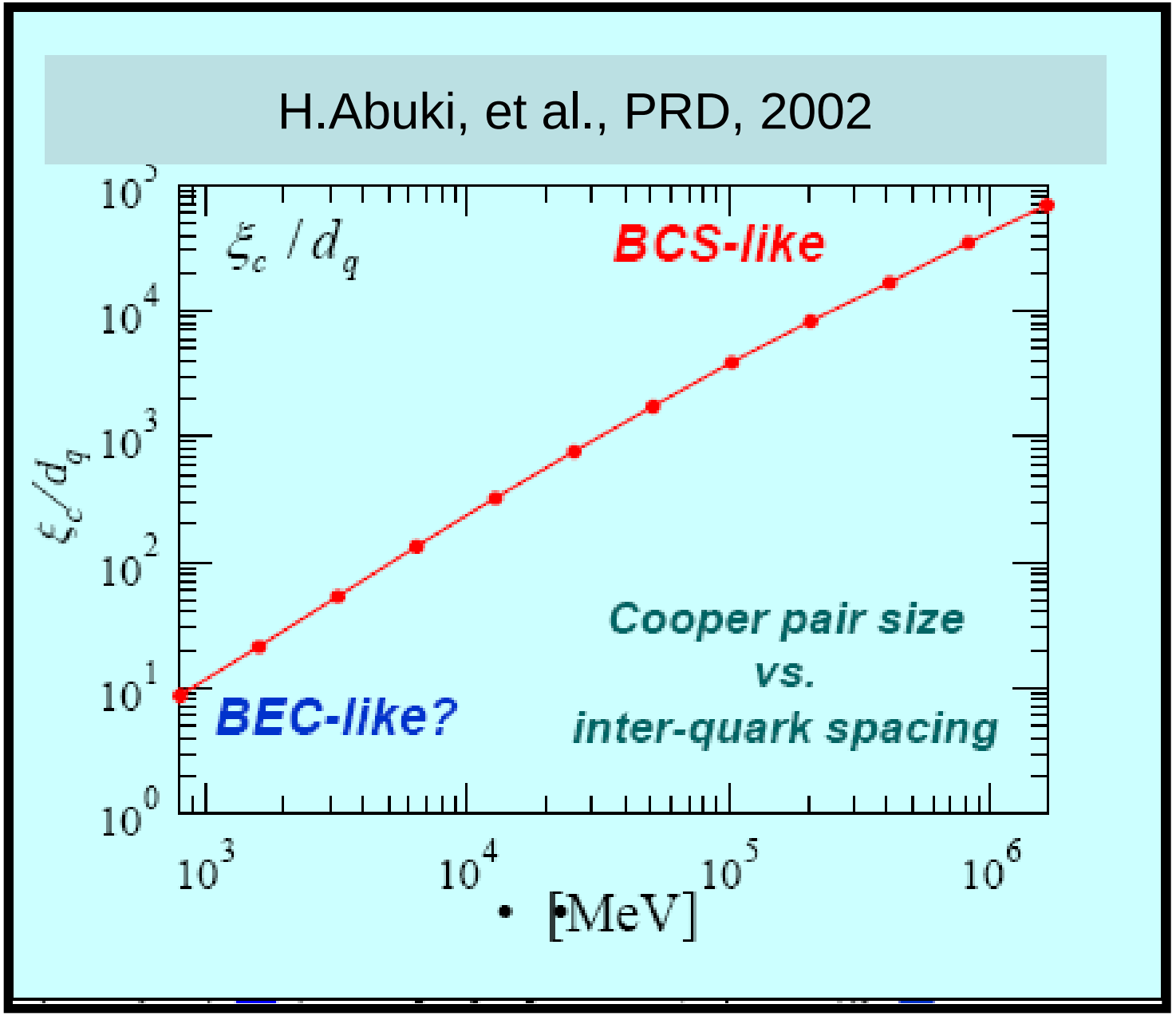
- Real QCD: Quark energy gap $\triangle$~100MeV at moderate density (μ ~ 400MeV) from phenomenological four-fermion models,

  large compared to the Fermi energy $\triangle/E_F$~0.2!!

  M. Alford et al., PLB,1998
- Small Cooper pair size compared to the inter-quark distance for quark chemical potential down to 1GeV

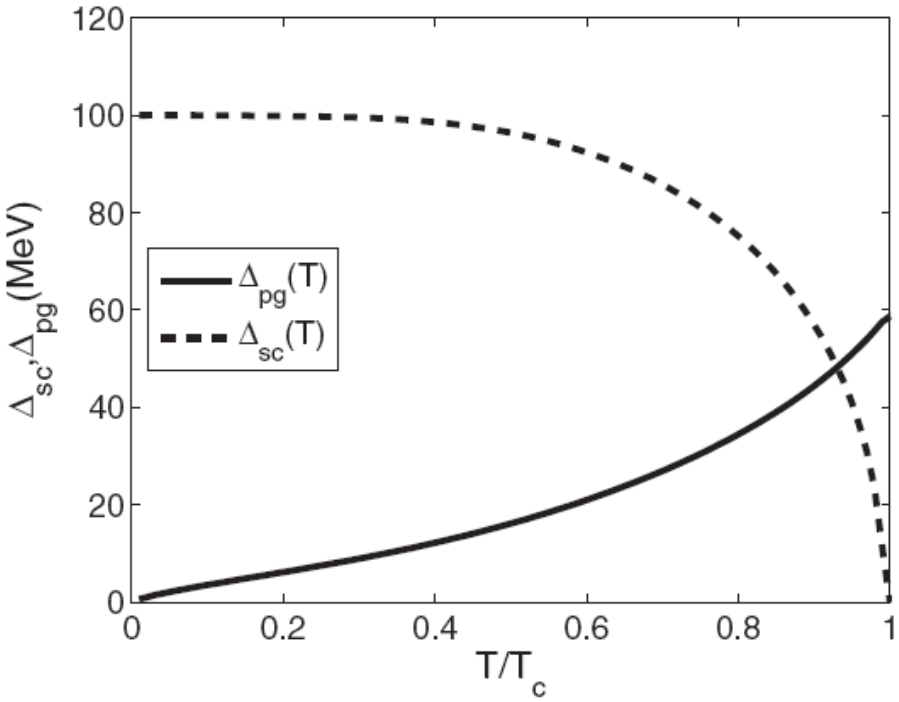
  H. Abuki et al., PRD, 2002

H.Abuki, et al., PRD, 2002



BEC-BCS Crossover in QCD?

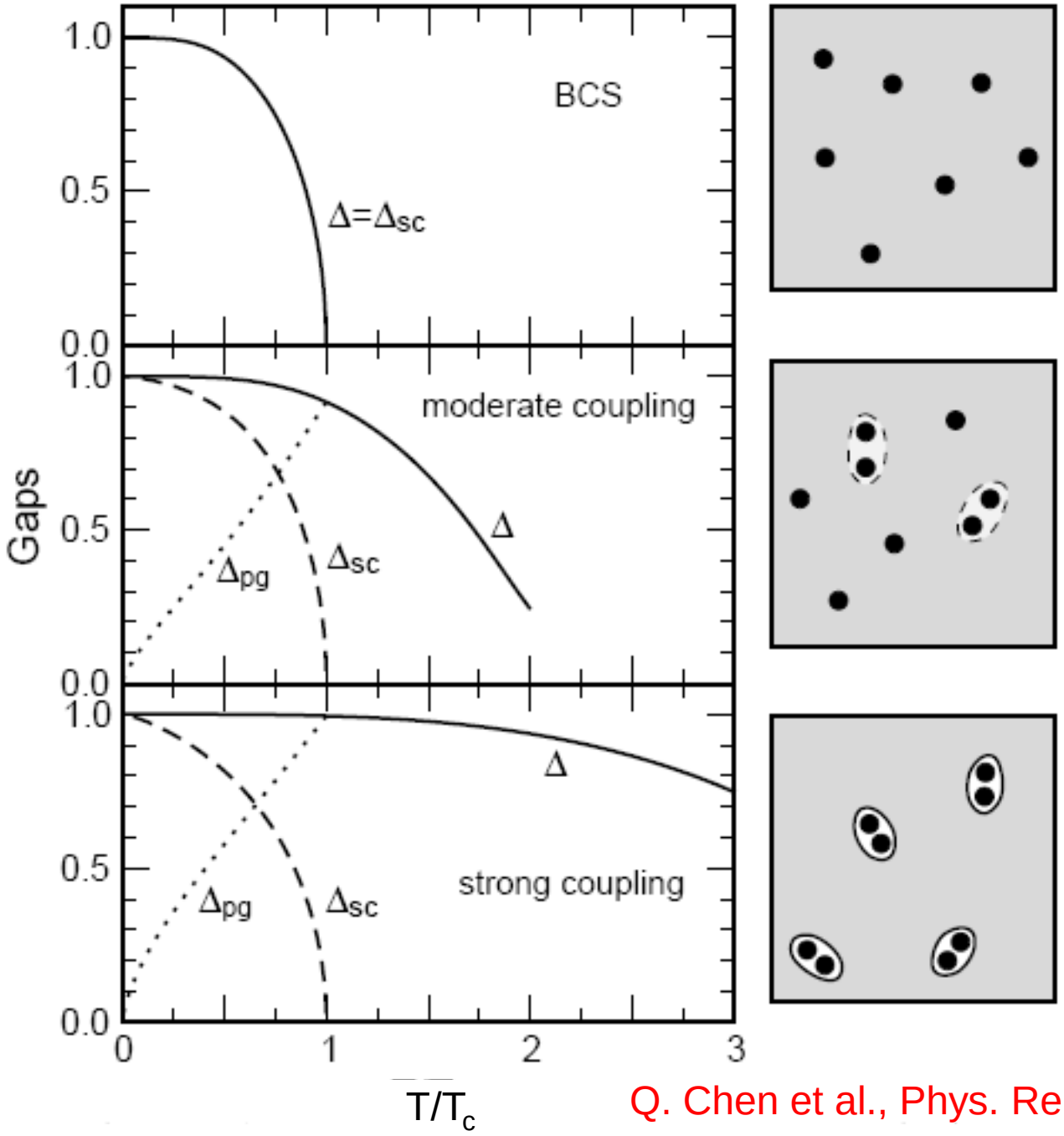
- Pseudogap size compared to the zero temperature pairing gap



LH & P. Zhuang, PRD, 2007

Using a self-consistent T-matrix theory from Q. Chen et al., Phys. Rept., 2005

$$\Delta^2 = \Delta_{\rm sc}^2 + \Delta_{\rm pg}^2$$



Q. Chen et al., Phys. Rept., 2005

NJL Model Description

- QCD-like theories: e.g., two-color QCD

$$\mathcal{L}_{\mathrm{NJL}} = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m_0)q + G[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau}q)^2 + (\bar{q}i\gamma_5\tau_2 t_2 q_c)(\bar{q}_c i\gamma_5\tau_2 t_2 q)],$$

m_0=0: $SU(2N_f)$=SU(4)

μ_B nonzero: $SU_L(N_f)XSU_R(N_f)XU_B(1)$

Quantum phase transition at $\mu_B = m_\pi$

C. Ratti & W. Weise, PRD, 2004;

T. Brauner et al., PRD, 2009; LH, PRD, 2010

NJL Model Description

- Weakly interacting Bose condensate at low density

$$V_{\mathrm{GL}}[\Delta(x)] = \int dx \Big[\Delta^{\dagger}(x) \Big(-\delta \frac{\partial^2}{\partial \tau^2} + \kappa \frac{\partial}{\partial \tau} - \gamma \nabla^2 \Big) \Delta(x) + \alpha |\Delta(x)|^2 + \frac{1}{2} \beta |\Delta(x)|^4 \Big],$$

$$\Psi(x) = \sqrt{2 m_{\pi} \mathcal{I}}\, \Delta(x)$$

$$V_{\mathrm{GP}}[\Psi(x)] = \int dx \Big[\Psi^{\dagger}(x) \Big(\frac{\partial}{\partial \tau} - \frac{\nabla^2}{2 m_{\pi}} \Big) \Psi(x) - \mu_d |\Psi(x)|^2 + \frac{1}{2} g_0 |\Psi(x)|^4 \Big],$$

$$\mu_d = \mu_B - m_{\pi}$$

LH, PRD, 2010

$$g_0 = 4\pi a_{\mathrm{dd}} / m_{\pi} \qquad a_{\mathrm{dd}} = \frac{m_{\pi}}{16\pi f_{\pi}^2}$$

NJL Model Description

- Characterize the BEC-BCS crossover

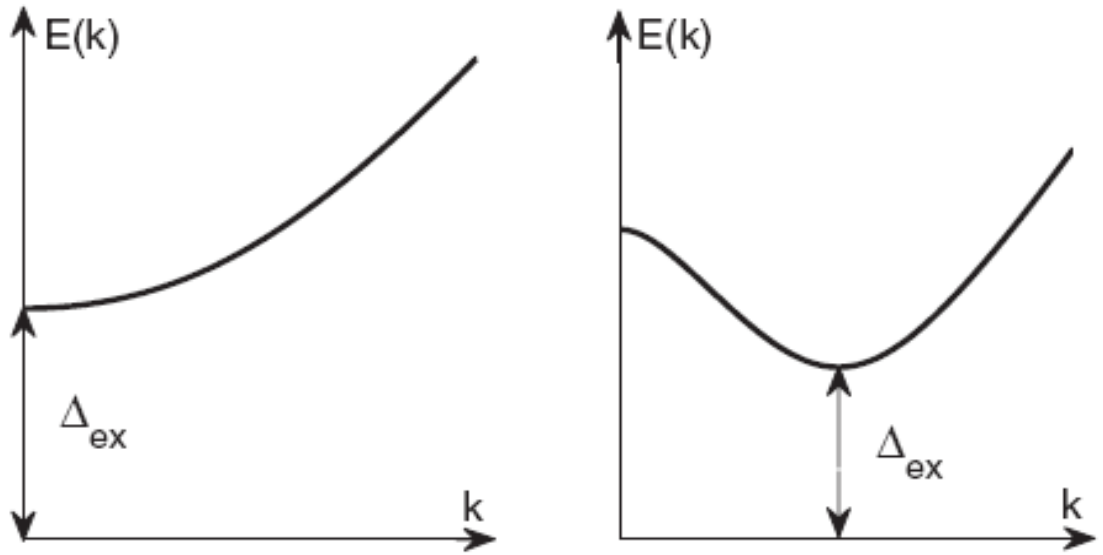


FIG. A schematic plot of the fermionic excitation spectrum in the BEC state (left) and the BCS state (right).

NJL Model Description

- BEC-BCS crossover point

$$E_{\mathbf{k}}^{\pm} = \sqrt{(E_{\mathbf{k}} \pm \mu_B/2)^2 + |\Delta|^2}$$

$$E_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + M^2} \qquad \Delta_{\mathrm{ex}} = \begin{cases} \sqrt{(M - \frac{\mu_B}{2})^2 + |\Delta|^2} & \mu_B < \mu_0 \\ |\Delta| & \mu_B > \mu_0 \end{cases}$$

Crossover: $\mu_B/2 = M(\mu_B)$

LH, M. Jin, and P. Zhuang, PRD, 2006
G. Sun, LH, and P. Zhuang, PRD, 2007
LH, PRD, 2010

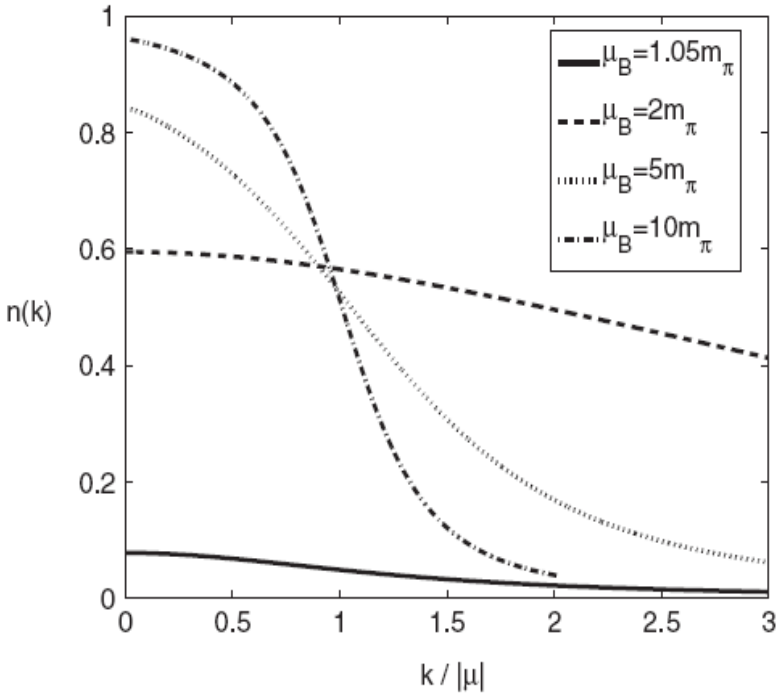
$$\frac{\mu_0}{m_\pi} \simeq \left(\frac{m_\sigma}{m_\pi}\right)^{1/3} \sim (1.6\text{-}2)$$

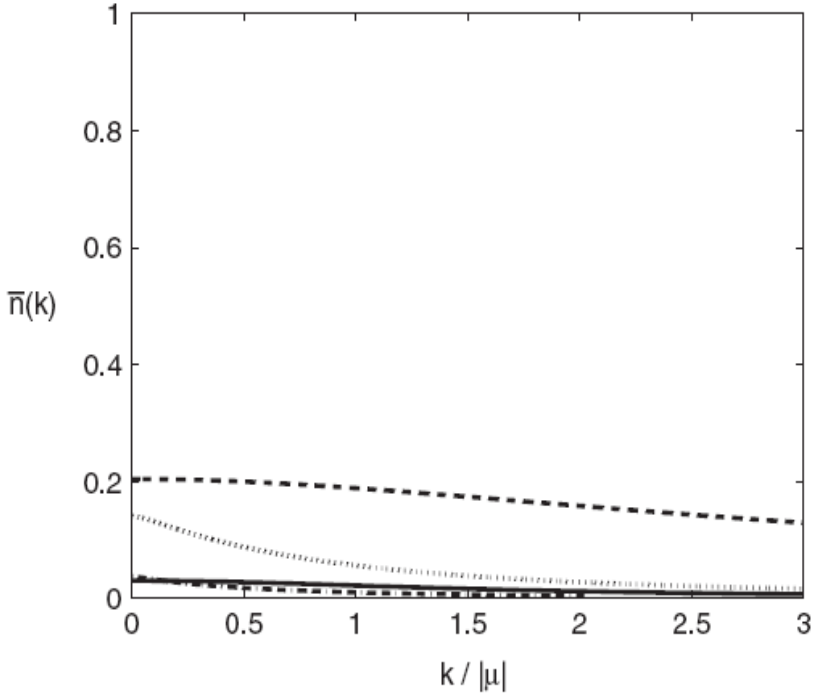
Quantitatively consistent with recent Lattice result: $\mu_0 = 1.47\, m_\pi$

S.Hands, et al., PRD(R), 2010

NJL Model Description

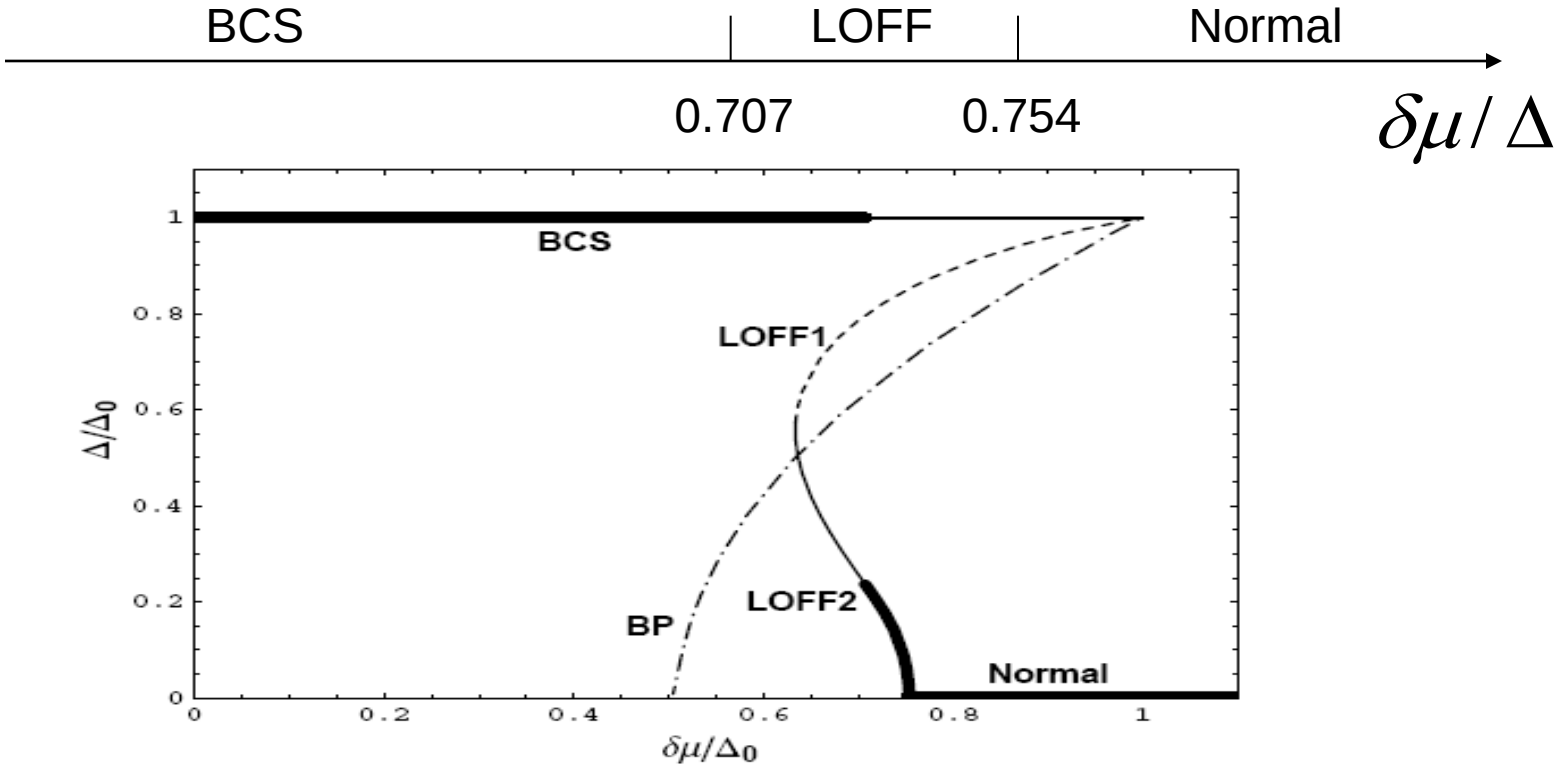
- Momentum distribution



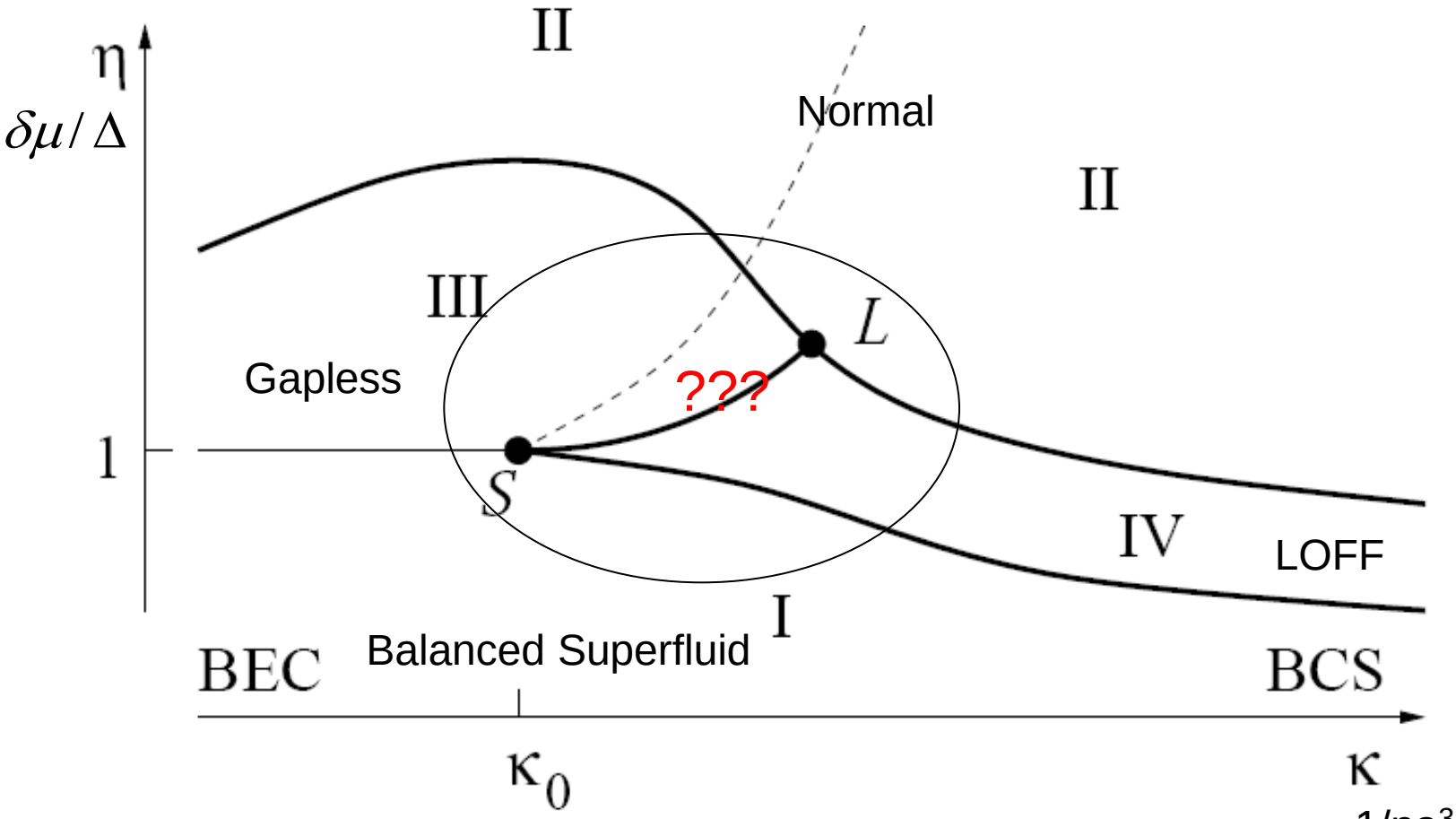


Stressed Pairing: Weak Coupling

- Pairing of fermions under stress, or imbalanced Fermi surfaces
- Phase structure:



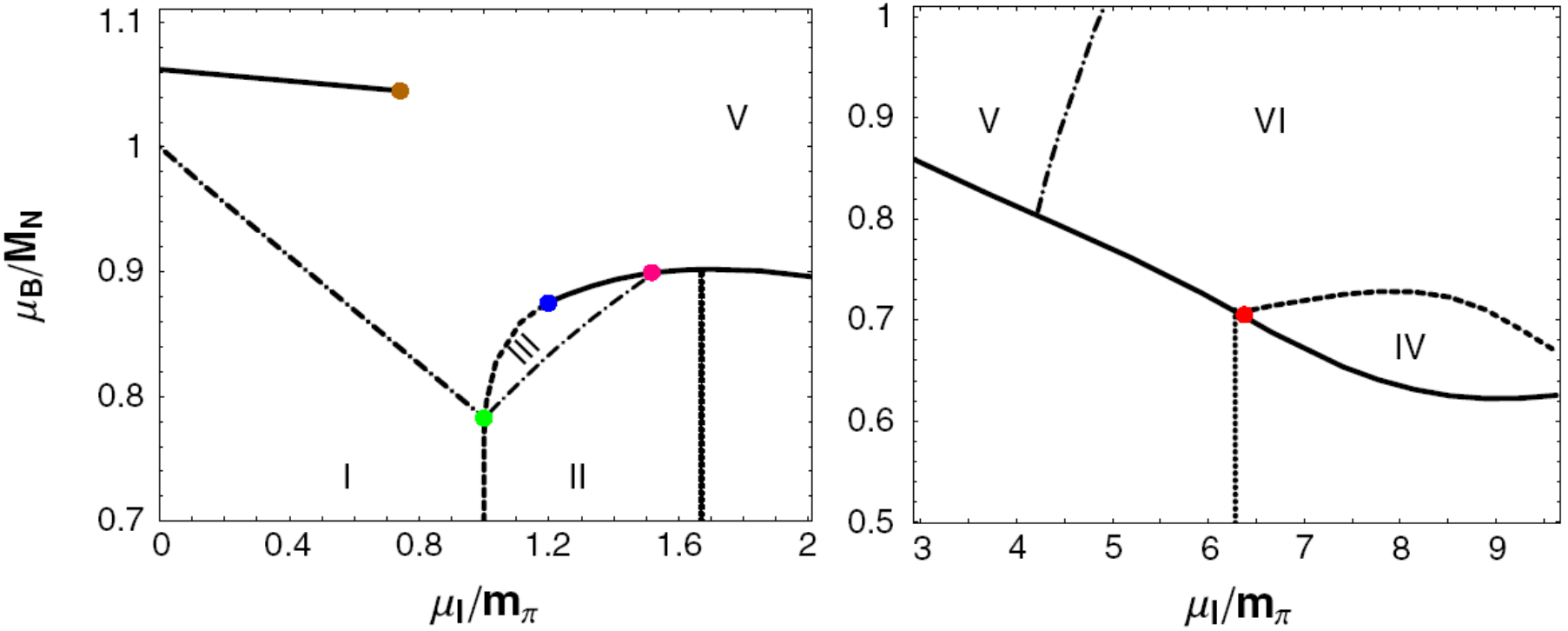
Stressed Pairing: BEC-BCS Crossover



D. T. Son & M. A. Stephanov, PRA, 2006

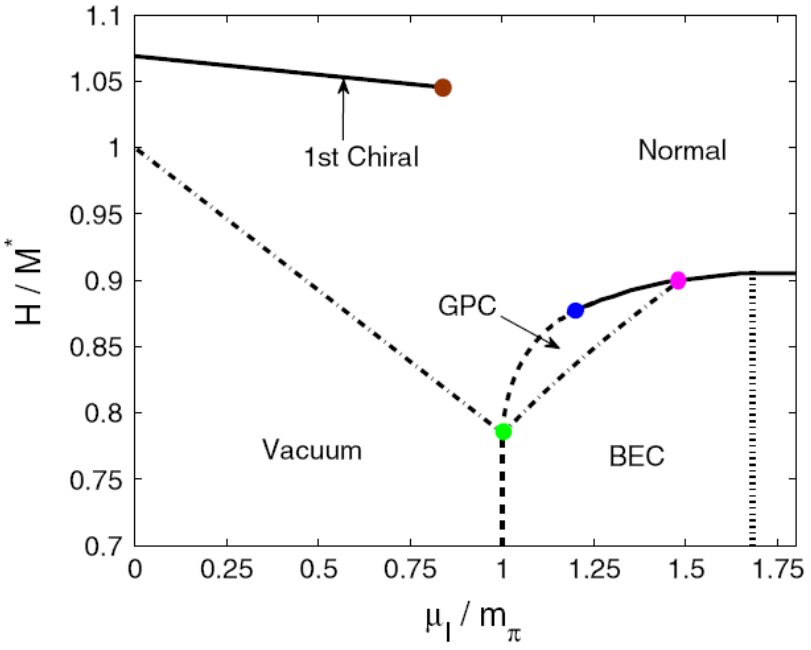
μ_B - μ_I Phase Diagram

- Phase Diagram from NJL Model: N_c=3, N_f=2

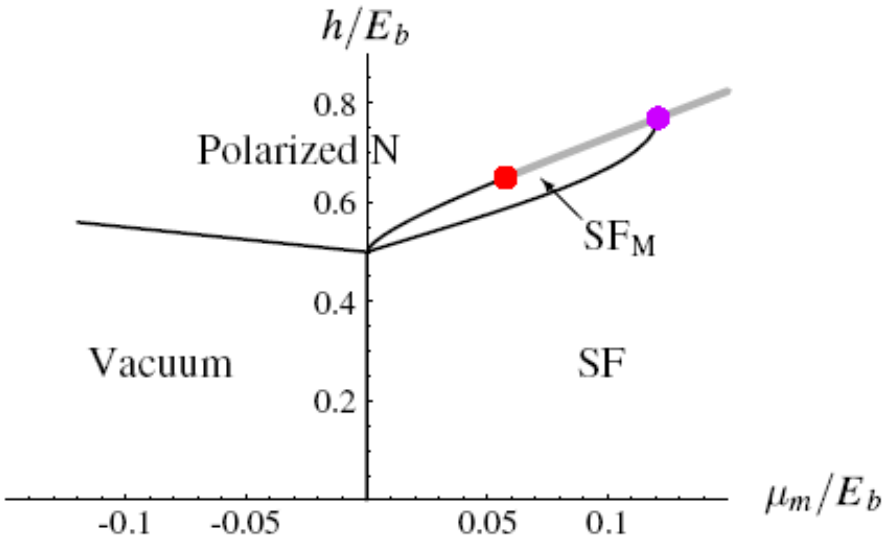


C. Mu, LH, and Y. Liu, PRD, 2010

QCD vs Cold Atoms



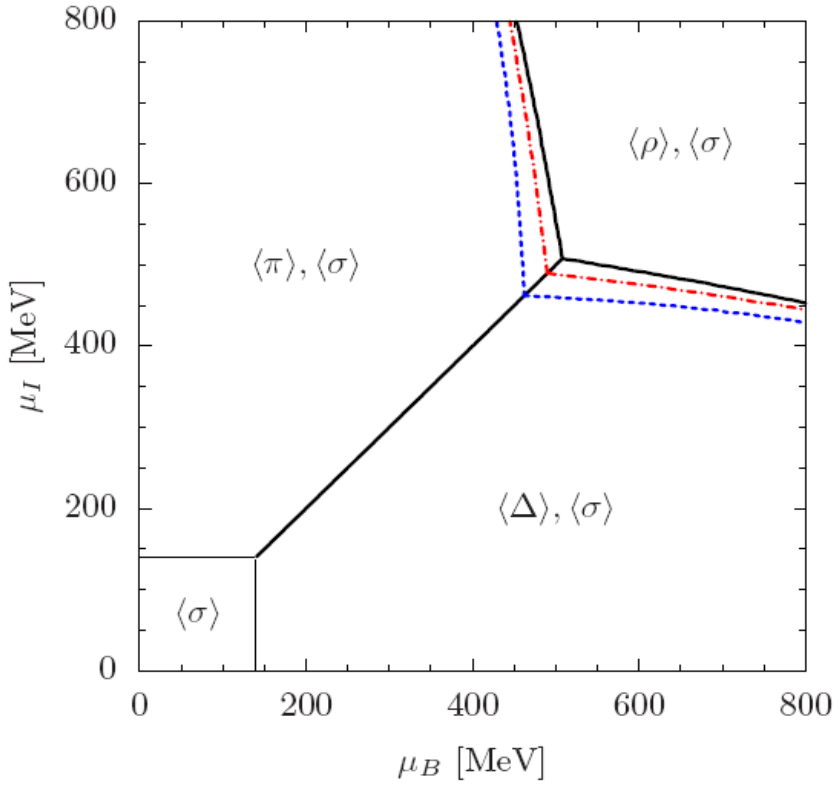
C. Mu, LH, and Y. Liu, PRD, 2010

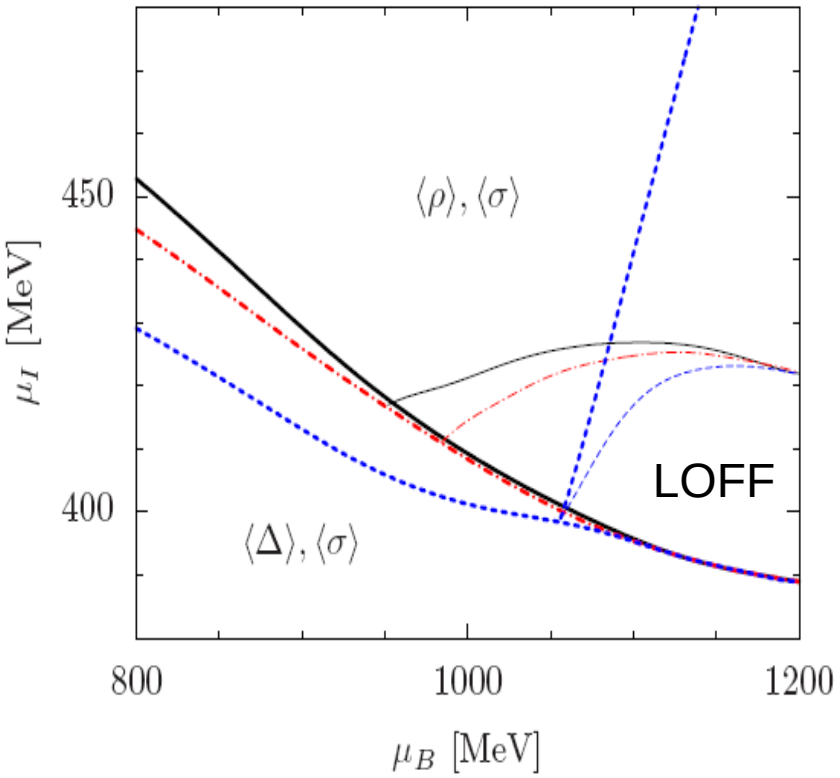


D. E. Sheehy and L. Radzikovsky, PRL, 2006; Ann. Phys., 2007

μ_B - μ_I Phase Diagram

- Phase Diagram from NJL Model: N_c=2, N_f=2

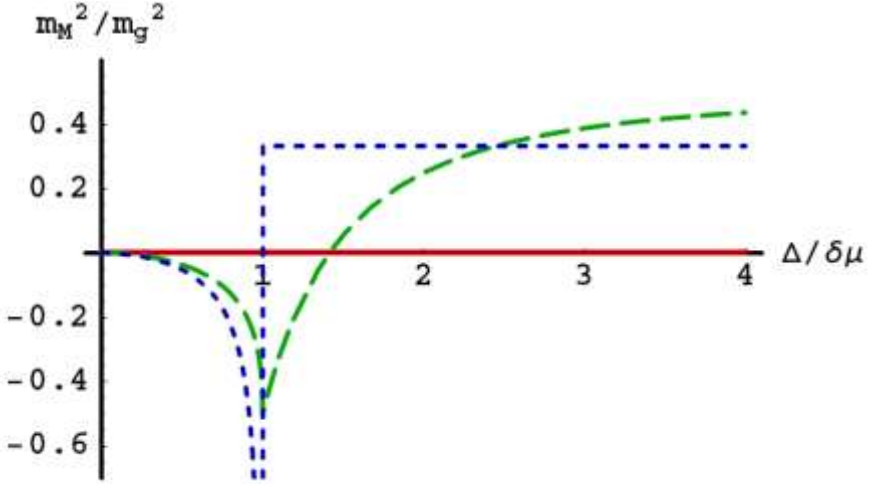




J. O. Andersen & T. Brauner, PRD, 2010

Stressed Pairing in CSC: Chromomagnetic Instability

- Chromomagnetic Instability in 2SC/g2SC



M. Huang & I. A. Shovkovy
PRD(R), 2004

- Solving the problem using cold atoms?

  Same instability in a cold atom model with QCD symmetry

  LH, M. Jin & P.Zhuang, PRD, 2006

Summary

- BEC-BCS crossover in QCD-like theories can be successfully described by the NJL model.
- BEC-BCS crossover plus stressed pairing leads to rich quantum phase diagrams of dense QCD: evolution from gapless superfluid phase to LOFF phase.
- Mimic dense matter using cold atoms?

Thanks!