Transport properties in bulk channel from lattice QCD

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in collaboration with Harvey Meyer

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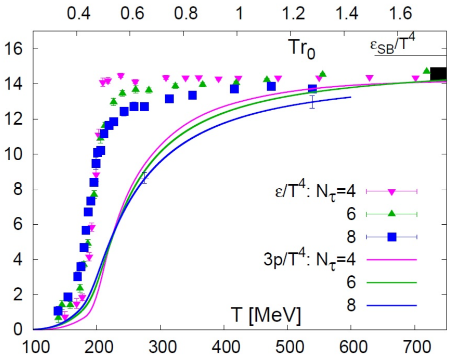


- Thermodynamics: crossover
- Transport: real time properties

[Bazavov et al 0903.4379]

Outline

- Introduction:
- Transport property: Bulk channel from lattice
- Comparison next-to-leading order perturbative calculation with lattice
- Conclusion

Introduction

Macroscopic form of the energy-momentum tensor:

$$
\begin{aligned}
T^{\mu\nu} &= -Pg^{\mu\nu} + (e+p)u^\mu u^\nu + \Delta T^{\mu\nu} \\
\Delta T^{\mu\nu} &= -\eta(\nabla_i u_k + \nabla_k u_i - \frac{2}{3}\delta_{ik}\boldsymbol{\nabla}\cdot\mathbf{u}) + \zeta\delta_{ik}\boldsymbol{\nabla}\cdot\mathbf{u}
\end{aligned}
$$

Kubo formulae: matching hydrodynamics with linear response description:

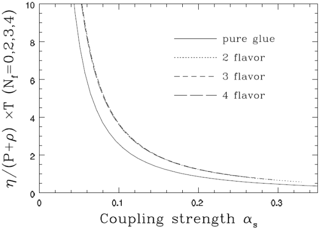
$$
\text{Shear viscosity}: \quad \eta(T) = \pi \lim_{\omega\to 0} \frac{\rho^{13,13}(\omega,\mathbf{0},T)}{\omega}
$$

$$
\text{Bulk viscosity}: \quad \zeta(T) = \frac{\pi}{9} \lim_{\omega\to 0} \frac{\rho^{ii,jj}(\omega,\mathbf{0},T)}{\omega}
$$

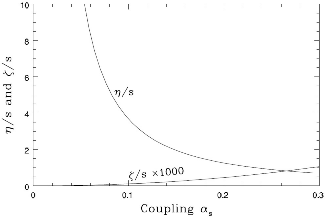
$\rho^{\mu\nu,\rho\sigma} = ImG_R^{\mu\nu,\rho\sigma}$, where $G_R^{\mu\nu,\rho\sigma} \equiv i\int_0^\infty dt e^{i\omega t}\int d^3x \, \langle[T^{\mu\nu}(0,\mathbf{x})T^{\rho\sigma}(0,\mathbf{0})]\rangle$

Introduction

perturbative QCD results



Arnold, Moore, Yaffe ‘04



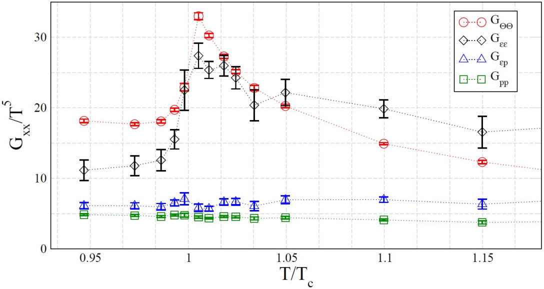
Arnold, Dogan, Moore ‘06

for $\alpha_s = 0.25$, $\eta/s \simeq 1; \zeta \simeq \eta/1000$.

However, the bulk viscosity is large in the vicinity of phase transition.

Introduction

lattice calculation: (SU(2), $N_\tau = 4$, $V \to \infty$)



[Huebner, Karsch, Pica '08]

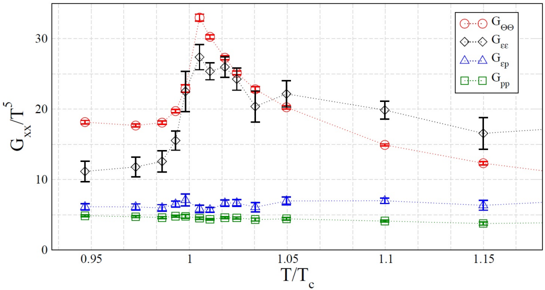
Euclidean correlator of trace anomaly $\theta \equiv T_{\mu\mu}$

$$C_\theta(t, \mathbf{q}, T) = \int d^3x e^{i\mathbf{q}\cdot\mathbf{x}} \left[\langle \theta(0)\theta(t,\mathbf{x}) \rangle_T - \langle \theta(0)\theta(t,\mathbf{x}) \rangle_0 \right]$$

$$C_\theta(t, \mathbf{q}, T) = \int_0^\infty \rho_\theta(\omega, \mathbf{q}, T) \frac{\cosh \omega(1/2T - t)}{\sinh(\omega/2T)}$$

Introduction

lattice calculation: (SU(2), $N_\tau = 4$, $V \to \infty$)



[Huebner, Karsch, Pica '08]

correlator receive a constant contribution from δ function

$$\frac{\rho_\theta}{\omega} = \frac{9\zeta}{\pi} + \frac{e+p}{c_s^2}(3c_s^2 - 1)^2 \delta(\omega)$$

Lattice setup

SU(3) pure gauge

$N_\sigma^3 \times N_t$	β	ξ	T/T_c	N_{conf}
$48^3 \times 16$	6.15	1.72425	0.96	7370
	6.50	1.75833	1.58	9840
	6.80	1.78018	2.30	6098
	7.20	1.80723	3.76	10300

anisotropic lattices: $S_G = \beta_\sigma S_\sigma + \beta_\tau S_\tau = \beta/\xi_0 S_\sigma + \beta\xi_0 S_\tau$

- anisotropy $\xi = a_\sigma/a_\tau = 2$
- renomalization factor for spacial and temporal part of energy momentum tensor needs to be determined

measure correlators with various momentum:

- (0,0,0),(0,0,1),(0,1,1),(0,0,2),(0,1,2),(0,2,2),(0,0,3) $\cdots$

Correlators at finite momentum

leading order in perturbation theory:

$$\rho_\theta(\omega, \mathbf{0}, T) = \frac{d_A}{4(4\pi)^2}\left(\frac{11\alpha_s N_c}{6\pi}\right)^2 \frac{\omega^4}{\tanh \omega/4T}$$

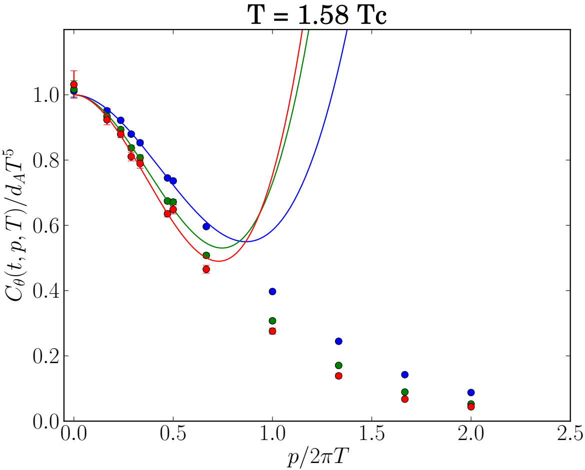
$$\rho_\theta(\omega, \mathbf{q}, T) = \cdots (\omega^2 - q^2)^2 \left[-\frac{\omega}{q}\theta(q-\omega) + \frac{2T}{q}\log\frac{\sinh(\omega+q)/4T}{\sinh|\omega-q|/4T}\right]$$

- high frequency contribution largely to the correlators

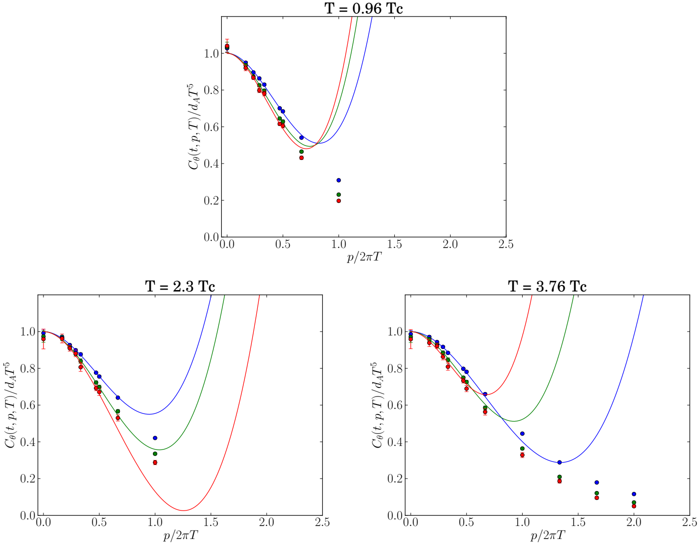
functional form of the Euclidean correlator at finite momentum

$$C(t, \mathbf{q}, T)/T^5 \propto a_0 + a_2\hat{q}^2 + a_3\hat{q}^3 + O(\hat{q}^4) \qquad \hat{q} = \mathbf{q}/\pi T$$

fit correlator using cubic ansatz:

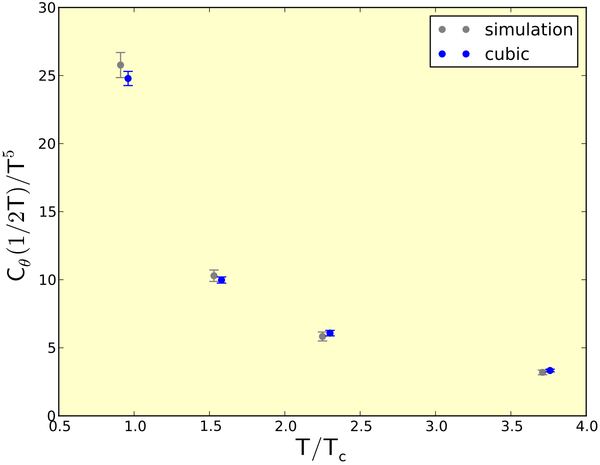


different color represents correlators of different time slice



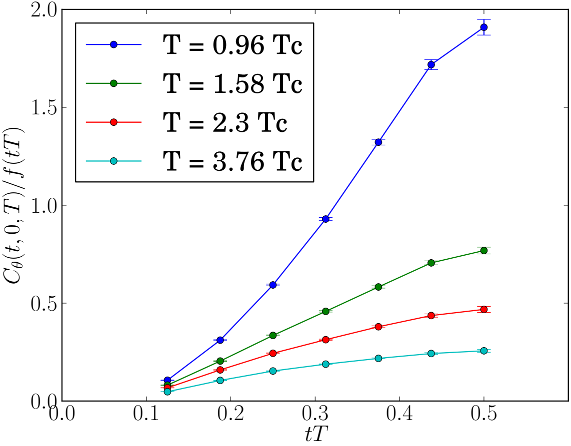
T = 0.96 Tc
T = 2.3 Tc
T = 3.76 Tc
$C_\theta(t,p,T)/d_A T^5$
$p/2\pi T$
0.0
0.5
1.0
1.5
2.0
2.5
0.2
0.4
0.6
0.8

Correlator at central time slice with zero momentum



fitting helps to reduce errors of correlators at zero momentum by 50%

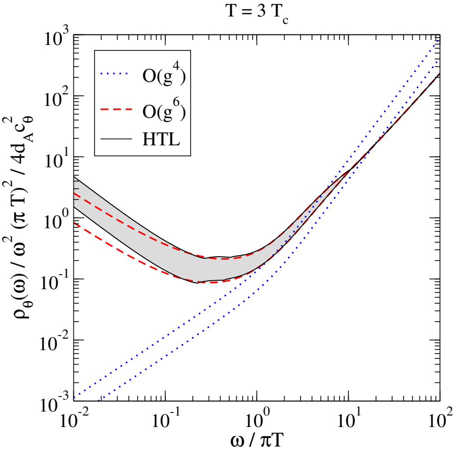
Correlator at zero momentum



leading order: $C(t, \mathbf{0}, T) = \frac{d_A}{4(4\pi)^2} \left(\frac{11\alpha_s N_c}{6\pi}\right)^2 f(tT)$

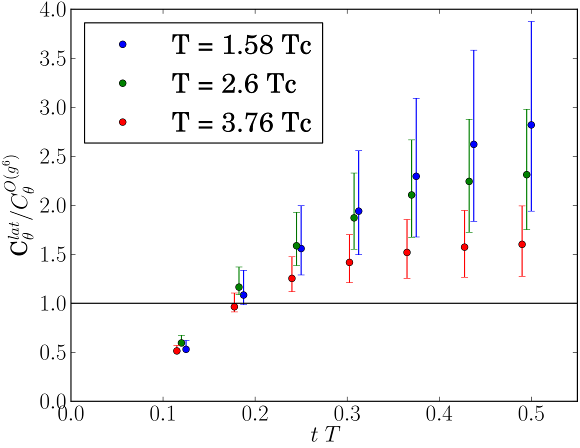
Perturbative: next to leading order

talks: Vuorinen, 14:30, Fri.
Zhu, 14:45, Fri.



$$\frac{\rho_\theta(\omega)}{4d_A c_\theta^2} = \frac{\pi\omega^4}{4\pi^2}(1+2n_{\frac{\omega}{2}})\left\{g^4 + \frac{g^6 N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar{\mu}^2}{\omega^2} + \frac{73}{3} + 8\phi_T(\omega)\right]\right\} + O(g^8)$$

Comparison: lattice vs perturbation



Conclusion

- For searching QGP and CEP, transport properties are very important.
- Correlators at finite momenta helpful for extrapolating to zero momentum, error bar improved by 50%
- Perturbation calculation at $O(g^6)$ converge better to lattice data.